Ground-based estimation of the aircraft mass, adaptive vs. least squares method

Richard Alligier, David Gianazza, Nicolas Durand

Ground-based Estimation of the Aircraft Mass, Adaptive vs. Least Squares Method

R. Alligier, D. Gianazza, N. Durand
ENAC, MAIAA, F-31055 Toulouse, France
Univ. de Toulouse, IRIT/APO, F-31400 Toulouse, France

***Abstract*—This paper focuses on the estimation of the aircraft mass in ground-based applications. Mass is a key parameter for climb prediction. It is currently not available to ground-based trajectory predictors because it is considered a competitive parameter by many airlines. There is hope that the aircraft mass might become widely available someday, but in the meantime it is possible to estimate an *equivalent mass* from the data already available, assuming the thrust to be known (maximum or reduced climb thrust for example).**

In this paper, we compare the performances of two mass estimation methods proposed in recent publications. Both methods estimate the aircraft mass by fitting the modeled energy rate (i.e. the power of the forces acting on the aircraft) with the energy rate observed at several points of the past trajectory. The first method, proposed by Schultz *et al.* ([1]), dynamically adjusts the weight parameter so as to fit the energy rate, using an adaptive sensitivity parameter to weight each observation. The second method, introduced in one of our previous publications ([2]), estimates the mass by minimizing the quadratic error on the observed energy rate, taking advantage of the polynomial expression of the modeled power when using the BADA model. The robustness of both methods to the observation errors is assessed, using simulated data with various distributions of the noise added to the observed state variables. The results show that both methods are able to find mass estimates that are very close to the "actual" mass, with slightly better performances for the least squares method.

Keywords: aircraft trajectory prediction, mass estimation, BADA, energy rate, specific power

INTRODUCTION

With the emergence of new operational concepts ([3], [4]) centered on trajectory-based operations, predicting aircraft trajectories with great accuracy has become a key issue for most ground-based applications in Air Traffic Management and Control (ATM/ATC). Some of the most recent algorithms applied to ATM/ATC problems require to test a large number of alternative trajectories. As an example, in [5] an iterative quasi-Newton method is used to find trajectories for departing aircraft, minimizing the noise annoyance. Another example is [6] where Monte Carlo simulations are used to estimate the risk of conflict between trajectories, in a stochastic environment. Some of the automated tools currently being developped for ATC/ATM can detect and solve conflicts between trajectories, using Genetic Algorithms ([7][1]), or Differential Evolution or Particle Swarm Optimization ([9]).

To be efficient, all these methods require a fast and accurate trajectory prediction, and the capability to test a large number of "what-if" trajectories. Such requirements forbid the sole use of on-board trajectory prediction, which is certainly the most accurate, but is not sufficient for these most promising applications. Even with the existing (or future) datalink capabilities that could transmit the on-board prediction to ground systems, there remains a need for a fast and accurate ground-based prediction.

Most trajectory predictors rely on a point-mass model to describe the aircraft dynamics. The aircraft is simply modeled as a point with a mass, and the second Newton's law is applied to relate the forces acting on the aircraft to the inertial acceleration of its center of mass. Such a model is formulated as a set of differential algebraic equations that must be integrated over a time interval in order to predict the successive aircraft positions, knowing the aircraft initial state (mass, current thrust setting, position, velocity, bank angle, etc.), atmospheric conditions (wind, temperature), and aircraft intent (thrust profile, speed profile, route).

Unfortunately, the data that is currently available to ground-based systems for trajectory prediction purposes is of fairly poor quality. The speed intent and aircraft mass, being considered competitive parameters by many airline operators, are not transmitted to ground systems. The actual thrust setting of the engines (nominal, reduced, or other, depending on the throttle's position) is unknown. There are uncertainties or noise in the Weather and Radar data. Some studies ([10], [11], [12]) detail the potential benefits that would be provided by additional or more accurate input data. In other works, the aircraft intent is formalized through the definition of an Aircraft Intent Description Language ([13], [14]) that could be used in air-ground data links to transmit some useful data to ground-based applications. There is hope that, in the future, all the necessary data required to predict aircraft trajectories will be available. In the meantime, we propose to learn some of the unknown parameters of the point-mass model – typically the aircraft mass – from the data that is already available.

Focusing on the aircraft climb, we are interested in this paper in estimating the aircraft mass, which is one of the key parameters for climb performance, using the past trajectory points. This approach, where some unknown parameters are adjusted by fitting the model to the observed past trajectory, is not new. The past publications following this path ([15], [16], [17], [18], [1], [2], [19]) propose several methods, with

[1]These algorithms are at the root of the strategic deconfliction through speed adjustments developped in the European ERASMUS project ([8]). A more recent application is the SESAR 4.7.2 (*Separation Task in En Route Trajectory-based Environment*) project, where lateral and vertical maneuvers are also used.

different choices for the adjusted parameter (mass, or thrust, for example), the modeled variable that is fitted on past observations (rate of climb, energy rate), and the algorithm that is applied (stochastic method, adaptive mechanism, least squares, etc.).

Among the publications dealing with mass estimation, let us cite [15], where Warren and Ebrahimi propose an *equivalent weight* as a workaround to use a point-mass model without knowing the actual aircraft mass. Nominal thrust and drag profiles are assumed. The equivalent mass is found by minimizing the gap between the computed and observed vertical rates. A second study ([16]) raises doubts about the reliability of the vertical rate for this purpose, and suggests to use the energy rate instead. The proposed method is tested on simulated trajectories only. In more recent works, Schultz, Thipphavong, and Erzberger ([1]) introduce an adaptive mechanism where the modeled mass is adjusted by fitting the modeled energy rate with the observed energy rate. This adaptive method provides good results on simulated traffic and the authors plan to try it on real data. In [2], we use a Quasi-Newton algorithm (BFGS) combined to a mass estimation method to learn the thrust profile minimizing the error between the modeled and observed energy rate. The thrust law, once learned on historical data, is used to predict the future trajectory of any new aircraft, together with the mass estimated on the past trajectory points. This method has been tested on two months of real data, showing good results. Concerning the mass estimation method, we showed that, when using the BADA[2] model of the forces (or a similar model), the aircraft mass can be estimated at any past point of the trajectory by solving a polynomial equation, knowing the thrust setting at this point. When using several points, and assuming a constant mass over the whole trajectory segment, the mass can be estimated by minimizing the quadratic error on the energy rate.

In the current paper, we propose an improvement of this least squares method. The mass is no longer assumed to be constant during the climb. It follows the fuel law given by the Eurocontrol Base of Aircraft DAta (BADA) model. We compare the performances of the two mass estimation methods: the proposed least squares method and the adaptive mechanism introduced by Schultz *et al.* in [1]. As the actual aircraft mass is not available in the real data that we have collected, we use simulated trajectories. In order to mimic the diversity and volatility of the real Radar tracks, the simulated trajectories are produced by sampling a number of parameters (mass, calibrated airspeed, Mach number, temperature differential) according to some given distributions, and a Gaussian noise is added independently to some state variables (altitude, true airspeed, rate of climb, acceleration, temperature) for each trajectory point. The robustness of both mass estimation methods to the noise added to each state variable is studied.

The rest of this paper is organized as follows: Section I describes the forces' model and the equations governing the aircraft dynamics. Section II describes the two mass estimation methods. The data and experimental setup are detailed in section III, and the results are shown and discussed in section IV, before the conclusion.

[2]BADA: the Eurocontrol Base of Aircraft DAta

I. Models and Equations

A. Aircraft Dynamics with the Effect of Wind

Ground-based trajectory predictors used for air traffic management and control purposes usually rely on a simplified point-mass model to predict aircraft trajectories. In such a model, all forces acting on the aircraft body are exerted at the center of mass, making several simplifying approximations. The inertial moments and angular accelerations of the aircraft around its center of gravity are not included in the model. The aircraft is modeled as a point of mass m, subject to the second Newton's law that gives us the inertial acceleration $\overrightarrow{a_i} = \frac{d\overrightarrow{V_i}}{dt} = \dot{\overrightarrow{V_i}}$ of the center of mass (the dot above a vector denotes the time derivative of this vector):

$$m\dot{\overrightarrow{V_i}} = \overrightarrow{Thr} + \overrightarrow{D} + \overrightarrow{L} + m\overrightarrow{g} \tag{1}$$

In equation (1), mass is considered a stationary variable[3] for what concerns its impact on the aircraft dynamics. At a larger scale, though, the fuel burn and the consequent loss of mass must be taken into account when integrating the equations to predict the future trajectory. Concerning the forces, it is assumed that the thrust $\overrightarrow{Thr}$ exerted by the aircraft engines is aligned to the airspeed vector $\overrightarrow{V_a}$, and in the same direction. The drag $\overrightarrow{D}$ exerted by the relative wind on the flying airframe is also aligned to $\overrightarrow{V_a}$, by definition, and in the opposite direction. The lift force $\overrightarrow{L}$ caused by the motion of the airframe through the air is perpendicular to these vectors and in the plane of symmetry of the aircraft. The flight is assumed to be symmetric and there is no aerodynamic sideforce. The effects of Earth rotation on the aircraft dynamics are neglected (flat Earth approximation).

The effect of wind $\overrightarrow{W}$ on the aircraft velocity and acceleration cannot be neglected, however. It can be written as follows:

$$\overrightarrow{V_i} = \overrightarrow{V_a} + \overrightarrow{W} \tag{2a}$$

$$\overrightarrow{a_i} = \dot{\overrightarrow{V_a}} + \dot{\overrightarrow{W}} \tag{2b}$$

We can project equation (1) onto the airspeed vector $\overrightarrow{V_a}$ axis. This gives us the following equation, where "." denotes the dot product of two vectors:

$$m\overrightarrow{V_a}.\frac{d\overrightarrow{V_i}}{dt} = \left(\overrightarrow{Thr} + \overrightarrow{D} + \overrightarrow{L} + m\overrightarrow{g}\right).\overrightarrow{V_a} \tag{3}$$

Combining equations (2) and (3), and introducing h the geodetic height of the aircraft, and $\dot{h} = \frac{dh}{dt}$ the inertial vertical velocity (counted positive upward), equation (3) can be reformulated as a law governing the total energy rate, denoting W_{Up} the upward component of the wind:

$$\underbrace{\left(\frac{Thr - D}{m}\right)V_a}_{\text{specific power}} = \underbrace{V_a\dot{V_a} + g\dot{h}}_{\text{specific energy rate}} + \underbrace{(\dot{\overrightarrow{W}}.\overrightarrow{V_a} - gW_{Up})}_{\text{wind effect}} \tag{4}$$

[3]We assume in fact that $\frac{d}{dt}(mV_i) = m\dot{V_i}$, and neglect the impact of $\dot{m}$ on the acceleration.

Expressing the power of the forces acting along the true airspeed axis, and the total energy (kinetic and potential) of the aircraft gives us an interesting insight to equation (4). We can see how the aircraft dynamics are governed by the specific power (i.e. power per unit of mass) and energy rate:

$$\mathit{Power} = (\mathit{Thr} - D)\,V_a \tag{5a}$$

$$\mathit{Energy} = \frac{1}{2}mV_a^2 + mgh \tag{5b}$$

$$\frac{\mathit{Power}}{m} = \frac{d}{dt}\left(\frac{\mathit{Energy}}{m}\right) + (\dot{\overrightarrow{W}}.\overrightarrow{V_a} - gW_{Up}) \tag{5c}$$

For historical and technical reasons, the geodetic altitude h and the inertial vertical velocity $\dot{h}$ are not much used in air traffic control operations. Instead, a *pressure altitude* H_p (also called *geopotential pressure altitude* in [20]) is computed on board the aircraft and transmitted to ground systems by Mode-C or Mode-S transponders. The relationship between the pressure altitude and the geodetic altitude is the following, with T denoting the air temperature, and ΔT is the difference with the temperature that would occur using the International Standard Atmosphere (ISA) model:

$$g\dot{h} = g_0\left(\frac{T}{T - \Delta T}\right)\frac{dH_p}{dt} \tag{6}$$

Neglecting the vertical component of the wind W_{UP} and using the relationship between $\dot{h}$ and $\dot{H}_p$ stated in equation (6), equation (4) can be re-written as follows, introducing $\frac{dH_p}{dt}$, the rate of climb or descent (ROCD), g_0 the gravitational acceleration *at mean sea level*, and a corrective factor related to the temperature:

$$\underbrace{\frac{\mathit{Thr} - D}{m}V_a}_{\text{specific power}} = \underbrace{V_a\frac{dV_a}{dt} + g_0\left(\frac{T}{T - \Delta T}\right)\frac{dH_p}{dt}}_{\text{specific energy rate}} + \underbrace{\frac{d\overrightarrow{W}}{dt}.\overrightarrow{V_a}}_{\text{wind effect}} \tag{7}$$

Considering an aircraft trajectory picked up from historical data, the energy rate and wind effect (right-hand part of equation (7)) can be computed at any point of the observed trajectory. The specific power (left-hand part) is a function of the mass m and the thrust and drag forces (Thr and D).

In the rest of this paper, we focus on estimating the mass for climbing aircraft, using equation (7). In the two methods presented in section II, the mass is adjusted so that equation (7) is satisfied. This requires a model of the thrust and drag forces.

B. Modeling the Forces

Using equation (7) to actually compute a trajectory requires a model of the aerodynamic drag D of the airframe flying through the air. We also need a computational model of the engines' thrust *Thr*. In our experiments, we used version 3.9 of the Eurocontrol Base of Aircraft Data (see [21]) to compute these forces.

The BADA model provides different parametric models of the thrust force *Thr* for jet, turboprop, and piston engines (see section 3.7 of [21]). These models are tuned by regression using manufacturers' data. They allow us to compute the standard maximum climb thrust $\mathit{Thr}_{\text{max climb}}$ as a function of H_p, ΔT, and V_a:

$$\mathit{Thr}_{\text{max climb}} = f_1(H_p, V_a, \Delta T) \tag{8}$$

The dimensionless lift and drag coefficients are defined as follows:

$$C_L = \frac{2mg_0}{\rho V_a S\cos\Phi} \tag{9a}$$

$$C_D = a_D + b_D C_L^2 \tag{9b}$$

where S is the wing surface, Φ is the bank angle, and a_D and b_D are values depending on the phase of flight (landing gear up or down, flaps extended, etc.).

Given these coefficients (experimentally found), the equation for the drag D is the following:

$$D = \frac{C_D\rho V_a^2 S}{2} \tag{10}$$

With the atmosphere model and the equations of [20], the air density ρ and temperature T can be expressed as a function of the temperature differential ΔT. So the drag is as a function of the aircraft mass m, the true air speed V_a, the geopotential pressure altitude H_p and the temperature differential ΔT. Moreover, one can notice that the drag D is a polynomial of the second degree with respect to the mass that has the following form:

$$D = f_2(H_p, V_a, \Delta T) + m^2 \times f_3(H_p, V_a, \Delta T, \Phi) \tag{11}$$

C. Fuel consumption

A fuel consumption model is also required when computing a full trajectory. In climbing phase, the fuel consumption is modeled by equation (12), where the mass variation $\frac{dm}{dt}$ is described as a function of H_p, V_a and ΔT.

$$\frac{dm}{dt} = -f_4(V_a, H_p, \Delta T) \tag{12}$$

II. Mass Estimation

The two mass estimation methods compared here rely on the idea of adjusting the mass m in order to equalize the specific power and the specific energy rate.

In order to be more specific, let us introduce P and Q, defined as follows, considering equations (5) and (7) :

$$P = \mathit{Power} - m \times \underbrace{\left[\frac{d}{dt}\left(\frac{\mathit{Energy}}{m}\right) + (\dot{\overrightarrow{W}}.\overrightarrow{V_a})\right]}_{Q} \tag{13a}$$

$$Q = V_a\frac{dV_a}{dt} + g_0\left(\frac{T}{T - \Delta T}\right)\frac{dH_p}{dt} + \frac{d\overrightarrow{W}}{dt}.\overrightarrow{V_a} \tag{13b}$$

The quantity Q is the sum of the energy rate and wind effect. It can be computed at any point of the past trajectory using the recorded Radar track, Weather data, and equations (2). Considering the forces model given by equations (8) and (10) in section I-B, only the mass m is missing to compute the power. Thus, at each point i of the trajectory, the power is a function $Power(m_i)$ of the mass m_i at point i. The total energy model equation (7) becomes:

$$\frac{P_i(m_i)}{m_i} = 0 \quad \Leftrightarrow \quad Power_i(m_i) = m_i Q_i \tag{14}$$

A. The Adaptive Method

The idea of the adaptive method introduced by Schultz *et al.* in [1] is to dynamically adjust the weight mg so that the modeled energy rate (i.e. the power of the forces acting on the aircraft) fits the observed energy rate. The weight is adjusted for each new trajectory point and the weight update depends on a sensitivity parameter which is dynamically adapted, comparing the energy rate error of the new observation to the average value over the five last points. Small values of the sensitivity parameter compensate for the volatility of radar track data, giving less importance to the outliers (i.e. the points that differ too much from the average), whereas high values allow the algorithm to better follow the energy rate variations.

Let us now describe more formally the two parts of this adaptive algorithm: the weight adjustment and the sensitivity parameter adaptation. Due to our choice of notations and to the form of our equation (7), and also because we adjust the mass m instead of the weight mg, our description of the adaptive method is slightly different from the one given by Schultz *et al.*. Otherwise, the mechanism is exactly the same.

In the dynamic weight adjustment, the power at point i is modeled using the previous mass m_{i-1}. The current mass m_i is then obtained by applying equation (14), using Q_i the energy rate and wind effect observed at point i:

$$m_i = \frac{Power_i(m_{i-1})}{Q_i} \tag{15}$$

Introducing $Q_i - \frac{Power_i(m_{i-1})}{m_{i-1}} = -\frac{P_i(m_{i-1})}{m_{i-1}}$, the error made on the energy rate when modeling the power at point i using the previous mass m_{i-1}, equation (15) can be rewritten as follows:

$$\begin{aligned} m_i =& \frac{Power_i(m_{i-1})}{Q_i} \\ =& \frac{Power_i(m_{i-1})}{\frac{Power_i(m_{i-1})}{m_{i-1}} + (Q_i - \frac{Power_i(m_{i-1})}{m_{i-1}})} \\ =& \frac{1}{\frac{1}{m_{i-1}} + \frac{1}{Power_i(m_{i-1})} \underbrace{\left(Q_i - \frac{Power_i(m_{i-1})}{m_{i-1}}\right)}_{\text{error on the energy rate when using } m_{i-1}}} \\ =& m_{i-1}\left(1 - \frac{P_i(m_{i-1})}{Power_i(m_{i-1})}\right)^{-1} \end{aligned} \tag{16}$$

For the reasons explained at the begining of this section, a sensitivity parameter β_i is introduced in the update term of equation (16). Finally, the mass is updated using the following equation:

$$m_i = m_{i-1}\left[1 + \beta_i \left(\frac{-P_i(m_{i-1})}{Power_i(m_{i-1})}\right)\right]^{-1} \tag{17}$$

The sensitivity parameter β_i is adapted by comparing the observed variations of the energy rate, given by $P_i(m_{i-1})$ in equation (17), to the average variation over the five previous points. The adaptation rule given in [1] is the following, where $\Delta\dot{E}_i = \frac{P_i(m_{i-1})}{m_{i-1} g V_a}$ (with our notations):

$$\begin{aligned} &\text{if } i > 0 \text{ and } \Delta\dot{E}_i > 0.0001 \\ &\qquad \text{and } \left|\frac{\Delta\dot{E}_i - \Delta\dot{E}_{avg}}{\Delta\dot{E}_{avg}}\right| < 3 \\ &\text{then} \\ &\qquad \beta_i = max(0.205, \beta_{i-1} + 0.05) \\ &\text{else} \\ &\qquad \beta_i = 0.005 \end{aligned} \tag{18a}$$

In equation (18), $\Delta\dot{E}_{avg}$ is the average value of $\Delta\dot{E}_i$ over the last five previous points. Note that there might be less than five points when the algorithm "warms up", at the beginning of the trajectory.

With this mechanism, if $\Delta\dot{E}_i$ is repeatedly high in the same order of magnitude, β_i will increase, strengthening the adaptation. Otherwise, β_i has a low value. As a consequence, an isolated high $\Delta\dot{E}_i$ does not have a great impact on the adaptation. This improves the robustness of this mass estimation process.

The algorithm starts with an initial mass m_0 (typically the reference mass given by the BADA model). The mass variation at each iteration is bounded: in our experiments, it is limited to 2%[4] of the reference mass. During the whole process, the estimated mass is bounded within 80% and 120% of the reference mass.

B. Least Squares Method

In the adaptive method presented in section II-A, the mass is iteratively updated with each new trajectory point. The algorithm starts with an initial mass m_0 and ends up with a final mass m_n after n iterations.

In the least squares method, the mass is directly estimated by minimizing the sum of the squared errors over n points. The total error $\mathcal{E}$ being minimized is the following:

$$\mathcal{E}(m_1, \ldots, m_n) = \sum_{i=1}^{n} \left(\frac{Power_i(m_i)}{m_i} - Q_i\right)^2 \tag{19a}$$

$$= \sum_{i=1}^{n} \left(\frac{P_i(m_i)}{m_i}\right)^2 \tag{19b}$$

Note that in equation (19), the error function is related to the modeled *specific power* $\frac{Power_i(m_i)}{m_i}$ (i.e. power per unit of mass), and not the power (which might have given simpler expressions later on in this section). This choice is motivated by the trajectory prediction purpose of the mass estimation: when trying to predict the pressure altitude H_p and true airspeed V_a of a climbing aircraft, one has to integrate the total energy model equation (7), or alternatively an equation in

[4]This value differs from the one given in [1], but it gives better results in our experiments.

the form $\frac{d\overrightarrow{V}}{dt} = \frac{\sum \overrightarrow{Forces}}{m}$. The final expression of this integral involves the specific power, not the power.

We introduced the least squares mass estimation method in [2], where it was associated to another method in order to learn the thrust profile of a given aircraft type from historical data. When considering fresh trajectory inputs, the learned thrust profile and the mass estimated on the past trajectory points were used to predict the future values of the energy rate. In a more recent publication currently under review, we showed that the proposed method could improve the altitude prediction accuracy up to 50% when compared to the standard BADA model. These studies were conducted on real data (Mode-C Radar tracks and Weather data).

In these previous works, we assumed that the mass m remains constant over the n points used for the estimation. We now improve the proposed method by taking the fuel burn into account. The mass variation is ruled by equation (12) (see section I-C).

With this equation, the mass m_i at point i and time t_i can be written as a simple function of the final mass m_n, knowing the values of the state variables (temperature, altitude, velocity, etc) observed at point i. The mass at point i is the following, with f_4 modeling the fuel burn:

$$
\begin{aligned}
m_i &= m_n + \int_{t_i}^{t_n} f_4(V_a(t), H_p(t), \Delta T(t))dt && \text{(20a)}\\
&\simeq m_n + \sum_{k=i}^{n-1} \frac{f_4(t_{k+1}) + f_4(t_k)}{2}(t_{k+1} - t_k) && \text{(20b)}\\
&= m_n + \delta_i && \text{(20c)}
\end{aligned}
$$

The quantity $\delta_i = \sum_{k=i}^{n-1} \frac{f_4(t_{k+1})+f_4(t_k)}{2}(t_{k+1} - t_k)$ can be computed from the available data for every point i of the observed past trajectory. Therefore, the sum of squares error $\mathcal{E}$ can be rewritten as follows:

$$
\begin{aligned}
\tilde{P}_i(m_n) &= P_i(m_n + \delta_i) && \text{(21a)}\\
\mathcal{E}(m_n) &= \sum_{i=1}^{n} \left(\frac{\tilde{P}_i(m_n)}{(m_n + \delta_i)} \right)^2 && \text{(21b)}
\end{aligned}
$$

The aircraft mass is estimated by minimizing $\mathcal{E}(m_n)$ given by equation (21b). This minimization can be done efficiently when using the model of forces provided by BADA. With this model, the power ($Power_i(m_i)$) can be expressed as a second-degree polynomial of the mass m_i, using the functions f_1, f_2, and f_3 (see section I-B for the model of the forces):

$$
\begin{aligned}
Power_i(m_i) = &- m_i^2 \times f_3(H_{p_i}, V_{a_i}, \Delta T_i, \Phi_i)\\
&+ f_1(H_{p_i}, V_{a_i}, \Delta T_i)\\
&- f_2(H_{p_i}, V_{a_i}, \Delta T_i)
\end{aligned} \tag{22}
$$

Consequently, $\tilde{P}_i(m_n) = Power_i(m_n+\delta_i) - (m_n+\delta_i)Q_i$ is a second-degree polynomial of the final mass m_n. The overall error $\mathcal{E}$ is a sum of rational terms (i.e. ratios of polynomial functions). The minimum m^* of this function satisfies the equation $\mathcal{E}'(m^*) = \frac{d\mathcal{E}}{dm}(m^*) = 0$. When introducing a common denominator in $\mathcal{E}'$, the equality $\mathcal{E}'(m^*) = 0$ becomes a polynomial equation of degree at most $3(n-1)+4$. Solving such a high degree polynomial might be a difficult task due to numerical issues [22]. Therefore, instead of minimizing $\mathcal{E}$ we minimize an approximation $\mathcal{E}_{approx}$ as defined by equation (23) below:

$$
\begin{aligned}
F_{avg}(m_n) &= \frac{1}{n} \sum_{i=1}^{n} (m_n + \delta_i) && \text{(23a)}\\
\mathcal{E}_{approx}(m_n) &= \sum_{i=1}^{n} \left(\frac{\tilde{P}_i(m_n)}{F_{avg}(m_n)} \right)^2 && \text{(23b)}
\end{aligned}
$$

With this approximation, the derivative of the error function is given by the following equation (24):

$$
\begin{aligned}
&\mathcal{E}'_{approx}(m_n) =\\
&\frac{2}{(F_{avg}(m_n))^3} \sum_{i=1}^{n} \tilde{P}_i(m_n) \left[\tilde{P}_i{}'(m_n) F_{avg}(m_n) - \tilde{P}_i(m_n) F_{avg}{}'(m_n) \right]
\end{aligned} \tag{24}
$$

With the above equation (24), the optimal mass m^* must satisfy the fourth-degree polynomial equation (25) below, in order to cancel out $\mathcal{E}'_{approx}$.

$$
\sum_{i=1}^{n} \tilde{P}_i(m^*) \left[\tilde{P}_i{}'(m^*) F_{avg}(m^*) - \tilde{P}_i(m^*) F_{avg}{}'(m^*) \right] = 0 \tag{25}
$$

One can solve analytically this fourth-degree polynomial equation using Ferrari's method. However, even for a third-degree polynomial, analytical methods might not be numerically stable [23]. In our experiments, we used the numerical method[5] provided by the GNU Scientific Library. This numerical method appears to be as fast as the analytical method in our experiments. Among the four potential solutions given by this numerical method, we select the solution[6] in $]0; +\infty[$ minimizing $\mathcal{E}_{approx}$. The obtained value is the estimated aircraft mass m^* at point n.

III. Data and Experimental Setup

A. Aircraft Trajectories

The two mass estimation methods (adaptive and least squares) are tested on simulated trajectories. The version 3.9 of the BADA model is used to produce 4-minute long climb segments, assuming a *max climb* thrust. The synthesized trajectories start at altitude 12,000 ft. Three different aircraft types are considered: the A320 which is a short-range aircraft, the A333 which is a medium-range aircraft, and the B744 which is a long-range aircraft.

When the thrust law is fixed, the climb trajectory depends only on the mass, the speed profile $(CAS, Mach)$ and the temperature differential ΔT.

[5]This method of the GNU scientific library uses a balanced-QR reduction of the companion matrix.

[6]Actually, under reasonable hypotheses on the observed variables, one can prove that there is exactly one solution in $]0; +\infty[$ that cancels out $\mathcal{E}'_{approx}$.

Our set of simulated trajectories is created by sampling these four parameters independently, using uniform laws. The parameters of these uniform laws are summarized in table I. Such a uniform distribution is not realistic, but it is sufficient for our purpose, which is to test the robustness of both methods on a variety of trajectories. The useful state variables $T, H_p, V_{TAS}, \frac{dH_p}{dt}, \frac{dV_{TAS}}{dt}$ are assumed to be observed every 12 seconds, giving us 21 points per trajectory. Each dataset used in our experiments comprises 1000 climb segments of 21 points.

parameter	distribution
CAS	$CAS_{ref} + uniform([-30; 30])$
Mach	$Mach_{ref} + uniform([-0.03; 0.03])$
ΔT	$uniform([-20; 20])$
mass	$mass_{ref} \times uniform([0.8; 1.2])$

Table I: The distribution of the parameters used to generate our trajectories.

B. Adding a Gaussian Noise

Assuming we used only the BADA model, without noise, to produce our dataset, the resulting trajectories would be smooth. Such trajectories would not be very representive of the real-life radar data, which is much more noisy and volatile.

Our aim is to assess the robustness of both methods to the observation errors. To that purpose, we add a Gaussian noise to the state variables associated to each trajectory point. This is done independently for each of the following five state variables: temperature T, pressure altitude H_p, true airspeed V_a, acceleration $\frac{dV_a}{dt}$, and rate of climb $\frac{dH_p}{dt}$.

We create separate trajectory datasets for these five variables, adding a Gaussian noise to only one variable in a given dataset. For a given climb segment, the random draws are made in an independent way: we draw a random noise from the chosen distribution for each trajectory point. Several values of the standard deviation of the Gaussian distribution are tested for each variable. For instance, if we want to test n different values of the standard deviation for each observed variable, we create $5 \times n$ datasets of 1000 trajectories each.

IV. Results

A. Robustness to Observation Errors

The results are assessed by computing the root mean square of the relative error, for each dataset. This relative error for a given trajectory is simply $100 \times \frac{mass_{estimated} - mass_{actual}}{mass_{actual}}$. The root mean square errors (RMSE) are plotted on figures 1 to 5 for each variable and for different values of the Gaussian noise's standard deviation.

Concerning figures 2 to 5, the ranges chosen for the standard deviation are inspired from the worst cases of the Eurocontrol document [24]. Assuming a number of primary and secondary surveillance radars, this document describes different scenarios, with the associated errors in position and velocity.

Looking at figures 1 to 5, we see that both methods estimate the mass with good accuracy. In all cases, the RMSE increases with the input noise, which is not very surprising. From the maximum ranges observed for the RMSE (only a few percents in all cases), we can say that both methods are quite robust to the noise introduced in the temperature, altitude, velocity, acceleration, and rate of climb.

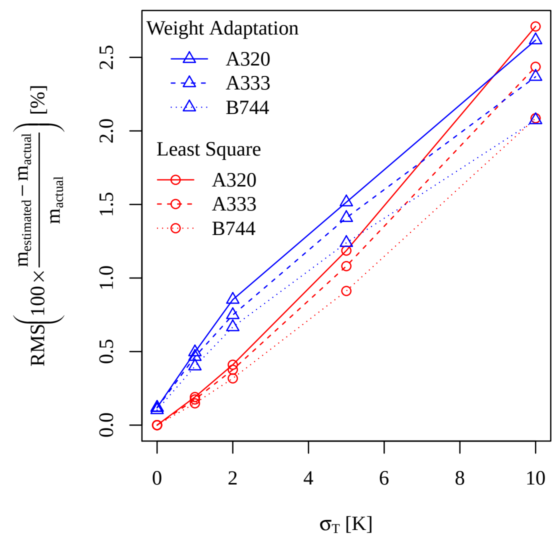

Figure 1: Sensitivity of the mass estimation methods to the noise in the temperature T.

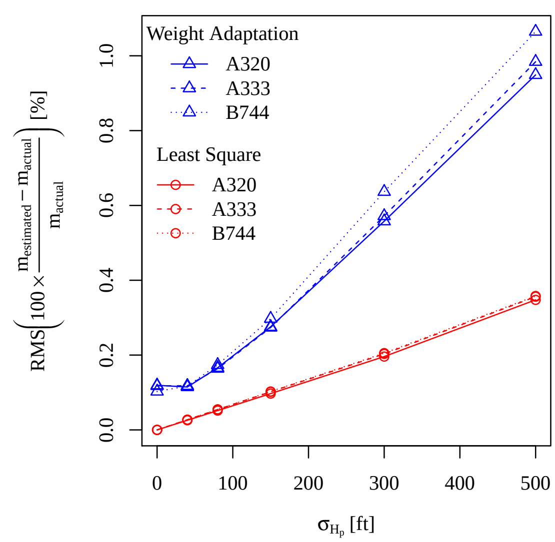

Figure 2: Sensitivity of the mass estimation methods to the noise in the pressure altitude H_p.

Surprisingly, the estimated mass is relatively insensitive to the noise in the pressure altitude H_p, according to the RMSE values displayed on figure 2. This is especially true for the least squares method.

The errors on the true airspeed V_a, acceleration $\frac{dV_a}{dt}$ and rate of climb $\frac{dH_p}{dt}$ are more sensitive to the input error, as can be seen on figures 4 to 5. The highest errors are observed when introducing a noise in the acceleration and rate of climb. This is not very significant, however, as we may have chosen too high standard deviations for the noise introduced in these

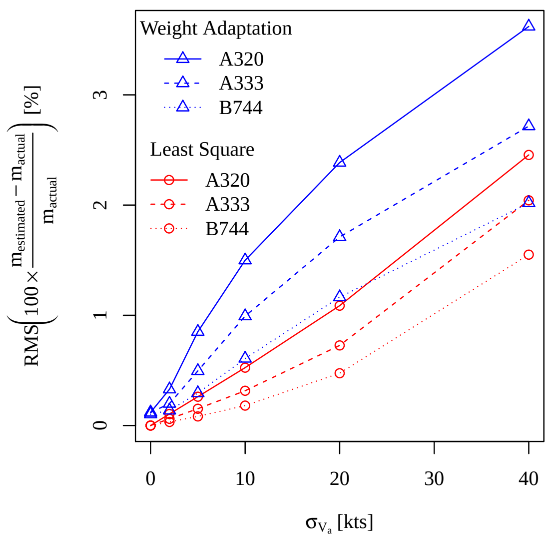

Figure 3: Sensitivity of the mass estimation methods to the noise in the true airspeed V_a.

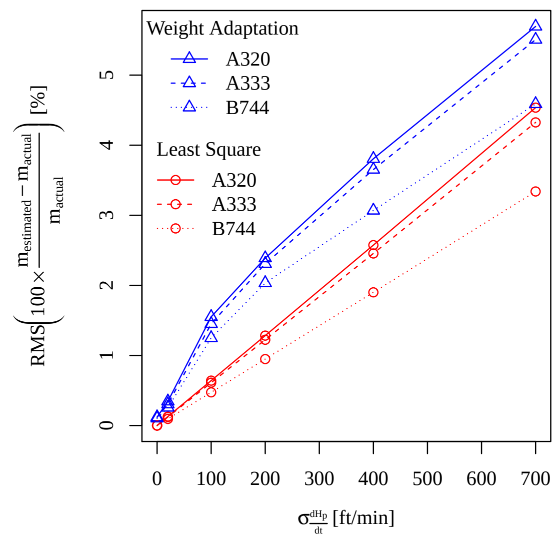

Figure 5: Sensitivity of the mass estimation methods to the noise in the rate of climb $\frac{dH_p}{dt}$.

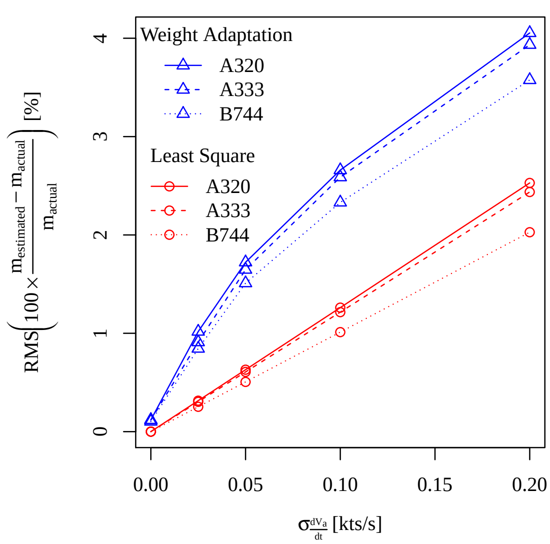

Figure 4: Sensitivity of the mass estimation methods to the noise in the true airspeed $\frac{dV_a}{dt}$.

variables. For example, a standard deviation of $0.2\ kts.s^{-1}$ for the acceleration is a high value: the acceleration $\frac{dV_a}{dt}$ in our experiments is in a range from $-0.08\ kts.s^{-1}$ to $0.44\ kts.s^{-1}$ for the three considered aircraft types.

The behavior of the two methods is consistently the same for all aircraft types, even if some differences can be observed between the three aircraft types that were tested: the mass estimation is slightly more sensitive to the noise for short-range aircraft than for long-range aircraft.

In all figures, the least squares methods exhibits a better RMSE than the adaptive method, except maybe for the noise in the temperature, where the performances of the two methods are fairly close. For the true airspeed V_a, acceleration $\frac{dV_a}{dt}$, and rate of climb $\frac{dH_p}{dt}$, the RMSE obtained with the least squares method is about 20 to 50 percent less than the RMSE obtained with the adaptive method. When considering the H_p variable, the order of magnitude of the difference in RMSE goes up to 60 to 70 percent in favor of the least squares.

Overall, for what concerns the robustness to the observation noise and with the parameter settings chosen for the algorithms (number of points, thresholds, etc), the least squares method seems to perform a little better than the adaptive method. One must keep in mind, however, that all errors remain in a range of a few percents only, for both methods.

B. Influence of ΔT on Mass Estimation Errors

Our datasets were generated by sampling random values for $(CAS, Mach, \Delta T, mass)$. Looking at how the error is distributed among these various samples, we can observe some differences, depending on the parameter values. This is particularly true for the temperature differential ΔT.

As an illustration, we have plotted the individual errors with respect to ΔT for both methods, with the observation noise added to the temperature T. The results are shown on figures 6 and 7. On these figures, we can observe a much higher variance of the errors for the samples with high values of ΔT. There is clearly a threshold for this ΔT parameter, above which the mass estimation is much more sensitive to input errors.

This can be explained as follows: according to the BADA model, when ΔT is superior to a given threshold ($C_{Tc,4}$[7]), the maximum climb thrust drastically decreases when the temperature increases, as shown on figure 8. Consequently, when the atmospheric conditions are hot (ΔT is superior a

[7]This constant is introduced by the BADA model. It depends on the aircraft type.

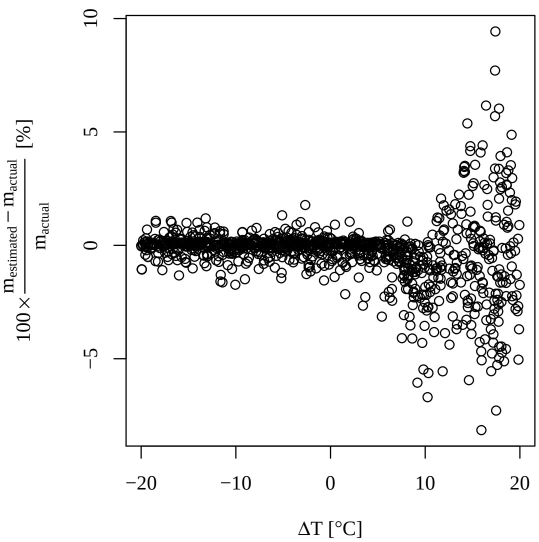

Figure 6: Influence of ΔT on the mass estimation error for the adaptive method, when introducing a Gaussian noise in the temperature T (A320, $\sigma_T = 5$ K).

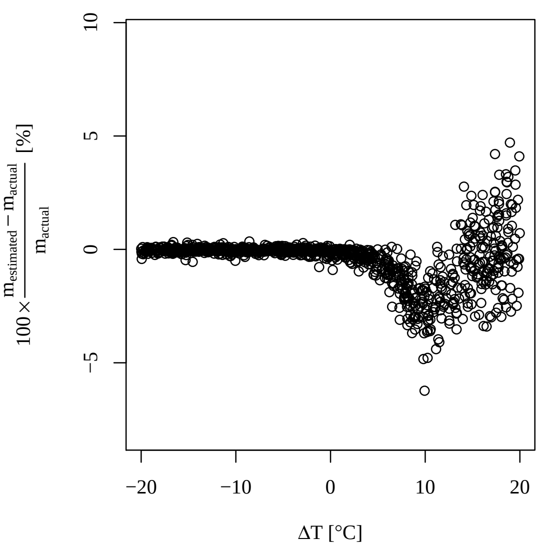

Figure 7: Influence of ΔT on the mass estimation error for the least squares method, when introducing a Gaussian noise in the temperature T (A320, $\sigma_T = 5$ K).

certain threshold), the mass estimation methods become much more sensitive to temperature errors.

C. Discussion on the Two Methods

We have seen in section IV-A that both methods, adaptive and least squares, are quite robust to the errors introduced in the observed trajectory, with a slight advantage to the least squares method that seems to give more accurate and robust mass estimations.

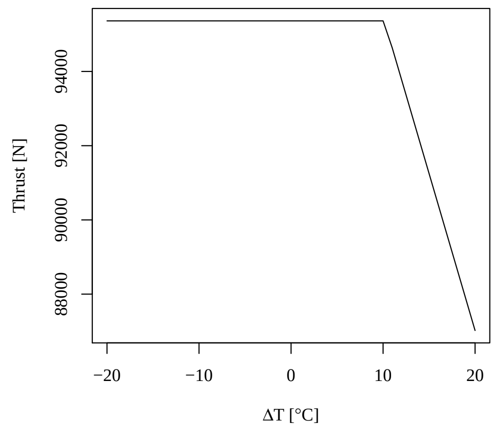

Figure 8: Thrust as a function of ΔT (BADA model, Airbus A320, FL180).

When choosing one method or the other, we must also consider the other characteristics of the two methods, discussed in this section. The weight adaptation method proposed by Schultz *et. al.* does not rely on a specific model of the forces, and a black-box model of the power can be used. The least squares method takes advantage of the fact that the specific power is a polynomial function, when using the Eurocontrol BADA equations to model the forces. Therefore, the least squares method is model-dependent, which might be considered as a drawback. However, other models of the forces might be compliant with this method. For instance, the Enhanced Jet Performance Model ([25]) seems to be compliant with the least squares method.

In both methods, the mass (or the weight) is adjusted by fitting the modeled specific power to the observed specific energy rate, assuming a given thrust law during climb. However, the mass variation during climb is guided by very different laws in the two methods. In the adaptive method, the weight update computed at each iteration is bounded so as to remain within a "reasonable" domain (2% of the reference mass, in our experiments). This mechanism is necessary in this method: the weight adaptation gives poor results without it. Due to this mechanism, a large number of iterations is required in order to possibly reach every mass within $[80\%; 120\%]$ of the reference mass. Apart from this bounding mechanism, the mass variations are free, so as to track the energy rate variations as best as possible. In the least squares method, the mass variations follow the fuel consumption law provided by the BADA model all along the observed climb segment.

As a consequence of the last remark, it is more difficult to dissociate the respective influence of the thrust and mass variations with the adaptive method than with the least squares method. The adaptive method dynamically adjusts an *equivalent weight* so as to follow the energy rate variations, that are caused by variations of both the thrust and actual mass (assuming all the other parameters to be known), making it difficult to dissociate these two sources of variation. In the least squares method, the mass variation is ruled by the fuel consumption model provided by BADA. If all other parameters are perfectly known, the only way to explain a variation of specific power is a change in the thrust law.

Introducing an additional constraint (the fuel consumption law, here) in the mass estimation method allows us to handle separately the thrust law and the mass estimation. In [2], we proposed to learn a typical thrust profile from historical data that minimizes the overall energy rate error, using an optimization algorithm combined with the mass estimation method. More detailed results on this approach can be expected in a publication currently under review.

D. Limitations of our Study

For the reasons explained in the introduction, we have used simulated data in this work, to compare the two mass estimation methods described in section II. The purpose of this study was not to compare the climb prediction performances of both methods on real data, but to check if they could find a close estimation of the "actual" mass, and if this estimation was robust to the errors introduced in the observed trajectories. These trajectory errors were artificially introduced by adding a noise to some state variables observed at each trajectory point.

Concerning the simulated data, we are aware that the uniform distributions from which the values of the BADA input parameters (CAS, $Mach$, "actual mass", etc.) were sampled are unlikely to be observed in real traffic. The uncorrelated Gaussian noise that was added to the state variables, independently for each trajectory point, might not be realistic either. Actually, some studies suggest that, in real-life, there are some correlated and systematic errors in the position and speed measurements ([24]).

The simulated traffic served our purpose, however, and we showed that for both methods the relative error on the estimated mass is low, considering the high values of standard deviation that were tested for the observation noise. We also showed how the estimation error varies with some of the input parameters: for instance, when ΔT is superior to a threshold, the mass estimation methods are much more sensitive to the noise in the temperature.

Another limitation of our study is that we have to know the thrust profile, because we want to find a meaningful estimation of the mass: our aim, in this study, is to compare this estimation with the "actual" mass used to simulate the observed trajectory. In this work, we assumed a maximum climb thrust, both when simulating the trajectories and when estimating the mass.

In operations, when trying to predict real trajectories, the thrust profile (past or future) is not known, and many aircraft use partial thrust instead of maximum climb thrust. Actually, the fact that both mass and thrust are unknown is what motivated the dynamic adjustment of modeled parameters such as the *equivalent weight* or the modeled thrust ([17]). Considering the equations governing the energy rate, one can either adjust the mass, assuming a constant thrust, or adjust the thrust, assuming a constant mass. In any case, the modelled mass and thrust will most likely be different from the real ones, but they can be tuned so as to improve the overall trajectory prediction.

Considering the limitations discussed above, it is difficult to draw some definitive conclusions from our results, as to how the two mass estimation methods would compare when using real data. To this end, we would need some real data containing the actual aircraft masses and thrust profiles. Such data is not available for the time being, so one can only expect to assess the overall climb performance of prediction methods that combine an adjusted (resp. assumed) mass with an assumed (resp. adjusted) thrust. Alternatively, typical thrust profiles can be learned from historical data. In [2], using two months of real data, we demonstrated that the least squares mass estimation method combined with a learned thrust profile actually improves the prediction of the energy rate. It would be interesting to compare this approach to the adaptive method on real data. Such a comparative study of the overall climb performance is not in the scope of the current paper, which is only a first step toward this objective.

Conclusion

To conclude, let us summarize our approach and findings, before giving a few perspectives on future works. In this study, we compare two mass estimation methods (adaptive and least squares), using simulated data. The adaptive method, recently introduced by Schultz *et al.* in [1], dynamically adjusts the weight to fit the modeled energy rate to the observation. The least squares method is a refinement of the analytical method that we proposed in [2]. This method minimizes the sum of squared errors on the energy rate, using several points of the past trajectory. It takes advantage of the fact that the specific power is a polynomial function of the mass when modeling the thrust and drag forces with the BADA model. Although it is model-dependent, we believe that the least squares method could be extended to some other point-mass models. As an improvement to the analytical method introduced in [2], the least squares method takes into account the fuel consumption.

The two mass estimation methods are tested on different sets of simulated trajectories. For that purpose, the values of the input parameters used to produce the simulated data (with the BADA model) are sampled from uniform distributions. Some noise, sampled from a Gaussian distribution, is introduced in the state variables of the resulting trajectories, so as to simulate observation errors. Several datasets are used, considering each variable in turn, with several values for the noise's standard deviation.

The results show that both methods are quite robust to the errors on the observed trajectory. Even when sampling the noise from distributions with large standard deviations, the estimated mass falls within a few percents of the "actual mass" that was initially used to produce the simulated trajectory. With the current parameter settings chosen for both algorithms, the least squares method proves slightly more efficient than the adaptive method when estimating the mass in noisy conditions.

The results presented in this paper prove that it is possible to accurately estimate the aircraft mass from noisy observations, at least when using simulated data and knowing the thrust profile. Some previous results ([2]) with real Weather and Mode-C Radar data prove that the mass estimation combined with a typical thrust profile learned from historical data can highly improve the overall performance of the trajectory prediction. Thus, with these two studies, we have a body of evidence that mass estimation can be successfully applied to real trajectories.

From an operational point of view, the resulting improvement in the climb prediction accuracy would certainly benefit air traffic controllers, especially in the vertical separation task as shown in [1].

In future works, it could be interesting to compare the two methods, adaptive and least squares, on Radar track records instead of simulated data. As discussed before, we cannot expect to find the actual aircraft mass in this case. Actually, we can only evaluate the overall performance of the trajectory prediction, using one method or the other to estimate an *equivalent mass* (or weight). Ghasemi *et al.* ([26], [27]) have applied machine learning techniques to the trajectory prediction problem. We intend to use the estimated mass as input to standard Machine Learning techniques (neural networks, linear regression, etc). Some preliminary experiments show that such techniques already give good results without even using the mass. Finally, we plan to compare the Machine Learning approach to the point-mass model with adjusted parameters (estimated mass, learned thrust profile).

References

[1] C. Schultz, D. Thipphavong, and H. Erzberger. Adaptive trajectory prediction algorithm for climbing flights. In *AIAA Guidance, Navigation, and Control (GNC) Conference*, August 2012.

[2] R. Alligier, D. Gianazza, and N. Durand. Energy Rate Prediction Using an Equivalent Thrust Setting Profile (regular paper). In *International Conference on Research in Air Transportation (ICRAT), Berkeley, California, 22/05/12-25/05/12*, page (on line), http://www.icrat.org, may 2012. ICRAT.

[3] SESAR Consortium. Milestone Deliverable D3: The ATM Target Concept. Technical report, 2007.

[4] H. Swenson, R. Barhydt, and M. Landis. Next Generation Air Transportation System (NGATS) Air Traffic Management (ATM)-Airspace Project. Technical report, National Aeronautics and Space Administration, 2006.

[5] X. Prats, V. Puig, J. Quevedo, and F. Nejjari. Multi-objective optimisation for aircraft departure trajectories minimising noise annoyance. *Transportation Research Part C*, 18(6):975–989, 2010.

[6] G. Chaloulos, E. Crück, and J. Lygeros. A simulation based study of subliminal control for air traffic management. *Transportation Research Part C*, 18(6):963–974, 2010.

[7] N. Durand, J.M. Alliot, and J. Noailles. Automatic aircraft conflict resolution using genetic algorithms. In *Proceedings of the Symposium on Applied Computing, Philadelphia*. ACM, 1996.

[8] F. Drogoul, P. Averty, and R. Weber. Erasmus strategic deconfliction to benefit sesar. In *Proceedings of the 8th USA/Europe Air Traffic Management R&D Seminar*, June-July 2009.

[9] C. Vanaret, D. Gianazza, N. Durand, and J.B. Gotteland. Benchmark for conflict resolution (regular paper). In *International Conference on Research in Air Transportation (ICRAT), Berkeley, California, 22/05/12-25/05/12*, page (on line), http://www.icrat.org, may 2012. ICRAT.

[10] Study of the acquisition of data from aircraft operators to aid trajectory prediction calculation. Technical report, EUROCONTROL Experimental Center, 1998.

[11] ADAPT2. aircraft data aiming at predicting the trajectory. data analysis report. Technical report, EUROCONTROL Experimental Center, 2009.

[12] R. A. Coppenbarger. Climb trajectory prediction enhancement using airline flight-planning information. In *AIAA Guidance, Navigation, and Control Conference*, 1999.

[13] J. Lopez-Leones, M.A. Vilaplana, E. Gallo, F.A. Navarro, and C. Querejeta. The aircraft intent description language: A key enabler for air-ground synchronization in trajectory-based operations. In *Proceedings of the 26th IEEE/AIAA Digital Avionics Systems Conference*. DASC, 2007.

[14] J. Lopes-Leonés. *The Aircraft Intent Description Language*. PhD thesis, University of Glasgow, 2007.

[15] A.W. Warren and Y.S. Ebrahimi. Vertical path trajectory prediction for next generation atm. In *Digital Avionics Systems Conference, 1998. Proceedings., 17th DASC. The AIAA/IEEE/SAE*, volume 2, pages F11/1 –F11/8 vol.2, oct-7 nov 1998.

[16] A.W. Warren. Trajectory prediction concepts for next generation air traffic management. In *3rd USA/Europe ATM R&D Seminar*, June 2000.

[17] G. L. Slater. Adaptive improvement of aircraft climb performance for air traffic control applications. In *Proceedings of the 2002 IEEE International Symposium on Intelligent Control*. IEEE conference publications, October 2002.

[18] I. Lymperopoulos, J. Lygeros, and A. Lecchini Visintini. Model Based Aircraft Trajectory Prediction during Takeoff. In *AIAA Guidance, Navigation and Control Conference and Exhibit*, Keystone, Colorado, August 2006.

[19] A. Hadjaz, , G. Marceau, P. Savéant, and M. Schoenauer. Online learning for ground trajectory prediction. In *Proceedings of the SESAR Innovation Days (2012)*. EUROCONTROL, 2012.

[20] D. Poles. Revision of atmosphere model in bada aircraft performance model. Technical report, EUROCONTROL, 2010.

[21] A. Nuic. User manual for base of aircarft data (bada) rev.3.9. Technical report, EUROCONTROL, 2011.

[22] James H. Wilkinson. *Rounding Errors in Algebraic Processes*. Dover Publications, Incorporated, 1994.

[23] N. J. Higham. *Accuracy and Stability of Numerical Algorithms*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, second edition, 2002.

[24] Eurocontrol standard document for radar surveillance in en-route airspace and major terminal areas. Technical report, Eurocontrol, March 1997.

[25] M. Kaiser, M. Schultz, and H. Fricke. Enhanced jet performance model for high precision 4d flight path prediction. In *Proceedings of the 1st International Conference on Application and Theory of Automation in Command and Control Systems*, ATACCS '11, pages 33–40, Toulouse, France, France, 2011. IRIT Press.

[26] M. Ghasemi Hamed, M. Serrurier, and N. Durand. Possibilistic knn regression using tolerance intervals. In *IPMU 2012, Catania, Italy, July*, volume 299 of *Communications in Computer and Information Science*. Springer, 2012.

[27] M. Ghasemi Hamed, M. Serrurier, and N. Durand. Simultaneous interval regression for k-nearest neighbor. In *Australasian Conference on Artificial Intelligence*, pages 602–613, 2012.

Biographies

Richard Alligier is a PhD student at the MAIAA lab. of the the french university of civil aviation (ENAC). He received his engineer's degrees (IEEAC, 2010) from the ENAC and his M.Sc. (2010) in computer science from the University of Toulouse, France.

David Gianazza received his two engineer degrees (1986, 1996) from the french university of civil aviation (ENAC) and his M.Sc. (1996) and Ph.D. (2004) in Computer Science from the "Institut National Polytechnique de Toulouse" (INPT). He has held various positions in the french civil aviation administration, successively as an engineer in ATC operations, technical manager, and researcher. He is currently associate professor at the ENAC, Toulouse.

Nicolas Durand graduated from the Ecole polytechnique de Paris in 1990 and the Ecole Nationale de l'Aviation Civile (ENAC) in 1992. He has been a design engineer at the Centre d'Etudes de la Navigation Aérienne (then DSNA/DTI R&D) since 1992, holds a Ph.D. in Computer Science (1996) and got his HDR (french equivalent of tenure) in 2004. He is currenlty professor at the ENAC/MAIAA lab.