Air traffic conflict resolution via light propagation modeling

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1. Introduction

The analysis of air traffic growth expects a doubling in the flights number over the next 20 years. The Air Traffic Management (ATM) will therefore have to absorb this additional burden and to increase the airspace capacity, while ensuring at least equivalent standards of safety.

The European project SESAR was initiated to propose solutions to this problem. It relies on a new concept of air traffic control, known as 4D (3D + time) trajectory planning, which consists in exploiting the new Flight Management System (FMS) abilities that ensure that the aircraft is at a given position at a given moment. For each flight, a reference trajectory, called Reference Business Trajectory (RBT), is requested by the operating airline. During the flight, conflict situations may nevertheless occur, in which two or several aircraft can dangerously approach each other. In this case, it is necessary to modify one or more trajectories to ensure that minimum separation standards (currently 5 Nm horizontally and 1000 ft vertically) are still satisfied. Moreover, it is desirable that proposed new trajectories deviate as little as possible from RBTs.

Several methods have been tested to find an optimal solution to adress this problem including genetic algorithm[1] and navigation function based approach[2]. The first approach can not gurantee a feasible (conflict-free) solution for a given time computing. The second one does not take into account the constraints imposed by ATM, such as bounded velocity.

2. Light Modeling Algorithm

We propose a new methodology, based on an optical analogy, which seeks to ensure sufficient separation between aircraft while producing flyable trajectories.

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2.1 Problem

The objective of our approach is to find for each aircraft a feasible (relevant to ATM constraints) optimal 4D trajectory, avoiding conflicts and wich minimizes a criterion based on a local metric. We consider here the following simplified problem: we want to determine the trajectory of one aircraft given that we know about the surrounding aircraft trajectories. In order to exploit future FMS capabilities,we represent an aircraft trajectory by a sequence of 4D points connected by line segments and by velocity 3D vectors (one such vector for each 4D point).

2.2 Light Modeling

We use light propagation analogy. Light propagates in space under *Descartes law* [5]: the trajectory of a light ray is the shortest path in time. The distance and travel time are correlated by a local metric called index. The analogy we use is to replace the index by a cost function for the aircraft trajectory: we consider the refractive index as a measure of congestion or so-called *traffic complexity*. We select a barrier index value in the prohibited areas, such as military areas, and in the protection volumes surrounding each aircraft. We compute the environment index associated to a given congested area (detail can be found in [3]). The optimal trajectory will be computed using a technique of ray tracing. The light will be slowed down in congested areas, but despite this, it can pass through. However, it will be completely blocked by aircraft protection volumes, which ensures conflict free-situations.The idea of our methodology consists in launching several light rays in various directions from the departure point of the aircraft, then the path of the first ray that reaches the arrival point corresponds to a geodesic approximation, hence a good flyable trajectory for the controlled aircraft.

2.3 Branch and Bound Algorithm

In order to compute this trajectory, we use a wavefront propagation algorithm in 3D with a time discretization (the wave propagation is done with a time step dt) from the departure point.

We implement the propagation with a *branch-and-bound algorithm*(B&B) [4], a classical framework for solving discrete optimization problems. The initial step of a B&B is to consider the set of all possible solutions, represented by the root of an enumeration tree. Procedures to obtain lower and upper bounds for the optimal value of our obective function (trajectory time travel) are applied to the root. If these two bounds are equals, then the optimal solution is found, and the algorithm stops. Otherwise, the solution set is partitioned into several sub-problems (new nodes). The method is then applied recursively on these sub-problems, generating a tree. If an optimal solution is found for a sub-problem, it is feasible but not necessarily optimal for the original problem. But, as a feasible solution, it can be used to eliminate partial solutions. The search goes on until all the nodes are explored or eliminated.

For the implementation of our light propagation case, a lower approximate bound for a given node is obtained as follows: we first compute a duration, "*TimeToDest*", for the remaining

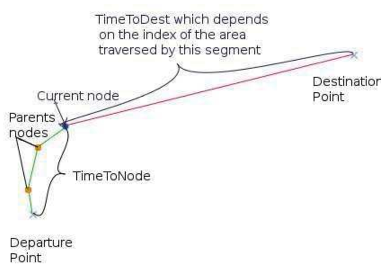

Figure 1. The lower bound computing.

time to reach the destination. This duration is a weighted sum of two terms (Formula 1 with α a weighting parameter). The first one, "*integTime*", is the time to reach destination considering the refractive index along the direct route. The second one, "*maxSpeedTime*", is the time needed to reach destination in direct route with the maximum speed.

$$TimeToDest := \alpha * integTime + (1 - \alpha) * maxSpeedTime. \tag{1}$$

The lower approximate bound is then the summation of *TimeToDest* and the time needed to reach the node from the origin (*TimeToNode*). It is given by (see Figure 1):

$$lowerBound := TimeToNode + TimeToDest.$$

Branching, in our context, involves launching rays as straight lines in a spatial cone of given radius dt, given steps $d\theta$ horizantally and $d\varphi$ vertically oriented towards the arrival point.

Browsing the search tree can be done in different ways. We choose a strategy whose priority is to find quickly a feasible solution (depth-first search or DFS). Here a node for which children have not yet been generated, with deepest level in the search tree, is chosen for exploration. DFS is then combined with a selection strategy. This consists in selecting the node that has the best lower bound among the nodes at the same level in the search tree (combination of DFS as the overall principle and best first search as a secondary selection criterion).

1. Set TrajSolution := null. Set upperBound := ∞

2. Discretize the cone towards the destination, whose center is Departure point and the radius is dt, with an angle steps $d\theta$ horizontally and an angle steps $d\varphi$ vertically.

3. While there is still unexplored nodes in the tree do:

- Choose a node N. If distance (N, destination point)$\leq \epsilon$ then TrajSolution := Set of points that leads to N and upperBound:= value of node N.
- Relaunch rays from node N in the cone towards the destination: For any light ray, if the light beam goes from a region with index n_1 into a region with index n_2 with an angle i_1, let it continue with a new angle i_2 such that n_1 sin(i_1) = n_2 sin(i_2) and with a velocity of $v = \frac{c}{n_2}$ where c is the light speed.
- Remove node N from the tree. Calculate node N's son values. Add them to the tree.

3. Numerical Results

Let us test our approach on a simplified instance of the problem, first in 2D then in 2D+time.

We use a coordinate system that is scaled with separation standards. Thus, we use an (x, y) grid with a standard horizontal separation (5 Nm) unit. We set the radius dt of the cone to the required time to travel a half standard separation distance. The cone maximum angle is set to $\frac{\Pi}{3}$. And the sampling angle $d\theta$ is set to $\frac{\Pi}{10}$. The weighting coefficients in the formula (1), is set to $\alpha := 0.9$.

3.1 Results in 2D

We first test our methodology on a 2D space instance to show it does find geodesics in simple cases.

Several refractive index functions were tested. For instance, index function used in Figures 2 is $\sum_{i=0}^{4} e^{-((x-a_i)^2+(y-b_i)^2)/k}$. It is a continuous function. High values (congested areas) are represented in red and low values (involving little traffic) in blue.

As can be seen in grey on Figure 2, the trajectory generated by our B&B algorithm avoids high index area and passes through "valleys", as one would expects. Thus, the aircraft avoids automatically congested areas.

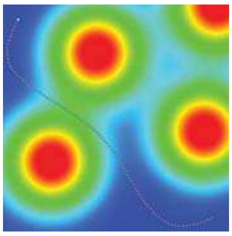

Figure 2. Resulting trajectory in 2D space.

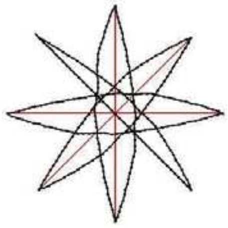

Figure 3. Conflict resolution with 8 aircraft

3.2 Results in 2D + time

Let us now consider a a 2D + time instance involving P aircraft which are initially positioned along a circle of radius 100 Nm, converging at identical speed (450 knots velocity) towards the circle center. At any time, each of the P aircraft has a position $(\overrightarrow{X_i})$. For any space point $\overrightarrow{Y}$, let us denate $\alpha := ||\overrightarrow{X_i} - \overrightarrow{Y}||$.

The used refractive index function we shall define must take into account avoidance of other aircraft protection zones. In order to ensure that the aircraft controlled by the algorithm avoids the other aircraft, we represent them by disks (whose radius is the standard distance separation),and we set the index function, n, to a very high constant value N inside these disks and we make it decrease rapidly outside the disk. The index function n is given by the following formula at any point $\overrightarrow{Y} \in \mathbf{R}^3$:

$$n(\overrightarrow{Y}) = \begin{cases} N & if \quad \alpha \leq R \\ 1 + \frac{N-1}{1+(\alpha-R)^q} & \quad otherwise. \end{cases}$$

with R the standard distance separation and q is a parameter that determines the speed with which the index decreases outside the separation zone. Our algorithm is sequentially applied to each aircraft until there is no conflict any more with P:= 8, N:= 2 and q:= 2.

We obtained a conflict free situation with the last aircraft that does not deviate from its direct route as displayed on Figure 3.

4. Conclusion

Our overall original light modelling methodology seems viable as it managed to resolve an academic conflict situation in (2D + time). Future work will concentrate on real-world instances and implementing a (3D + time) version of the algorithm.

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