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An Optimization Approach for Long Term Planning of Multimodal Freight Transportation Systems

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Abstract
In this communication is considered the problem of long term forecasting of the freight traffic growth in a large transportation network. This problem is crucial when planning the necessary investments in terminals, transportation links and fleets. One of the main difficulty of this task is related with the estimation of future demand over the network which has direct influence on the operational conditions and costs. The proposed approach makes use of two different optimization models: One model is devoted to freight demand forecasting, the other one defines the global transport supply according with a profit maximization behavior for the whole freight transport sector . The freight demand forecasting process is based a new entropy maximization approach to determine the distribution of origin-destination matrices. A two level solution technique considering vehicle flows at the first level and the freight flows at the second level is introduced. The proposed solution scheme is composed of an iterative process between the current solution for freight demand forecasting and the supply optimization problem: the entropy maximizing freight distribution problem provides the freight origin-destination matrix given a fare structure, while the supply optimization problem provides the fare structure given a freight origin-destination matrix.

Keywords: Planning, Network Flows, Bi-level Programming

1. Introduction

In this communication is considered the problem of long term forecasting of the freight traffic growth in a large multimodal transportation system. This problem is crucial when planning the necessary investments in terminals, networks and fleet resources. One of the main difficulty of this task is related with the estimation of future demand over the network which has direct influence on the operational conditions and costs.

The proposed approach makes use of two different optimization models: One model is devoted to freight demand forecasting, the other one defines the transport supply according with a profit maximization behavior for the whole sector operating in this area.

The supply optimization model considers simultaneously two classes of flows: vhicle flows providing transportation capacity and passengers flows generating revenues to the operators. Hence, no classical flows in networks optimization technique is available to solve this problem while two level solution techniques considering vehicle flows at the first level and freight flows at the other can be considered. Then a global optimization problem is associated to demand and operations costs.

Each of the two optimization problems, taken separately is convex, however, the whole problem, through the definition of revenue and of global cost constraints is non convex.

A proposed solution schemes is composed of an iterative process between the current solutions of the demand and the supply optimization problems: the entropy maximizing problem provides the freight origin-destination matrix given a fare structure, while the supply optimization problem provides the fare structure given a freight origin-destination matrix. To enforce convergence while maintaining convexity of the two problems, the effective-potential demand level constraints of the supply optimization problem are modified.

2. Distribution of Demand

To perform the prevision of demand, it is supposed that an a priori trip distribution is available. This information may be the result of a prediction of trip distribution for a previous time period or the result of local forecasting studies. It is also supposed [1] that predictions of generation and attraction potential levels $O_i\ ,\ i \in \{1, ..., N\}$ and $D_j\ ,\ j \in \{1, ..., N\}$, are available.

2.1 Local cost elasticities

We consider that the demand for a given origin destination i-j is such that there exists a positive constant λ_{ij} with :

$$T_{ij}(\overline{\pi}_{ij}) = T_{ij}(\overline{\pi}_{ij}^0)\, e^{-\lambda_{ij}(\overline{\pi}_{ij} - \overline{\pi}_{ij}^0)} \tag{1}$$

where $\overline{\pi}_{ij}$ is a mean cost between *i* and *j*. Considering that the potential origin and destination levels $O_i\ ,\ i \in \{1, ..., N\}$ and $D_j\ ,\ j \in \{1, ..., N\}$ are such as :

$$\sum_{j=1, j\neq i}^{N} T_{ij}(0) = O_i \quad i = 1\ \grave{a}\ N \tag{2}$$

$$\sum_{i=1, i\neq j}^{N} T_{ij}(0) = D_j \quad j = 1\ \grave{a}\ N \tag{3}$$

we have the following constraints:

$$\sum_{j=1, j\neq i}^{N} T_{ij}\, e^{\lambda \overline{\pi}_{ij}} = O_i \quad i = 1\ \grave{a}\ N \tag{4}$$

$$\sum_{i=1, i\neq j}^{N} T_{ij}\, e^{\lambda \overline{\pi}_{ij}} = D_j \quad j = 1\ \grave{a}\ N \tag{5}$$

Observe that a more general relation for (1) could be:

$$T_{ij}(\overline{\pi}_{ij})\, f_{ij}(\overline{\pi}_{ij}) = T_{ij}(\overline{\pi}_{ij}^0)\, f_{ij}(\overline{\pi}_{ij}^0) \tag{6}$$

where f_{ij} is a continuous positive monotonous decreasing function such as:

$$f_{ij}(0) = 1 \ \text{ and } \lim_{x \to +\infty} f_{ij}(x) = 0 \tag{7}$$

2.2 Estimation of demand distribution

The demand distribution estimation problem is taken here as a constrained entropy maximization problem. Then to a choice of an instance $I=\{[\hat{T}_{ij}],\ O_i\ ,\ i \in \{1, ..., N\},\ D_j\ ,\ j \in \{1, ..., N\}, [\lambda_{ij}]\}$ is associated the following maximization problem, Problem I:

$$\max \quad -\sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} T_{ij}\, Ln\left(T_{ij} / \hat{T}_{ij}\right) \tag{8}$$

under constraints (4), (5) and

$$T_{ij} \geq 0\ ,\ i = 1\ \grave{a}\ N,\ j = 1\ \grave{a}\ N,\ i \neq j \tag{9}$$

The adopted optimization criteria, a conditional entropy function, is representative of the global distortion between the a priori and the predicted demand distributions.

The instance is said to be *consistent* if the following condition is fulfilled :

$$\sum_{i=1}^{N} O_i = \sum_{j=1}^{N} D_j = T \tag{10}$$

2.3 Analysis of the solution

The above optimization problem being convex it is useful to introduce the Lagrangian associated to this problem :

$$L = -\sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} T_{ij} \; Ln\left(T_{ij} / \hat{T}_{ij}\right) + \sum_{i=1}^{N} \alpha_i \; (\sum_{j=1, j\neq i}^{N} T_{ij} e^{\lambda \, \overline{\pi}_{ij}} - O_i) + \sum_{j=1}^{N} \beta_j \; (\sum_{i=1, i\neq j}^{N} T_{ij} e^{\lambda \, \overline{\pi}_{ij}} - D_j) \tag{11}$$

The first order optimality conditions are such as :

$$\frac{\partial L}{\partial T_{ij}} = 0 \qquad i = 1 \; à \; N, \; j = 1 \; à \; N, \; i \neq j \tag{12}$$

or :

$$-1 - Ln\,(T_{ij} / \hat{T}_{ij}) + \alpha_i \; e^{\lambda_{ij} \overline{\pi}_{ij}} + \beta_j \; e^{\lambda \, \overline{\pi}_{ij}} = 0 \qquad i = 1 \; à \; N, \; j = 1 \; à \; N, \; i \neq j \tag{13}$$

Then the solution is such as :

$$T_{ij}^{*} = \hat{T}_{ij} \; e^{-(1+(\alpha_i+\beta_j) e^{\lambda_{ij}\overline{\pi}_{ij}})} \qquad i = 1 \; à \; N, \; j = 1 \; à \; N, \; i \neq j \tag{14}$$

In the case that $\lambda_{ij} \; \overline{\pi}_{ij}$ is very small, this expression can be approximated by:

$$T_{ij}^{*} = \hat{T}_{ij} \; e^{-(1+(\alpha_i+\beta_j)(1+\lambda_{ij} \; \overline{\pi}_{ij}))} \qquad i = 1 \; à \; N, \; j = 1 \; à \; N, \; i \neq j \tag{15}$$

The dual variables are such as:

$$\sum_{j=1, j\neq i}^{N} \theta_{ij} \;\; p_{ij}^{(\alpha_i + \beta_j)} = O_i \qquad i = 1 \; to \; N \tag{16}$$

$$\sum_{i=1, i\neq j}^{N} \theta_{ij} \;\; p_{ij}^{(\alpha_i + \beta_j)} = D_j \qquad j = 1 \; to \; N \tag{17}$$

where

$$\theta_{ij} = \hat{T}_{ij} e^{(\lambda_{ij} \; \overline{\pi}_{ij} - 1)} \quad \text{and} \quad p_{ij} = e^{e^{\lambda_{ij}\overline{\pi}_{ij}}} \tag{18}$$

however, the numerical solution of an instance of Problem I will be obtained using linear –convex algorithm such as the Simplex-Convex or Frank-Wolfe algorithm [1]. Another interesting direction for numerical resolution is through the solution of the geometric primal associated to this problem [2].

3. Freight Supply Model

The proposed model takes into account two types of flows and their corresponding constraints [3], [4]: vehicle flows along the network according to the available fleets and freight flows using the resulting transport capacity [6]. The fleet of vehicle and its operation generate fixed and variable costs, while freight flows are the main source of revenue for the freight transportation sector.

3.1 Vehicle Flow Network Model

Considering that freight fleet is composed of M different vehicles classes. To each freight terminal i, $i \in \boldsymbol{A}$, and to each freight modality is associated the set of freight terminals directly reachable with this modality : $\boldsymbol{A_C}(i)$. Then M freight transportation networks can be introduced:

$$\boldsymbol{R}_{\mathrm{m}} = [\boldsymbol{G}_m, [\, f_{ij}^{m} \,]] \qquad m \in \{1, \cdots, M\} \tag{19}$$

where $\boldsymbol{G_m}$ is the graph $[\boldsymbol{A}, \boldsymbol{\Gamma_m}]$ where the successor function $\boldsymbol{\Gamma_m}$ is such as :

$$\boldsymbol{\Gamma_m}(i) = \boldsymbol{A_m}(i) \qquad i \in \boldsymbol{A} \tag{20}$$

$[\, f_{ij}^{m} \,]$ is the flow of freight vehicles over $\boldsymbol{G_m}$. It satisfies to conservation and positive ness constraints:

$$\sum_{j=1, j\neq i}^{N} f_{ji}^{m} = \sum_{k=1,k\neq i}^{N} f_{ik}^{m} \quad m \in \{1,\cdots, M\} \tag{21}$$

$$f_{ij}^{m} \geq 0\,, \quad i,j \in \boldsymbol{A} \quad m \in \{1,\cdots, M\} \tag{22}$$

Here flows integrity constraints are not taken into account since no scheduling or routing problem will be formulated according to these flows which should only provide a global view of the future development of the air transportation network. However for sake of realism, fleet capacity constraints are introduced:

$$\sum_{i=1}^{N} \sum_{j\in A_m(i)} f_{ij}^{m}\, d_{ij}^{m} \leq D_m\, F_m \tag{23}$$

where d_{ij}^{m} is the block time for a freight vehicle of class m to go from terminals i and j , D_m is the medium time availability of a vehicles of class m.
It can be also of interest to introduce terminal capacities such as:

$$\sum_{j\in\Gamma_m^{-1}(i)} \omega_{am} f_{ji}^{m} + \sum_{j\in\Gamma_m(i)} \omega_{dm} f_{ji}^{m} \leq K_i \qquad i\in A \tag{24}$$

where K_i is the capacity at terminal *i j*. Here ω_{am} and ω_{dm} are capacity parameters attached to the different types of operations (arrival or departure) and fleets.

3.2 Freight Flows Modeling

The *M* freight networks provide the physical support for the freight flows network defined as :

$$\boldsymbol{R_f} = [\boldsymbol{G_f}, [\phi_{ij}]] \tag{25}$$

where $\boldsymbol{G_f} = [\boldsymbol{A}, \boldsymbol{\Gamma_f}]$ and $\boldsymbol{\Gamma_f}$ is such as:

$$\boldsymbol{\Gamma_f}(i) = \bigcup_{m\in\{1,\cdots,M\}} A_m(i) \qquad i\in\boldsymbol{A} \tag{26}$$

[ϕ_{ij}]is the freight flows over this network. It obeys to the following capacity constraints :

$$0 \leq \phi_{ij} \leq \sum_{m=1}^{M} C_m\, f_{ij}^{m} \quad i,\, j \in A \tag{27}$$

Within graph $\boldsymbol{G_f}$, for each origin-destination pair, a set of concurrent paths composed of a succession of trips is retained according to directness criteria:

$$Ch_{ij} = \{ Ch^{n}_{ij} \text{ , n =1 to n}_{\max}\} \quad \text{i, j} \in \boldsymbol{A} \tag{28}$$

Here it is considered that freight firms assign demand for a given O-D pair between these different possible paths according with their available capacity. This approach is acceptable when available capacity is close to demand levels.

Let [$\alpha(i,j,n,k,l)$] be the incidence matrix between path Ch^{n}_{ij} and arc (k,l) of $\boldsymbol{G_{pax}}$: $\alpha(i,j,n,k,l) = 1$ if $(k,l)\in Ch^{n}_{ij}$ $\alpha(i,j,n,k,l) = 0$ else. Then the freight flow between terminals i and j is given by :

$$\phi_{ij} = \sum_{k=1}^{N} \sum_{l=1,l\neq k}^{N} (\sum_{n=1}^{N_{ij}} \alpha(k,l,n,i,j)\, \theta_{k\,\mathrm{ln}}) \qquad i,j \in A,\; i \neq j \tag{29}$$

where θ_{ijn} is the flow of freight between terminals i and j using the n[th] path between them.

The mean fare for the *i-j* origin-destination pair is then given by :

$$\bar{\pi}_{ij} = (\sum_{n=1}^{N_{ij}} \pi_{ij}^{n}\, \theta_{ijn}) / (\sum_{n=1}^{N_{ij}} \theta_{ijn}) \qquad i,j \in A \tag{30}$$

4. Supply Optimization Model

Given a distribution of potential demand $[T_{ij}^*]$, associated to $[T_{ij}]$ O_i , $i \in \{1, ..., N\}$, D_j , $j \in \{1, ..., N\}$, and $[\lambda_{ij}]$, as well as to mean air transportation fares $[\overline{\pi}_{ij}]$, the optimization of the supply (capacities and fares) by the freight firms can be considered: Problem II is concerned with the optimization of the global economic performance over a future period of time of the firms operating the freight transportation network.

Here the decision variables are the vehicle flows (f_{ij}^m, $m \in \{1, \cdots, M\}$) between the different terminals and the fares (π_{ij}^n) applied to each selected path between the terminals. To solve this problem it is also necessary to introduce the effective flows of freight (θ_{ijn}) associated to each selected path between the terminals.

The optimization criterion of Problem II is given by:

$$\max_{f_{ij}, \pi_{ij}^n, \theta_{ij}} \left(\sum_{i=1}^{N} \sum_{j \in \Gamma_{pax}} \sum_{n=1}^{N_{ij}} \pi_{ij}^n \, \theta_{ijn}\right) / (1+\tau_r) - \left(\sum_{m=1}^{M} (c_{mf} + c_{mv} F_m + \sum_{i=1}^{N} \sum_{j \in \Gamma_c} c_{ij}^m f_{ij}^m) \right) \tag{31}$$

where τ is a return rate and the c_{mv} are coefficients related with fixed and variable fleet and vehicle flows costs.

Problem II must satisfy the following constraints:

$$\sum_{j=1, j \neq i}^{N} f_{ji}^m = \sum_{k=1, k \neq i}^{N} f_{ik}^m \quad m \in \{1, \cdots, M\} \quad i, j \in \boldsymbol{A} \tag{32}$$

$$\sum_{i=1}^{N} \sum_{j \in A_m(i)} f_{ij}^m \, d_{ij}^m \leq D_m \, F_m \tag{33}$$

$$\sum_{k=1}^{N} \sum_{l=1, l \neq k}^{N} \sum_{n=1}^{N_{kl}} \alpha(k,l,n,i,j) \, \theta_{k\ln} \leq \sum_{m=1}^{M} C_m \, f_{ij}^m \quad i, j \in \boldsymbol{A} \tag{34}$$

$$\sum_{n=1}^{N_{ij}} \theta_{ijn} \leq T_{ij}^* \quad i, j \in \boldsymbol{A} , i \neq j \tag{35}$$

with the positiveness conditions:

$$\theta_{k\ln} \geq 0 \quad k, l \in \boldsymbol{A}, \, n \in N_{kl} \tag{36}$$

$$f_{ij}^m \geq 0 \quad m \in \{1, \cdots, M\} \; i, j \in \boldsymbol{A} \tag{37}$$

$$\pi_{ij}^n \geq 0 \qquad n \in N_{ij}, i, j \in \boldsymbol{A} \tag{38}$$

5. Global Solution Scheme

It appears that Problem I and Problem II are strongly interdependent: while Problem I provides to Problem II potential levels of demand $[T_{ij}^*]$ (constraint (35)), Problem II provides mean fare values $[\overline{\pi}_{ij}]$ (relation (30)) to Problem I.

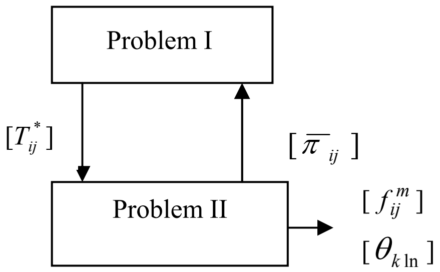

Figure 1: Interaction between problems

Each of the two optimization problems, taken separately is a convex programming problem, however, the whole problem when integrating Problem I into Problem II through constraint (35) is non convex. Then it seems interesting to solve numerically these problems separately and design an interactive process towards equilibrium. In this case, some questions are of interest:

- the easiness to solve numerically each of the optimization problems,
- the guarantee of convergence of the iterative process,
- the speed of convergence,
- the quality of the limit solution.

These two linked problems constitute a non standard bilevel programming problem [7] where Problem II is the leader's problem while Problem I is the follower's problem. This results from the adoption of a deregulated point of view in which firms operate the freight transport network according to their direct economic interest without considering any social surplus.

Constraint (35) plays a central role in the articulation of the two problems and the convergence of their solutions towards a common global solution. Since this constraint transmits to the revenue optimization problem the reaction of demand with respect to changes in mean fares between the different origins and destinations, it is useful to make apparent this effect so that limited fare values will be provided by the solutions of Problem II. However, the relation between origin-destinations flows $[T^*_{ij}]$ and fare levels $[\bar{\pi}_{ij}]$ is quite complex and at least non linear. So, to maintain the convexity of Problem II, this constraint is replaced by its first order approximation where the reference values are the solutions of the two problems at the previous iteration:

$$\sum_{n=1}^{N_{ij}} \theta_{ijn} \leq T_{ij}^{*(k-1)} + \sum_{l=1}^{N} \sum_{m=1, m\neq l}^{N} \sum_{n=1}^{N_{lm}} \frac{\partial T^*_{ij}}{\partial \pi^n_{lm}}\Big|_{k-1} (\pi^n_{lm} - \bar{\pi}_{lm}^{n\,(k-1)}) \qquad (39)$$

$$i, j \in A, \quad i \neq j$$

The partial derivatives present in relation (39) can be computed by differentiating relations (15), (16),(17) and (18) at the solutions of Problem I and II at iteration k-1 and solving linear systems of equations.

The following inequalities are guaranteed:

$$\left.\begin{array}{l} \dfrac{\partial T^*_{ij}}{\partial \pi^n_{ij}}\Big|_{k-1} \leq 0, \quad \dfrac{\partial T^*_{ij}}{\partial \pi^n_{lm}}\Big|_{k-1} \geq 0 \\ \left|\dfrac{}{\partial \pi^n_{lm}}\Big|_{k-1}\right| << \left|\dfrac{\partial T^*_{ij}}{\partial \pi^n_{ij}}\Big|_{k-1}\right| \end{array}\right\} \qquad (40)$$

$$i, j \in A, \quad i \neq j, l, m \in A, \quad l \neq m, \ (l, m) \neq (i, j)$$

So it appears that starting from low fares, an increase of fares on trips linking origin i and destination j at solution of Problem II at iteration k-1 will imply a decrease of potential demand on the same origin-destination pair at the same iteration of Problem I and then

a negative effect on the increase of airlines revenue. Fares will then be increased by this process until no more improvement of revenue is obtained. Starting from high level fares, the inverse process will be obtained.
This iterative process can be displayed in a very simple example of optimization problem:

$$\left.\begin{array}{l}\max\limits_{\pi,\theta}\ \pi\,\theta - c\,\theta \\ \\ \text{under}\quad \theta \le \theta_0\, e^{-\lambda\,\pi}\end{array}\right\} \qquad (41)$$

where θ_0 and λ are positive constants. Its exact solution is given by: $\pi^* =(1+c\,\lambda)/\lambda$ and $\theta^* = \theta_0\, e^{-(1+\lambda\, c)}$. Similarly to (51), its feasible set can be approximated around $(\pi_{k-1}, \theta_{k-1})$ by:

$$\theta \le \theta_{k-1}(1+\lambda\,\pi_{k-1}) - \lambda\,\theta_{k-1}\,\pi \qquad (42)$$

and the solution of problem $\{ \max\limits_{\pi,\theta}\ \pi\,\theta - c\,\theta$ under (42)$\}$ is :

$$\begin{cases}\pi_k = (1+\lambda\,\pi_{k-1} + \lambda\, c)/(2\,\lambda) \\ \theta_k = \theta_0\ \exp(-\lambda\,\pi_k)\end{cases} \qquad (43)$$

It is clear that the limit when $k \to +\infty$ of (π_k, θ_k) is $((1+c\,\lambda)/\lambda,\ \theta_0\, e^{-(1+c\,\lambda)})$. Here, the convergence rate is such as:

$$\pi_k - \pi_{k-1} = (1/2)\,(\pi_{k-1} - \pi_{k-2}) \qquad (44)$$

Other bilevel schemes have been considered in [7] and [8] for different transportation problems while numerical convergence conditions have been discussed in [9].

6. Conclusion

This communication has considered the problem of long term forecasting of the flows and traffic growth in a freight transportation network.
The proposed approach has introduced two different optimization models: One model devoted to demand forecasting and the other describing a profit maximization supply behavior by freight firms. An entropy maximization approach is used to determine origin-destination matrices. The formulation of this problem introduces in a new way elasticity of demand with respect to fares. The supply optimization model considers simultaneously two classes of flows: vehicle flows providing freight transportation capacity and goods flows generating revenues to the freight operators. Then a global optimization problem is associated to each scenario with respect to demand and operations costs.
The proposed solution scheme is composed of an iterative process between the current solutions of the demand and the supply optimization problems: the entropy maximizing problem provides the passengers origin-destination matrix given a fare structure, while the supply optimization problem provides the fare structure given a passengers origin-destination matrix. Convergence conditions are discussed for this iterative process between two problems which can be seen as inverse of each other.

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