Aircraft local wind estimation from radar tracker data

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***Abstract*—Accurate wind magnitude and direction estimation is essential for aircraft trajectory prediction. For instance, based on these data, one may compute entry and exit times from a sector or detect potential conflict between aircraft. Since the flight path has to be computed and updated on real time for such applications, wind information has to be available in real time too.The wind data which are currently available through meteorological service broadcast suffer from small measurement rate with respect to location and time. In this paper, a new wind estimation method based on radar track measures is proposed. When on board true air speed measures are available, a linear model is developed for which a Kalman filter is used to produce high quality wind estimate. When only aircraft position measures are available, an observability analysis shows that wind may be estimated only if trajectories have one or two turns depending of the number of aircraft located in a given area. Based on this observability conditions, closed forms of the wind has been developed for the one and two aircraft cases. By this mean, each aircraft can be seen as a wind sensor when it is turning. After performing evaluations in realistic frameworks, our approach is able to estimate the wind vectors accurately.**

***Index Terms*—Wind estimation, Speed triangle, Trajectory prediction, Kalman filter, Air traffic management.**

I. Introduction

When an air traffic controller observes its traffic on a radar screen, he tries to identify convergent aircraft which may be in conflict in a near future, in order to apply maneuvers that will separate them. The problem is to estimate where the aircraft will be located in this near future (5-10 minutes); this process is call trajectory prediction. The trajectory prediction depends mainly on the residual noise after filtering (see [4]): the weight of the aircraft, the temperature and the wind. The residual noise is integrated with time with a growing covariance matrix indicating that the estimated position is less and less accurate. The weight of the aircraft is relevant in the flight dynamic model but is still a raw data. The engines of aircraft are sensitive to the air temperature and such a data is very useful to model the trust of the aircraft but it is also very difficult to measure on real time. Finally, the wind influences strongly the cinematic of the aircraft and limits also the trajectory prediction. Based on the available accuracy, the actual limit of the trajectory prediction is about 20 minutes. It means that after 20 minutes the uncertainty is so big that the estimated position is no more useful for any ATM purposes.

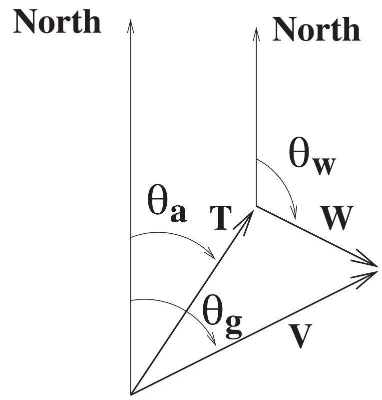

Fig. 1. Speed vectors relations

Several efforts have been tried to improved the trajectory prediction by estimating or suppressing the wind [8], [2], [3]. The present paper, proposes a new method for estimating the wind around aircraft by the mean of observations of the radar tracks and some down linked data. One goal of this work is to show how it is possible to extract wind information from the radar observations.

The paper is organized as follow : the first part presents the relation between air speed vector, ground speed vector and the wind. The second part gives the observability condition of the wind based on the available measures. The third part presents the different models which have been used for the wind extraction. Finally, the fourth part presents some results and compares the performance of our models.

II. General relations between speed vectors

The following notations will be used in the paper. Vectors and matrices are shown with underlined symbols.

Speeds

$$\underline{V} = \begin{bmatrix} v_x \\ v_y \end{bmatrix} \quad V = \sqrt{v_x^2 + v_y^2} \quad \theta_g = \arctan\left(\frac{v_x}{v_y}\right)$$

Where θ_g is the The route angle (with reference to north).

The same notation will be used for the air speed $(\underline{T}, t_x, t_y, \theta_a)$ and the wind $(\underline{W}, w_x, w_y, \theta_w)$. We have :$\underline{V} = \underline{T} + \underline{W}$ This relation is shown on figure 1.

Air Turning Rate $\omega_a = \frac{d\theta_a(t)}{dt}$

Ground Turning Rate

$$\boxed{\omega_g = \frac{d\theta_g(t)}{dt} = \frac{\gamma_x.v_y - v_x.\gamma_y}{V^2}} \quad (1)$$

with $\gamma_x = \frac{dv_x}{dt}$ and $\gamma_y = \frac{dv_y}{dt}$. It can be shown that the air turning rate (ω_a) and the ground turning rate (ω_g) are related by the following expression :

$$\boxed{\omega_g = \left(\frac{T^2 + T*W*\cos(\theta_a(t) - \theta_g(t))}{T^2 + W^2 + 2.T.W\cos(\theta_a(t) - \theta_g(t))}\right)\omega_a} \quad (2)$$

A. Hypothesis

We consider en-route traffic (traffic away from airports), with aircraft flying in cruise phase. Aircraft are supposed to fly at constant air speed ($T = C^{te}$) and turn with constant air turning rate ($\omega_a = C^{te}$).

The average wind is supposed to be constant in the neighborhood of the aircraft.

III. Observability Conditions

A. Mode S Radar

Modee S radars [7] are able to access on board parameters and especially airspeed vector. Having such measures the system is fully determined at any time as it is shown in the following equations.

$$\begin{cases} v_x = Tsin(\theta_a) + w_x \\ v_y = Tcos(\theta_a) + w_y \end{cases} \quad (3)$$

In this system, there are two equations and two unknowns (w_x, w_y); the other parameters are given by the radar (v_x, v_y, T, θ_a).

A Kalman filter [5], [11] will be used to extract the wind by removing the noise.

B. Classical Radar

If measures come from classical radars, only position measures are available (x, y) and two situations have to be taken into account.

1) First situation: One aircraft: In the case where radar measures come from one aircraft only, wind may be observed only after two asymmetric turns (meaning three straight lines separated by two turns). As a mater of fact, the first segment brings the following system of equations ;

$$v_{1x} = Tsin(\theta_{a1}) + w_x \quad v_{1y} = Tcos(\theta_{a1}) + w_y \quad (4)$$

In this system (4), there are 4 unknowns $(T, \theta_{a1}, w_x, w_y)$ and two equations, so two equations are missing.

After the first turn two new equations are added to this system with one extra unknown (θ_{a2}) :

$$\begin{cases} v_{1x} = Tsin(\theta_{a1}) + w_x & v_{1y} = Tcos(\theta_{a1}) + w_y \\ v_{2x} = Tsin(\theta_{a2}) + w_x & v_{2y} = Tcos(\theta_{a2}) + w_y \end{cases} \quad (5)$$

In this new system (5), there are 5 unknowns $(T, \theta_{a1}, \theta_{a2}, w_x, w_y)$ and only 4 equations. It is only after the second turn that the system is fully determined (6 unknowns $(T, \theta_{a1}, \theta_{a2}, \theta_{a3}, w_x, w_y)$ and 6 equations) :

$$\begin{cases} v_{1x} = Tsin(\theta_{a1}) + w_x & v_{1y} = Tcos(\theta_{a1}) + w_y \\ v_{2x} = Tsin(\theta_{a2}) + w_x & v_{2y} = Tcos(\theta_{a2}) + w_y \\ v_{3x} = Tsin(\theta_{a3}) + w_x & v_{3y} = Tcos(\theta_{a3}) + w_y \end{cases} \quad (6)$$

This system has a closed form solution for which the wind is given by :

$$\boxed{w_x = \frac{(v_{3y} - v_{2y})V_1^2 + (v_{1y} - v_{3y})V_2^2 + (v_{2y} - v_{1y})V_3^2}{2\{v_{1y}(v_{2x} - v_{3x}) + v_{2y}(v_{3x} - v_{1x}) + v_{3y}(v_{1x} - v_{2x})\}}} \quad (7)$$

$$\boxed{w_y = \frac{(v_{2x} - v_{3x})V_1^2 + (v_{3x} - v_{1x})V_2^2 + (v_{1x} - v_{2x})V_3^2}{2\{v_{1y}(v_{2x} - v_{3x}) + v_{2y}(v_{3x} - v_{1x}) + v_{3y}(v_{1x} - v_{2x})\}}} \quad (8)$$

with

$$V_1 = \sqrt{v_{1x}^2 + v_{1y}^2} \; V_2 = \sqrt{v_{2x}^2 + v_{2y}^2} \; V_3 = \sqrt{v_{3x}^2 + v_{3y}^2}$$

For both expressions (w_x, w_y) the denominators must not be equal to zero meaning that turns have to be asymmetric ($\underline{V}_1 \neq \underline{V}_2 \neq \underline{V}_3$)

Having expressions for w_x and w_y it is very easy to extract the other unknowns :

$$\begin{cases} t_{1x} = v_{1x} - w_x & t_{1y} = v_{1y} - w_y \\ t_{2x} = v_{2x} - w_x & t_{2y} = v_{2y} - w_y \\ t_{3x} = v_{3x} - w_x & t_{3y} = v_{3y} - w_y \end{cases}$$

2) Second situation : Two aircraft: When radar measures are available for two aircraft (a and b), only one turn for both trajectories is needed to have enough information for wind estimation. The two first segments bring the following system of equations with 6 unknowns $(T_a, T_b, \theta_{a_{a1}}, \theta_{a_{b1}}, w_x, w_y)$ and 4 equations :

$$\begin{cases} va_{1x} = T_a\sin(\theta_{a_{a1}}) + w_x & va_{1y} = T_a\cos(\theta_{a_{a1}}) + w_y \\ vb_{1x} = T_b\sin(\theta_{a_{b1}}) + w_x & vb_{1y} = T_b\cos(\theta_{a_{b1}}) + w_y \end{cases} \quad (9)$$

After the first turn, the new systems is fully determined with 8 unknowns$(T_a, T_b, \theta_{a_{a1}}, \theta_{a_{b1}}, \theta_{a_{a2}}, \theta_{a_{b2}}, w_x, w_y)$ and 8 equations :

$$\begin{cases} va_{1x} = T_a\sin(\theta_{a_{a1}}) + w_x & va_{1y} = T_a\cos(\theta_{a_{a1}}) + w_y \\ vb_{1x} = T_b\sin(\theta_{a_{b1}}) + w_x & vb_{1y} = T_b\cos(\theta_{a_{b1}}) + w_y \\ va_{2x} = T_a\sin(\theta_{a_{a2}}) + w_x & va_{2y} = T_a\cos(\theta_{a_{a2}}) + w_y \\ vb_{2x} = T_b\sin(\theta_{a_{b2}}) + w_x & vb_{2y} = T_b\cos(\theta_{a_{b2}}) + w_y \end{cases} \quad (10)$$

The associated closed form of the wind is given by :

$$\boxed{w_x = \frac{(vb_{1y} - vb_{2y})(Va_1^2 - Va_2^2) + (va_{2y} - va_{1y})(Vb_1^2 - Vb_2^2)}{2\{(va_{1x} - va_{2x})(vb_{1y} - vb_{2y}) - (va_{1y} - va_{2y})(vb_{1x} - vb_{2x})\}}} \quad (11)$$

$$\boxed{w_y = \frac{(vb_{2x} - vb_{1x})(Va_1^2 - Va_2^2) + (va_{1x} - va_{2x})(Vb_1^2 - Vb_2^2)}{2\{(va_{1x} - va_{2x})(vb_{1y} - vb_{2y}) - (va_{1y} - va_{2y})(vb_{1x} - vb_{2x})\}}} \quad (12)$$

with

$$Va_1 = \sqrt{va_{1x}^2 + va_{1y}^2} \quad Va_2 = \sqrt{va_{2x}^2 + va_{2y}^2}$$
$$Vb_1 = \sqrt{vb_{1x}^2 + vb_{1y}^2} \quad Vb_2 = \sqrt{vb_{2x}^2 + vb_{2y}^2}$$

and ($\underline{V}a_1 \neq \underline{V}a_2$;$\underline{V}b_1 \neq \underline{V}b_2$)

Like for the one aircraft case, it is very easy to extract the other unknowns :

$$\begin{cases} ta_{1x} = va_{1x} - w_x & ta_{1y} = va_{1y} - w_y \\ tb_{1x} = vb_{1x} - w_x & tb_{1y} = vb_{1y} - w_y \\ ta_{2x} = va_{2x} - w_x & ta_{2y} = va_{2y} - w_y \\ tb_{2x} = vb_{2x} - w_x & tb_{2y} = vb_{2y} - w_y \end{cases}$$

Based on those observability conditions, several wind estimation models have been developed which are now in the following section.

IV. Models

A. Model with Air Speed and Air Turning Rate (model 1)

The state vector used in the Kalman filter is given by :

$$\underline{X}(k) = [x(k)\, y(k)\, t_x(k)\, t_y(k)\, w_x(k)\, w_y(k)]^T \tag{13}$$

where $x(k), y(k)$ is the position, $t_x(k), t_y(k)$ the True Air Speed (TAS) and $w_x(k), w_y(k)$ the wind.

The measure vector consists in the radar position and the true air speed :

$$\underline{Z}(k) = [x_m(k)\, y_m(k)\, t_{x_m}(k)\, t_{y_m}(k)]^T \tag{14}$$

Having access to the air turning rate (ω_a), it can be included in the prediction matrix. The structure of the system is the following :

$$\underline{X}(k+1) = \underline{F}(k).\underline{X}(k) + \underline{v}(k) \tag{15}$$

where

$$\underline{F}(k) = \begin{bmatrix} 1 & 0 & C_1(\omega_a) & C_2(\omega_a) & \Delta_t & 0 \\ 0 & 1 & -C_2(\omega_a) & C_1(\omega_a) & 0 & \Delta_t \\ 0 & 0 & C_3(\omega_a) & C_4(\omega_a) & 0 & 0 \\ 0 & 0 & -C_4(\omega_a) & C_3(\omega_a) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

$$\underline{Z}(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} .\underline{X}(k) + \underline{w}(k) \tag{17}$$

where

$$\begin{cases} C_1(\omega_a(k)) = \frac{\sin(\omega_a(k)\Delta_t)}{\omega_a(k)} \\ C_2(\omega_a(k)) = \frac{1-\cos(\omega_a(k)\Delta_t)}{\omega_a(k)} \\ C_3(\omega_a(k)) = \cos(\omega_a(k)\Delta_t) \\ C_4(\omega_a(k)) = \sin(\omega_a(k)\Delta_t) \end{cases} \tag{18}$$

The model being exact, the model noise covariance matrix $\underline{R} = \underline{0}$. The measure noise covariance matrix $\underline{R}$ is given by :

$$\underline{R} = \begin{bmatrix} \sigma_p^2 & 0 & 0 & 0 \\ 0 & \sigma_p^2 & 0 & 0 \\ 0 & 0 & \sigma_T^2 & 0 \\ 0 & 0 & 0 & \sigma_T^2 \end{bmatrix}$$

with $\sigma_p = 100$ meters and $\sigma_T = 0.2$ kts.

B. Model with Air Speed only (model 2)

For this model, the state vector and the measure vector are the same as in the first model.

The prediction matrix is now given by :

$$\underline{F}(k) = \begin{bmatrix} 1 & 0 & \Delta_t & 0 & \Delta_t & 0 \\ 0 & 1 & 0 & \Delta_t & 0 & \Delta_t \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

The measure matrix $\underline{H}$ and the associated covariance matrix $\underline{R}$ are the same as in the first model. This model is linear but is false for the air speed vector evolution. In order to take into account this model error, the following covariance matrix (Q) is included in the filter : $\underline{Q}$ is given by :

$$\underline{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

C. Model with Turning Rate only (model 3)

This model is the same as the first one with the following measure vector ;

$$\underline{Z}(k) = [x_m(k)\, y_m(k)]^T \tag{20}$$

and the associated measure equation :

$$\underline{Z}(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} .\underline{X}(k) + \underline{w}(k) \tag{21}$$

D. Models Without Command

1) Model using Kalman Filter (model 4): Having no access to the turning rate, this parameter may be extracted by a Kalman filter (ω_a included in the state vector). If the turning rate is included in the state vector, the evolution of the system is not linear and an EKF [9] or an UKF [6] has to be used to manage such state vector.

2) Model using turns: As it has been shown in the previous section (observability conditions), wind may be extracted by observing ground radar track during turns.

The key element of this approach is the turning rate detector based on equation (1). When ω_g is greater than a given threshold, the aircraft is considered to be in turn. Based on this turn detector, straight line segments are easily identified for which ground speed vector averages are computed. Speed vector estimates in straight lines are given by the framework of figure 2. In this figure, the turns are first detected and a counter is then used to select the right averaging process in order to compute ground speed estimates. When two aircraft are used for wind estimation, the same framework is duplicated and counters select only two averaging blocks in order to produce the following estimates :$\underline{V}_{a1}, \underline{V}_{a2}, \underline{V}_{b1}, \underline{V}_{b2}$.

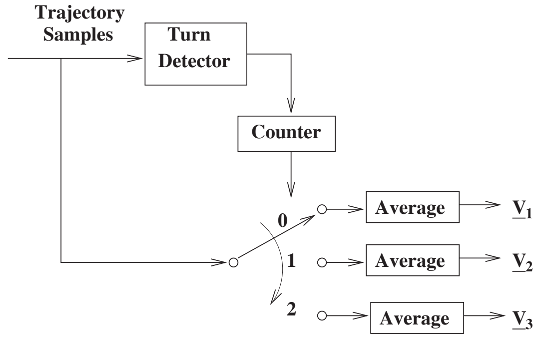

Fig. 2. This figure presents the framework used to estimate the average speed in straight line between turn.

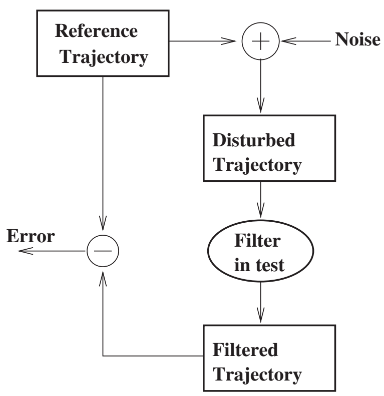

Fig. 3. Test Framework

V. RESULTS

A. Simulation framework

In order to test and compare our models, a radar tracker simulator has been used to produce the reference trajectory which has been disturbed with a Gaussian noise. The disturbed trajectory is then filtered by a Kalman filters which generates the estimated trajectory which is then compared to the reference trajectory. This framework is summarized on figure 3. For all experiments, a wind of 40 kts has been used with $\theta_w = 240^o \Rightarrow \underline{W} = [-34.64kts, -20kts]^T$= $[-17.82m/s, -10.28m/s]^T$.

B. Models with Mode-S radar data (model 1,2 and 3)

The first trajectory used for our experiments is built with 3 straight lines (20 minutes for each) connected with turns as it can be seen on figure 4.This trajectory has been disturbed by a Gaussian noises for which the means are zero and the standard deviations are the following :$\sigma_{position} = 100m$ and $\sigma_{TAS} = 0.2kts$. Those values are given by the performance of the actual radar trackers.

1) Results for models 1 and 2: Models 1 and 2 have been tried on this trajectory for which the residual errors on the wind estimates are given on figures 5 and 6. The figure 5 shows the residual error for the wind strength estimate after the convergence phase which last 2 minutes. Figure 6 shows the same kind of results for the residual wind angle estimates.

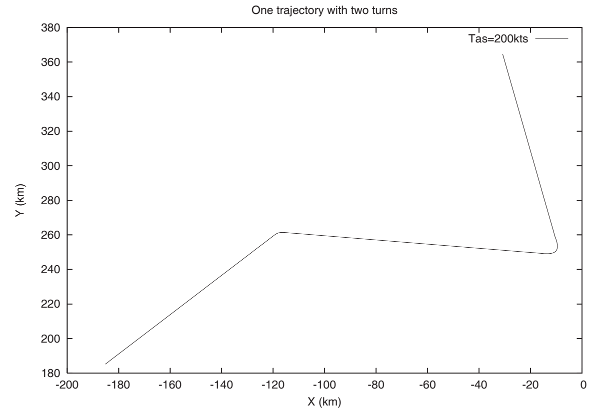

Fig. 4. First trajectory. Each straight lines last 20 minutes. The turning rate of the first turn is 1 deg and -1 deg for the second turn. The aircraft is considered to fly on cruise phase (vertical speed equal zero).

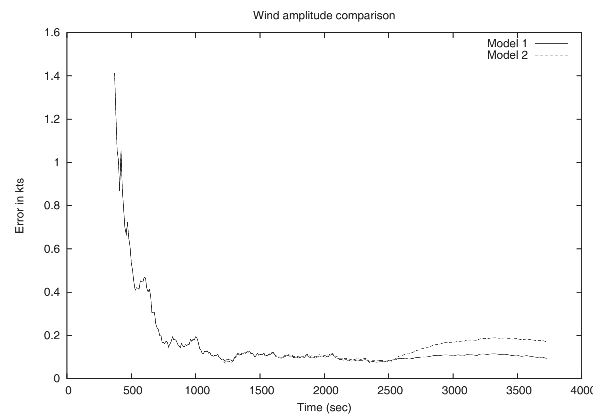

Fig. 5. Residual wind strength error for model 1 and 2

The models 1 and 2 have the same evolution during the convergence phase. After this convergence phase, the wind strength estimate error stays below 0.2kts for both models 5. The model 1 has a more accurate behavior at the end of the simulation with an error which stay below 0.1kts. The wind angle residual error stays below 1 degree (in absolute value) for both model 6. As for the wind strength estimation, the second

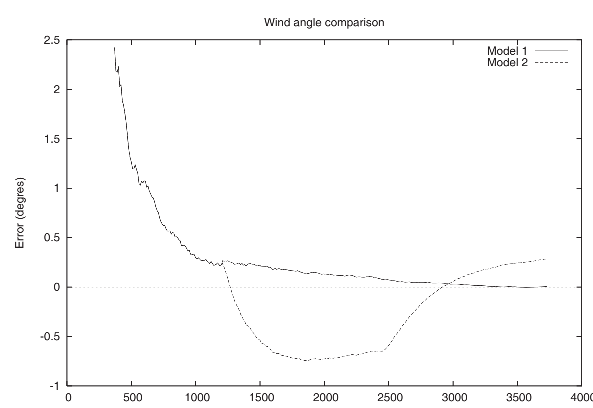

Fig. 6. Residual wind angle error for model 1 and 2

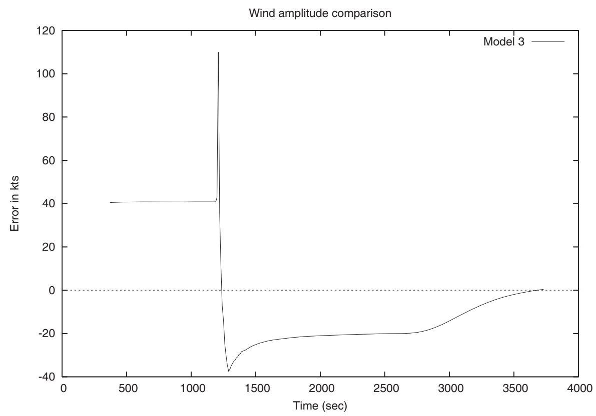

Fig. 7. Residual wind strength error for model 3

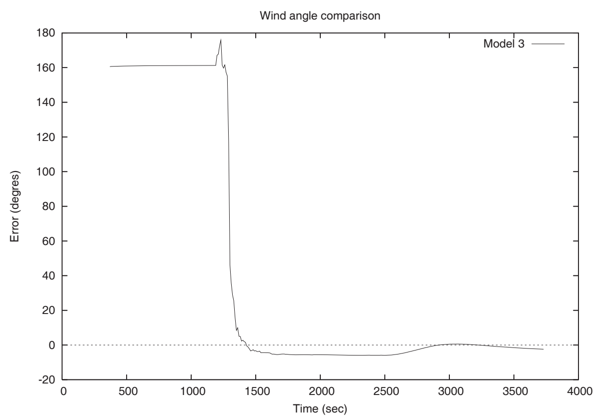

Fig. 8. Residual wind angle error for model 3

model (with turning rate) produce a better estimate. The first model is disturbed by the turns. Both models model are able to produce accurate wind estimate all along the trajectory (after the convergence phase) even during the turns.

2) Results for models 3: When air speed is removed from measures, model 1 is not able to converge and model 3 (which is models 2 without air speed measures) has to wait the second turn to be able to produce an estimate of the wind (see figure **??** and 8). During the convergence phase, model 3 reduces the wind strength error till 40kts which is the limit for such estimation because the Kalman filter is not able to distinguish in the position measures the part coming from the air speed and the part coming from the wind. After the convergence phase (see figure 7) the filter has to wait the second turn in order to be able to produce a reliable estimate of the wind strength (as it has been previously shown (observability conditions), it is only after the second turn that the wind observability conditions are met). As for the wind strength error, the wind angle error reach zero after the second turn. During the convergence phase the wind angle error reach 160 deg (see figure 8).

C. Models with classical radar data (model 4 and 5)

1) Results for models 4: When on board measures (TAS, heading, turning rate) are not available, the turning rate has to be included in the state vector inducing a non linear prediction

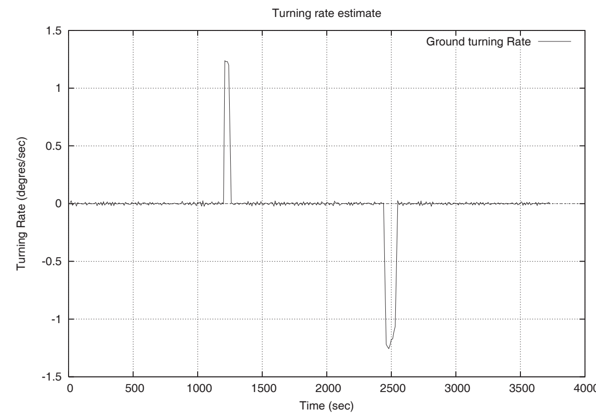

Fig. 9. Ground turning rate estimate for the first trajectory.

equation of the filter. In order to address such non linear evolution, an EKF and an UKF has been developed and tested on the first trajectory. Both filters are not able to estimate the wind vector even after the second turn. This behavior is due to the residual error of the Taylor expansion used in the EKF or in the UKF. Based on the observation (positions) the filter is not able to find the part due to the wind and the one due to the error coming from the Taylor expansion.

2) Results for model 5: Based on the turning rate estimate given by equation 1, it is very easy to build a turn detector when $|\omega_g| > threshold$. The ground vector speed has been disturbed by a Gaussian noise with zero mean and 0.2kts standard deviation (performance of the actual trackers). This method has been used on the first trajectory. The ground turning rate estimate is given by figure 9. The two turns are well identified with a value of about +/- 1.3 degree/sec in turns (it must be noticed that the ground turning rate is different from the air turning rate). The results of this approach are given in the following table :

Wind Est	Strength and Angle	Error
$w_x = -17.6798 m/s$	$39.65 kts$	$0.35 kts$
$w_y = -10.1831 m/s$	$240.053 deg$	$0.053 deg$

Those results has been computed by using equations (7), (8) and the following ground speed estimates coming from the averaging process :

$v_{1x} = 54.4818$ m/s	$v_{1y} = 61.9523$ m/s
$v_{2x} = 84.3536$ m/s	$v_{2y} = -10.2142$ m/s
$v_{3x} = -17.6780$ m/s	$v_{3y} = 91.8504$ m/s

When two aircraft trajectories are available, only one turn for each is necessary. For such a model, the trajectories given on figure 10 has been used. The turning rate estimates are given on figure 11. The results of this approach are given in the following table :

Wind Est	Strength Angle	Error
$w_x = -17.6485 m/s$	$39.64 kts$	$0.36 kts$
$w_y = -10.2227 m/s$	$239.918 deg$	$0.082 deg$

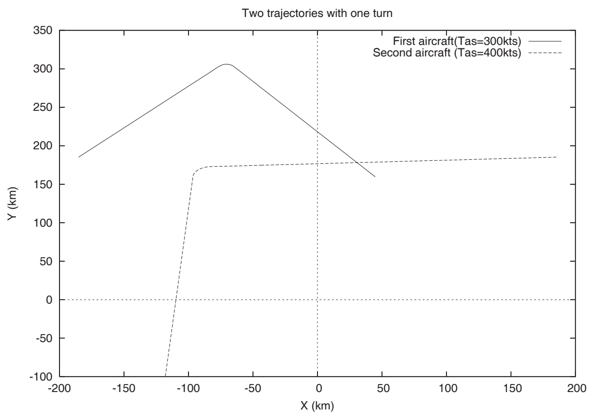

Fig. 10. Second trajectory set. Each straight lines last 20 minutes. The turning rate in the first trajectory is 1 deg and -1 deg for the second one. The true air speed of the first aircraft is 300kts and 400 for the second one.

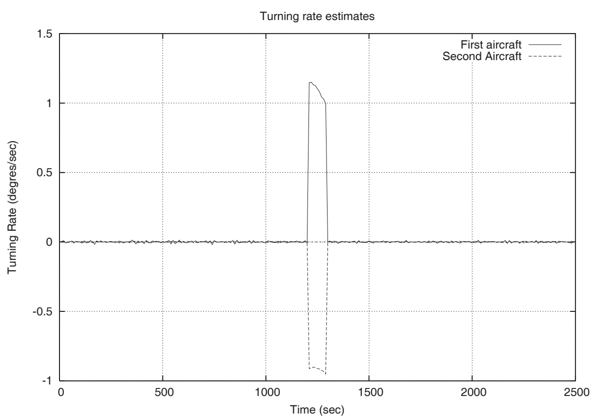

Fig. 11. Turning rate estimates of both aircraft

Those results has been computed by the using equations 11 12 and the ground speed estimates coming from the averaging process :

$$\boxed{\begin{array}{l} va_{1x} = 90.5494 \text{ m/s } va_{1y} = 98.0082 \text{ m/s} \\ va_{2x} = 90.5552 \text{ m/s } va_{2y} = -118.4478 \text{ m/s} \\ vb_{1x} = -221.7254 \text{ m/s } vb_{1y} = -10.2111 \text{ m/s} \\ vb_{2x} = -17.6796 \text{ m/s } vb_{2y} = -214.2995 \text{ m/s} \end{array}}$$

Having now some wind estimates on some points in the airspace (where the aircraft are located), the following step consists in the global wind field interpolation based on the meteorological model called ”Shallow-Water”. This model is described in [1] and the associated discret expansion in given in [10].

VI. Conclusion

This paper has given a new approach for extracting the wind information from the radar tracks. Two approaches has been presented. When True Airspeed measures are available, linear models may be used with regular Kalman filter. In this first approach, wind estimate are available all along the aircraft trajectory. When only position measures are available, an observability analysis has shown that wind may be estimated only after turns (one turn for two aircraft or two turns for one aircraft). For such approach, a closed form of the wind has been developed. Then, those models have been validated with realistic simulations.

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