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# Competitive Assessment of an Oligopolistic Market Open to International Trade with Incomplete Data

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**Abstract:** This paper is aimed at detecting whether an oligopolistic industry facing an international competition can sustain collusion or whether international trade disciplines it to competition. To do so we consider a differentiated-products oligopoly model and we estimate both demand and supply sides of the industry under investigation. The empirical analysis is performed in four steps. In the first step, we build a mathematical representation of the working of the industry. The second step is devoted to the estimation of the model under the assumption that the market is competitive. The third step provides an estimation of the model when it is assumed that the main firms on the market are forming a cartel and, more specifically, are behaving like a monopoly. Finally, the fourth step consists in comparing the two estimations in their capacity to represent reality, i.e., detecting which of the conducts – competition or collusion – is the most statistically adequate to represent the working of the market. The main result is that, in the industry under consideration and regarding the period of interest, the competition model statistically performs better than the collusion model. The very innovative part of the analysis here lies in the use of a limited amount of information. Indeed we perform the estimation of the model based on data from only one firm of the oligopoly and some aggregated data, thanks to the specification and the role of the international side of the market.

**Keyword:**

**JEL Classification:**

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## 1. Introduction

Since the Porter's seminal article published in 1983, relatively few empirical studies related to the workings of cartels are publicly available, mainly because of cartels' illegality and confidentiality constraints imposed to competent authorities. This paper contributes to this literature by providing a test which is aimed at detecting whether an oligopolistic industry facing an international competition can sustain collusion or whether international trade disciplines it to competition.

The competitive constraint exerted by the international trade also plays a crucial role in the Röller and Steen's analysis of the Norwegian cement industry that experienced a period during which a cartel was legally organized cartel. For this reason, Röller and Steen (2006) have access to all relevant data to study the effectiveness of the cartel defined as "the ability to achieve profit maximization in light of a particular sharing rule." Taking into account this rule and using an estimation of demand, they recovering the cartel's marginal costs, which are then the basis for measuring the cartel's effectiveness and its impact on consumers and welfare.

Our approach differs in that we consider a differentiated-products oligopoly model and we estimate both demand and supply sides of the industry under investigation before testing for collusion against competition. The confidentiality of the situation under competitive assessments prevents us to name the industry and firms involved. For this reason; we refer herein to Country 1 as the domestic country, Firms A, B and C as the three firms acting in Country 1, and Product X as the main product type of the oligopoly.

The empirical analysis is performed in four steps. In the first step, we build a mathematical representation of the working of the industry. The model differs depending on the level of competition prevailing on the market. The crucial point of the analysis is the assumption related to the status of capacities: full or excess. The confrontation between the level of production and the available capacities does lead to conclude with certainty about the real constraint exerted by capacities. We will then consider the two cases of binding and non-binding capacity constraint and select the assumption that better matches the reality of the industry. The second step is devoted to the estimation of the model under the assumption that the market is competitive. Under this assumption we estimate the model in the two capacity conditions. The third step provides an estimation of the model when it is assumed that the main firms on the market are forming a cartel and, more specifically, are behaving like a

monopoly. Again, under the assumption of collusion we estimate the model in the two cases of capacities constraint. The final stage of these two previous steps consists in comparing the estimations under the two different capacity conditions and determining their ability to represent reality. That is which of the assumption, full or excess capacity, is the most statistically adequate to represent the working of the industry. Whatever the functioning of the market in terms of competition, the main conclusion at this stage of the analysis is that full capacity might be preferred to excess capacity. On this basis, the final step consists in comparing which of the conducts – competition or collusion – is the most statistically adequate to represent the working of the market. The main result is that, in the industry under consideration, regarding the period of interest and given full capacity conditions, the competition model statistically performs better than the collusion model.

We differ in three ways from previous studies on cartel economics. First, the literature mainly focuses on the cartel participants' incentives to respect their collusive agreement as in Röller and Steen. For instance, Davidson and Deneckere (1990) show that, under the assumption on tacit collusion on prices and competition on capacity, there exist some equilibrium where firms invest in excess capacity. This results in cartel inefficiencies. The reason for this inefficiency lies in the cartel sharing rules by which the shares are allocated on the basis of production capacity. Second, several articles investigate the stability of cartels following Porter (1983). (See Ellison, 1994, and Vasconcelos, 2000.) Our approach is static which avoids us to deal with the question of separating collusion's and competition's periods in the time series, a task which is hardly endogeneized by the model. Third, we develop a structural model comprising demand and pricing equations. Focusing also on the cement industry, Steen and Lars Sörgard (1999) use a reduced form model to analyze the effectiveness of the cartel, but it prevents them from explaining the working of the cartel. Röller and Steen (2006) use a structural approach to estimate the marginal costs as a first step to assess the welfare gains from the cartel. However they do not account for the horizontal or vertical differentiation of products.

Now the main innovative part of our analysis lies in the use of a limited amount of information. Indeed we perform the estimation of the model based on data from only one firm of the oligopoly, on the price and quantity of imports, and on some aggregated data related to the market, thanks to the specification and the role of the international side of the market. In other words, it is a further example of a relatively rich econometric model that can be estimated without a complete data set and that allow implementing a test on antitrust practices which is critical in competition policy.

In general, the analyst must pay the price of a limited amount of information by introducing a set of more or less strong identifying restrictions. Our analysis is not immune of such assumptions that are discussed in the sequel. At this point, we can already indicate that the data on imports somewhat facilitates our task here. Indeed, by assuming that the imports play the role of the outside alternative for the domestic customers and are provided by an importer with no strategic capacity in terms of price competition –which, we consider, are mild assumptions- we easily reach identification of a static Nash equilibrium based on a logit-type demand model. This adds to the originality and the specific interest of the proposed model.

Section 2 describes the industry and Section 3 provides a descriptive analysis which is aimed at motivating the hypotheses which serves to build the economic model presented and estimated in Section 4. Here we present the results of tests of collusion versus competition.

## **2. Main features of the industry under investigation**

On the demand side, the Country 1 industry comprises three domestic producers having no financial interests between them, namely FIRM A, FIRM B, FIRM C, and some importers. They all provide the market with a number of products that are used for producing other consumer goods. We consider the market for one product only, namely Product X, as it accounts roughly for eighty-five percent of industry total sales, because, the two other products are complementary products to Product X.<sup>1</sup> Consequently, in terms of profit, this choice leads to the analysis of firm's profit restricted to Product X production. The strategic behaviour of domestic firms is described in the sequel.

There is also an international market for Product X and the firms sell on this market taking the international price as given. In other words, none of the Country 1 firms is able to act as a price-maker on the international market. Country 1 also imports Product X. Without loss of generality, all the importers are aggregated in one firm, the Importer.<sup>2</sup> The latter has here no strategic behaviour: it acts as a competitive fringe.<sup>3</sup>

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<sup>1</sup> There is an additional constraint due to data availability: The costs of production for the two other products are missing as well as the level of their imports. The level of exports is available for the full period under consideration for FIRM A and Product X only.

<sup>2</sup> When it is not explicitly mentioned, we always refer to the market or the industry of Country 1 in the sequel.

<sup>3</sup> In the appendices, the different producers are referred by the index 1 for FIRM A, 2 for FIRM C, 3 for FIRM B and 0 for the Importer.

On the demand side, a representative customer can buy from any of the producers.<sup>4</sup> Although Product X is a quite homogenous product which must satisfy some standards, there could be some differentiation among firms in terms of their commercial networks and marketing activities. So, knowing the quantity it wants to buy, the customer has mainly to compare the different offers it can receive from the producers in terms of quality and prices. In these conditions, the market share of any firm corresponds to the probability that the customer chooses one of the firms on the market to satisfy its demand.<sup>5</sup> (See Appendices 1 and 2 for presentation of the demand model.)

On the supply side, the equilibrium conditions are determined by the combination of two basic elements: The behaviour of the firms playing on the market - competitive or collusive - and the capacity conditions – excess or full capacity. Facing demands for Product X from domestic and overseas customers, how do the firms react? This is where the conduct of firms enters the picture. In the first place, we assume that the market is competitive; in the second place, we then consider that the three firms behave like a monopoly. In each case, we derive the strategic behaviour of each domestic firm.<sup>6</sup>

Assume first that the market is competitive. The revenues of the firm come from two sources: the domestic sales evaluated at the domestic price and the exports evaluated at the international price which the firm takes as given. The firm produces Product X for the domestic and overseas markets using a production technology for which the main input is the Raw Material Y. The price of this input is the main driver of production costs.

Our view is that each firm maximizes its profit using two instruments: The price of its product and the amount of its exports. Firms behave strategically, which means that they consider that each competitor also maximizes its profit using the same available instruments. In the case of full capacity, the level of exports is obtained from the capacity constraint as soon as the level of domestic production is determined and vice versa. The firm has to arbitrage between selling on the domestic market and exporting overseas. To make its decision, the firm must choose a domestic price so that the marginal revenue from selling on the domestic market is equal to the international price. In other words, the international price, which represents the opportunity costs of not exporting, plays the role of marginal costs. Under full capacity and if the capacities are different among firms, one should expect differences in market shares, whatever the differentiation level of the product.

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<sup>4</sup> Given data availability, one cannot account for the heterogeneity of Country 1 customers, *i.e.* for the differences among customers in terms of preferences and willingness-to-pay.

<sup>5</sup> We followed Berry (1994) for the description of the demand side of the industry.

<sup>6</sup> The mathematical presentation of the model is postponed in Appendix 3. (See Ivaldi and Verboven, 2005.)

In the case of excess capacity, the firm faces the same arbitrage as in the case of full capacity, and so, again, chooses a domestic price so that the marginal revenue from selling on the domestic market is equal to the international price. However, this time, the amount of exports is defined so that the marginal cost of producing one additional unit of Product X, whether it is for the domestic market or for overseas, is just equal to the international price. Note that, under excess capacity, if there is not much differentiation among firms so that each firm's individual demand is similar to the demand of its competitors, one should expect that the industry converges to a symmetric situation where all the domestic firms have roughly the same market share and put the same price, since they all take the international price as a benchmark.

Now assume that the domestic firms coordinate their strategies. They can do it in different ways in practice. Here we are considering the extreme case where they all together behave as a monopoly. In other words, the coordination among them is so strong that they define one unique domestic price and the overall level of exports. That is, there is only one firm in the industry which has basically the same choices to perform than the individual firm in the competitive case, except now that quantities are defined at the industry level rather than at the individual firm level. The decisions are different whether there is excess capacity or that the capacity constraint is binding. In this latter case, the monopoly must equalize its domestic marginal revenue to the international price to determine its domestic price, and so its domestic sales (for the whole industry) and in turn, its exports. In the first case, the monopoly equalizes its marginal revenue to its domestic price to determine its domestic production, while the level of exports is defined thanks to the equality between marginal cost and international price level.

### **3. Descriptive analysis**

Available data, which covers the period 2002-2008, are fairly complete for FIRM A and for the whole industry. They bear on the domestic prices of FIRM A, its domestic production levels and exports, its production capacities, the prices of Raw Material Y, the international Product X prices, the production capacities of competitors, and the level of Country 1 exports of Product X. Some approximations of the market size are also available.

However, we do not observe the prices of Product X sold by competitors, neither one is able to measure any differences in terms of quality between the different firms. This mainly drives the choice of the joint-profit maximization assumption to approximate the coordinated

solution among the domestic firms. Below we indeed assume that the FIRM A’s observed price corresponds to the cartel price if the firms in the industry maximize their joint profit. On the one side, this hypothesis (to which we refer as the monopoly case herein) can be deemed too strong; on the other side, it is convenient since the prices of other firms in the industry than the FIRM A are not necessary for the quantitative analysis.<sup>7</sup>

3.1. Market figures

The domestic market shares on the period under investigation for the three main competitors are reported in Table 1. FIRM A’s market share is the highest during the period under consideration, waving between 35% in 2005 and 42% in 2008. The yearly difference in terms of market share between Firm A and the second producer, namely Firm B, is about 10%, except for 2005. The third producer, FIRM C, experiences a jump in its production level in 2004-2005 following the increase in its production capacities. Up to 2004, FIRM C market share is lower than the market share of importers. Since 2005, this trend is reversed. Finally, observe a sharp drop in imports at the end of period.

The differences in market shares across competitors might be indicative of differentiation among each other products. In the absence of differentiation, the demand faced by the competitors would be similar and the industry would converge to a symmetric equilibrium.

Table 1. Product X market shares

Undertaking	2002	2003	2004	2005	2006	2007	2008
FIRM A	40%	36%	37%	35%	37%	40%	42%
FIRM B	31%	34%	33%	33%	27%	30%	25%
FIRM C	8%	8%	13%	21%	18%	23%	28%
IMPORTS	20%	22%	17%	11%	18%	7%	4%

Source: Confidential data on the industry

Two types of prices are available for FIRM A: the list price and the final price. Their evolutions over the period into consideration are fairly similar, as shown in Figure 1, but the list prices are significantly higher than the final prices. As the final prices are the prices

<sup>7</sup> Nonetheless missing information limits our capacity to test for the robustness of our estimations.



Country 2 and Country 3 are the main Product X importers and they compete with domestic firms products. Although the final prices of FIRM A are often higher than the prices of importers, there remains a positive demand for the domestic products. The domestic firms might take advantage of their domestic position, differentiating their production of Product X probably through commercial networks and marketing activities. In the sequel, we aggregate the importers in one unique firm (which we refer to the Representative Importer in the sequel) and we use a weighted average of importers prices for the price of importations.<sup>8</sup>

Figure 1 compares the evolution of the two relevant prices on Product X market, namely FIRM A domestic price (i.e. final price) and the average import price, with the export price. The latest is the international price and is given for exporter firms. Over the whole period, 2002 to 2008, the price of domestic Product X charged by FIRM A is higher than the export price.

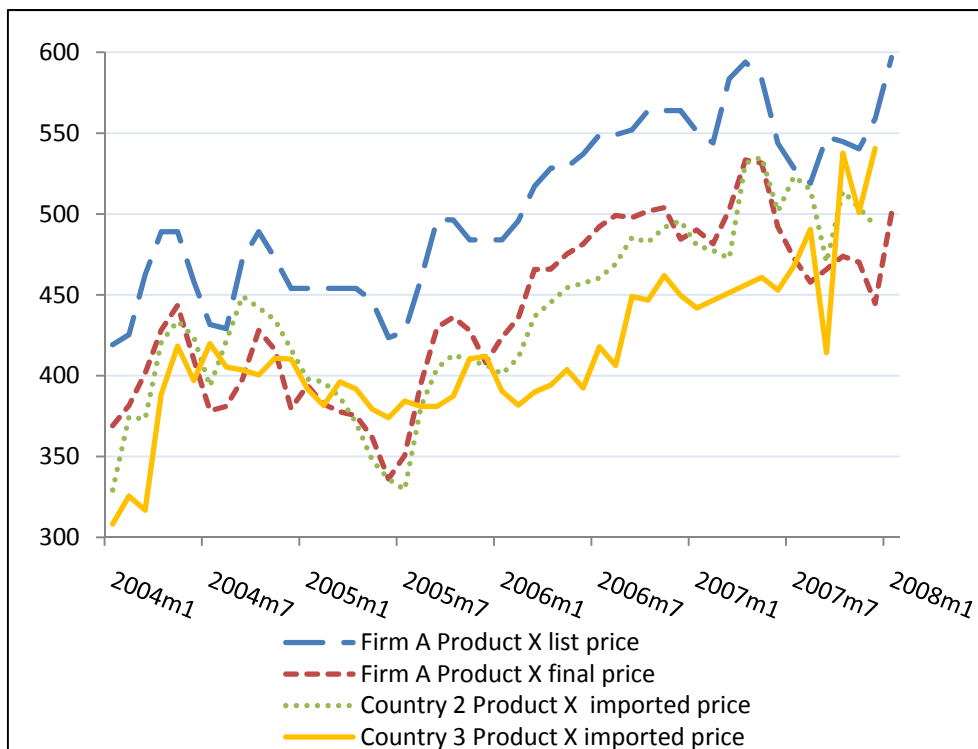
When considering the unit cost of Product X production, the export price is often lower than the unit cost of production. In 2004, 2005 and end 2007, it can append that the unit cost of production is higher than the domestic price charged by FIRM A, leading to negative margins. This phenomenon might be explained by the need to reduce inventories while the year ends.

Figure 1 finally highlights the singularity of year 2008, corresponding to the beginning of the international downturn. This latter year is characterized by high increase in prices and costs. This has to be kept in mind for the forthcoming econometric analysis.

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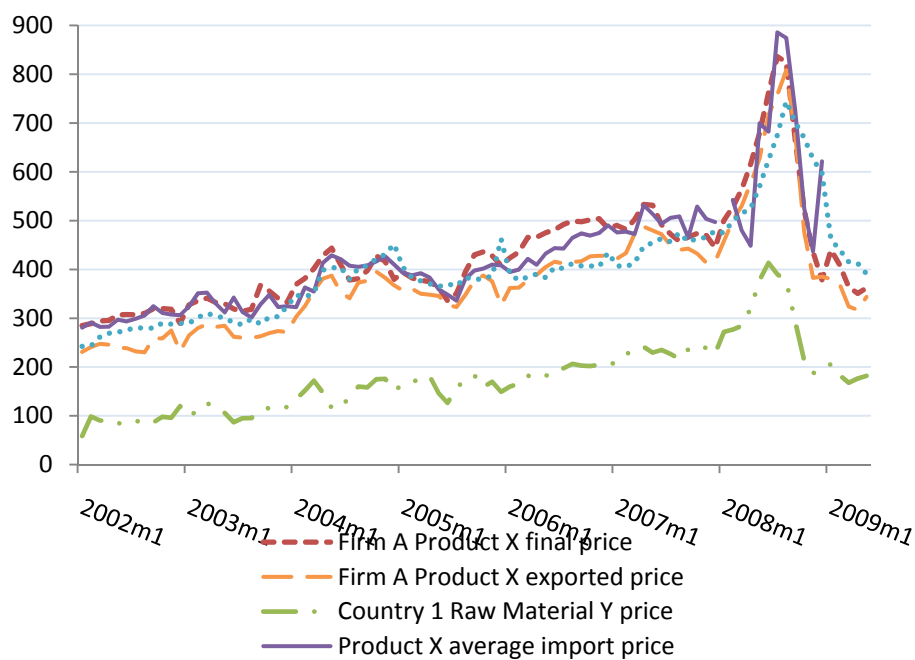
<sup>8</sup> We could have used the lowest price of importers prices. As a matter this does not affect the results.

Figure 1. Final prices versus list and imported prices



Source: Confidential data on the industry. The unit is euro per ton.

Figure 2. Firm A Product X final and exported prices, Product X average import price, Firm A Product X unit cost of production and Raw Material Y price (€/ton)



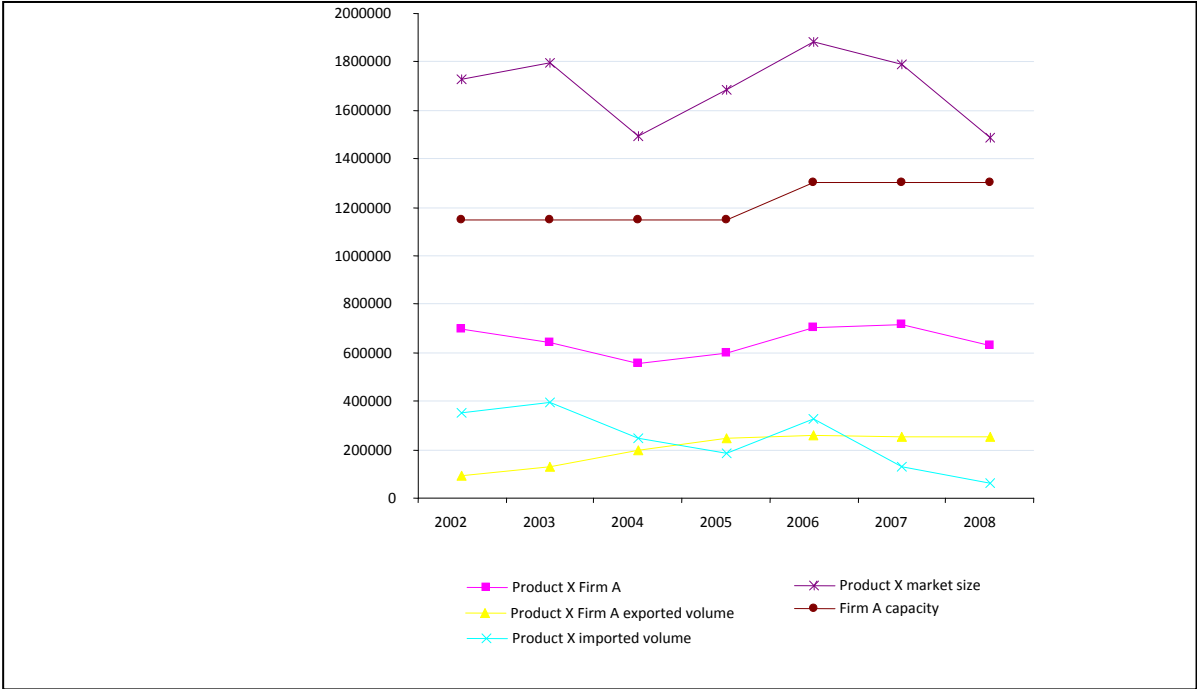
Source: Confidential data on the industry

Consider now the quantities. Assuming that the market is competitive, the relevant quantities are the FIRM A’s productions, both for domestic and international markets. Under this assumption, we conclude that FIRM A’s domestic production decreases until 2004, then increases until 2007 (See Figure 2). In year 2008 starts the previously described downturn. The level of imports and the total market size show roughly the same evolution on the period under consideration, while the exports increase until 2005 and remain stable since then. FIRM A’s production capacities increase in 2006.

Assuming now coordinated practices across firms, the relevant quantities are the combined productions of the three firms, both for domestic and international markets.

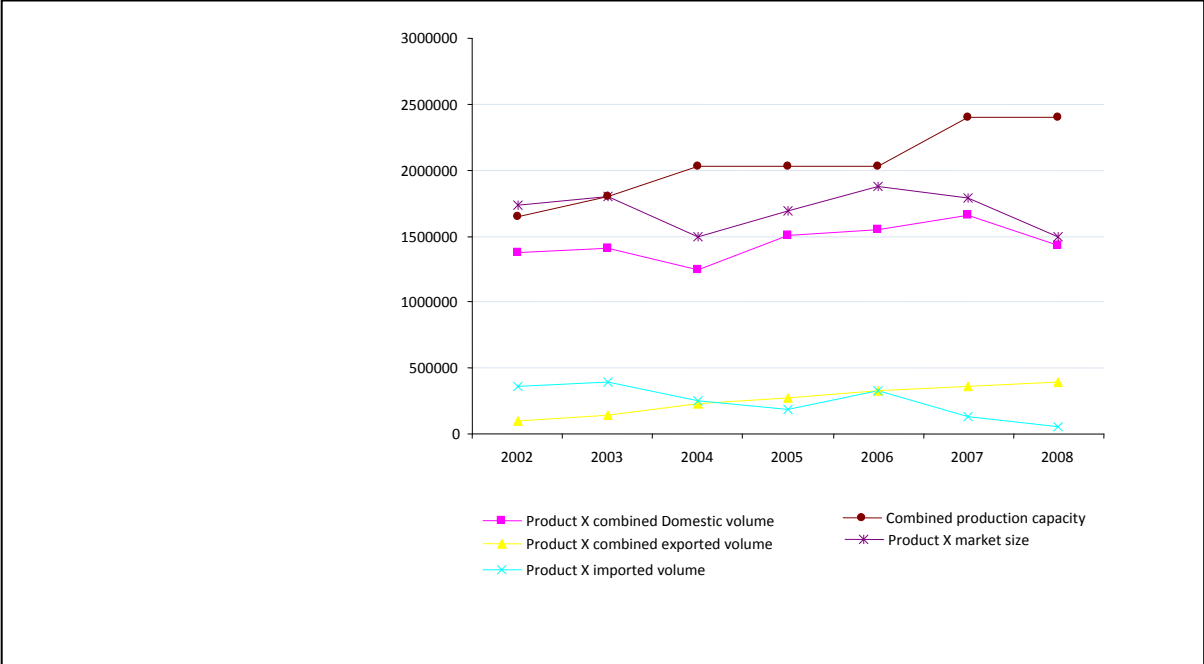
Figure 3 draws the evolution over the period under consideration of the relevant figures related to production, importations, capacities and market size. The differences between the market size of Product X and the combined domestic volumes decrease over the period. The overall evolution of domestic production is quite similar to the evolution of FIRM A’s domestic production. The combined exportations keep increasing over the period.

Figure 3. Domestic market size, level of productions and capacities



Source: Confidential data on the industry. The volumes are expressed in tons.

Figure 4. Domestic levels of production and importations



Source: Confidential data on the industry. The volumes are expressed in tons.

### 3.2. Discussion on capacity constraint<sup>9</sup>

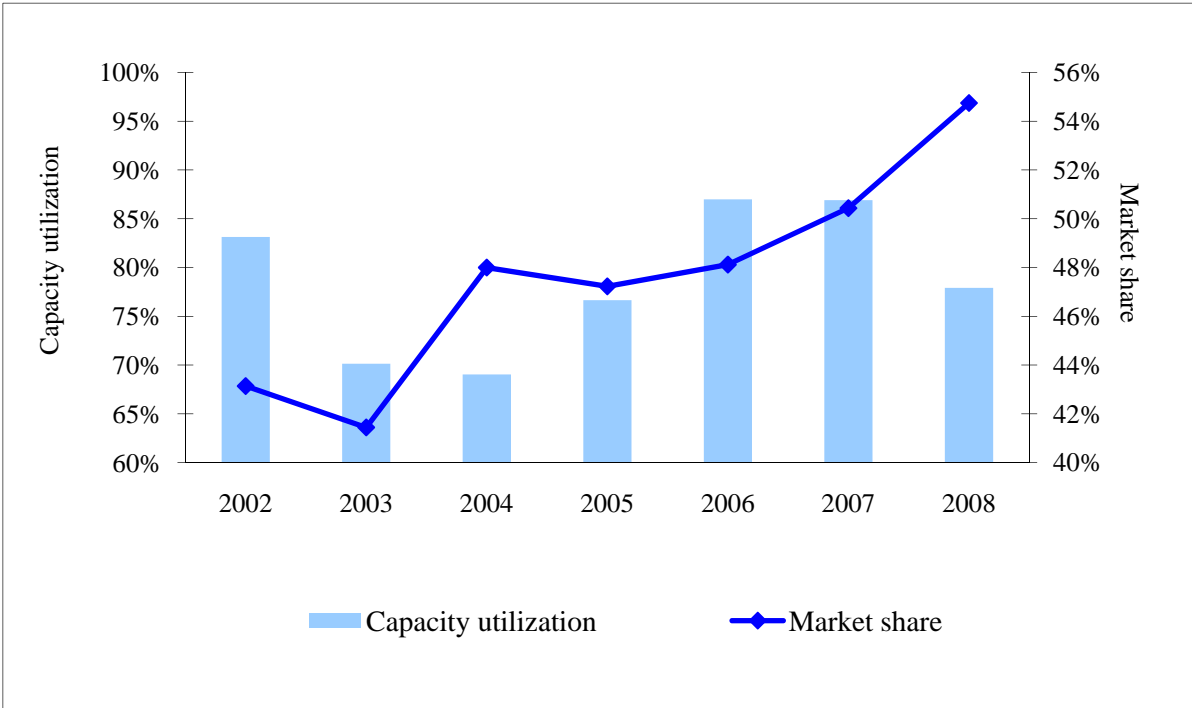
A crucial issue to identify the equilibrium of the market is to characterize the level of capacity utilization. If one looks at the situation of FIRM A as presented in Figure 5, the level of capacity utilization is higher than 75 percent, which is considered as the efficient level of production, in all years except in 2003 and 2004; these latter years experience excess capacity.

Now, Figure 6 displays how the capacity utilization has evolved during the period under investigation at the industry level. We observe only one year where there are excess capacities, specifically 2004. With 76.09 percent capacity utilized, the year 2008 is just at the threshold for full capacity utilization. These are two specific years: 2004 follows a period of economic boom and 2008 is the beginning of the international downturn.

Given this observation of periods of excess and full capacities, one should use the adequate model for each different period, given that we restrict attention to static equilibriums. However, it would raise identification issues of time effects. Hence we estimate the model under the assumption that there is full or excess capacity over the whole period.

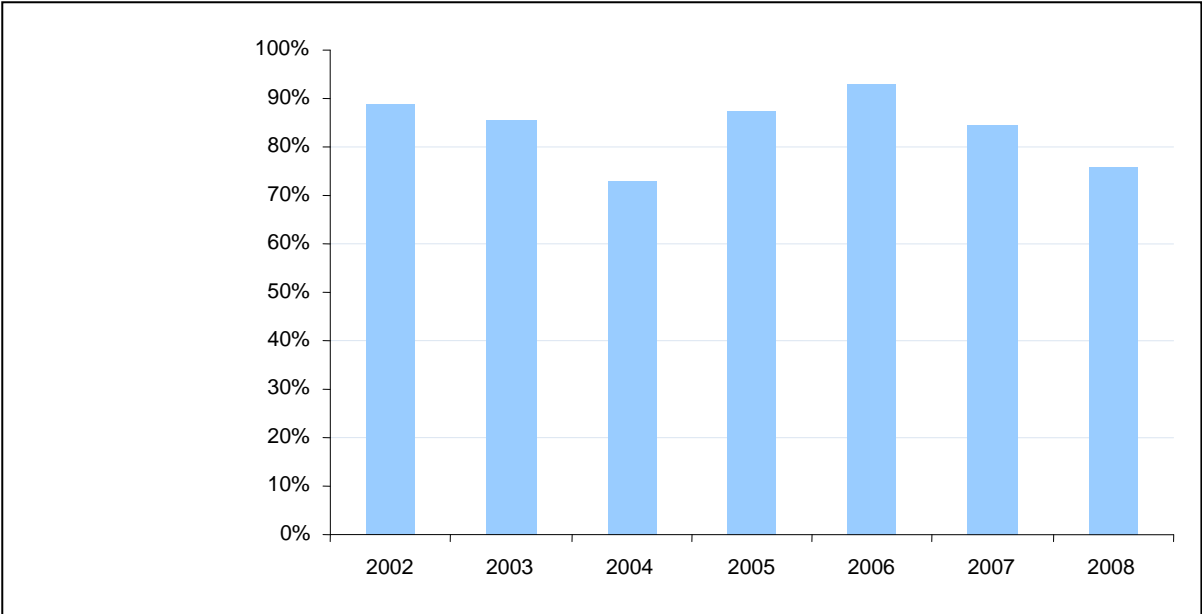
<sup>9</sup> Here we consider total capacity referring to physical capacities that can be directed to domestic sales or to exports.

Figure 5. Capacity utilization and market share for FIRM A



Source: Confidential data on the industry.

Figure 6. Capacity utilization at the industry level



Source: Confidential data on the industry

#### 4. Model specification

This section introduces the econometric model based on the framework discussed so far: i) An oligopoly facing an international competition; ii) Data mainly available for one of the firms, namely FIRM A.

##### 4.1. Demand side

Let index  $i$  be equal to A or to 0 when it refers to FIRM A or to the Representative Importer.<sup>10</sup> Based on a logit-type specification, the demand function addressed to firm  $i$  is expressed as follows:

$$\ln(s_i) - \ln(s_0) = \delta_i - \alpha p_i + \zeta_i, \quad (1)$$

where  $s_i$  is the market share of firm  $i$  on the Product X domestic market,  $s_0$  is the market share of the Product X Importer on this market,  $p_i$  is the price charged by firm  $i$  for one unit of the product,  $\delta_i$  measures the observed “quality” of product X sold by firm  $i$ ,  $\zeta_i$  measures the unobserved “quality” of product X sold by firm  $i$ , and  $\alpha$  is the marginal utility of income. The market share of firm  $i$  is measured as:

$$s_i = \frac{q_i}{q_0 + q_d}, \quad (2)$$

where  $q_i$  is the production level of firm  $i$ ,  $q_d$  is the total domestic production (i.e., the sum of production levels of all domestic firms) on the market, and  $q_0$  is the total amount of imports of Product X.

Hence the demand for product X results from a trade-off between “quality” and price. The term denoted by  $\delta$  allows for a differentiation across undertakings, which might be interpreted as reflecting any own firm’s characteristics, in particular its capacity to react to new economic conditions. In the case of an exogenous shock, the demand would shift due to change in this term. As a matter of fact, this term is expressed as a function of a dummy variable which accounts for the specific aspects of year 2008 compared to the other years, as previously pointed out. Specifically, we write:

$$\delta_i = (\delta_{i,cst} + \delta_{i,2008} * dummy_{2008}), \quad (3)$$

where  $\delta_{i,cst}$  and  $dummy_{2008}$  are parameter to be estimated.

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<sup>10</sup> The mathematical specification of the demand equation is precisely described in Appendix 2.

The parameter  $\alpha$  related to the price, defined as the marginal utility of income, might be interpreted as an exchange rate between one unit of quality and one monetary unit. Note that, by specifying this parameter as a function of Country 1's GDP, that is to say,

$$\alpha = (\alpha_{cst} + \alpha_{GDP} * GDP), \quad (4)$$

where  $\alpha_{cst}$  and  $\alpha_{GDP}$  are parameter to be estimated, we expect it to be positive and decreasing with GDP, reflecting a wealth effect, namely that a richer country is less sensitive to any change in money value.

Whatever the assumption, competition or collusion, two demand functions are estimated jointly in the model, the first one for firm  $i$ , FIRM A or the monopoly respectively, the second one for the Importer.<sup>11</sup>

The supply side of the economy is characterized by the mark-up of the firm, either FIRM A or the monopoly depending on the assumption related to the level of competition. One can show that the mark-up is a function of the own price elasticity of demand; thus the mark-up is in particular a function of the parameter  $\alpha$  described in the demand equation.<sup>12</sup>

In our case, each firm maximizes its profit defined as the difference between revenues and cost, where the revenues come from domestic sales and exports and where the cost is a function of the total production. On the domestic market, we assume that the firms compete in price. This choice is driven by the observation of firms' behaviours and is meaningful given the Product X homogeneity given firms' characteristics. On the international market, the firms take prices as given and compete in quantities.

Deriving the maximization program of the firm, one can show that the mark-up is expressed in terms of the Product X export price rather than in terms of marginal costs of production. The expression of the mark-up is the following:

$$p_i - W = \frac{1}{\alpha(1 - s_i)}, \quad (4)$$

where:

- $p_i$ ,  $s_i$  are respectively the price charged by firm  $i$  and its market share on Product X domestic market;

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<sup>11</sup> The index  $i$  equals 1 in the first case and 0 in the second. The estimated parameters are reported in Table 9 and Table 10 of Appendix 4.

<sup>12</sup> Appendix 3 describes the supply equations as well as the equations resulting from the firms' maximisation program.

- $\alpha$  is the marginal utility of income and its expression is identical to the one in the demand function;
- $W$  is the price for exports, which is given for firm  $i$ .

Whatever the assumption, competition or collusion, the mark-up related to firm  $i$  is the only mark-up included in the model to be estimated.

Under the assumption of binding capacity constraint, the mark-up equation perfectly describes the behaviour of the firm and allows us determining the optimal level of domestic production, while the level of exportation is deduced from the full capacity condition.

However under the assumption of non binding capacity constraint, an additional equation is necessary to describe the strategic behaviour of the firm, as both the level of domestic production and the level of exports have to be determined. This additional equation expresses that the marginal cost of production equals the export price. The marginal cost of production is expressed as a linear function of the Raw Material Y price and the total level of production (domestic production and exports). From an econometric perspective, both the equation related to the marginal cost of production and the equation related to the total cost of production will be included in the system to be estimated.

The specification for the total cost function is the following:

$$TC(y_i) = \beta_{cst} + \beta_{RM} \cdot p_{RM} + \beta_{RM,Y} \cdot p_{RM} \cdot y_i + \frac{1}{2} \beta_Y (y_i)^2 \quad (5)$$

where

- $y_i$  is the total production from of Product X, that is the sum over domestic production,  $q_i$ , and exports  $x_i$ ;
- $TC(y_i)$  is the total cost to produce the quantity  $y_i$ ;
- $p_{RM}$  is the Raw Material (RM) Y price.

Given the specification of the total cost function, the marginal cost function is expressed:

$$MC(y_i) = \beta_{RM,Y} \cdot p_{RM} + \beta_Y y_i \quad (6)$$

The equilibrium is characterized by equality between marginal cost and export price. Thus, under the assumption of non binding capacity constraint the following relationship holds for firm  $i$ :



$$W = \beta_{RM,Y} \cdot P_{RM} + \beta_Y y_i$$

7)

As explained previously, both this latter equation and the total cost function related to firm  $i$  are included in the model to be estimated.

The equilibrium on the Product X market is characterized by a system of equations related to both demand and supply. Given the identification constraints of the model, we can show that the model might be perfectly identified thanks to the description of demand and supply of a unique firm belonging to the domestic oligopoly and the demand of the Importers on the domestic market. Because we observe the production, price and cost of Firm A, we use Firm A's data for the description and the estimation of the model.

## 1.2. Empirical results

The main results of the estimations are reported in the Table 10 and Table 11 in Appendix 4. Under the assumption of binding capacity constraint, the equilibrium is described by three equations: the demand addressed to FIRM A or to the monopoly, the demand addressed to the Importer, and the mark-up. Under the assumption of non binding capacity constraint the equilibrium is described by four main equations: the demand addressed to FIRM A or to the monopoly, the demand addressed to the Importer, the mark-up and the marginal cost equation; plus the additional total cost equation.

We focus here on the main economic indicators that allow us selecting between the different models and assumptions, *i.e.* whether the capacity constraint is or not binding and whether firms' behaviour is competitive or collusive. The economic indicators are the estimated own price elasticity of demand, the level of the estimated margin and the estimated marginal cost in the case of non-binding capacity constraint. Moreover, the sign of some of the parameters has to be check for economic consistency, for instance the marginal utility of income has to be positive.

First, we discuss the estimated model under the assumption of competition on the market. We compare the level of the relevant economic indicators obtained under the two capacity conditions: full and excess capacity. Second, we discuss the results of the models estimated under the assumption of coordinated practices and we compare the level of the

relevant economic indicators obtained under the two capacity conditions. Then, we present the results of a statistical standard test, the Vuong test, which is a specification test allowing the evaluation of the assumptions on conduct that better represents the observed data. The two tests implemented at first lead to the selection of the most relevant capacity condition, given the market observations and the two possible market conditions, competition or collusion. The results of these two preliminary tests drive the last step of the analysis involving the comparison of behavioural strategies under the assumption of binding capacity constraint.

i. Competitive behaviour

Under the assumption of competitive behaviour, the estimated parameters of the full or excess capacity models are reported in Table 10 of appendix 4. All the parameters are statistically significant at 1%, 5% or 10% level whatever the capacity condition, except the parameter  $\beta_{Scrap,Y}$  in the cost function. This parameter corresponds to the cross effect between the price of raw materials and the level of total production. The parameters  $\delta_i$ ,  $i=0,1$ , are positive, and the parameter  $\alpha_{GDP}$ , related to the GDP, is negative as expected.

Moreover the marginal utility of income,  $\alpha$ , is positive. Table 2 reports its values for the two models with binding or non-binding capacity assumption. The marginal utility of income is decreasing over the period into consideration in the full capacity model. Although marginal utility of income is constant, due to structural assumptions, in the excess capacity model. The marginal utility of income is lower in the model with non-binding capacity assumption than in the model with binding capacity assumption.

The own price elasticities of demand belong to the interval [-9.6;-6.17] in the case of binding capacity constraint (see Table 3). They are lower in absolute value in the case of excess capacity where they belong to the interval [-4.8;-2.4].

The estimated margins are reported in Table 4. Under full capacity assumption, the estimated margins belong to the interval [11.40, 16.40]. The margins are the lowest in 2006. Under the assumption of excess capacity, the estimated margins belong to the interval [22.70, 40.90]. They exhibit a higher yearly variability than in the case of binding capacity constraint. Finally the estimated marginal costs belong to the interval [24.45, 66.12]. The lowest value is taken in 2008.

At this stage of our analysis, the relative values of own price elasticity of demand and the relative values of the estimated margins do not favour the assumption of binding or non-binding capacity assumption as a better approximation of the data.

With respect to the main indicators, one can conclude that the two models estimated under competition and full or excess capacities are coherent with the main constraints imposed by the economic theory.

ii. Collusive behaviour

The collusive behaviour is based on the assumption that the firms act all together like a monopoly. This assumption is in some sense stronger than the assumption of joint profit maximization. Recall that the choice to describe the collusive behaviour by using a monopoly model has been dictated by the lack of data on Product X prices set by the two competitors of FIRM A.

The estimated parameters of the competitive and coordinated models are reported in Table 11 of Appendix 4. All the parameters are statistically significant at 1% level whatever the binding or non-binding assumption considered, except the constant,  $\beta_{cst}$  (significant at 10% level), in the total cost function and the parameter related to the interaction between Raw Material Y price and Product X production in the total cost function,  $\beta_{RM,Y}$ , which is not significant. The parameters  $\delta_i$ ,  $i=0,1$ , are positive, as well as the marginal utility of income,  $\alpha$ . As well as under competition, the marginal utility of income is decreasing over time in the case of full capacity and constant and lower, up to 2006, in the case of excess capacity. From 2007 the marginal utility of income becomes higher in the excess capacity model.

The own price elasticities of demand are reported in Table 7. They belong to the interval  $[-38.7;-8.5]$  in the case of binding capacity constraint and exhibit a high variability for each year into consideration. The own price elasticities of demand are lower in absolute value in the case of non-binding capacity assumption as they belong to the interval  $[-3.2, -1.00]$ .

The estimated margins are reported in Table 8. Under the assumption of binding capacity constraint, the estimated margins belong to the interval  $[2.8, 26.7]$ . The margins increase during the period under consideration, except in 2003 to 2006. Under the assumption of non-binding capacity constraint, the estimated margins are on average higher: they belong to the interval  $[34\%, 217\%]$ . They do not exhibit any regular trend during the overall period.

Finally the estimated marginal costs are reported in Table 9. They are on average higher than in the case of competitive behaviour.

The analysis of the economic indicator estimated under the assumption of coordinated practices shows that when considering the own price elasticity values the non-binding capacity constraint seems to be more relevant. Although when considering the margins, the binding capacity constraint leads to more realistic results. These results do not favour the assumption of binding or non-binding capacity assumption as a better approximation of the data.

Table 2. Estimated values of the marginal utility of income under the assumption of competitive behaviour

Year	2002	2003	2004	2005	2006	2007	2008
<b>Competition binding</b>	0.034	0.033	0.032	0.031	0.030	0.028	0.027
<b>Competition non-binding</b>	0.014	0.014	0.014	0.014	0.014	0.014	0.014

Source: Confidential data on the industry

Table 3. Own price elasticities of demand under the assumption of competition

YEAR	2002	2003	2004	2005	2006	2007	2008
# Obs	12	12	12	12	12	12	12
<b>Competition binding</b>							
Mean	-6.170	-7.112	-8.045	-7.799	-8.855	-8.241	-9.536
Std	0.542	0.735	1.198	1.214	0.928	1.051	3.311
Min	-7.006	-8.215	-9.663	-9.101	-10.970	-10.482	-18.331
Max	-5.359	-5.590	-5.661	-5.196	-7.552	-6.483	-5.841
<b>Competition non binding</b>							
Mean	-2.462	-2.928	-3.429	-3.439	-4.077	-3.969	-4.781
Std	0.216	0.302	0.510	0.535	0.427	0.506	1.660
Min	-2.796	-3.382	-4.118	-4.013	-5.051	-5.048	-9.191
Max	-2.139	-2.301	-2.413	-2.291	-3.477	-3.123	-2.929

Source: Confidential data on the industry

Table 4. Estimated margins (expressed as percentage) under the assumption of competition

YEAR	2002	2003	2004	2005	2006	2007	2008
# Obs	12	12	12	12	12	12	12/11
<b>Competition binding</b>							
Mean	16.320%	14.209%	12.729%	13.174%	11.402%	12.313%	11.405%
Std	1.404%	1.582%	2.217%	2.491%	1.149%	1.548%	3.108%
Min	14.274%	12.172%	10.349%	10.988%	9.115%	9.541%	5.455%
Max	18.660%	17.889%	17.665%	19.247%	13.241%	15.424%	17.120%
<b>Competition non binding</b>							
Mean	40.896%	34.515%	29.868%	29.878%	24.764%	25.564%	22.747%
Std	3.519%	3.842%	5.201%	5.649%	2.495%	3.213%	6.198%
Min	35.768%	29.568%	24.283%	24.921%	19.797%	19.808%	10.880%
Max	46.760%	43.455%	41.449%	43.652%	28.757%	32.025%	34.146%

Source: Confidential data on the industry

Table 5. Estimated marginal costs under the assumption of non binding capacity constraint and competition

YEAR	2002	2003	2004	2005	2006	2007	2008
# Obs	12	12	12	12	12	12	12
<b>Competition non binding</b>							
Mean	66.112	59.655	47.759	54.368	60.134	51.588	24.459
Std	14.672	17.059	19.818	22.651	16.018	15.304	28.636
Min	31.646	38.652	29.919	14.843	30.167	31.675	-49.119
Max	81.952	95.751	83.976	92.852	80.570	87.107	55.600

Source: Confidential data on the industry

Table 6. Estimated values of the marginal utility of income under the assumption of collusive practices

Year	2002	2003	2004	2005	2006	2007	2008
<b>Monopoly binding</b>	0.562	0.524	0.483	0.445	0.397	0.350	0.310
<b>Monopoly non-binding</b>	0.038	0.038	0.038	0.038	0.038	0.038	0.038

Source: Confidential data on the industry

Table 7. Own price elasticities of demand under the assumption of collusive practices

YEAR	2002	2003	2004	2005	2006	2007	2008
# Obs	12	12	12	12	12	12	12
<b>Monopoly binding</b>							
Mean	-35.012	-38.624	-32.238	-19.202	-33.328	-12.569	-8.512
Std	11.378	9.811	11.768	7.697	10.348	7.436	9.103
Min	-50.031	-51.440	-52.186	-36.107	-47.101	-28.416	-31.268
Max	-12.468	-19.862	-11.199	-7.969	-16.118	-1.246	0.000
<b>Monopoly non binding</b>							
Mean	-2.375	-2.809	-2.543	-1.646	-3.198	-1.368	-1.047
Std	0.772	0.714	0.928	0.660	0.993	0.809	1.120
Min	-3.394	-3.742	-4.116	-3.095	-4.519	-3.093	-3.846
Max	-0.846	-1.445	-0.883	-0.683	-1.546	-0.136	0.000

Source: Confidential data on the industry

Table 8. Margins (expressed in %) under the assumption of collusive practices

YEAR	2002	2003	2004	2005	2006	2007	2008
# Obs	12	12	12	12	12	12	12
<b>Monopoly binding</b>							
Mean	3.315%	2.815%	3.653%	6.137%	3.354%	14.720%	26.628%
Std	1.693%	1.000%	1.885%	2.819%	1.293%	20.873%	25.401%
Min	1.999%	1.944%	1.916%	2.770%	2.123%	3.519%	3.198%
Max	8.021%	5.035%	8.930%	12.548%	6.204%	80.251%	76.351%
<b>Monopoly non binding</b>							
Mean	48.857%	38.703%	46.317%	71.590%	34.958%	135.233%	216.501%
Std	24.958%	13.750%	23.896%	32.887%	13.474%	191.769%	206.526%
Min	29.461%	26.726%	24.295%	32.308%	22.128%	32.332%	26.003%
Max	118.221%	69.217%	113.216%	146.385%	64.664%	737.285%	620.788%

Source: Confidential data on the industry

Table 9. Estimated marginal costs under the assumption of non binding capacity constraint and collusive practices

YEAR	2002	2003	2004	2005	2006	2007	2008
# Obs	12	12	12	12	12	12	12
<b>Monopoly non binding</b>							
Mean	124.938	112.816	90.516	103.019	113.976	97.984	47.050
Std	27.651	32.152	37.368	42.688	30.202	28.855	53.833
Min	60.043	73.224	56.831	28.515	57.522	60.474	-91.447
Max	154.792	180.883	158.831	175.565	152.480	164.977	105.681

Source: Confidential data on the industry

iii. Vuong test as a test of model selection

A standard statistical test to determine which of two models better fit the data is the Vuong test. (See Gasmi, Laffont and Vuong, 1992.) This test is based on the comparisons of likelihood values of the estimated models, taking into account the variance-covariance matrix of the parameters and the estimated errors of the models. In our specific case, the Vuong test allows us comparing the assumption of binding and non binding capacity constraints and the assumption of competition and coordinated practices across firms.

When testing assumption A against assumption B, if the Vuong test statistics is higher than 2, then the assumption A is a better approximation of the data. If the Vuong test statistics is lower than -2, then the assumption B is a better approximation of the data. Finally, if the

Vuong test statistics belongs to  $[-2; 2]$ , none of the assumption can be statistically preferred to the other.

Three Vuong tests have been implemented to determine which are the capacity conditions and type of competition that prevail on the market.

First, we have tested the assumption of binding capacity constraint against non binding capacity constraint in the case of competitive market. The Vuong statistics takes the value 9.70. As it is higher than 2, under the assumption of competition between firms, we accept the assumption of binding capacity constraint.

Then we have tested the assumption of binding capacity constraint against non binding capacity constraint in the case of coordinated practices. The Vuong statistics takes the value 3.15. As it is higher than 2, under the assumption of collusion between firms, we accept the assumption of binding capacity constraint.

Finally we have tested under the assumption of binding capacity constraint we have tested the assumption of competition against the assumption of coordinated practices. The Vuong statistics takes the value 2.20. As it is higher than 2, we can conclude that the competition between firms is the market condition which better fits the observations over the period into consideration.

Under the assumption of binding capacity constraint, the competition prevails over coordination.

## **2. Conclusion**

The crucial point while implementing the econometric model is the assumption related to full or excess capacities. If the level of 75 percent in capacity utilization is accepted as being the full capacity threshold, the observation of the capacity utilization over the period under consideration leads to the conclusion of full capacity. This is confirmed statistically by the Vuong tests implemented in the two possible market conditions, competition and coordination between firms. This conclusion might be valid both for FIRM A and at the industry level under the assumption of coordinated practices across the firms.

We decided to implement the econometric model either for full capacities or for excess capacities. The Vuong test which is a test of model validation leads to the conclusion that the binding capacity constraint better fits the reality of the market.



Under this more realistic assumption of full capacities, the question of market functioning is answered again through a Vuong test implementation. The Vuong test leads to prefer the competition rather than the collusion for a better fit of the working of the observed market.

To conclude, if the full capacity assumption is recognised as the most realistic, which the observed level of production seems to indicate on Product X market, then the findings of our econometric analysis leads to the acceptance of competition between the three main players on the market under consideration.

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## Appendix 1: Derivation of equilibrium conditions

### Notations

The Product X is produced by three firms noted by  $i = 1, 2, I = 3$  and by an importer indexed by the number 0. The main notations are as follows. We define  $\bar{Q}_i$  as the total domestic production capacity,  $q_i$  the domestic production of firm  $i$  for product X,  $p_i$  the domestic price of product X,  $x_i$  the export of firm  $i$ ,  $x_0$  the total volume of exports of all domestic firms,  $q_d = \sum_{i=1}^3 q_i$ , the total domestic production,  $q_0$  the imports,  $p_i$  the import price,  $W$  the international price,  $Q$  the domestic market size (i.e., the sum of the domestic production and the imports).

### Demand elasticities

For the logit specification of the demand introduced in the text the own-price elasticity is given by

$$\varepsilon_{ii} = -\frac{p_i}{q_i} \frac{\partial q_i}{\partial p_i} = \alpha p_i (1 - s_i), \quad (\text{A.1})$$

where  $s_i$  is the market share as defined above, and the cross-price elasticity is obtained as:

$$\varepsilon_{ij} = -\frac{p_j}{q_i} \frac{\partial q_i}{\partial p_j} = \alpha p_j s_j. \quad (\text{A.2})$$

### Equilibrium conditions under competition

Assume that the industry is competitive. In this case, each firm sets her price and the quantity exported so as to maximize her profit, knowing that the other competitors are doing the same. Then the profit maximization is as follows:

$$\text{Max}_{p_i, x_i} p_i q_i + W x_i - C(q_i + x_i) \quad \text{st} \quad q_i + x_i \leq \bar{Q}_i. \quad (\text{A.3})$$

That is to say, each firm maximizes its profit defined as the difference between revenues and cost, where the revenues come from domestic sales and exports and where the cost is a function of the total production.

The first order conditions are:

$$q_i + \left( p_i - \frac{\partial C}{\partial q_i} - \lambda \right) \frac{\partial q_i}{\partial p_i} = 0, \quad (\text{A.4a})$$

$$W - \frac{\partial C}{\partial x_i} - \lambda = 0, \quad (\text{A.4b})$$

$$(\bar{Q}_i - q_i - x_i) \lambda = 0. \quad (\text{A.4c})$$

where  $\lambda$  is the multiplier associated with the capacity condition.

When the capacity constraint is binding,  $p_i$  is chosen so that:

$$\frac{(p_i - W)}{p_i} = \frac{1}{-\frac{p_i}{q_i} \frac{\partial q_i}{\partial p_i}}, \quad (\text{A.5})$$

since  $W = \frac{\partial C}{\partial x_i} + \lambda = \frac{\partial C}{\partial q_i} + \lambda$ , and  $x_i$  is chosen so as  $x_i = \bar{Q}_i - q_i$ .

When the capacity constraint is not binding, then  $\lambda = 0$  and  $p_i$  and  $x_i$  are chosen so that

$$\frac{(p_i - W)}{p_i} = \frac{1}{-\frac{p_i \partial q_i}{q_i \partial p_i}} = \frac{1}{\alpha p_i (1 - s_i)}, \quad (\text{A.6})$$

and

$$\frac{\partial C}{\partial x_i} = \frac{\partial C}{\partial q_i} = \frac{\partial C}{\partial (q_i + x_i)} = W. \quad (\text{A.7})$$

### *Equilibrium conditions under coordination*

Assume now that the firms in the industry behave as a monopoly, i.e., maximize their joint profit. In this case, the monopoly sets its prices and its exports by solving the following programme:

$$\text{Max}_{p,x} \sum_{i=1}^3 p_i q_i + W x_0 - \sum_{i=1}^3 C(q_i + x_i) \quad \text{st} \quad q_d + x_0 \leq \bar{Q}. \quad (\text{A.8})$$

When the capacity constraint is not binding, and using a result presented in Appendix 2 below, the first-order conditions yield the following equilibrium conditions:

$$\frac{(p_i - W)}{p_i} = \frac{1}{\alpha p_i s_0}, \quad (\text{A.9})$$

and

$$\frac{\partial C}{\partial x_i} = \frac{\partial C}{\partial q_i} = \frac{\partial C}{\partial (q_i + x_i)} = W. \quad (\text{A.10})$$

### *Structure of the econometric model*

The typical econometric model comprises four equations: The demand that firm A faces, the demand for imported product X, the pricing equation (for instance Equation (A.6) or Equation (A.9)), the cost function of firm A, as it is introduced in the text.

## Appendix 2: Note on the joint-profit maximization case in a generic example

Consider a set of firms that maximizes their joint profit. Assume that the technology exhibits constant returns to scale.

$$\text{Max}_{p,x} \sum_{i=1}^3 (p_i - c_i) q_i .$$

The first-order conditions are:

$$q_i + (p_i - c_i) \frac{\partial q_i}{\partial p_i} + \sum_{j \neq i} (p_j - c_j) \frac{\partial q_j}{\partial p_i} = 0 .$$

Rearranging yields:

$$q_i + (p_i - c_i) \frac{p_i}{q_i} \frac{\partial q_i}{\partial p_i} \frac{q_i}{p_i} + \sum_{j \neq i} (p_j - c_j) \frac{p_j}{q_j} \frac{\partial q_j}{\partial p_i} \frac{q_j}{p_i} = 0 .$$

Replacing by the expressions of the elasticities from the logit specification of the demand, we obtain:

$$q_i + (p_i - c_i) \alpha (1 - s_i) q_i + \sum_{j \neq i} (p_j - c_j) \alpha s_j q_j = 0 .$$

Rearranging yields:

$$q_i - (p_i - c_i) \alpha q_i + \alpha s_i \sum_k (p_k - c_k) q_k = 0 .$$

Dividing by the market size obtains:

$$s_i - (p_i - c_i) \alpha s_i + \alpha s_i \sum_k (p_k - c_k) s_k = 0 ,$$

which can be simplified by  $s_i$ . After rearranging again we observe that:

$$p_i - c_i = \frac{1}{\alpha} + \sum_k (p_k - c_k) s_k ,$$

which means that all margins are equal. So we can write:

$$(p_i - c_i) \left( 1 - \sum_i s_i \right) = \frac{1}{\alpha} .$$

Hence:

$$p_i - c_i = \frac{1}{\alpha s_0} ,$$

since  $s_0 = 1 - \sum_i s_i$ .

## Appendix 4 – Estimated parameters of the models and main relevant test statistics

Table 10. Estimated parameters under the assumption of competitive practices

Parameter	<i>Competition binding</i>					<i>Competition non-binding</i>				
	Estimate	Approx. Std Err	t Value	Approx.Pr >  t	1st Stage R-Square	Estimate	Approx. Std Err	t Value	Approx.Pr >  t	1st Stage R-Square
$\delta_0$	12.143	0.711	17.08	<.0001	1.000	5.454	1.001	14.95	<.0001	1.000
$\delta_{0,2008}$	5.274	1.566	3.37	0.0012	1.000	2.611	1.272	8.44	<.0001	1.000
$\delta_1$	13.357	0.702	19.04	<.0001	1.000	6.477	1.041	15.35	<.0001	1.000
$\delta_{1,2008}$	6.750	1.561	4.32	<.0001	1.000	3.886	1.368	7.23	<.0001	1.000
$\alpha_{cst}$	0.049	0.009	5.70	<.0001	0.893	0.014	0.002	16.67	<.0001	0.781
$\alpha_{GDP}$	-7.1E-07	3.1E-07	-2.29	0.0244	0.935					
$\beta_{cst}$						5742536	2386451	1.91	0.060	1.000
$\beta_{scrap}$						124769	36872	3.54	0.001	1.000
$\beta_{scrap,Y}$						-0.252	0.475	-0.99	0.324	0.858
$\beta_Y$						0.001	0.001	3.79	0.000	0.401
Number of Observations										
Used	83					82				
Missing	1					2				
Statistics for System										
Objective	1.508					3.414				
Objective*N	125.151					279.941				

Notes: If  $Approx.Pr > |t| < 0.01$  the corresponding parameter is statistically significant at level 1%;  
if  $0.001 < Approx.Pr > |t| < 0.05$  the corresponding parameter is statistically significant at level 5%;  
If  $Approx.Pr > |t| > 0.1$  the corresponding parameter is not statistically significant.

Table 11. Estimated parameters under the assumption of collusive practices

Parameter	<i>Monopoly binding</i>					<i>Monopoly non-binding</i>				
	Estimate	Approx. Std Err	t Value	Approx. Pr >  t	1st Stage R-Square	Estimate	Approx. Std Err	t Value	Approx. Pr >  t	1st Stage R-Square
$\delta_0$	171.918	14.663	11.72	<.0001	1.000	14.964	1.001	14.95	<.0001	1.000
$\delta_{0,2008}$	37.994	12.725	2.99	0.004	1.000	10.739	1.272	8.44	<.0001	1.000
$\delta_1$	176.755	14.955	11.82	<.0001	1.000	15.981	1.041	15.35	<.0001	1.000
$\delta_{1,2008}$	29.823	12.734	2.34	0.022	1.000	9.892	1.368	7.23	<.0001	1.000
$\alpha_{cst}$	1.097	0.140	7.85	<.0001	0.827	0.038	0.002	16.67	<.0001	0.781
$\alpha_{GDP}$	-0.00003	4.266E-6	-6.00	<.0001	0.867					
$\beta_{cst}$						4556365	2386451	1.91	0.060	1.000
$\beta_{scrap}$						130427	36872	3.54	0.001	1.000
$\beta_{scrap,Y}$						-0.471	0.475	-0.99	0.324	0.858
$\beta_Y$						0.003	0.001	3.79	0.000	0.401
Number of Observations										
Used	83					82				
Missing	1					2				
Statistics for System										
Objective	1.821					2.509				
Objective*N	151.140					205.727				

Notes: If  $Approx.Pr > |t| < 0.01$  the corresponding parameter is statistically significant at level 1%;  
if  $0.001 < Approx.Pr > |t| < 0.05$  the corresponding parameter is statistically significant at level 5%;  
If  $Approx.Pr > |t| > 0.1$  the corresponding parameter is not statistically significant.