Pricing of ATC/ATM services through bilevel programming approaches

Rabah Guettaf, Catherine Mancel, Moussa Larbani, Felix Mora-Camino

INVITED PAPER

PRICING OF ATC/ATM SERVICES THROUGH BILEVEL PROGRAMMING APPROACHES

Rabah Guettaf
Faculty of Sciences, Mouloud Mammeri University, Tizi-Ouzou, Algeria

Catherine Mancel
LARA, Ecole Nationale de l'Aviation Civile, University of Toulouse, France

Moussa Larbani
Business Administration Department, IIUM University, Kuala Lumpur, Malaysia

Félix Mora-Camino
LARA, Ecole Nationale de l'Aviation Civile, University of Toulouse, France

Received: April 27, 2010; Accepted: May 20, 2010

ABSTRACT

In this study a multilevel framework is proposed to analyze the charges definition problem for an ATC/ATM service provider. Two cases are considered: a case in which the ATC/ATM service provider is a public company whose final objective is, while guaranteeing safety, to promote the level of end users demand, and a case in which the ATC/ATM service provider is a private company whose objective is to maximize her profit. In both cases this leads to the formulation of a bi-level optimization program involving the ATC/ATM service provider as the leader and the whole airline sector as the follower. Direct optimal solutions are obtained in the simplistic considered one dimensional case. Then in both cases, public or private, natural negotiation schemes between the ATC/ATM service provider and the airline sector are introduced. Sufficient conditions are established so that the proposed negotiation processes converge towards the respective optimal solutions while a compensation scheme is proposed to make the two solutions identical. This study provides a basis for the study of the ATC/ATM charge definition problem for more complex airspace market structures.

Keywords: Pricing; Air Traffic Management; Air Transportation; Optimization; Bi-Level Programming.

1. INTRODUCTION

The quality of Air Traffic Control (ATC) and Air Traffic Management (ATM) services is critical for the development of the air transportation sector since increased traffic levels induce higher needs for traffic control and management actions to enforce required safety levels (ICAO, 2004 and European Community, 2005) over airspaces and airports where saturation situations appear. ATC/ATM activity requires high cost investments in equipements and highly skilled manpower, then the organization and the funding of this activity is a crucial issue (PriceWatehouseCoopers, 2001 and Cordle et al, 2005).

Along the last decades, many studies in the fields of Operations Research, Systems Management and Applied Economics have been devoted to air transportation planning, tariffs and operations related questions (Deschinkel et al, 2002 and Raffarin, 2002). In general the analysis which have been performed are limited to direct effects so that the scope of the adopted models are in general too limited. This implies that feedback phenomena between the different actors and involved air transportation activities cannot be fully taken into account to perform a comprehensive analysis and to design efficient plans and policies. In this study a multilevel approach following some ideas from (Labbé et al, 1998), is developed.

The objective assumed for airlines in this study is of a pure economic nature: profit maximization. The main concern of this study being with the definition of efficient ATC/ATM charges, the whole airline sector is taken as a whole, so that market competition between airlines is not contemplated. This is a limitation of the study, which is done in sake of limited complexity, since in fact, ATC/ATM charges may have some influence on the equilibrium state of different air transportation markets. However, it is also worth to observe that these airlines are in general represented by a unique entity during negotiations with ATC/ATM authorities. In the case of a public ATC/ATM service provider, it is considered that the main objective is to promote air transportation for end users, i.e. the passengers (freight is not considered explicitly in this study), through a safe and efficient transportation supply by airlines and ATM authorities. The objective assumed for a private ATC/ATM service provider as well as the airlines sector, is profit maximization. Hence a passenger's demand model, reactive to airlines tariffs is introduced to take into account indirect influence of ATC/ATM charges on passengers demand levels over different air transportation markets. In both cases, private or public, it is supposed that the economic performance of the airline sector is not impaired by the retained levels of ATC/ATM charges. Also, ATC/ATM costs related directly with the current traffic situation (investments costs leading to enlarged ATC/ATM capacity to face future traffic situations are not considered) should be adequately covered. This results in bi-level optimization problems (Bard, 1998, Brotcorne et al, 2000 and Dempe, 2002) which are of the upper linear-lower linear class.

To illustrate these two cases a one dimensional system is considered. In that case it is possible to establish directly its optimal solutions while sufficient conditions are established so that two negotiation processes between ATM authorities and the airline sector lead to these optimal solutions. There, the problem is split in two dependent problems: one where ATM authorities determine ATC/ATM charges for given airlines tariffs and one where the airline sector determines the tariffs and seat capacity supplies over the different markets for given ATC/ATM charge levels.

2. BASIC ASSUMPTIONS

Here is considered the case of an elementary air transportation system composed of a single pair of airports linked by a single air route. There is a unique ATC/ATM operator and a unique airline operating between these two airports.

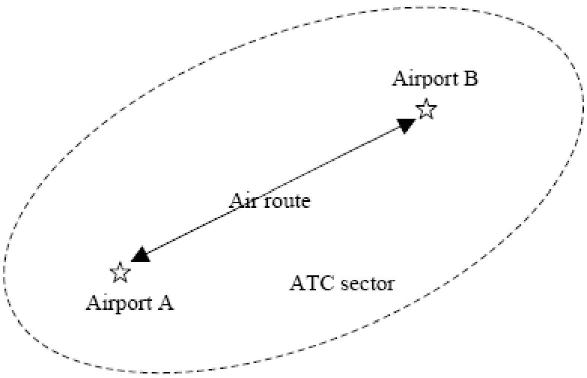

Figure 1 - The elementary ATC/ATM case

The potential demand is supposed to be composed of round trips and to obey, for the sake of simplicity and the ability to develop clear analytical and graphical results, to the following affine demand function:

$$\phi = D_0 - \rho\,\pi(1+\lambda) \quad (1)$$

where ϕ is the effective level of potential demand. Here it is supposed that there is a unique class of travelers and a unique apparent price $\tilde{\pi} = \pi(1+\lambda)$ is adopted for round trips. D0 is an absolute potential demand and ρ is a constant positive parameter characteristic of the response of the market to price changes. The parameter λ represents a tax index applied to each trip ticket. Other demand models such as the exponential one with constant price elasticity:

$$\phi = D_0\, e^{-\rho\,\tilde{\pi}} \quad (2)$$

or either unspecified models such as:

$$\phi = D(\tilde{\pi}) \quad (3)$$

with adequate assumptions such as:

$$\partial D / \partial \tilde{\pi} \le 0 \quad \text{and} \quad \partial^2 D / \partial \tilde{\pi}^2 \ge 0 \quad (4)$$

could have been adopted.

Beyond the ATC charges, an additional way to fund the ATC services which appears natural, is to assign a proportion $\alpha\,(\alpha \in [0,1])$ of the above tax, which is paid by the final users, to the ATC service provider.

The transport capacity of the airline is given by the maximum affordable frequency of service $f_{\max}$ which is related with the size of the fleet of the airline. Here for simplicity and considering that for the given time period *f* can be a high number, *f* will be taken as real. When a frequency of service *f* is adopted, the operations costs are supposed to be given by:

$$(c+v)\, f + C_{ALN}^F \quad (5)$$

here:

- *c* is a positive parameter (a mean variable cost with respect to frequency). It is related with the price of fuel, the cost of the crew and the length of the flights.
- C_{ALN}^F is a fixed cost related with the sizes of the fleet and the crews of the airline as well as with the characteristics of the operated network.
- *v* is the ATC tariff applied to a round flight, including en route control and airport control. Here no distinction will be made between airport taxes and approach and en route charges.

The available seat capacity is given by:

$$q\ f \quad (6)$$

where *q* is the mean seat capacity of an aircraft of the fleet of the airline. The operating costs of the ATC supplier are given by:

$$\sigma\ f + C_{ATC}^F \quad (7)$$

Here σ is a positive parameter (a mean variable cost with respect to frequency). It is related mainly with the length of the flights. C_{ATC}^F is a fixed cost related with the characteristics of the controlled airspace and with the size of the ATC staff. No saturation effects with consequences over the cost functions of the ATC service provider and the airline are considered in this study.

3. OPTIMIZING CHARGES FOR A PUBLIC ATC/ATM SERVICE SUPPLIER

Here it is supposed that the final objective of the public ATC/ATM service supplier is to maximize the satisfied demand while guaranteeing a minimum economic return for the ATC/ATM services, R_{ATC}, and a minimum economic return R_{ALN} for the airline. It is supposed also that the airline tries to maximize her benefit taking into account her cost function and the ATC/ATM tariff.

3.1. A bilivel program for public ATC/ATM pricing

According to the above assumptions, a bilivel program can be established:

$$\max_{v \geq 0} \quad \phi \quad (8)$$

with

$$vf + \alpha\lambda\pi\phi - (\sigma\, f + C_{ATC}^{F}) \geq R_{ATC} \quad (9)$$

where λ is the tax rate applied to air travelers, and the airline's profit constraint:

$$\pi\,\phi - ((c+v)\,f + C_{ALN}^{F}) \geq R_{ALN} \quad (10)$$

where π and f are given by the solution of:

$$\max_{\pi,\, f} \quad \pi\,\phi - ((c+v)\,f + C_{ALN}^{F}) \quad (11)$$

with

$$\phi = \max\{0, \min\{q\ f\,,\, D_0 - \rho\,\pi(1+\lambda)\}\} \quad (12)$$

and

$$0 \leq f \leq f_{\max} \quad (13)$$

3.2. Solving the airlines profit maximization problem

To solve the airline's profit maximization, two cases must be considered with respect to the effective level of passengers demand. Either:

$$\phi = q\ f \quad (14)$$

or:

$$\phi = D_0 - \rho\,\pi(1+\lambda) \quad (15)$$

3.2.a Case in which effective demand is determined by the seat capacity

In this first case, we have:

$$q\ f \leq D_0 - \rho\,\pi(1+\lambda) = D_0 - \tilde{\rho}\,\pi \quad (16)$$

where

$$\tilde{\rho} = (1+\lambda)\,\rho \quad (17)$$

where p_{ALN} is a chosen level of profit for the airline. Changing the value of p_{ALN} and considering constraint (13) in the (π, f) plane, we get hyperbola arcs for the profit level curves.

It appears clearly in figure 2 that the maximum profit is obtained when constraint (13) reduces to equality. This result is also valid (see figure 3) when the tangency of a profit level curve and the demand ligne provides a frequency above f_{max}.

In the case of no active fleet constraints, we get a double solution for the following equation:

$$\frac{p_{ALN} + C_{ALN}^{F}}{q\pi - (c+v)} = \frac{1}{q}(D_0 - \tilde{\rho}\,\pi) \quad (19)$$

when p_{ALN} reaches the value:

$$p_{ALN}^{\max} = \frac{1}{\tilde{\rho}}\left(\frac{(c+v)\tilde{\rho} + qD_0}{2q}\right)^2 - (c+v)D_0/q - C_{ALN}^{F} \quad (20)$$

Then:

$$f^* = \max\left\{0, \min\left\{f_{\max}, \frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c+v)\right\}\right\} \quad (21)$$

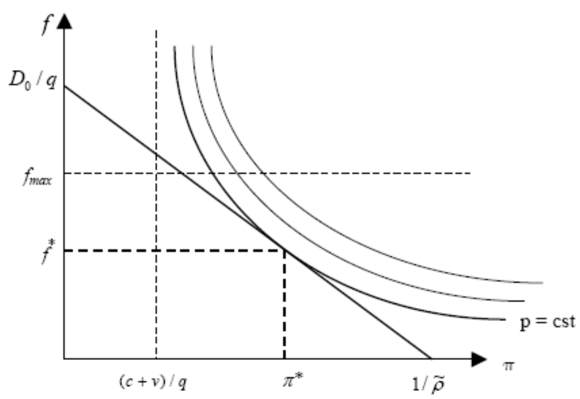

Figure 2 - Optimization of airline profit, no active fleet constraint, full capacity.

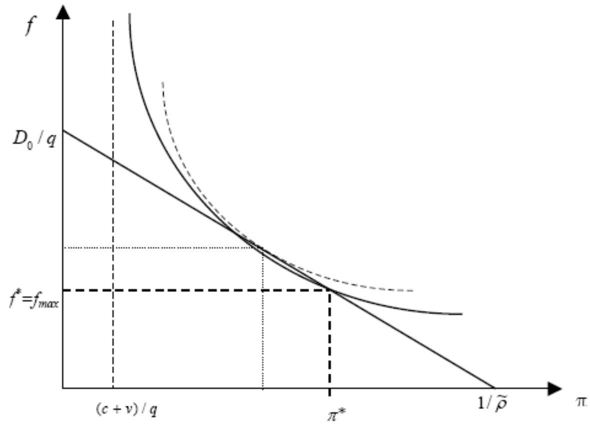

Figure 3 - Optimization of airline profit with an active fleet constraint at full capacity.

and either (no activation of the fleet constraint):

$$\pi^* = \frac{c+v}{2q} + \frac{D_0}{2\tilde{\rho}} \quad (22)$$

or (activation of the fleet constraint):

$$\pi^* = \frac{D_0}{\rho} - \frac{q}{\tilde{\rho}} f_{\max} \quad (23)$$

while the optimal airline profit is given either by $p^*_{ALN} = p^{\max}_{ALN}$ (relation (16)) or by:

$$p^*_{ALN} = (\frac{q}{\tilde{\rho}} D_0 - c) f_{\max} - \frac{q^2}{\tilde{\rho}} f_{\max}{}^2 - C^F_{ALN} \quad (24)$$

3.2.b Case in which effective demand is determined by the price level

In this second case, we have:

$$q\ f \geq D_0 - \tilde{\rho}\pi \quad (25)$$

Now, considering airline's profit level curves p_{ALN}, we get:

$$f = \frac{D_0}{c+v}(1-\tilde{\rho}\pi)\pi - \frac{p_{ALN} + C^F_{ALN}}{c+v} \quad (26)$$

where p_{ALN} is a chosen level of profit for the airline. Changing the value of p_{ALN} and considering constraint (25) in the (π, f) plane, we get parabola arcs for the profit level curves.

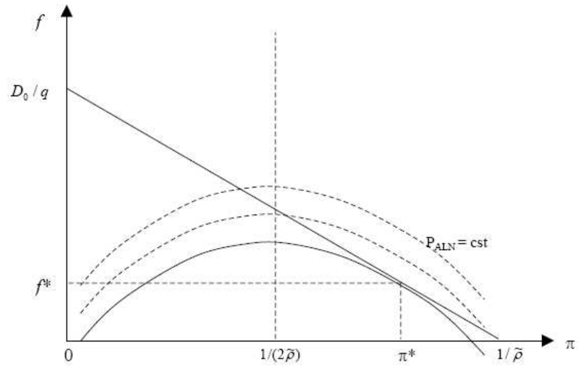

Figure 4 - Optimization of airline profit with priced constrained demand.

It appears that here again the optimal solution is given by relations:

$$f^* = \frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c+v) \quad (27)$$

with again (20), (22) and:

$$\phi = \frac{D_0}{2} - \frac{\tilde{\rho}}{2q}(c+v) \quad (28)$$

Relation (22) reveals the influence of ATC/ATM tolls over air transportation tariff and consequently over satisfied demand (relation (28)). In the following, it will be assumed that the fleet constraint remains inactive.

3.3. Solving the public ATC/ATM service provider problem

Considering the solution of the airline profit maximization problem, problem (8), (9), (10) with (11), (12) and (13) becomes:

$$\max_{v \geq 0} \quad \frac{D_0}{2} - \frac{\tilde{\rho}}{2q}(c+v) \quad (29)$$

under the constraints:

$$\frac{D_0 v}{2q} - \frac{\tilde{\rho} v}{2q^2}(c+v) + \alpha\lambda(\frac{c+v}{2q} + \frac{D_0}{2\tilde{\rho}})(\frac{D_0}{2} - \frac{\tilde{\rho}}{2q}(c+v)) + \sigma(\frac{\tilde{\rho}}{2q^2}(c+v) - \frac{D_0}{2q}) - C_{ATC}^F \geq R_{ATC} \quad (30)$$

$$(\frac{c+v}{2q} + \frac{D_0}{2\tilde{\rho}})(\frac{D_0}{2} - \frac{\tilde{\rho}}{2q}(c+v)) - (c+v)(\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c+v)) - C_{ALN}^F \geq R_{ALN} \quad (31)$$

and

$$0 \leq v \leq (q/\tilde{\rho})D_0 - c \quad (32)$$

This last condition insures that there is some satisfied demand.

This problem can be rewritten as:

$$\min_x \quad x \quad (33)$$

under the constraints:

$$(x-c)(\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}x) + \alpha\lambda(\frac{x}{2q} + \frac{D_0}{2\tilde{\rho}})(\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q}x) + \sigma(\frac{\tilde{\rho}}{2q^2}x - \frac{D_0}{2q}) - C_{ATC}^F \geq R_{ATC} \quad (34)$$

$$(\frac{x}{2q} + \frac{D_0}{2\tilde{\rho}})(\frac{D_0}{2} - \frac{\tilde{\rho}}{2q}x) - x(\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}x) - C_{ALN}^F \geq R_{ALN} \quad (35)$$

and

$$x_{\min} = c \leq x \leq (q/\tilde{\rho})D_0 = x_{\max} \quad (36)$$

Constraints (34) and (35) can be rewritten as:

$$-ax^2 + bx - c \geq 0 \quad (37)$$

with

$$\begin{cases} a = \dfrac{\tilde{\rho}}{2q^2}(1+\alpha\lambda/2) \\ b = \dfrac{\tilde{\rho}}{2q^2}(\sigma + c + \dfrac{q}{\tilde{\rho}}D_0) \\ c = \dfrac{D_0}{2q}(c + \sigma + \dfrac{\alpha\lambda}{2\tilde{\rho}}D_0) + C_{ATC}^F + R_{ATC} \end{cases} \quad (38)$$

and

$$\frac{\tilde{\rho}}{4q^2}x^2 - \frac{D_0}{2q}x + \frac{D_0{}^2}{4\tilde{\rho}} - C_{ALN}^F - R_{ALN} \geq 0 \quad (39)$$

The equation:

$$-ax^2 + bx - c = 0 \quad (40)$$

presents real roots which are then positive, when:

$$b \geq 2\sqrt{a\ c} \quad (41)$$

or

$$C_{ATC}^F + R_{ATC} \leq \frac{\rho(1+\lambda)}{8q^2}\frac{(c+\sigma+(q/\rho(1+\lambda))D_0)^2}{1+\alpha\,\lambda/2} - \frac{D_0}{2q}(c+\sigma+\frac{\alpha\lambda}{2\rho(1+\lambda)}D_0) \quad (42)$$

When $\alpha = 0$, this condition reduces to:

$$C_{ATC}^F + R_{ATC} \leq \frac{\rho(1+\lambda)}{8q}(\frac{D_0}{\rho(1+\lambda)} - \frac{c+\sigma}{q})^2 \quad (43)$$

Let x_1 be the minimum real root of (40) when it exists:

$$x_1 = \frac{(\sigma + c + (q/\tilde{\rho})D_0) - \sqrt{\Delta(\alpha)}}{2(1+\alpha\lambda/2)}$$

with

$$\Delta(\alpha) = (c + \sigma + (q/\tilde{\rho})D_0)^2 - 4(1+\alpha\lambda/2)((q/\tilde{\rho})D_0(c+\sigma+(\alpha\lambda/(2\tilde{\rho}))D_0) + (2q^2/\tilde{\rho})(C_{ATC}^F + R_{ATC})) \quad (44)$$

when α =0, we get:

$$x_1 = \frac{(\sigma + c + (q/\tilde{\rho})D_0) - \sqrt{\Delta(0)}}{2} \quad (45)$$

with

$$\Delta(0) = (c+\sigma+(q/\tilde{\rho})D_0)^2 - 4((q/\tilde{\rho})D_0(c+\sigma) + (2q^2/\tilde{\rho})(C_{ATC}^F + R_{ATC})) \quad (46)$$

and x_1 is then less than x_{max} if:

$$(q/\tilde{\rho})D_0 + \sqrt{\Delta(0)} - (c+\sigma) \geq 0 \quad (47)$$

It will be supposed in the following that α, λ and R_{ATC} are chosen so that x_1 is less than x_{max}

The roots of equation:

$$\frac{\tilde{\rho}}{4q^2}x^2 - \frac{D_0}{2q}x + \frac{D_0^2}{4\tilde{\rho}} - C_{ALN}^F - R_{ALN} = 0 \quad (48)$$

are always real, one of them being always positive:

$$x_2^+ = \frac{q}{\tilde{\rho}}(D_0 + \sqrt{\tilde{\rho}(C_{ALN}^F + R_{ALN})}) \quad (49)$$

The smallest of these roots, x_2^-, is such as:

$$x_2^- = \frac{q}{\tilde{\rho}}(D_0 - \sqrt{\tilde{\rho}(C_{ALN}^F + R_{ALN})}) \quad (50)$$

Considering (36), x_2^- is such as:

$$x_2^- < x_{\max} \quad (51)$$

Here also, it will be supposed in the following that λ and R_{ALN} are chosen so that x_2 is less than x_{max}. By inspection of all the possibilities, only two situations lead to a solution for problem (33), (34), (35) and (36).

They are represented graphically bellow:

- In the case in which x_2 is positive superior to c, to have a solution , x_1 must be inferior or equal to x_2 and superior to c, then the solution x^* is equal to x_1.
- In the case in which x_2^- is less than c, there is a solution given by $x^* = \max\{x_1, x_2^+\}$ when:

$$c \leq \max\{x_1, x_2^+\} \leq x_{\max} \quad (52)$$

The ordinates in figure 5 and figure 6 below represent the profits above the guaranteed values R_{ATC} and R_{ANL}.

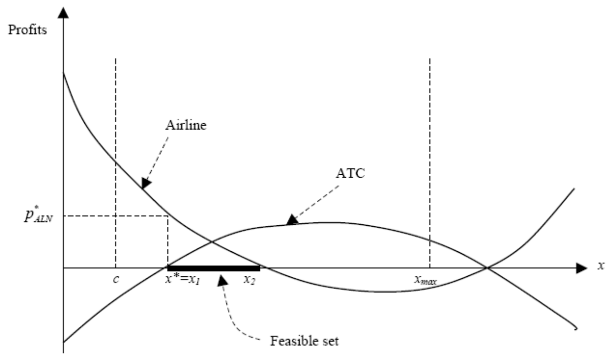

Figure 5 - Solution of the public ATC/ATM supplier problem (case in which x_2^- is superior to c).

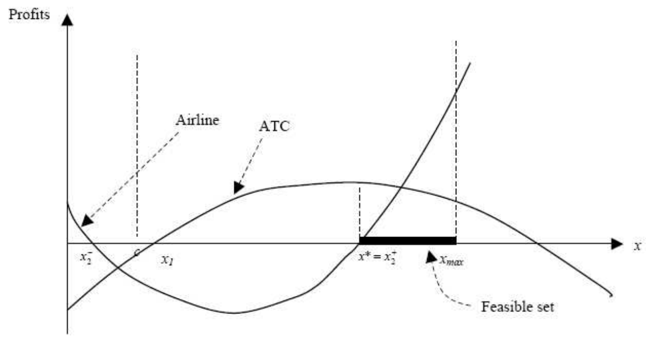

Figure 6 - Solution of the public ATC/ATM supplier problem (case in which x_2^- is inferior to c).

4. GLOBAL SOLUTION FOR PUBLIC ATC/ATM SERVICE PROVIDER

4.1. The global solution scheme

It will be supposed for the following that λ and R_{ALN} are chosen so that $x_2^- \geq c$ (figure 5):

$$0 \leq \lambda \leq (\frac{q}{c\rho} D_0 + \frac{q^2}{2\rho c^2}(C_{ALN}^F + R_{ALN} - \sqrt{\frac{4c}{q} D_0 (C_{ALN}^F + R_{ALN}) + (C_{ALN}^F + R_{ALN})^2}) - 1 \quad (53)$$

then, only the first case is to be considered and the solution of the whole problem (8) to (13) is given by:

$$v^* = \frac{(\sigma + c + (q/\tilde{\rho})D_0) - \sqrt{\Delta(\alpha)}}{2(1+\alpha\lambda/2)} - c \quad (54)$$

The level of the satisfied demand is given by:

$$\phi^* = \frac{D_0}{2} - \frac{\rho(1+\lambda)}{2q}(c + v^*) \quad (55)$$

The profit of the public ATC supplier is equal (see figure 5) to the minimum guaranteed level R_{ATC} while the profit of the airline is given by:

$$p^* = \frac{1}{\rho(1+\lambda)}(\frac{(c+v^*)\rho(1+\lambda) + qD_0}{2q})^2 - (c+v^*)D_0/q - C_{ALN}^F \quad (56)$$

which is superior or equal (see figure 4) to the minimum guaranteed level R_{ALN}.

From relations (54) and (55), it is easy to show $(\partial v^*/\partial \alpha < 0)$ that for a given value of λ, the value of α which maximizes ϕ is $\alpha^* = 1$.

Then, a further step towards the optimization of the sector would be to choose efficiently the rate of the tax applied to the trips. In that case, considering (54) with α =1, we should define $v^*(\lambda)$ by:

$$v^*(\lambda) = \frac{1}{2}((\sigma - c + (q/\tilde{\rho}(\lambda))D_0) - \sqrt{(c + \sigma - qD_0/\tilde{\rho}(\lambda)) - 8(C_{ATC}^F + R_{ATC}))q^2/\tilde{\rho}(\lambda))} \quad (57)$$

and the best value of λ would be solution of:

$$\max_{\lambda} \frac{D_0}{2} - \frac{\rho(1+\lambda)}{2q}(c + v^*(\lambda)) \quad (58)$$

with

$$\frac{1}{\rho(1+\lambda)}(\frac{(c+v^*(\lambda))\rho(1+\lambda) + qD_0}{2q})^2 - (c+v^*(\lambda))D_0/q - C_{ALN}^F \geq R_{ALN} \quad (59)$$

and

$$0 \leq \lambda \leq (D_0 - (2qc + \sqrt{4\rho(C_{ALN}^F + R_{ALN})}) \quad (60)$$

However, the value of parameter λ is the result of an exogenous choice process where overall economic as well as political considerations are taken into account.

4.2 Optimality of the pricing negotiation process

Since the considered pricing problem involves two major economic agents, the ATC/ATM service provider and the airline sector, a solution of the considered bi-level problem resulting from the hypothesis with respect to the nature, public or private, and the goals of the ATC/ATM service provider, through a negotiation process can be of interest.

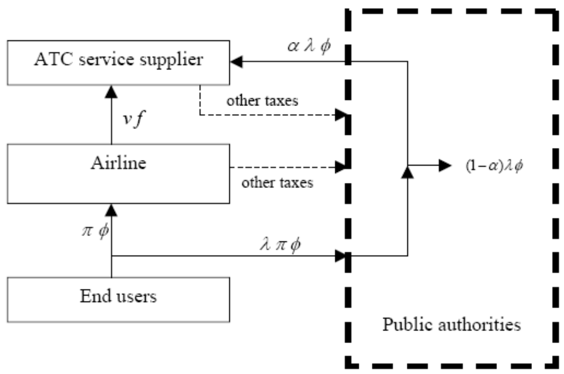

Figure 7 - Financial flows and activity levels with public ATC/ATM service supplier.

Following the general formulation (5) to (10), a natural negotiation process based on the objectives of the involved economic agents could be the following:

The public ATC/ATM service supplier solves at iteration n+1 the following problem with respect to v, given π^n and f^n:

$$v^{n+1} = \arg\{ \max_{v \geq 0} \quad \phi(v) \ \} \quad (61)$$

with

$$vf^n + \alpha\lambda\pi^n \phi^n - (\sigma f^n + C^F_{ATC}) \geq R_{ATC} \quad (62)$$

and the airline's profit constraint:

$$\pi^n \ \phi(v) - ((c+v) f^n + C^F_{ALN}) \geq R_{ALN} \quad (63)$$

where π^n and f^n are provided by the airline sector which solves the following problem given v^n:

$$\max_{\pi,\, f} \quad \pi \min\{q\, f, D_0 - \tilde{\rho}\,\pi\} - ((c+v^n) f + C^F_{ALN}) \quad (64)$$

with

$$0 \leq f \leq f_{\max} \quad (65)$$

This process is represented in figure 8.

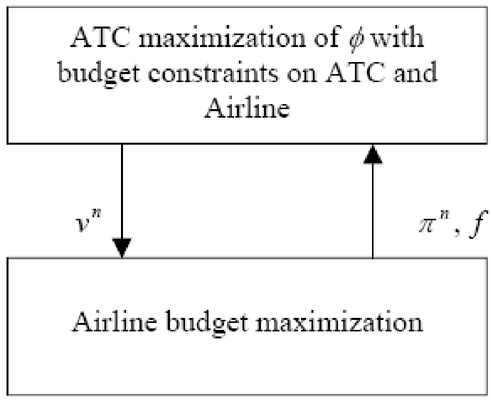

Figure 8- Negotiation process between public ATC/ATM and airlines.

Since $\phi(v) = \dfrac{D_0}{2} - \dfrac{\tilde{\rho}}{2q}(c+v)$, the ATC/ATM service supplier has to satisfy the two budget constraints with the minimum value for v, since the profit of the airline is a decreasing function of v, the solution of the problem corresponds to the saturation of his own budget constraint, so that:

$$v^{n+1} = \frac{R_{ATC} + C^F_{ATC} + \sigma f^n + \alpha\lambda\pi^n((\tilde{\rho}\, c/q) - D_0)/2}{f^n - (\tilde{\rho}/2q)\alpha\lambda\pi^n} \quad (66)$$

Here we consider the case in which the solution of the airline problem is given by:

$$f^n = \frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c+v^n) \quad (67)$$

with

$$\pi^n = \frac{c+v^n}{2q} + \frac{D_0}{2\tilde{\rho}} \quad (68)$$

then we get a recurrent formula for v^n:

$$v^{n+1} = \frac{R_{ATC} + C_{ATC}^{F} + \sigma\left(\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c+v^n)\right) + \alpha\lambda\left(\frac{c+v^n}{2q} + \frac{D_0}{2\tilde{\rho}}\right)((\tilde{\rho}\, c/q) - D_0)/2}{\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c+v^n) - (\tilde{\rho}/2q)\alpha\lambda\left(\frac{c+v^n}{2q} + \frac{D_0}{2\tilde{\rho}}\right)} \quad (69)$$

For simplicity we consider here only the case in which α = 0, then:

$$v^{n+1} = \frac{R_{ATC} + C_{ATC}^{F} + \sigma\left(\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c+v^n)\right)}{\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c+v^n)} = \sigma + \frac{R_{ATC} + C_{ATC}^{F}}{\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c+v^n)} \quad (70)$$

or

$$v^{n+1} = \sigma + \frac{R_{ATC} + C_{ATC}^{F}}{\left(\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}c\right) - \frac{\tilde{\rho}}{2q^2}v^n} \quad (71)$$

Here we consider that the following condition is satisfied:

$$D_0 > \tilde{\rho}\, c/q \quad (72)$$

Then in figure 9, the convergence of the negotiation process is analyzed graphically. It appears that if:

$$(c + \sigma + (q/\tilde{\rho})D_0)^2 - 4((q/\tilde{\rho})D_0(c+\sigma) + (2q^2/\tilde{\rho})(C_{ATC}^{F} + R_{ATC})) > 0 \quad (73)$$

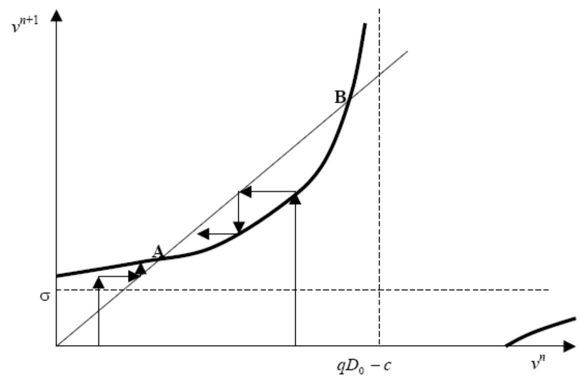

Figure 9 - Convergence of the public ATC/ATM-Airline negotiation process.

or

$$R_{ATC} < \frac{(c+\sigma+(q/\tilde{\rho})D_0)^2}{8q^2/\tilde{\rho}} - C_{ATC}^{F} - \frac{D_0(c+\sigma)}{2q} \quad (74)$$

then there are two equilibrium points: point A which is a stable equilibrium solution and point B which is an unstable equilibrium solution. It appears that point A corresponds to the optimal solution of the bi-level problem treated in section 3.3. In that case the proposed negotiation schemes leads to the optimal solution.

5. OPTIMAL PRICING FOR A PRIVATE ATC/ATM SERVICE SUPPLIER

It is supposed in this case that the objective of the private ATC/ATM service supplier is to maximize his profit while guaranteeing a minimum economic return R_{ALN} for the airlines sector so that he maintains his activity. It is assumed that at the same time the airline tries to maximize her benefit taking into account her cost function and the ATC/ATM tariff. The public authority collects a tax (rate λ) over each trip and may refund with an amount μ the ATC/ATM service supplier. The operating costs of the private ATC/ATM service supplier are given now by $s\, f + Q_{ATC}^{F}$ where s is a positive parameter (a mean variable cost with respect to frequency). It is related mainly with the length of the flights. Q_{ATC}^F is a fixed cost related with the sizes of the airspace and of the ATC/ATM staff. According to the above assumptions, a bi-level program can be established:

$$\max_{v\geq 0} vf-(s\ f+Q_{ATC}^F) \quad (75)$$

where the profit of the ATC/ATM service provider is given by:

$$r_{ATC}=vf-(s\ f+Q_{ATC}^F)+\mu \quad (76)$$

with an exogenous refunding constraint:

$$\mu \leq \lambda\ \pi\ \phi \quad (77)$$

and the airline's profit constraint:

$$\pi\ \phi-((c+v)\ f+C_{ALN}^F)\geq R_{ALN} \quad (78)$$

where π and f are given by the solution of the optimization problem defined by relations:

$$\max_{\pi,\ f} \quad \pi\ \phi-((c+v)\ f+C_{ALN}^F) \quad (79)$$

with

$$\phi=\max\{0,\min\{q\ f\ ,D_0-\rho\ \pi(1+\lambda)\}\ \}$$

and

$$0\leq f\leq f_{\max} \quad (80)$$

Taking into account the behavior of the airline, the optimization problem of the private ATC/ATM service supplier becomes:

$$\max_{v\geq 0} \quad (v-s)(\frac{D_0}{2q}-\frac{\tilde{\rho}}{2q^2}(c+v))-Q_{ATC}^F \quad (81)$$

under

$$(\frac{c+v}{2q}+\frac{D_0}{2\tilde{\rho}})(\frac{D_0}{2}-\frac{\tilde{\rho}}{2q}(c+v))-((\frac{D_0}{2q}-\frac{\tilde{\rho}}{2q^2}(c+v))(c+v)+C_{ALN}^F)\geq R_{ALN} \quad (82)$$

and

$$0\leq v\leq (q/\rho)\tilde{D}_0-c \quad (83)$$

The unconstrained solution of the above problem is given by:

$$\hat{v}=\frac{s}{2}+\frac{q}{2\tilde{\rho}}D_0 \quad (84)$$

Let us consider the roots of equation:

$$av^2-bv+c=0 \quad (85)$$

with:

$$\begin{cases} a=\tilde{\rho}/(4q^2), \\ b=(D_0-c\ \tilde{\rho}/q)/(2q) \\ c=\dfrac{D_0{}^2}{4\tilde{\rho}}+\dfrac{\tilde{\rho}\,c^2}{4q^2}-\dfrac{cD_0}{2q}-C_{ALN}^F-R_{ALN} \end{cases} \quad (86)$$

Here we assume that R_{ALN} and λ are chosen such as $p_{ALN}(0)\geq R_{ALN}$. If the following conditions $4\,a\,c-b^{\ 2}>0$ and $0\leq\hat{v}\leq(q/\tilde{\rho})D_0-c$ are satisfied, then (see figure 10):

$$v^*=\hat{v} \quad (87)$$

where $\hat{v}$ which is given by (81) is solution of the optimization problem of the private ATC service supplier. If condition $p_{ALN}(0)\geq R_{ALN}$ is satisfied while $0\leq\hat{v}\leq(q/\tilde{\rho})D_0-c$ is not satisfied, the solution of the optimization problem of the private ATC service supplier is given by:

$$v^*=v_{\max}=(q/\tilde{\rho})D_0-c \quad (88)$$

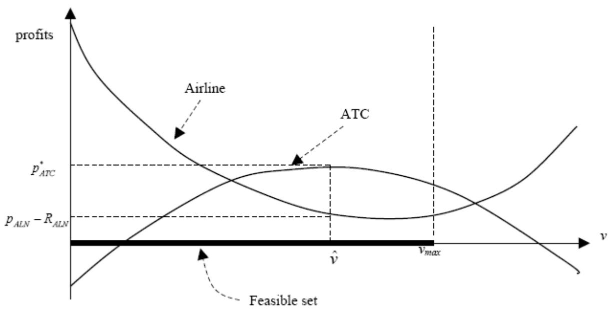

Figure 10 - Solution of the private ATC/ATM supplier problem (no real roots for (85)).

When condition $p_{ALN}(0)\geq R_{ALN}$ is satisfied while condition $4\,a\,c-b^{\ 2}>0$ is not satisfied, equation (85) s two positive roots

v^- and v^+ with $v^- \le v^+$. Here we can consider the three different cases (see figure 11):

if $v^- \le \hat{v}$ and $v^+ \ge \hat{v}$ then

$$if \quad v^+ > v_{\max} \; then \quad v^* = v^- \quad (89\text{-a})$$

$$if \quad v^+ \le v_{\max} \; then \quad v^* = \max_{v \in \{v^-, v^+\}} p_{ATC}(v) \quad (89\text{-b})$$

if $v^- \le \hat{v}$ and $v^+ \le \hat{v}$ then

$$v^* = \hat{v} \quad (90)$$

if $v^- \ge \hat{v}$ then

$$v^* = v^- \quad (91)$$

where:

$$v^- = (b - \sqrt{b^2 - 4ac})/(2a)$$

and

$$v^+ = (b + \sqrt{b^2 - 4ac})/(2a) \quad (92)$$

The level of the satisfied demand is given by:

$$\phi^* = \frac{D_0}{2} - \frac{\rho(1+\lambda)}{2q}(c + v^*) \quad (93)$$

The profit of the private ATC service supplier is equal to:

$$p^*_{ATC}(\mu) = -\frac{\tilde{\rho}}{2q^2}v^{*2} + (\frac{D_0}{2q} + \frac{s\,\tilde{\rho}}{2q^2})v^* + (\frac{\tilde{\rho}\,s\,c}{2q^2} - \frac{s\,D_0}{2q} - Q^F_{ATC} + \mu) \quad (94)$$

Refunding μ is chosen in order to guarantee that $p^*_{ATC} \ge R_{ATC}$.

$$if \quad (-\frac{\tilde{\rho}}{2q^2}v^{*2} + (\frac{D_0}{2q} + \frac{s\,\tilde{\rho}}{2q^2})v^* + (\frac{\tilde{\rho}\,s\,c}{2q^2} - \frac{s\,D_0}{2q} - Q^F_{ATC})) \ge R_{ATC} \quad then \quad \mu = 0 \quad (95)$$

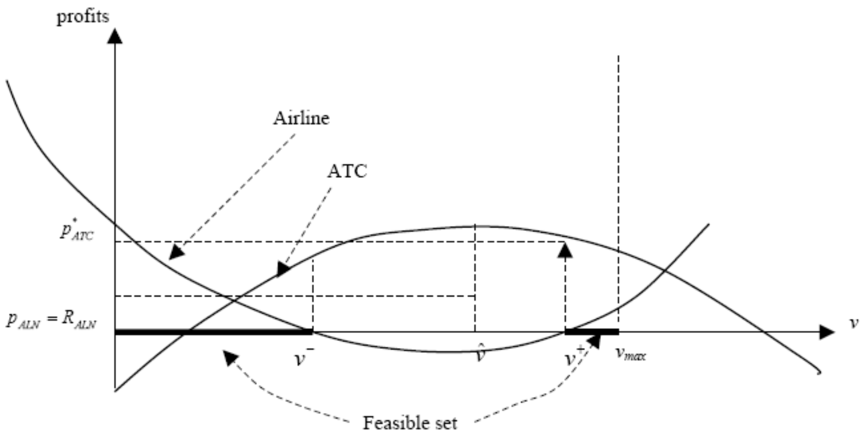

Figure 11 - Solution of the private ATC/ATM supplier problem (two real roots for (85)).

$$if \quad (-\frac{\tilde{\rho}}{2q^2}v^{*2} + (\frac{D_0}{2q} + \frac{s\,\tilde{\rho}}{2q^2})v^* + (\frac{\tilde{\rho}\,s\,c}{2q^2} - \frac{s\,D_0}{2q} - Q^F_{ATC})) < R_{ATC} \quad (96)$$

$$then \quad \mu = \min\{R_{ATC} - p^*_{ATC}(0), \lambda\phi^*\}$$

where $\phi^* = \frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c + v^*)$. Here, the profit of the airline is given by:

$$p^* = \frac{1}{\rho(1+\lambda)}(\frac{(c+v^*)\rho + qD_0}{2q})^2 - (c+v^*)D_0/q - C^F_{ALN} \quad (97)$$

which is superior or equal (see figures 10 and 11) to the minimum guaranteed level R_{ALN}.

6. REGULATION SCHEME WITH A PRIVATE ATC/ATM SERVICE PROVIDER

6.1. The regulation scheme

From the solution of the airline's profit maximization problem it appears that to get the same level of satisfied demand in the two cases (public or private ATC/ATM service providers), it is necessary to apply the same ATC/ATM tariff to the airline operations. The ATC/ATM tariff in the public supplier case is given by:

$$v_{pub} = \frac{(\sigma + c + (q/\tilde{\rho})D_0) - \sqrt{\Delta(1)}}{2+\lambda} - c \quad (98)$$

while in the private case, it is given by:

$$v_{pri} = v^* \quad (99)$$

where, according to the case, as seen in section V, v^* adopts different expressions. In general the resulting tariffs will be such as:

$$v_{pri} \geq v_{pub} \quad (100)$$

The proposed refunding scheme in section V has in fact no effect on the level of the tariff adopted by the private ATC/ATM service provider and then on the resulting level of the satisfied demand. Then another scheme must be conceived. What is proposed here is to provide a subvention to the airline by lowering the effective tariff paid to the private ATC/ATM service provider, the difference being compensated by the state up to the amount collected directly from the passengers through the passenger tax.

Let w be the part of the ATC/ATM tariff which is subventionned by the state. This value should be such as:

$$w = v_{pri} - v_{pub} \quad (101)$$

with the condition:

$$w\ f^* + \mu \leq \lambda\ \phi^* \quad (102)$$

or

$$w\,(\frac{D_0}{2q} - \frac{\rho(1+\lambda)}{2q^2}(c+v_{pub})) + \mu \leq \lambda(\frac{D_0}{2} - \frac{\rho(1+\lambda)}{2q}(c+v_{pub})) \quad (103)$$

Then, it will be possible to get the same level for the satisfied demand if:

$$\lambda \geq (v_{pri} - v_{pub})/q \quad (104)$$

Then we get the structure for financial flows displayed on figure 12.

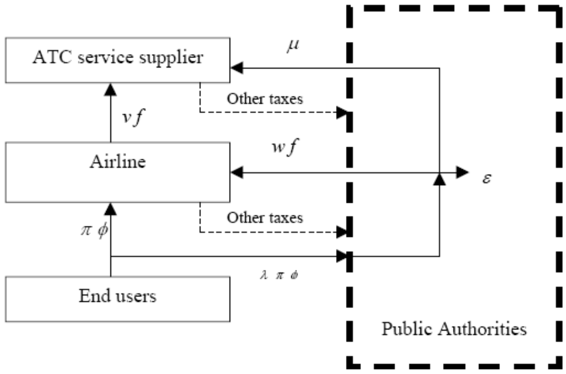

Figure 12 - Financial flows and activity levels with private supplier with regulation.

6.2 THE CONVERGENCE OF THE NEGOTIATION PROCESS

Following the general formulation, a natural negotiation process based on the objectives of the involved economic agents could be the following:

The private ATC/ATM service supplier solves at iteration n+1 the following problem with respect to v, given π^n and f^n:f

$$v^{n+1} = \arg\{ \max_{v} \quad v\,f^n + \alpha\lambda\pi^n\phi^n - (s\ f^n + Q^F_{ATC}) \} \quad (105)$$

with the airline's profit constraint:

$$\pi^n\ \phi(v) - ((c+v)\,f^n + C^F_{ALN}) \geq R_{ALN} \quad (106)$$

where π^n and f^n are provided by the airline sector which solves problem (11), (12) and (13) given v^n. This process is represented in figure 13.

Supposing that v will be chosen superior to s, the private ATC supplier problem reduces here to the finding of the maximum value of v which satisfies the airline budget constraint:

$$v^{n+1} = \frac{\pi^n \phi^n - (R_{ALN} + C_{ALN}^F + c f^n)}{f^n} \quad (107)$$

Here also we consider the case in which the solution of the airline problem is given by (22) , (27) and (28). Then we get a recurrent formula for v^n:

$$v^{n+1} = \frac{1}{2}(D_0 \frac{q}{\tilde{\rho}} + c) + \frac{1}{2}v^n - \frac{(2q^2/\tilde{\rho})(R_{ALN} + C_{ALN}^F)}{(D_0 \frac{q}{\tilde{\rho}} - c) - v^n} \quad (108)$$

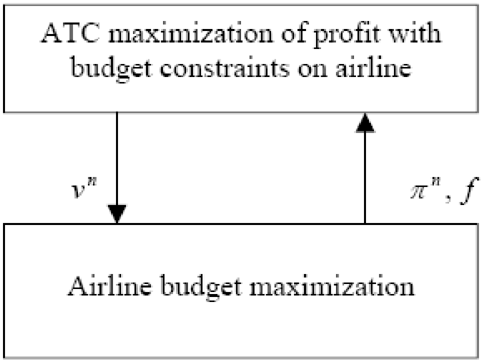

Figure 13 - Negotiation process between private ATC/ATM supplier and airlines.

Here also we consider that condition (96) is satisfied, and then in figure 14, the convergence of the negotiation process is analyzed graphically.

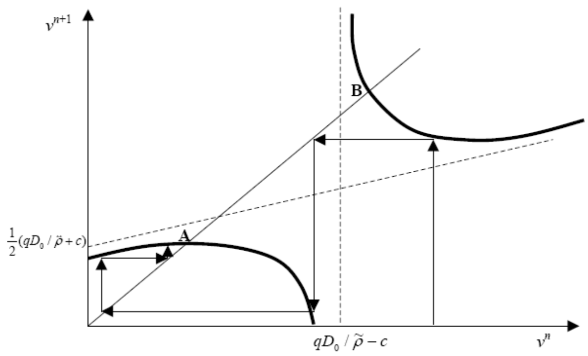

Figure 14 - Convergence of the private ATC/ATM-Airline negotiation process.

It appears that if

$$c > 2(q/\tilde{\rho})\sqrt{C_{ALN}^F + R_{ALN}} \quad (119)$$

or

$$R_{ALN} < c^2 \tilde{\rho}^2 / 4q^2 - C_{ALN}^F \quad (120)$$

then there are two equilibrium points: point A which is a stable equilibrium solution reached from lower v values and point B which is an unstable equilibrium solution where:

$$v_A = qD_0/\tilde{\rho} - \sqrt{c^2 - 4\frac{q^2}{\tilde{\rho}^2}(C_{ALN}^F + R_{ALN})}$$

and

$$v_B = qD_0/\tilde{\rho} + \sqrt{c^2 - 4\frac{q^2}{\tilde{\rho}^2}(C_{ALN}^F + R_{ALN})} \quad (121)$$

It appears that point A corresponds to the optimal solution of the bi-level problem found previously.. Moreover, it appears that condition (110) is satisfied by the problem parameters if a negative return is accepted by the private ATC service provider. When condition (110) is not satisfied, the negotiation process does not converge to an equilibrium point.

7. CONCLUSION

In this study a multilevel framework has been proposed to analyze the ATC/ATM charges definition problem in the cases of a public and a private ATC/ATM service provider a deregulated market. Here the one dimensional case has been considered. The objective which has been assumed for the ATC/ATM public service provider is to promote air transportation for end users while the objective of the airlines sector is profit maximization.

This has led to the formulation of two bi-level optimization programs involving ATC/ATM service provider as the leader and the whole airline sector as the follower. Direct optimal solutions have been obtained in the simplistic considered one dimensional case. Then two natural negotiation processes between the ATC/ATM service provider and the airline sector have been introduced, splitting these problems in two dependent problems: one where ATM authorities determine ATC/ATM charges for given airlines tariffs and one where the airline sector determines the tariffs and seat capacity supplies over the different markets for a given ATC/ATM charge level. Sufficient conditions have been established so that the proposed negotiation processes converge towards the respective optimal solution. Then it appears of interest to the service provider to adopt similar negotiation processes when dealing with the full scale networked pricing problem.

REFERENCES

BARD, J.F. (1998) *Practical bi level optimization: algorithms and applications*. (Kluwer Book Series: Non convex optimization and its applications, 30).

BROTCORNE L., LABBÉ, M., MARCOTTE, P. and SAVARD, G. (2000) A bilevel model for toll optimization on a multicommodity transportation network. *Transportation Science,* vol. 35, p. 1-14.

EUROPEAN COMMUNITY (2005) *Common requirements for air navigation services*. R2096/2005.

CORDLE, V. and POOLE, R. (2005) *Charging users would rescue air traffic control.* The Reason Foundation.

CORDLE, V. and POOLE, R. (2005) *Resolving the crisis in air traffic control funding*. The Reason Foundation.

DEMPE, S. (2002) Foundations of bilevel programming. Kluwer Academie Publishers, Dordrecht.

DESCHINKEL, K., FARGES, J.P. and DELAHAYE, D. (2002) Optimising and assigning price levels for air traffic management. *Transportation Research* Part E, p.221-237.

ICAO (2004). *ICAO's policies for airports and air navigation services*.

LABBÉ M., MARCOTTE, P. and SAVARD, G. (1998) A bilevel model of taxation and its application to optimal highway pricing. *Management Science,* vol. 44, p. 1595-1607.

PRICEWATERHOUSECOOPERS (2001) *Study of the terminal charges for air traffic control services*. Commission of the European Communities.

RAFFARIN, M. (2002) *Le contrôle aérien en France*. Paris. Thèse de doctorat.