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Ground Handling Management at Airports with Fuzzy Information

Salma Fitouri-Trabelsi*, Carlos Alberto Nunes Cosenza, Felix Mora-Camino ***

**Automation Research Group, MAIAA, Ecole Nationale de l'Aviation Civile (ENAC), 31055 Toulouse, France, (+33)562174358, fitouri_trabelsi_salma@hotmail.com, felix.mora@enac.fr*
***LabFuzzy, COPPE, Universidade Federal do Rio de Janeiro, Brazil CEP 21949-900, (+55)(21)25628286 Cosenza@pep.ufrj.br*

Abstract: In this communication the ground handling fleet management problem at airports is considered with the aim of improving aircraft service at arrival and departure while the operational cost of the ground service fleets is taken into account. The complexity of the considered problem, as well as, operational considerations lead to propose an on-line decentralized management structure where the criticality of each aircraft demand for service is evaluated using a fuzzy formalism. Then after detailing the proposed collaborative scheme between ground handling fleet managers, airlines and airport authorities, a heuristic approach is proposed to solve each fleet assignment problem. A case study considering a large airport is discussed.

1. INTRODUCTION

The ground handling operations represent the airside activities at airports in charge of processing passengers, cargo, facilities and supplies at and around parked aircraft. Most of these operations are performed by different service providers, using vehicles which are specific to each type of operation.

Ground handling is not a prominent activity within the air transportation system (ATS), however this activity is an important enabler for efficient airport operation and its management is an important issue. Over the last decades, the complexity of ATS has increased to face the worldwide growth of air traffic. Today the operation of this system involves global actors (airports, airlines, air traffic control (ATC), air traffic management (ATM)) as well as local actors (ground handlers, local suppliers…) whose coordination, while pursuing different and sometimes contradictory objectives, is difficult to achieve. The main objectives of traffic management at airports are to improve operational efficiency by reducing aircraft delays, to optimize the use of airport resources to reduce operation costs and to improve the predictability of air transportation operations (flight arrivals and departures).

So, to face the current situation with acceptable safety and efficiency standards a new concept has been developed: Airport Collaborative Decision Making (CDM) which tries to create a common ground for the different component of the ATS. This concept is based on an improved communication between the different actors of the airport. As an example, for Roissy-CDG airport, where CDM has been implemented since 2010, departure times are respected in more than 85% of the cases, against 80% before, the ground traffic is more fluid (taxiing times have been shortened by 2 to 4 minutes), a reduction of 14.5 tons of fuel in daily consumption and also a significant decrease in CO^2 emissions [2].

So far the management of the different service fleets which perform the turnaround process of ground handling has not been considered specifically in the CDM approach, even if it has an important part in the fluidity of aircraft ground movements, however [3] shows that 10% of all the flight delays are caused by the inefficient management of the different fleet of the ground handling operation.

This paper after identifying possible objectives and essential constraints for ground handling at airports, proposes a decentralized structure to cope with this multi-fleet assignment problem. Considering that many parameters are imperfectly known, the fuzzy modelling formalism is introduced to take into account this important characteristic of this ground handling multi-fleet (GHMF) problem.

2. WORK CONTEXT

Aircraft turnaround defines the process of servicing an aircraft while it is on the ground between two successive flights it operates. The turnaround term implies a fast sequence between an arrival and a departure, however for many air transport operations, in particular for long haul flights, a large time interval may be programmed between them. During the turnaround, an aircraft must undergo a complex process composed of a set of elementary ground handling activities such as landing / boarding, unloading / loading of luggage, fuelling, catering, cleaning, water and sanitation processes. Fig.1 describes the main ground handling operations taking place around a grounded aircraft as well as their precedence constraints. Arrival and departure ground handling tasks are distinguished.

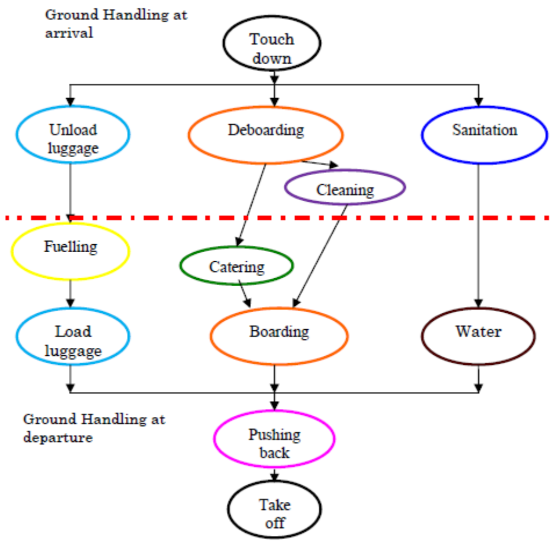


Fig. 1. Ground handling activities at arrival and departure

Ground handling operations are carried out by various service companies, using vehicle which are specific to each type of operation. To perform the turnaround process for each aircraft within the allocated time, these different companies have to coordinate between each other while respecting the constraints of scheduling tasks for each aircraft and the constraints related to the use of service vehicles. The duration of each ground handling operation is variable from one flight to another and depends in general of the type of aircraft, the volumes of passengers/luggage to be processed as well as of other external factors such as the current weather conditions at the airport. Then the large variability of elementary task durations should be taken into account when managing the different ground handling fleets. Each ground handling fleet type is supposed homogenous so that the same task can be performed with the same efficiency by any vehicle of each considered ground handling fleet. The duration of an elementary task t on aircraft $a(i)$ assigned to flight i can be estimated either by an airline ground station manager or the corresponding ground handling manager who has received information about the load of the flight from the airline. It is here supposed that this duration is given by a dual fuzzy number $\widetilde{d}_t^i = d_t^i + \varepsilon \cdot \delta_t^i$ where d_t^i is the current central value of the duration of task t and δ_t^i is the uncertainty range. See *appendix A* for an introduction to fuzzy dual numbers. A set of fuzzy rules can be built to generate these fuzzy dual task durations where the backbone is the nominal processing times with scaling factors and the fuzzy rules generate the dual part of the elementary task durations.

Each of the ground handling activities makes use of specialized equipment which must be turned available at the aircraft parking place at the right time to avoid delays. Some of the ground handling activities should be performed as soon as possible after the arrival of the aircraft at their parking stand and others must be performed only some time before departure from their parking stand. Depending of aircraft operation these two sub sets of activities can be performed in immediate sequence or are separated by an idle period of variable duration according to arrival and departure schedules of a given aircraft. Fig.2 displays a standard situation for an aircraft undergoing a turnaround process where space is a rather scare resource and some tasks cannot be performed simultaneously (mainly for safety reasons). It appears that the efficient operations of such complex process which repeats with each aircraft arrival or departure is very difficult to be achieved while it is a critical issue for airport operations performance. Then advanced management tools are necessary to cope in a satisfactory way with this problem.

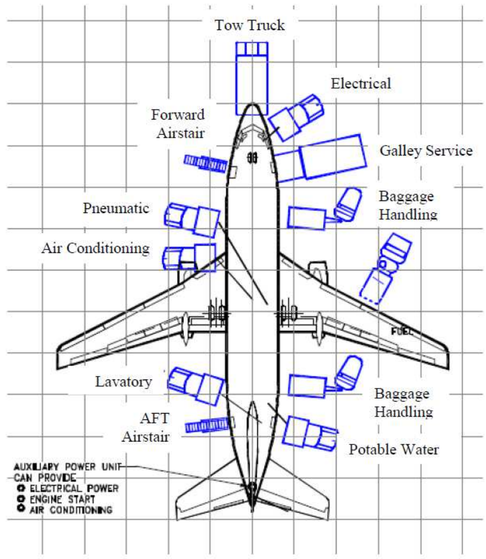


Fig. 2. Ground handling arrangement [2] .

Few publications covering Fuzzy VRP or Fuzzy Scheduling are available in the literature. The Fuzzy VRP has been introduced as a VRP problem with time window constraints where the customer demand, the service and the travel times are given by fuzzy numbers. In [15], a simple description of a VRP problem with fuzzy traveling times is introduced and its solution is obtained through a genetic algorithm. In [16] where the duration of the time window of a VRP problem is considered as a fuzzy variable, the solution has been computed with an Ant algorithm whose monitoring is based on the evolution of the entropy of the solution. With regard to Fuzzy Scheduling, [17] presents an overview of fuzzy approaches to scheduling and emphasizes the representation of preference profiles and the modelling of uncertainty distributions.

3. DECENTRALIZED MANAGEMENT OF GHMF

In this case, it is considered that airlines communicate with the ground handling fleet managers through their own ground station managers which are in charge of monitoring the ground handling activities at arrival or departure of each flight. For example, one of their objective with respect to flight arrivals is to minimize the waiting time for de-boarding passengers and luggage, another one is to make sure that passengers board the aircraft in due time before scheduled flight departure time. So, they will be in charge of requesting in due time the necessary ground handling resources for flight arrival or departure processing. In the case of a decentralized management of the different fleets of ground handling vehicles, the size of each fleet management problem is of course smaller. With respect to the definition of the corresponding decision problems, some objectives of the ground handling problem can be expressed as constraints at the individual fleet level. Once these constraints are set, a major objective for each ground handling fleet manager will consist in minimizing its ground handling variable costs related mainly to the fleet operations costs. This can be considered to be achieved by minimizing the travelled distance of the corresponding ground handling fleet, contributing also to airport environment protection (chemical emissions and noise).

To be feasible, a decentralized approach, nominal or on-line, must be cooperative. Each ground handling fleet assignment and scheduling (GHFAS) problem must be executed according to a sequence compatible with the organization of the ground handling activities (Fig.1). Then each GHFAS problem should integrate time constraints generated from the solution of the uphill GHFAS problems or from the updated expected flight arrival schedules. The ground handling services are delivered in a disturbed environment with many operational uncertainties. For example, the expected arrival times for flights are subject to frequent delays, the duration of ground handling tasks is sensitive to unexpected events such as additional travel time due to traffic congestion on airside service ways or machine breakdown.

Airport air traffic control services update the predicted arrival times which are forwarded to airport services, including airlines and ground handling. This starts the process of updating the assignment and scheduling of tasks for each ground handling fleet. In the case in which repeated aircraft arrival schedule perturbations are expected, according for instance to meteorology conditions, the horizon of fleet management can be commonly limited to some hours ahead, while ground handling resources computed from the daily nominal GHMF problem must remain on the lookout. Each ground handling fleet manager may solve the new instance of each GHFAS problem by taking into account the scheduling constraints generated and provided by the uphill GHFAS problems or from the updated aircraft arrival schedule and parking positions forwarded by the airline. The predicted completion time of his activities on each aircraft are sent to the other ground handling managers and the corresponding airline. When a fleet manager decides to generate a new plan he communicates the result to the downhill ground handling operators so that they update their plans. The immediate uphill ground handling managers will be able then to compute the estimated time margins for each task by comparing their processing time plus the nominal duration of their task with the earliest processing time of the following tasks in turnaround process. Then, if some vehicle is delayed but remains within the computed time margin, no delay warning is sent to the following task providers. When delays of vehicles overcome time margins new scheduling constraints are generated and the following ground handling fleet managers solve an updated version of their GHFAS problem.

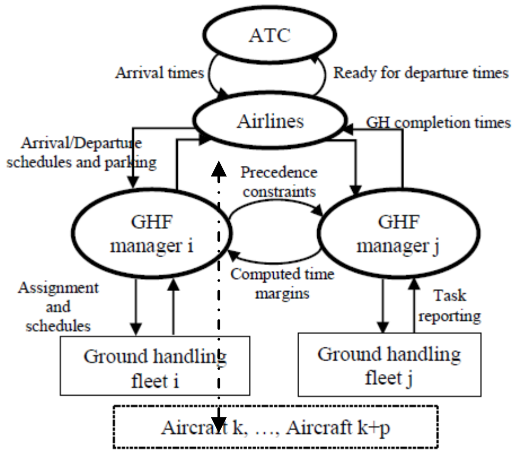


Fig. 3. Information flows structure

The solutions of the successive updated GHFAS problems are forwarded to the airlines which produce new estimates for the departure schedule of their aircraft. However, it appears that this approach generates a lot of communication between each fleet managers as well as a large amount of computation to update detailed assignment solutions. So in the next section a simplified approach to decentralized ground handling management is introduced with the corresponding heuristic to produce on-line solutions to the GHMF assignment problem.

4. A FUZZY HEURISTIC FOR ON-LINE GHMF ASSIGNMENT PROBLEM

The problem for each ground handling fleet is here to assign ground handling vehicles to arriving or departing aircraft so that each aircraft is serviced by a vehicle while, according to the current operational situation, no delay or a minimum delay is produced. For that, the airline ground station managers generate resources requests to the ground handling fleet managers. The produced schedules are based on the predicted arrival times as well as the scheduled departure times. These schedules take not only into consideration the possible variation of the ground handling tasks durations by using a fuzzy dual formalism [18, 19] , but consider also the criticality of the flight. This criticality depends on the current predicted delay as well as the operational consequences on other flights. Then more critical flights may get their ground handling solution treated before earlier less critical scheduled flights. The following notations are adopted: Each task of the turnaround process $t \in \{1,...,T\}$ is carried out on an aircraft $a(i)$ associated to a flight i, $i \in I$, ($I=I_A \cup I_D$, I_A is the set of arriving flights and I_D is the set of departing flights) by a specific service provider $k \in \{1,..., K\}$.

4.1 Fuzzy-based ranking of flights

The first step of the proposed heuristic consists in performing an initial ordering of the flights in accordance with their current predicted arrival time $\hat{t}_i^a$ at their assigned parking amended by considering their criticality. To each arriving flight $i \in I_A$, can be assigned the difference $\Delta t_i^a = \hat{t}_i^a - \bar{t}_i^a$ between the predicted arrival time $\hat{t}_i^a$ and the scheduled arrival time $\bar{t}_i^a$. Here $\hat{t}_i^a$ and $\bar{t}_i^a$ can be either real numbers or fuzzy dual numbers, where $\hat{t}_i^a$ is provided by the ATC. Each arriving flight must cope with two types of operational constraints:

- Connection constraints when arriving passengers must reach without delay another departing flight.
- Departure schedule when the arriving aircraft must be ready to start a new flight with a tight schedule.

When considering connection constraints, let C_i be the set of departing flights connected to arriving flight i. The time margin between fight i and each flight j in C_i is given by:

$$\widetilde{m}_{ij}^a = \bar{t}_j^d - \hat{t}_i^a - \max\left\{\widetilde{d}_{db}^i + \widetilde{T}_{ij}, \widetilde{d}_{ul}^i + \widetilde{\theta}_{ij}\right\} \quad j \in C_i \tag{1}$$

Here $\widetilde{T}_{ij}$ and $\widetilde{\theta}_{ij}$ are respectively the connecting delay for passengers and luggage between flights i and j. The margin between arrival flight i and departure flight j serviced in immediate succession by the same aircraft is:

$$\widetilde{m}_{ij}^a = \bar{t}_j^d - \hat{t}_i^a - \widetilde{D}_{ij} \quad \text{with} \quad j = \sigma(i) \tag{2}$$

where $\widetilde{D}_{ij}$ is the minimum fuzzy dual duration of ground handling around arrival of flight i and departure of flight j. Here $\sigma(i)$ provides the number of the next flight serviced by the aircraft operating flight i. Then:

$$\widetilde{D}_{ij} = \max\begin{Bmatrix} \widetilde{d}_{ul} + \widetilde{d}_{fu} + \widetilde{d}_{ll} \\ \widetilde{d}_{db} + \widetilde{d}_{ca} + \widetilde{d}_{bd} \\ \widetilde{d}_{db} + \widetilde{d}_{cl} + \widetilde{d}_{bd} \\ \widetilde{d}_{sa} + \widetilde{d}_{wa} \end{Bmatrix} + \widetilde{d}_{pb} \tag{3}$$

Then, the fuzzy margin of arriving aircraft i is given by:

$$\widetilde{m}_i^a = \min_{j \in C_i \cup \sigma(i)} \widetilde{m}_{ij}^a \tag{4}$$

The amended arrival time for flight i is then given by:

$$\widetilde{\widetilde{t}}_i^a = \hat{t}_i^a + \widetilde{m}_i^a \tag{5}$$

To each departing flight $i \in I_D$, can be assigned the difference $\Delta t_i^d = \hat{t}_i^d - \bar{t}_i^d$ between the predicted departure time $\hat{t}_i^d$ and the scheduled departure time $\bar{t}_i^d$. Here also, $\hat{t}_i^d$ and $\bar{t}_i^d$ can be either real numbers or fuzzy dual numbers. Symmetrically, each departing flight must cope with operational constraints related with successive flights by the same aircraft and flight connections for passengers and cargo.

In the case in which the ground handling tasks are relative to a departing flight j, the amended predicted time to start grand handling activities at the corresponding parking position is now given by:

$$\widetilde{\widetilde{t}}_j^d = \bar{t}_j^d - \min_{i \mid j \in C_i and\, i = \sigma^{-1}(j)} \widetilde{m}_{ij}^a \tag{6}$$

with
$$\widetilde{m}_{i\sigma(i)}^a = \max\begin{Bmatrix} \widetilde{d}_{fu} + \widetilde{d}_{ll} \\ \widetilde{d}_{ca} + \widetilde{d}_{bd} \\ \widetilde{d}_{wa} \end{Bmatrix} + \widetilde{d}_{pb} \tag{7}$$

Then, to each flight i, either arriving or departing, is assigned a time parameter τ_i such as:

$$\tau_i = \left\| \widetilde{\widetilde{t}}_i^a \right\| \text{ for arriving flights} \tag{8.a}$$

$$\tau_i = \left\| \widetilde{\widetilde{t}}_i^d \right\| \text{ for departing flights} \tag{8.b}$$

where $\| \ \|$ is the fuzzy dual pseudo norm defined in the appendix. Then the flights, either arriving or departing, present in the considered period of operation can be ranked according to an increasing τ_i index. Let the integer $r_a(i)$ be the amended rank of flight i.

4.2 Ground Handling Fleets assignment to flights

Then flights are processed in the produced order $r_a(i)$ where ground handling vehicles are assigned to the corresponding aircraft. In the case of an arriving flight, ground handling arrival tasks (unloading luggage, de-boarding, cleaning and sanitation) are coped with by assigning the corresponding vehicles in accordance to their previous assigned tasks with other aircraft, their current availability, and their current distance to the considered aircraft. Here the common reference time schedule for the ground handling arrival tasks is $\hat{t}_i^a, i \in I_A$.

In the case of a departing flight, ground handling departure tasks (fuelling, catering, luggage loading, boarding, water and push back) are also coped with by assigning the corresponding vehicles in accordance to their previous assigned tasks with other aircraft, their current availability, and their current distance to the considered aircraft. Here the common reference time schedule for the ground handling departure tasks is $B^{low}(\widetilde{\widetilde{t}}_i^d), i \in I_D$.

In both cases it is considered that the whole set of different ground handling vehicles necessary at arrival or departure is assigned by considering the common reference time schedule. This assignment of vehicles to flights either arriving or departing is performed on a greedy base by considering the closest vehicle available to perform the required task. This will make that at the start of ground handling activities for an arrival or departure flight, all necessary resources will be nearby the parking place and that scheduling constraints between elementary ground handling tasks (see fig. 1) will be coped with locally without need of communication between the different ground handling fleet managers. This is a rather simple greedy heuristic which provides for each fleet facing the current service demand a complete solution through a reduced computational effort. So there is no limitation in calling back this solution process any time a significant perturbation occurs.

5. CASE STUDY

To validate the proposed cooperation scheme and the associated heuristics real traffic data from Palma de Mallorca (PDM) Airport has been considered. PDM Airport is, with respect to aircraft and passengers traffic, the third largest Spanish airport. During the summer period it is one of the busiest airports in Europe, with 22.7 million of passengers in 2011. The airport is the main base for the Spanish carrier Air Europa and also a focus airport for German carrier Air Berlin. It occupies an area of 6.3 km2 (2.4 sq mi). Due to rapid growth of aircraft traffic and passenger flows along the last decades, additional infrastructure has been added to the two original terminals A (1965) and B (1972). PDM Airport is composed now of two runways, four terminals and 180 parking stands (27 of them at aprons) [20]. It can handle up to 25 million passengers per year, with a capacity to dispatch 12,000 passengers per hour. Figure 4 displays the hourly traffic of arriving and departing aircraft on a typical summer day at this airport. It appears that aircraft traffic remains intense from early morning until the beginning of night hours.

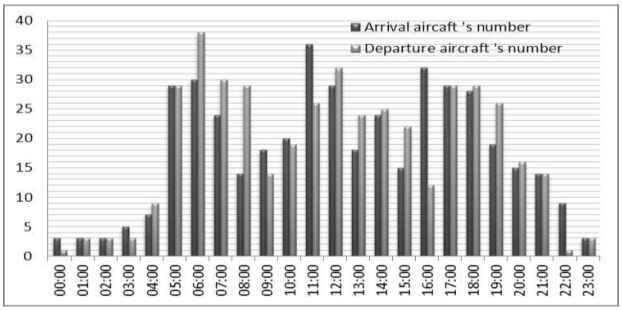


Fig.4. 01/08/2007 PDM Airport Aircraft Hourly Traffic

The evaluation of the proposed decentralized approach has been performed using aircraft traffic data for a 24h period with ground handling activities taking place at the four parking areas related with the four terminals of PDM Airport. Except for aircraft staying at night at the airport, a large majority of ground handling operations are done in the context of fast turnaround operations. Different sizes for each of the ground handling fleets have been considered in various scenarios. Fig.5 displays one of the considered compositions for ground handling fleets. Perturbations have been also introduced for some arriving aircraft with updated predictions available with fifteen minutes ahead.

The proposed heuristic approach has been tested for the aircraft traffic at 1st of August, 2007 (345 aircraft turnarounds on that day). The resulting earliest departure time for aircraft have been compared with the real time departure data, showing that with rather reduced ground handling fleets, available at each terminal, the proposed decentralized heuristic, does not generate additional delays. The application of the proposed heuristic approach has led to delays with respect to departure schedule involving only 36 aircraft, with a maximum delay of 16 minutes.

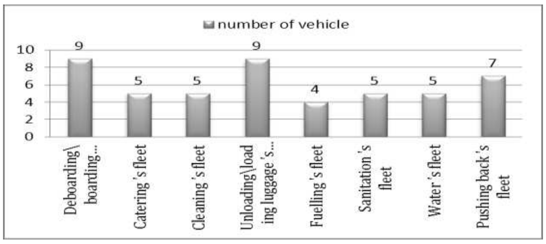
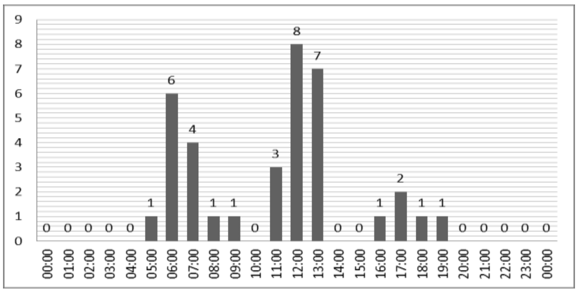


Fig.5. Example of composition of ground handling fleets

The average delay among delayed aircraft has been of 7 minutes. Historical data from 01/08/2007 at Palma de Mallorca Airport indicate that about 200 aircraft departures where delayed for multiple reasons, including one of the main reasons, ground handling delays. Figure 6 displays the hourly distribution of delayed aircraft at departure resulting from the application of the proposed decentralized approach. Clearly, the occurrence of these delays corresponds to the busiest aircraft traffic periods at the airport.

Fig.6. Hourly distribution of resulting delays



6. CONCLUSION

In this communication the problem of managing in a decentralized way airport ground handling has been considered. Then, adopting a decentralized management structure, where airline station managers and ground handling fleet managers interact, an heuristic taking explicitly into account the uncertainty about elementary processing times has been developed. This heuristic is based on the cooperation between the different tactical decision makers, providing an efficient reactive ground handling multi fleet management structure. This cooperation scheme appears to be compatible with an overall collaborative decision making approach for the airside management at airports. A case study considering aircraft and ground handling traffics at PDM Airport during a typical summer day has been developed through simulation, showing the interest of the proposed approach.

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APPENDIX A

Fuzzy dual variables and basic calculus

The set of dual numbers is the set $\tilde{\Delta}$ of numbers (Cheng, 1994) of the form $a+\varepsilon b$ such as $a \in R, b \in R^+$ where a is the primal part and b is the dual part of the fuzzy dual number. Here ε is the unity pure dual number. A crisp fuzzy dual number will be such as b is zero, it loses both its dual and its fuzzy attributes. The lower and upper bounds of $a+\varepsilon b$ are given by $B^{low}(a+\varepsilon \cdot b)=a-b$ and $B^{high}(a+\varepsilon \cdot b)=a+b$. graphical representation of a fuzzy dual number is given below where μ is a symmetrical membership function defined over R:

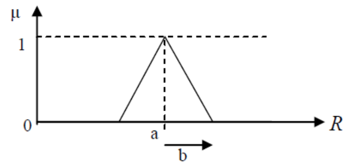

Fig. A 1. Example of dual representation of a fuzzy number

The fuzzy dual (FD) addition of dual fuzzy numbers, written $\tilde{+}$ is given by:

$(x_1+\varepsilon y_1)\tilde{+}(x_2+\varepsilon y_2)=(x_1+x_2)+\varepsilon(y_1+y_2)$. Its neutral element is $(0+0\varepsilon)$, written $\tilde{0}$. The pseudo norm of a dual fuzzy number is given by $\|a+\varepsilon b\|=|a|+\rho b \in R^+$, where ρ is a shape parameter. Figure A.2 displays standard fuzzy dual numbers with different shape parameters.

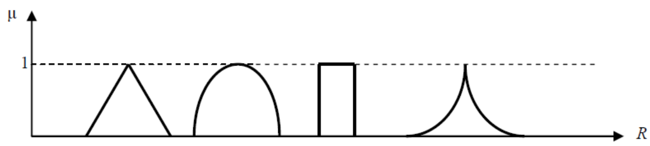

Fig. A 2. Examples of fuzzy dual numbers

The shape parameter is given by: $\rho=(1/b)\int_{-b}^{+b}\mu(u)\,du$

Let $c+\varepsilon \cdot \gamma=\max\{a+\varepsilon \cdot \alpha, b+\varepsilon \cdot \beta\}$ with $a,b \in R, \alpha,\beta \in R^+$ then $c=\max\{a,b\}$ and $\gamma=\max\{a+\rho_a \cdot \alpha, b+\rho_b \cdot \beta\}-\max\{a,b\}$.

Let $d+\varepsilon \cdot \delta=\min\{a+\varepsilon \cdot \alpha, b+\varepsilon \cdot \beta\}$ with $a,b \in R, \alpha,\beta \in R^+$ then $d=\min\{a,b\}$ and $\delta=\min\{a+\rho_a \cdot \alpha, b+\rho_b \cdot \beta\}-\min\{a,b\}$.

Observe that here the max and min operators produce new fuzzy dual numbers.