Differential flat control for rotorcraft trajectory tracking

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Abstract—**The purpose of this communication is to investigate the usefulness of the differential flatness control approach to solve the trajectory tracking problem for a four rotor aircraft. After introducing simplifying assumptions, the flight dynamics equations for the four rotor aircraft are considered. A trajectory tracking control structure based on a two layer non linear approach is then proposed. A supervision level is introduced to take into account the actuators limitations.**

Index Terms—**differential flatness, guidance supervision, rotorcraft flight mechanics, trajectory tracking.**

I. Introduction

In the last years a large interest has risen for the four rotor concept since it appears to present simultaneously hovering, orientation and trajectory tracking capabilities of interest in many practical applications [1]. The flight mechanics of rotorcraft are highly non linear and different control approaches (integral LQR techniques, integral sliding mode control [2], reinforcement learning [3]) have been considered with little success to achieve not only autonomous hovering and orientation, but also trajectory tracking In this paper, after introducing some simplifying assumptions, the flight dynamics equations for a four rotor aircraft with fixed pitch blades are considered.

The purpose of this study is to investigate the usefulness of the differential flatness control approach to solve the trajectory tracking problem for this class of rotorcraft. This approach has been already considered in the case of aircraft trajectory tracking by different authors [8, 9]. It appears that the flight dynamics of the considered rotorcraft present a two level affine structure which is made apparent by the definition of a new set of equivalent inputs. It can be shown then that the rotorcraft flight dynamics are composed of a differentially flat structure followed by a non linear invertible structure. This allows to introduce a new non linear control structure devoted to orientation and trajectory tracking.

N. Zhang (nzhang@laas.fr) and G. Andrei (andrei_geanina@yahoo.com) are PhD students at the LARA Research Laboratory of the Air Transport Department of ENAC, the French Civil Aviation Institute, 7 avenue Edouard Belin, 31055 Toulouse, France.

A. Drouin (drouin@recherche.enac.fr) is research engineer at LARA and URI-Drônes (UAV research team at ENAC).

F. Mora-Camino (corresponding author, phone:0033562174358, fax: 0033562174403, felix.mora@enac.fr) is Professor at ENAC and Haedtd of LARA.

N. Zhang and F. Mora-Camino are also respectively junior and senior researchers with LAAS du CNRS, 7 avenue du Colonel Roche, 31077, Toulouse, France.

II. Rotorcraft Flight Dynamics

The considered system is shown in figure 1 where rotors one and three are clockwise while rotors two and four are counter clockwise. Annex 1 describes the rotor dynamics.

The main simplifying assumptions adopted with respect to flight dynamics in this study are a rigid cross structure, no wind, negligible aerodynamic contributions resulting from translational speed, no ground effect as well as negligible air density effects and very small rotor response times. It is then possible to write simplified rotorcraft flight equations [1].

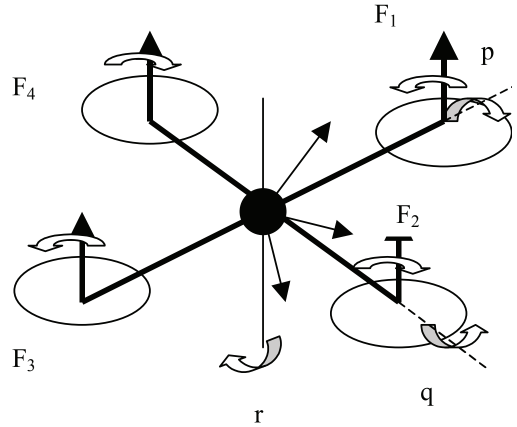

Fig 1. Four engine rotorcraft

The moment equations can be written as:

$$\begin{cases} \dot{p} = (a(F_4 - F_2) + k_2\, q\, r - k_p p) / I_{xx} \\ \dot{q} = (a(F_1 - F_3) + k_4 p\, r - k_q q) / I_{yy} \\ \dot{r} = (k(F_2 - F_1 + F_4 - F_3) - k_r r) / I_{zz} \end{cases} \quad (1)$$

where p, q, r are the components of the body angular velocity, with $k_2 = (I_{zz} - I_{yy})$ and $k_4 = (I_{xx} - I_{zz})$, I_{xx}, I_{yy} and I_{zz} being the moments of inertia in body-axis and m the total mass of the rotorcraft. Here the terms $k_p p$, $k_q q$, $k_r r$ are related with the drag of the whole structure.

The Euler equations are given by:

$$\begin{cases} \dot{\phi} = p + \tan(\theta)\sin(\phi)\, q + \tan(\theta)\cos(\phi)\, r \\ \dot{\theta} = \cos(\phi)\, q - \sin(\phi)\, r \\ \dot{\psi} = ((\sin(\phi)/\cos(\theta))\, q + (\cos(\phi)/\cos(\theta))\, r \end{cases} \quad (2)$$

where θ, ϕ, and ψ are respectively the pitch, bank and heading angles.

The acceleration equations written directly in the local Earth reference system are such as:

$$\begin{cases} \ddot{x} = (1/m)((\cos(\psi)\sin(\theta)\cos(\phi) + \sin(\psi)\sin(\phi))\ F - k_x\dot{x}) \\ \ddot{y} = (1/m)((\sin(\psi)\sin(\theta)\cos(\phi) - \cos(\psi)\sin(\phi))\ F - k_y\dot{y}) \\ \ddot{z} = -g + (1/m)(\cos(\theta)\cos(\phi)\ F - k_z\dot{z}) \end{cases} \quad (3)$$

where x, y and z are the centre of gravity coordinates and where :

$$F = F_1 + F_2 + F_3 + F_4 \quad (4)$$

The terms $k_x\dot{x}, k_y\dot{y}, k_z\dot{z}$ are also related with the drag of the whole structure.

The rotor forces satisfy the constraints:

$$0 \le F_{i_i} \le F_{\max} \quad i \in \{1,2,3,4\} \quad (5)$$

III. FLAT CONTROL LAW FOR ROTORCRAFT TTRAJECTORY TRACKING

A. Differential Flatness of Smooth Systems

A general nonlinear system given by:

$$\underline{\dot{X}} = f(\underline{X}, \underline{U}), \underline{X} \in \mathbf{R}^n, \underline{U} \in \mathbf{R}^m \quad (6)$$

where F is a smooth mapping, is said explicitly flat with respect to the output vector $\underline{Z}$, if $\underline{Z}$ is an m^{th} order vector which can be expressed analytically as a function of the current state, the current input and its derivatives, while the state and the input vectors can be expressed analytically as a function of $\underline{Z}$ and a finite number of its derivatives. Then there exists smooth mappings G_X, G_U, and G_Z such as:

$$\begin{cases} \underline{Z} = G_Z\left(\underline{X}, \underline{U}, \ldots, \underline{U}^{(n_z)}\right) & (7\text{-}1) \\ \underline{X} = G_X\left(\underline{Z}, \underline{\dot{Z}}, \ldots, \underline{Z}^{(n_x)}\right) & (7\text{-}2) \\ \underline{U} = G_U\left(\underline{Z}, \underline{Z}, \ldots, \underline{Z}^{(n_x+1)}\right) & (7\text{-}3) \end{cases}$$

where n_z and n_x are integer numbers. Vector $\underline{Z}$ is called a *flat output* for the nonlinear system given by equation (6). Although today there is no systematical way to determine flat outputs and eventually to prove its uniqueness, the flat outputs usually possess some physical meaning.

The explicit flatness property is of particular interest for the solution of control problems when physically meaningful flat outputs can be related with their objectives. In many situations, the control problem can be formulated as a flat output trajectory following problem. In general, for these cases, the flat output of equation (7-1) can be reduced, through state transformation, to a function of a single argument, the new system state itself:

$$\underline{Z} = G_Z(\underline{X}) \quad (8)$$

Then a possible control law providing to the flat outputs linear decoupled dynamics towards reference values is given by:

$$\underline{U} = G_U\left(\underline{Z}, \underline{Z}, \ldots, -\sum_{i=1}^{n_x} A_i \underline{Z}^{(i)} - A_0(\underline{Z} - \underline{Z}_c)\right) \quad (9)$$

where the A_i matrices are diagonal matrices such as the m polynomials :

$$s^{n_x+1} + \sum_{i=1}^{n_x} a_i(j,j)\ p^i + a_0(j,j) \quad j = 1,\ldots,m \quad (10)$$

B. Differential Flatness of Rotorcraft Dynamics

The equations of motion of the rotorcraft can be written in non linear state form as:

$$\underline{\dot{x}} = f(\underline{x}, \underline{u}) \quad (11\text{-}1)$$

where $\underline{x} = (p, q, r, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, x, y, z)'$ (11-2)

and $\underline{u} = (F_1 - F_3, F_2 - F_4, F_1 + F_3, F_2 + F_4)'$ (11-3)

Considering the output vector $\underline{Z} = (\phi, \theta, \psi, z)'$, the inversion of the Euler equations (2) provides expressions such as:

$$\begin{cases} p = p(\theta, \dot{\phi}, \dot{\psi}) \\ q = q(\phi, \theta, \dot{\theta}, \dot{\psi}) \\ r = r(\phi, \theta, \dot{\theta}, \dot{\psi}) \end{cases} \quad (12)$$

or more specifically:

$$\begin{cases} p = \dot{\phi} - \sin\theta\ \dot{\psi} & (12\text{-}1) \\ q = \cos\phi\ \dot{\theta} + \sin\phi\cos\theta\ \dot{\psi} & (12\text{-}2) \\ r = -\sin\phi\ \dot{\theta} + \cos\phi\cos\theta\ \dot{\psi} & (12\text{-}3) \end{cases}$$

while $\underline{u}$ can be expressed as:

$$\underline{u} = \underline{u}(\phi, \theta, \dot{\phi}, \dot{\theta}, \dot{\psi}, \ddot{\phi}, \ddot{\theta}, \ddot{\psi}, \dot{z}, \ddot{z}) \quad (13)$$

by inversion of the set of equations (1-1), (1-2), (1-3) and (3-3), or more specifically:

$$\begin{cases} u_1 = (I_{yy}\dot{q} - k_4 pr + k_q q)/a & (14\text{-}1) \\ u_2 = (-I_{xx}\dot{p} + k_2 qr - k_p p)/a & (14\text{-}2) \\ u_3 = (((\ddot{z}+g)m + k_z\dot{z})/(\cos\theta\cos\phi) - (I_{zz}\dot{r} + k_r r)/k)/2 & (14\text{-}3) \\ u_4 = (((\ddot{z}+g)m + k_z\dot{z})/(\cos\theta\cos\phi) + (I_{zz}\dot{r} + k_r r)/k)/2 & (14\text{-}4) \end{cases}$$

Then, it can be concluded that the attitude and heading dynamics as well as the vertical dynamics of the rotorcraft are differentially flat when considering the input-output relation between $\underline{u}$ and $\underline{Z}$.

C. Design of Flat Flight Control Laws

We adopt for the flat outputs second order dynamics such as:

$$\begin{cases} \ddot{\phi} = -2\,\zeta_\phi\ \omega_\phi\ \dot{\phi} - \omega_\phi^{\,2}(\phi - \phi_c) & (15\text{-}1) \\ \ddot{\theta} = -2\,\zeta_\theta\ \omega_\theta\ \dot{\theta} - \omega_\theta^{\,2}(\theta - \theta_c) & (15\text{-}2) \\ \ddot{\psi} = -2\,\zeta_\psi\ \omega_\psi\ \dot{\psi} - \omega_\psi^{\,2}(\psi - \psi_c) & (15\text{-}3) \\ \ddot{z} = -2\,\zeta_z\ \omega_z\ \dot{z} - \omega_z^{\,2}(z - z_c) & (15\text{-}4) \end{cases}$$

(8)

The control expressions of the control inputs in relations (14-1), (14-2), (14-3) and (14-4) are fed by p, q, r given by (12-1), (12-2) and (12-3) and by $\dot{p}, \dot{q}, \dot{r}$ given by:

$$\dot{p} = \ddot{\phi} - \cos\theta\, \dot{\theta}\, \dot{\psi} - \sin\theta\, \ddot{\psi} \qquad (16\text{-}1)$$

$$\dot{q} = \cos\phi\, \ddot{\theta} + \sin\phi \cos\theta\, \ddot{\psi} - \sin\phi\, (1+\sin\theta)\, \dot{\theta}\, \dot{\psi} + \cos\phi \cos\theta\, \dot{\phi}\, \dot{\psi} \qquad (16\text{-}2)$$

$$\dot{r} = -\sin\phi\, \ddot{\theta} + \cos\phi \cos\theta\, \ddot{\psi} - \cos\phi\, \dot{\phi}\, \dot{\theta} - \sin\phi \cos\theta\, \dot{\phi}\, \dot{\psi} - \cos\phi \sin\theta\, \dot{\theta}\, \dot{\psi} \qquad (16\text{-}3)$$

where $\ddot{\phi}, \ddot{\theta}, \ddot{\psi}$ are given by (15-1), (15-2) and (15-3).

Following the non linear inverse control approach (NLI), to insure that x and y follows second order dynamics such as:

$$\begin{cases} \ddot{x} + 2\,\zeta_x\, \omega_x\, \dot{x} + \omega_x^{\,2}(x - x_c) = 0 & (17\text{-}1) \\ \ddot{y} + 2\,\zeta_y\, \omega_y\, \dot{y} + \omega_y^{\,2}(y - y_c) = 0 & (17\text{-}2) \end{cases}$$

ϕ_c and θ_c are chosen such as:

$$(1/m)((\cos(\psi)\sin(\theta_c)\cos(\phi_c) + \sin(\psi)\sin(\phi_c))\, F - k_x \dot{x}) + 2\,\zeta_x\, \omega_x\, \dot{x} + \omega_x^{\,2}(x - x_c) = 0 \qquad (18\text{-}1)$$

$$(1/m)((\sin(\psi)\sin(\theta)\cos(\phi_c) - \cos(\psi)\sin(\phi_c))\, F - k_y \dot{y}) + 2\,\zeta_y\, \omega_y\, \dot{y} + \omega_y^{\,2}(y - y_c) = 0 \qquad (18\text{-}2)$$

Then : $\phi_c = \arcsin(\frac{m}{F} \sin\psi\; d_x - \cos\psi\; d_y)$ (19-1)

and $\theta_c = \arcsin(\frac{m}{F}(\cos\psi\; d_x + \sin\psi\; d_y) / \cos(\phi_c))$ (19-2)

where $d_x = (k_x - 2\,\zeta_x\, \omega_x)\, \dot{x} - \omega_x^{\,2}(x - x_c)$ (20-1)

$d_y = (k_y - 2\,\zeta_y\, \omega_y)\, \dot{y} - \omega_y^{\,2}(y - y_c)$ (20-2)

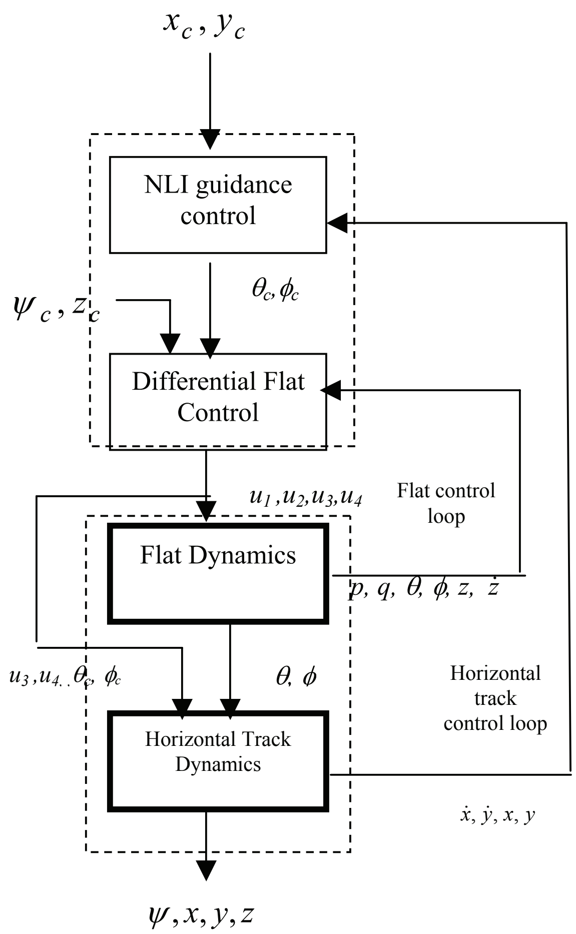

Fig 2. Proposed flat-NLI control structure

IV. SUPERVISION OF THE FLAT CONTROL LAW

The function of the control law supervision, is to turn feasible the control laws derived from the differential flatness approach.

Relation (19-1) is meaningful when :

$$-1 \le \frac{m}{F}(\sin\psi\; d_x - \cos\psi\; d_y) \le 1 \qquad (21)$$

In general $F/m \approx g$, then this condition is in general satisfied. A sufficient condition for (21) to be satisfied is:

$$((k_x - 2\varsigma_x\omega_x)\dot{x} - \omega_x^{\,2}(x - x_c))^2 + ((k_y - 2\varsigma_y\omega_y)\dot{y} - \omega_y^{\,2}(y - y_c))^2 = (F/m)^2 \qquad (22)$$

In that case defining angle b_y such as:

$$b_y = \arcsin(d_y m / F) \qquad (23)$$

then : either $\phi_c = \psi - b_y$ or $\phi_c = \psi - b_y + \pi$ (24)

To guarantee that :

$$-\pi/2 < -\phi_{\max} \le \phi_c \le \phi_{\max} < \pi/2 \qquad (25)$$

the parameters ζ_y, ω_y^2 and y_c must be chosen such as:

$$s_{\min} \le ((k_y - 2\,\zeta_y\, \omega_y)\, \dot{y} - \omega_y^{\,2}(y - y_c)) m/F \quad \text{and} \quad ((k_y - 2\,\zeta_y\, \omega_y)\, \dot{y} - \omega_y^{\,2}(y - y_c)) m/F \le s_{\max} \qquad (26)$$

where $s_{\min} = \min\{\sin(\psi + \phi_{\max}), \sin(\psi - \phi_{\max})\}$ (27-1)

and $s_{\max} = \max\{\sin(\psi + \phi_{\max}), \sin(\psi - \phi_{\max})\}$ (27-2)

Introducing upper bounds for the derivative of y as well as to the current error in y, sufficient conditions for the satisfaction of (26) are:

$$s_{\min} \le -(|\, k_y - 2\,\zeta_y\, \omega_y\, |\, \dot{y}_{\max} + \omega_y^{\,2} \Delta y_{\max}) m/F \qquad (28\text{-}1)$$

$$(|\, k_y - 2\,\zeta_y\, \omega_y\, |\, \dot{y}_{\max} + \omega_y^{\,2} \Delta y_{\max}) m/F \le s_{\max} \qquad (28\text{-}2)$$

Then considering b_x such as:

$$b_x = \arccos(d_x m / F) \qquad (29)$$

then : either $\phi_c = \psi - b_x$ or $\phi_c = \psi + b_x$ (30)

To guarantee again that :

$$-\pi/2 < -\phi_{\max} \le \phi_c \le \phi_{\max} < \pi/2 \qquad (31)$$

parameters ζ_x, ω_y^2 and x_c must be chosen such as:

$$c_{\min} \le (k_x - 2\,\zeta_x\, \omega_y)\, \dot{x} - \omega_x^{\,2}(x - x_c) m/F \quad \text{and} \quad (k_x - 2\,\zeta_x\, \omega_x)\, \dot{x} - \omega_x^{\,2}(x - x_c) m/F \le c_{\max} \qquad (32)$$

where

$$c_{\min} = \min\{\cos(\psi + \phi_{\max}), \cos(\psi - \phi_{\max})\} \qquad (33\text{-}1)$$

and

$$c_{\max} = \max\{\cos(\psi + \phi_{\max}), \cos(\psi - \phi_{\max})\} \qquad (33\text{-}2)$$

Introducing upper bounds for the derivatives of x as well as to current errors in x, sufficient conditions for the satisfaction of (32) are:

$$c_{\min} \le -(|\, k_x - 2\,\zeta_x\, \omega_x\, |\, \dot{x}_{\max} + \omega_x^{\,2} \Delta x_{\max}) m/F \qquad (34\text{-}1)$$

$$(|\, k_x - 2\,\zeta_x\, \omega_x\, |\, \dot{x}_{\max} + \omega_x^{\,2} \Delta x_{\max}) m/F \le c_{\max} \qquad (34\text{-}2)$$

Relation (19-2) can be written as:

$$\sin\theta_c\, \cos\phi_c = \cos(\psi - b) \qquad (35)$$

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where b is such as :

$$\sin b = d_y m / F \quad \text{and} \quad \cos b = d_x m / F \tag{36}$$

To guarantee that

$$-\pi / 2 < -\theta_{\max} \le \theta_c \le \theta_{\max} < \pi / 2 \tag{37}$$

the parameters ζ_x, ω_y , x_c , ζ_y, ω_y and y_c must be chosen such as:

$$\begin{aligned}&(\cos\psi((k_x - 2\,\zeta_x\,\omega_x)\,\dot{x} - \omega_x^2 (x - x_c))m / F + \\ &(\sin\psi((k_y - 2\,\zeta_y\,\omega_y)\,\dot{y} - \omega_y^2 (y - y_c))m / F \le \sin\theta_{\max}\cos\phi_{\max}\end{aligned} \tag{38-1}$$

and

$$\begin{aligned}&(\cos\psi((k_x - 2\,\zeta_x\,\omega_x)\,\dot{x} - \omega_x^2 (x - x_c))m / F + \\ &(\sin\psi((k_y - 2\,\zeta_y\,\omega_y)\,\dot{y} - \omega_y^2 (y - y_c))m / F \ge -\sin\theta_{\max}\cos\phi_{\max}\end{aligned} \tag{38-2}$$

with the stability conditions:

$$\varsigma_x > 0, \omega_x > 0, \varsigma_y > 0, \omega_y > 0 \tag{38-3}$$

A sufficient condition for (38-1) and (38-2) is:

$$\begin{aligned}&((\left|\ k_x - 2\,\zeta_x\,\omega_x\ \right|\dot{x}_{\max} + \omega_x^2 \Delta x_{\max})m / F + \\ &(\left|\ (k_y - 2\,\zeta_y\,\omega_y\ \right|\dot{y}_{\max} + \omega_y^2 \Delta y_{\max})m / F \le \sin\theta_{\max}\cos\phi_{\max}\end{aligned} \tag{39}$$

Finally parameters ζ_x, ω_y^2 , x_c , ζ_y, ω_y^2 and y_c must be chosen such that relations (22), (26), (32), (38-1), (38-2) and (34-3) are satisfied or introducing the sufficient conditions relations (22), (34-1), (34-2), (58-1) and (39).

Introducing upper bounds to $|\,\dot{\theta}\,|$, $|\,\dot{\phi}\,|$ and $|\,\dot{\psi}\,|$: $\dot{\theta}_{\max}$, $\dot{\phi}_{\max}$ and $\dot{\psi}_{\max}$, to insure that constraints:

$$-F_{\max} \le u_i \le F_{\max} \quad i = 1,2 \tag{40-1}$$

and

$$0 \le u_i \le 2\,F_{\max} \quad i = 3,4 \tag{40-2}$$

are satisfied the positive parameters ζ_θ, ζ_ϕ, ζ_ψ, ζ_z, ω_θ, ω_ϕ, ω_ψ, and ω_z must be chosen such as(sufficient conditions) :

$$\left|2I_{yy}\varsigma_\theta\omega_\theta - k_q\ \right|\dot{\theta}_{\max} + I_{yy}\omega_\theta^2\theta_{\max} \le a\,F_{\max} \tag{41-1}$$

$$\left|2I_{xx}\varsigma_\phi\omega_\phi - k_p\ \right|\dot{\phi}_{\max} + I_{xx}\omega_\phi^2\phi_{\max} \le a\,F_{\max} \tag{41-2}$$

$$\begin{aligned}&\left|k_z - 2\varsigma_z\omega_z m\right|\dot{z}_{\max} + m\omega_z^2\Delta z_{\max} + I_{zz}\left|k_r - 2\varsigma_\psi\omega_\psi\right|\dot{\psi}_{\max} \\ &+ I_{zz}\ \omega_\psi^2\Delta\psi_{\max} \le 2F_{\max}\cos\theta_{\max}\cos\phi_{\max} - mg\end{aligned} \tag{41-3}$$

$$\begin{aligned}&\left|2\varsigma_z\omega_z - k_z\right|\dot{z}_{\max} + \omega_z^2\Delta z_{\max} + \left|2I_{zz}\varsigma_\psi\omega_\psi - k_r\right|\dot{\psi}_{\max} / k \\ &+ I_{zz}\omega_\psi^2\Delta\psi_{\max} / k \le g\end{aligned} \tag{41-4}$$

where $\Delta\psi_{\max}$ and $\Delta z_{\max}$ are upper bounds for current desired changes in heading and flight height.

To insure that relations:

$$\begin{cases}\left|\dot{\phi}\right| \le \dot{\phi}_{\max},\quad \left|\dot{\theta}\right| \le \dot{\theta}_{\max},\quad \left|\dot{\psi}\right| \le \dot{\psi}_{\max} & (42\text{-}1)\\ \left|\dot{x}\right| \le \dot{x}_{\max},\quad \left|\dot{y}\right| \le \dot{y}_{\max},\quad \left|\dot{z}\right| \le \dot{z}_{\max} & (42\text{-}2)\\ \left|x - x_c\right| \le \Delta x_{\max},\quad \left|y - y_c\right| \le \Delta y_{\max} & (42\text{-}3)\\ \left|z - z_c\right| \le \Delta z_{\max},\quad \left|\psi - \psi_c\right| \le \Delta\psi_{\max} & (42\text{-}4)\end{cases}$$

are satisfied while tracking trajectory:

$$(x_{cc}(t), y_{cc}(t), z_{cc}(t), \psi_{cc}(t)), \quad t \ge 0$$

$x_c(t)$, $y_c(t)$, $z_c(t)$ and $\psi_c(t)$ can be chosen such as:

$$x_c(t) = x_{cc}(t) - \alpha\,(x_{cc}(t) - x(t)) \tag{43-1}$$

and

$$y_c(t) = y_{cc}(t) - \alpha\,(y_{cc}(t) - y(t)) \tag{43-2}$$

with $0 < \alpha < 1$.

With this choice, the instantaneous dynamics in x (or y) are such as:

$$\ddot{x} + 2\varsigma(\alpha)\,\omega(\alpha)\,\dot{x} + \omega(\alpha)^2 (x - x_{cc}) = 0 \tag{44-1}$$

$$\text{with } \varsigma(\alpha) = \varsigma_x / \sqrt{(1-\alpha)} \quad \text{and} \quad \omega(\alpha) = \sqrt{(1-\alpha)} \tag{44-2}$$

so that convergence can be expected. The same approach can be adopted with z and ψ:

$$z_c(t) = z_{cc}(t) - \beta\,(z_{cc}(t) - z(t)) \tag{45-1}$$

and

$$\psi_c(t) = \psi_{cc}(t) - \beta\,(\psi_{cc}(t) - \psi(t)) \tag{45-2}$$

with $0 < \alpha \le \beta < 1$.

V. CASE STUDIES

A. Heading Control at Hover

The objective is to hover at an initial position of coordinates x_0, y_0, z_0 while acquiring a new orientation ψ_1.

In this case we get the guidance control laws:

$$u_3 = \frac{1}{2}(m\,g - (I_{zz}\dot{r} + k_r r) / k) \tag{46-1}$$

$$u_4 = \frac{1}{2}(m\,g + (I_{zz}\dot{r} + k_r r) / k) \tag{46-2}$$

with the following reference values for the attitude angles:

$$\theta_c = 0 \quad \text{and} \quad \phi_c = 0 \tag{47}$$

Here the heading acceleration is given by:

$$\ddot{\psi} = -2\,\zeta_\psi\,\omega_\psi\psi - \omega_\psi^2 (\psi - \psi_c) \tag{48}$$

Starting from an horizontal attitude ($\theta(0)=0$, $\phi(0)=0$), attitude inputs u_1 and u_2 given by relation (14-1) and (14-2) remain equal to zero. Then, figures 3 and 4 display some simulation results:

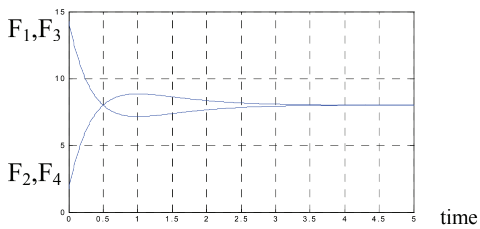

Fig 3. Hover control inputs

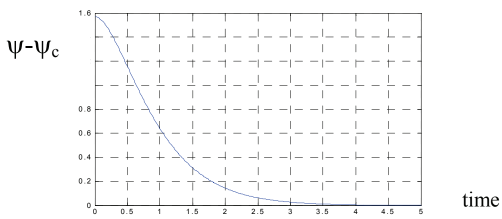

Fig 4. Heading response during hover

B. *Trajectory Tracking Case*

Here the rotorcraft is tracking the helicoïdal trajectory of equations:

$$\begin{cases} x_c(t) = \rho \cos \nu t \\ y_c(t) = \rho \sin \nu t \\ z_c = z_0 + \gamma t \\ \psi_c(t) = \nu t + \pi / 2 \end{cases} \qquad (49)$$

where ρ is a constant radius and γ is a constant path angle. In this case the guidance control laws tend towards:

$$u_1 = k_q \sin \phi_c / a \qquad (50\text{-}1)$$

$$u_2 = (k_2 \sin \phi_c - k_p) \cos \phi_c \; v / a \qquad (50\text{-}2)$$

$$u_3 = (((m + k_z \gamma / g)/(\cos\theta_c \cos\phi_c)) - k_r v / k)/2 \qquad (50\text{-}3)$$

and

$$u_4 = (((m + k_z \gamma / g)/(\cos\theta_c \cos\phi_c)) + k_r v / k)/2 \qquad (50\text{-}4)$$

Here, with $k_x = k_y$, the permanent reference values for the attitude angles are such as:

$$\theta_c = arctg(\frac{k_x v}{mg + k_z \gamma}) \qquad (51.1)$$

and

$$\phi_c = -arctg(\frac{m v^2}{\sqrt{{k_x}^2 v^2 + (mg + k_z \gamma)^2}}) \qquad (51.2)$$

In figures 5 to 7 simulation results are displayed where at initial time the rotorcraft is hovering:

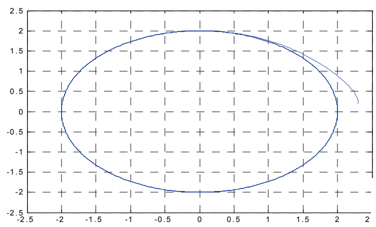

Fig 5. Evolution of rotorcraft horizontal track

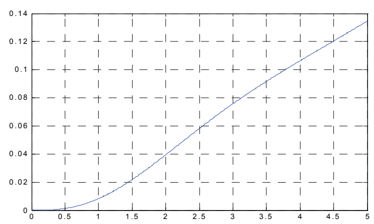

Fig 6. Evolution of rotorcraft altitude

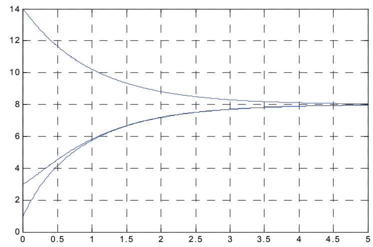

Fig 7. Rotorcraft trajectory tracking inputs

VI. Conclusion

In this communication the theoretical applicability of the differential flatness control technique to rotorcraft trajectory tracking has been investigated. It appears that this approach leads to the design of a two level control structure based on analytical laws. Considering the structure of the rotorcraft flight dynamics, other promising non linear control techniques are non linear inverse control [4] and back stepping control [10],[11].

A supervision layer has been designed to tackle with the limitations of the actuators of the rotorcraft and to avoid the uncontrolled effect of actuator saturation: sufficient conditions are have been established with respect to the convergence dynamics while a scaling scheme is proposed to define a current reference trajectory.

However, the robustness of these control laws with respect to the different aerodynamic effects which have been taken as negligible should be investigated. Since only very intricate theories are available to approach this problem, real flight tests appear, at this stage, to be unavoidable.

Appendix

The rotor engine dynamics are characterized by the relation between the input voltage V_a and the angular rate ω. A possible model of rotor dynamics is given by:

$$\dot{\omega}(t) = -\frac{1}{\tau} \omega(t) - K_Q \omega(t)^2 + (K_{V_a} / \tau) \, V_a(t) \qquad (A.1)$$

with $\omega(0) = \omega_0$, where τ, K_Q and K_{Va} are given positive parameters and where the voltage input is such as:

$$0 \le V_a \le V_{\max} \qquad (A.2)$$

with a negligible time response for the voltage generator. The step response (V_a =constant) of the rotor is solution of the scalar *Riccati* equation:

$$\dot{\omega}(t) = -\frac{1}{\tau} \omega(t) - K_Q \omega(t)^2 + (K_{V_a} / \tau) \, V_a \qquad (A.3)$$

with $\omega(0) = \omega_0$.

A particular solution ω_l of the associated differential equation is such as:

$$\omega_1 = \frac{1}{2 \tau K_Q} (\sqrt{1 + 4 K_{V_a} K_Q \tau V_a} - 1) \qquad (A.4)$$

In the general case, the solution of (A.3) can be written as

$$\omega(t) = \omega_1 + \frac{1}{\frac{1}{\omega(0)-\omega_1} + K_Q \tau'(1-e^{-t/\tau'})} e^{-t/\tau'} \quad \text{(A.5)}$$

with

$$\tau' = \tau / \sqrt{1 + 4 K_V K_Q \tau V_a} \quad \text{(A.6)}$$

and

$$\lim_{t \to +\infty} \omega(t) = \omega_1 \quad \text{(A.7)}$$

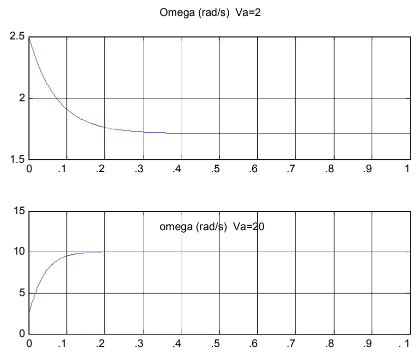

Fig. A1- Two examples of rotor step response

REFERENCES

[1] Hoffmann, G., Rajnarayan, D.G., Waslander, S. L., Dostal , D., Jang, J.S. and Tomlin, C. J., The Standford Tetsbed of Autonomous Rotorcraft for Multi-Agent Control, 23rd Digital Avionics Systems Conference, Salt Lake City, UT, November 2004.
[2] Hassan K. Khalil, Nonlinear Systems, Prentice Hall, 3rd Ed., 2002.
[3] Sutton R.S. and Barto, A. G., Reinforcement learning: an introduction, MIT Press, Cambridge, MA, 1998.
[4] Singh, S. N. and Schy, A. A., Nonlinear decoupled control synthesis for maneuvring aircraft, Proceedings of the 1978 IEEE conference on Decision and Control, Piscataway, NJ, 1978.
[5] Ghosh, R. and Tomlin,C. J., Nonlinear Inverse Dynamic Control for Model-based Flight, Proceeding of AIAA, 2000.
[6] Fliess,M., J. Lévine, P. Martin, and P. Rouchon 1995). Flatness and defect of non-linear systems: theory and examples. *International Journal of Control*, Vol. 61, No. 6, 1995, pp. 1327-1361.
[7] A. Drouin, A.B. Ramos, F. Mora-Camino (2007), Rotorcraft trajectory tracking by non linear inverse control, 6th Brazilian Conference on Dynamics, Control and their Applications,USP- São José do Rio Preto, Brazil.
[8] Lu, W.C., F. Mora-Camino and K. Achaibou (2004). Flight Mechanics and Differential Flatness. *Dynamics and Control Conference*. Ilha Solteira, Brasil, pp. 830-839, 2004.
[9] Martin, P. (1992). *Contribution à l'étude des systèmes differentiellement plats*. Ecole des Mines de Paris. Ph.D. Dissertation, Paris.
[10] Miquel, T. (2004), Contribution à la synthèse de lois de guidage relatif, approche non linéaire. PhD. Thesis, Université Paul Sabatier, Toulouse.
[11] A. Bento Moutinho (2007), Modeling and nonlinear control for airship autonomous flight, Instituto Superior Tecnico, PhD Dissertation, Lisbon.