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A traffic complexity approach through cluster analysis

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Abstract

The conflict resolution problem is quite simple as long as the number of aircraft involved is small. Many automation projects disregarded the problem of clusters and failed on real traffic tests because they were unable to deal with complex conflicts. The first definition of cluster appeared in the middle of the nineties when theoretical research started on conflict resolution. The n-aircraft conflict resolution problem is highly combinational and cannot be optimally solved using classical mathematical optimisation techniques. The set of admissible solutions is made of many unconnected subsets enclosing different local optima, but the subset enclosing the optimum cannot be found a priori. Using a priority order to solve a n-aircraft conflict is much easier but the solution is not optimal. However it is difficult to determine the best order or even a good order that ensures that a solution exists. In this paper, a theoretical study of the possible structures of clusters is presented. A simulation using French real traffic data compares the structure of clusters with direct and standard routes. The sensitivity of cluster sizes to uncertainties on trajectories forecast is studied.

Keywords: cluster, graphic sequences, traffic complexity, conflict resolution

Introduction

The conflict resolution problem is quite simple as long as the number of aircraft involved is small. Many automation projects disregarded the problem of clusters and failed on real traffic tests because they were unable to deal with complex conflicts. The American AERA project [Nie89] did not suggest any algorithm for the MOM[1] level or the AMPF[2] level, which were supposed to deliver two aircraft conflict to the ASF[3]. In Europe, ARC2000 [K$^+$89, FMT93] or FREER [DHN97] used 1-to-n strategies to solve conflicts which were unable to deal with large conflicts. A 1-to-n solver using an on board token allocation strategy was developed by Granger and Durand [DAG99], but is not efficient with high traffic densities [Gra02].

Theoretical approaches using potential fields models [TPS98, GT00], neural networks [DAN96a, DAM00], linear programming [M9́4], or semidefinite programming [FMF, Dod99] are limited by the size of the problem considered (never more than 5 aircraft). Furthermore, assumptions made on trajectories forecast are generally not realistic (use of constant speed, no uncertainty. . .).

A first definition of clusters was given by Durand and Alliot [DA95, DAN96b] in the middle of the nineties. A global solver using genetic algorithm was presented in the CATS/OPAS ([ABDM97, DA97]) simulator. It showed its efficiency on conflicts involving up to 30 aircraft.

The size of clusters widely depends on the detection window, the uncertainty on aircraft speed and the separation standards. Dealing with small clusters is essential to ensure the resolution efficiency.

In order to have a better understanding on the cluster problem, we first detail mathematically different possible cluster configurations. The different configurations are then tested with the CATS/OPAS arithmetic simulator on real traffic data in a second part. The size of clusters also depends on the routes followed by the aircraft and on uncertainties. In the third part, the influence of uncertainty on cluster size and the feedback clusters process are discussed.

1 Cluster analysis

1.1 Definitions

A cluster is a transitive closing on conflicting pairs of aircraft. For example (see figure 1), if aircraft A and B are in conflict and aircraft B and C are in conflict then A, B, C belong to the same cluster. A cluster cannot be defined

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[1] Maneuver Option Manager

[2] Airspace Manager Planning functions

[3] Automated Separation Function

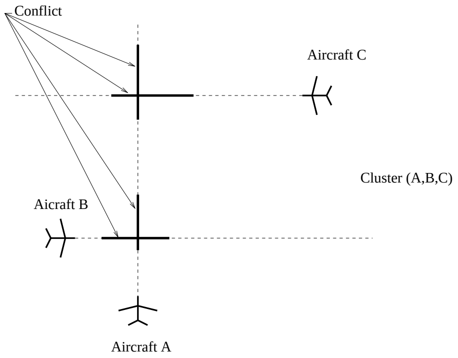

Figure 1: 3 aircraft cluster

only by its size. For example there are 2 sorts of 3 aircraft cluster (see figure 2).

A cluster can be represented by a graph : the nodes are the aircraft and the edges are the conflicts. A cluster or a graph can be characterized by:

- the number of aircraft or nodes of the graph;
- the number of conflicts or edges;
- the diameter of the cluster or the graph (see definition 1.2);
- the graphic sequence of the graph or the cluster (see definition 1.3).

Definition 1.1 – *The distance between two nodes of a graph is the minimum number of edges connecting the two nodes.*

Definition 1.2 – *The diameter of a graph is the maximum distance among every nodes pair.*

Theorem 1.1 – *The range of the diameter of a connected graph of size* $n > 1$ *is* $[1, n-1]$.

This result is self-evident.

Definition 1.3 – *A graphic sequence* $d = (d_1, d_2, ..., d_n)$ *is a sequence of numbers which can be the degree sequence of some graph.*

1.2 Theoretical results

Theorem 1.2 – *A graphic sequence* $d = (d_1, d_2, ..., d_n)$ *is a degree sequence of a connected graph if and only if* $\sum_{i=1}^{n} d_i \geq 2\,(n-1)$.

The proof of this last theorem can be found in [Gra02] and uses the following theorem [Die97].

Theorem 1.3 – *A connected* n *nodes graph is a tree if and only if it has exactly* $n-1$ *edges.*

nb of acft	nb conf min	nb conf max	nb graphic sequences	log	nb conf opt	nb graph sequ max
2	1	1	1	0	1	1
3	2	3	2	0.30	2/3	1
4	3	6	6	0.78	3/4	2
5	4	10	19	1.28	5/6/7	4
6	5	15	68	4.22	8	11
7	6	21	236	5.46	11	29
8	7	28	863	6.76	16	84
9	8	36	3137	8.05	20	253
10	9	45	11636	9.36	24	790
11	10	55	43306	10.68	29	2518
12	11	66	162728	12.00	35	8268
13	12	78	614142	13.33	41	27496
14	13	91	2330454	14.66	48	92800
15	14	105	8875656	16.00	55	317276
16	15	120	33924699	17.34	63	1095802
17	16	136	130038017	18.68	71	3823385
18	17	153	499753560	20.03	79	13444643
19	18	171	1924912505	21.38	89	47617067
20	19	190	7429159770	22.73	98	169797901
21	20	210	28723877046	24.08	108	608747098

Table 1: Number of possible graphic sequences as a function of the cluster size.

The following theorem proved by Erdos and Gallai [EG60] can be used to define an algorithm that counts the number of possible graphic sequences.

Theorem 1.4 – *A sequence degree* $d = (d_1, d_2, ..., d_n)$ *is graphic if and only if:*

$$\forall r \in [1, n-1], \sum_{i=1}^{r} d_i - r\,(r-1) \leq \sum_{i=r+1}^{n} min(r, d_i)$$

Table 1 details for a given cluster size the minimum and maximum number of conflicts, the total number of graphic sequences, its logarithm and the number of conflicts that gives the maximum number of graphic sequences. The growth of the number of graphic sequences is exponential. Even for a five aircraft cluster, there are 19 different possible configurations. This result gives a better understanding of the inability for human controller to handle clusters involving more than 2 or 3 aircraft.

2 Experimental results

Results presented in this part were computed with the CATS/OPAS traffic simulator [ABDM97] on a French loaded traffic day (May 21^{th} 1999). The uncertainties used

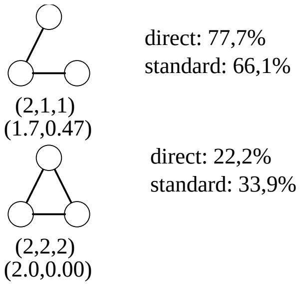

Figure 2: Possible graphic sequences for a 3 aircraft cluster

for the trajectory forecast were 5% on the horizontal speed and 15% on the climbing and descending rates. The time window was set to 8 minutes and the detection was done every 2 minutes.

Two scenari were tested: the first one used direct routes from origin to destination and the second one standard routes given by the flight plans. No pre-regulation is done on flight plans.

Table 2 and 3 detail the structure of different clusters with the standard routes scenario (2) and direct routes scenario (3). The possible and observed graphic sequences are given column 2 and 3. The ratio (percentage of observed/possible structures) is given column 4. The number and percentage of clusters of each size are detailed column 5 and 6. For standard routes, up to 5 aircraft, every configuration is represented. For direct routes, there are still 16 out of 19 different five aircraft clusters. It is important to notice that the graphic sequence of a cluster only measures the interactions between aircraft. No information on the cluster geometry is considered (number of leveled, climbing descending aircraft). If such criteria were taken into account, the number of different clusters would probably become huge and no expert system or human controller can handle so many situations. This explains why the CATS/OPAS global solver cannot be used as a decision making tool for big clusters. For such situations, it is most of the time too difficult for a human to understand the solution suggested by the solver.

Figures 2, 3 and 4 detail for 3, 4 and 5 aircraft the possible graphic sequences. The (mean/standard deviation) of the number of conflict per aircraft are given below each graphic sequence. The clusters are sorted according to the number of aircraft they contain. At the end of each line, the ratios of observed clusters of the line over the total number of clusters are given.

For the 3 sizes of clusters, the number of conflict is lower with direct routes than with standard routes.

Finally, the mean diameters of clusters are smaller with the standard route scenario than with the direct routes scenario (figure 5).

clus	Graphic sequences			clusters	
size	possible	obs	ratio	nb	ratio
2	1	1	100.00	18119	66.78
3	2	2	100.00	4952	18.25
4	6	6	100.00	1883	6.94
5	19	19	100.00	905	3.34
6	68	53	77.94	479	1.77
7	236	98	41.53	270	1.00
8	863	119	13.79	198	0.73
9	3137	92	2.93	112	0.41
10	11636	52	0.45	55	0.20
11	43306	56	0.13	59	0.22
12	162728	36	0.02	37	0.14
13	614142	27	0.00	27	0.10
14	2330454	10	0.00	10	0.04
15	8875656	8	0.00	8	0.03
16	33924699	9	0.00	9	0.03
17	130038017	4	0.00	4	0.01
18	499753560	3	0.00	3	0.01
21	28723877046	1	0.00	1	0.00

Table 2: Standard Routes - 5 and 15% of uncertainty - $T_w = 8mn$ - $\delta = 2mn$ - 21 mai 1999

clus	Graphic sequences			clusters	
size	possible	obs	ratio	nb	ratio
2	1	1	100.00	12118	80.52
3	2	2	100.00	2049	13.61
4	6	6	100.00	540	3.59
5	19	16	84.21	217	1.44
6	68	24	35.29	79	0.52
7	236	13	5.51	23	0.15
8	863	14	1.62	15	0.10
9	3137	8	0.26	8	0.05
11	43306	1	0.00	1	0.01

Table 3: Direct Routes - 5 and 15% of uncertainty - $T_w = 8mn$ - $\delta = 2mn$ - 21 mai 1999

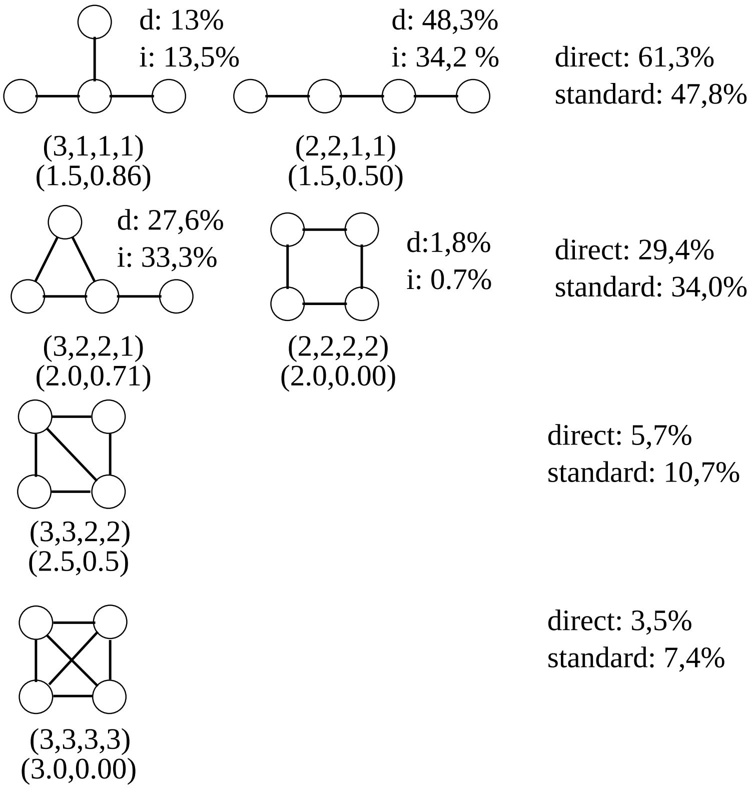

Figure 3: Possible graphic sequences for a 4 aircraft cluster

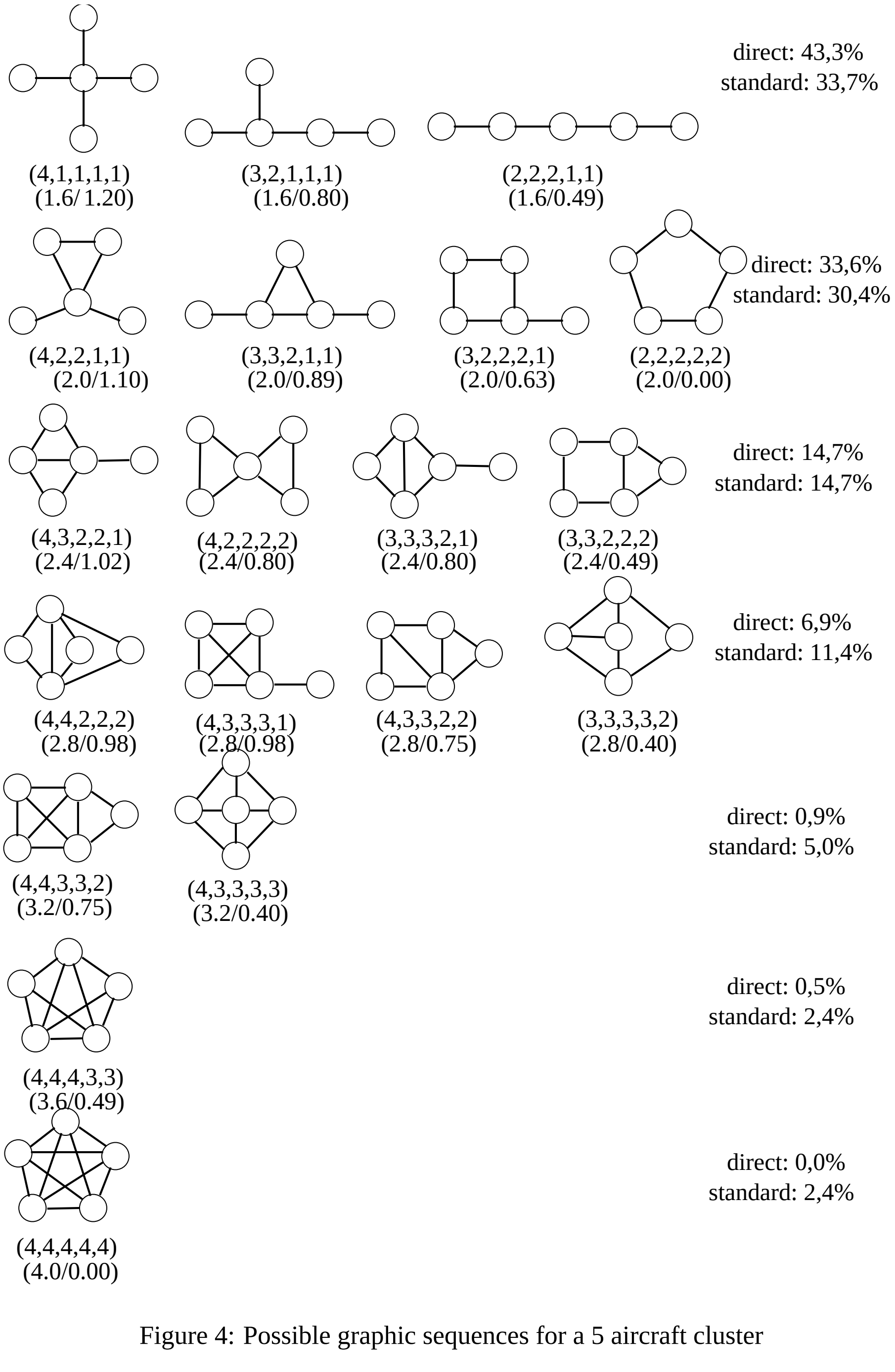

Figure 4: Possible graphic sequences for a 5 aircraft cluster

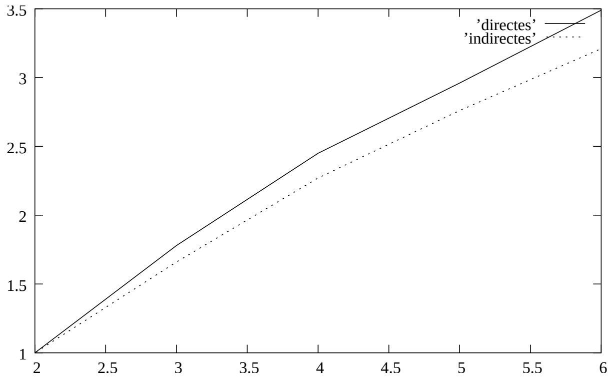

Figure 5: Cluster diameter as a function of the number of aircraft.

With direct routes, clusters are less dense and more spreaded out than with standard routes. The simulation does not include any regulation on the traffic, but these results show that the existing routes network may not be optimal for conflict resolution.

3 Sensitivity to uncertainties and resolution

3.1 Trajectories forecast uncertainties

When uncertainty is added on the horizontal speed and on the climbing and descending rates, more conflicts are detected and the size of clusters increases. Table 3.1 details the number of maneuvers, the size of the biggest cluster and the remaining conflicts for the different scenari. The global optimisation solver can easily handle clusters of size lower than 30 aircraft which is confirmed by the simulations. These results point up the necessity of a good trajectory forecast in order to limit the size of clusters and build an efficient solver.

Scenario/ uncertainties	Nb of maneuvers	Biggest cluster	Remaining conflicts
Direct 2% 5%	2461	16	0
Direct 5% 15%	3881	43	4
Direct 10% 30%	6819	97	20
Standard 2% 5%	5539	22	0
Standard 5% 15%	10088	54	66
Standard 10% 30%	19729	266	404

3.2 The feedback clusters effect

When a maneuver is decided for an aircraft, the solver must check that the new trajectory defined does not create a new conflict with other modified or existing trajectories. For example, if cluster $A - B - C$ has been solved by changing the trajectory of aircraft B, and if this new trajectory creates a conflict with aircraft D, then the solver has to consider the new cluster $A - B - C - D$. Clusters stemmed from this process will be called feedback clusters.

Table 4 gives for the two scenari the total number of clusters, the number of feedback clusters and the ratios feedback clusters/total clusters. The global ratio of feedback clusters/total clusters is close too 28% with standard routes and only 7% with direct routes. The ratio increases with the size of the clusters. As large clusters appear in dense areas the feedback process occurs more often.

The feedback cluster ratio increases with uncertainty: table 5 shows for the direct route scenario, with three hypothesis of uncertainties, the total number of clusters, the number of feedback clusters and the corresponding ratios. Uncertainty has an important influence on the feedback process. The more uncertainty, the more feedback clusters. With direct routes, when increasing the uncertainty on climbing and descending rates to 50% and on horizontal speed to 20% (see table 6), the total number of clusters and the ratios of feedback clusters become close to those observed with standard routes (with different uncertainties). However, for small clusters, the ratios of feedback clusters are lower for direct routes.

4 Conclusion

The diversity of cluster structures can be modelled by graphic sequences. The number of different clusters increases exponentially with the size of the cluster. Experimental results have shown that, up to 5 aircraft, all the structures were represented. For five aircraft, 19 different structures can be distinguished disregarding aircraft attitudes (climbing, descending, levelled). This diversity explains the structure of the existing air traffic control system: as a human controller cannot recognize and handle so many situations, different filters must guarantee that he will not have to manage more than 2 or 3 aircraft at a time in the same cluster.

The cluster problem must be taken into account when designing an automatic conflict solver. When clusters become too large (more than 20 to 30 aircraft), finding solutions becomes problematical.

Consequently, the key point for any future control system is to limit the cluster sizes of the problem considered. On the French example, it was shown that the routes network has an important influence on the size of clusters. Direct routes seem to be better than standard routes, but no other network has been tested yet. However, the most important point is to improve the trajectory forecast: actually,

cluster	Standard			Direct		
size	clusters	feedback	%age	clusters	feedback	%age
2	15922	3237	20.33	9970	527	5.29
3	4981	1556	31.24	1855	206	11.11
4	2244	860	38.32	586	93	15.87
5	1356	535	39.45	218	43	19.72
6	753	338	44.89	100	22	22.00
7	541	235	43.44	42	14	33.33
8	418	203	48.56	24	7	29.17
9	311	154	49.52	14	2	14.29
10	212	105	49.53	9	3	33.33
11	190	97	51.05	5	1	20.00
12	158	73	46.20	2	1	50.00
13	111	42	37.84	1	1	100.00
14	93	56	60.22	1	0	0.00
15	85	40	47.06	2	1	50.00
16	60	28	46.67	1	0	0.00
17	63	32	50.79	1	0	0.00
18	61	35	57.38			
19	57	35	61.40			
20	51	30	58.82			
21	34	15	44.12			
..	..	..	..			
52	5	1	20.00			
54	2	2	100.00			
55	1	0	0.00			
56	1	0	0.00			
62	1	0	0.00			
total	28152	7984	28.36	12831	921	7.18

Table 4: Ratios of feedback with 5 and 15% of uncertainty, for the two scenari

	30% et 10%			15% et 5%			5% et 2%		
size	clusters	feedback	%age	clusters	feedback	%age	clusters	feedback	%age
2	12859	1032	8.03	9970	527	5.29	7205	257	3.57
3	3326	562	16.90	1855	206	11.11	1021	78	7.64
4	1317	330	25.06	586	93	15.87	224	30	13.39
5	741	188	25.37	218	43	19.72	56	4	7.14
6	388	120	30.93	100	22	22.00	23	2	8.70
7	245	91	37.14	42	14	33.33	6	1	16.67
8	153	42	27.45	24	7	29.17	3	1	33.33
9	81	33	40.74	14	2	14.29	1	0	0.00
10	77	28	36.36	9	3	33.33			
11	45	19	42.22	5	1	20.00			
12	31	16	51.61	2	1	50.00			
13	27	9	33.33	1	1	100.00			
14	16	5	31.25	1	0	0.00			
15	18	8	44.44	2	1	50.00			
16	10	5	50.00	1	0	0.00			
17	10	8	80.00	1	0	0.00			
18	9	4	44.44						
19	10	4	40.00						
20	7	5	71.43						
21	5	1	20.00						
22	4	1	25.00						
23	1	0	0.00						
24	2	1	50.00						
25	1	1	100.00						
26	1	0	0.00						
28	2	1	50.00						
30	1	1	100.00						
32	2	0	0.00						
37	1	0	0.00						
tot	19390	2515	12.97	12831	921	7.18	8539	373	4.37

Table 5: Ratios of feedback for different uncertainties (direct routes)

a good trajectory forecast is essential to limit clusters sizes. With the help of datalink communications, aircraft will be able to give to the control system their future positions for the next ten minutes with a good accuracy. Regarding the current traffic increase, this seems the only way to limit clusters sizes.

There is still much to do on the cluster problem: some cluster structures are probably easier to solve than others but how can they be caracterized ? Some large clusters are easier to split into small ones but how can they be recognized ? Future models will have to take into account the aircraft attitudes (climbing, descending, leveled). The complexity will be increased, but will the information added be useful to design new resolution strategies ?

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	50% et 20%		
size	clusters	feedback	%age
2	15279	2399	15.70
3	5086	1356	26.66
4	2458	856	34.83
5	1523	602	39.53
6	1018	460	45.19
7	730	328	44.93
8	599	297	49.58
9	457	240	52.52
10	292	145	49.66
11	263	150	57.03
12	229	132	57.64
13	162	88	54.32
14	143	78	54.55
15	113	68	60.18
16	114	63	55.26
17	98	60	61.22
18	83	56	67.47
19	74	43	58.11
20	72	43	59.72
21	58	35	60.34
22	46	28	60.87
23	45	25	55.56
24	35	24	68.57
25	45	36	80.00
..	..	..	..
73	3	3	100.00
74	1	1	100.00
75	1	1	100.00
76	4	1	25.00
77	2	0	0.00
78	1	0	0.00
83	1	1	100.00
118	1	1	100.00
120	1	0	0.00
tot	29806	8095	27.16

Table 6: Direct routes: large uncertainties

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Biography

Géraud Granger graduated from the Ecole Nationale de l'Aviation Civile (ENAC) in 1998. He holds a Ph.D. in Computer Science at the Ecole Polytechnique de Paris (2002).

Nicolas Durand graduated from the Ecole Polytechnique de Paris in 1990 and from the Ecole Nationale de l'Aviation Civile (ENAC) in 1992. He has been a design engineer at the Centre d'Etudes de la Navigation Aérienne (CENA) since 1992 and holds a Ph.D. in computer Science (1996).