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A NEW AIR TRAFFIC COMPLEXITY METRIC BASED ON DYNAMICAL SYSTEM MODELIZATION

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Introduction

Air traffic control organizes air flows in order to ensure flight security (in terms of collision risks) and to increase the capacity of the route network planes are flying on. According to the nature of traffic, the three following types of control can be distinguished:

- airport control : management of taxi, taking-off and landing periods.
- approach control : traffic management immediately before landing or after take-off, in an a area close to the airport.
- en-route control : concerns mainly traffic cruising between airports.

Currently, about 7500 movements are detected over France, which is a crossroads for the whole European airspace. To deal with this traffic demand, airspace is divided into elementary sectors, which are managed by air navigation controllers. The role of controllers is to separate all aircraft in the sector while maintaining an efficient traffic flow. For several years, a constant increase of air traffic has provoked more and more congestion in control sectors. There are two possible approaches to reduce this congestion:

1. Adapting demand to existing capacity. This method consists in allocating take-off timeslots to planes to make sure that sector capacities are respected. At operational level, this allocation is done by the CFMU (Central Flow Management Unit).
2. Adapting capacity to demand. This method consists in modifying incrementally route network or sectorization whenever capacity is out passed in a given sector. The result is to maintain traffic flow in the sector until next out passing.

In the two preceding approaches, the capacity of a sector is measured by the number of aircraft flying across the sector during a given time period. The observation of controlled sectors shows that sometimes, controllers accept planes beyond the capacity threshold, while in other situations, they refuse traffic before capacity is reached. This phenomenon clearly shows that the operational metric alone cannot account for controller's workload. The goal of this study is to synthesize a traffic complexity indicator in order to better quantify the congestion in an air sector, which will be more relevant than a simple number of aircraft. More precisely, our objective is to build some metrics of the intrinsic complexity of the distribution of traffic in the airspace. Those metric must capture the level of disorder of any traffic distribution but they have to be able to identify any organization structure. Usually, metrics are focused on the speed vector distribution and the associated disorder metric captures a part of some potential traffic distribution disorder. As a matter of fact **the real objective of those works is to measure the disorder or the level of organization of a set of trajectories in a 4D space (3D for the space and 1D for the time).**

Those metrics are relevant for many applications in the air traffic management area. For instance, when a sectoring is designed [1], the sectors have to be balanced from the congestion point of view and for the time being, only the number of aircraft is used to reach this objective . Another example where a congestion metric is needed is the traffic assignment [2], [3] for which an optimal time of departure and a route are

searched in order to reduce the congestion in the air sector then a more precise measure of the congestion is needed to reach this goal. This complexity indicator may also be used to design new air networks, for the dynamic sectoring concept, to define some Free Flight areas etc ... .

Complexity metrics would enable to qualify and quantify the performance of the Air Traffic services providers and permit a more objective consultation between airlines and providers.

NASA has recently proposed a control workload model, obtained by statistical regression on radar traffic samples. The workload thus calculated uses more sophisticated indicators such as the number of planes on a steady course, climbing up, climbing down, the number of conflicts etc. The present study consists in inferring new complexity metrics from observation vector speeds in control sectors, thus completing the NASA model with measures of vector disorder.

Two class of metrics are then presented .The first class is based on geometrical observations and produce local disorder metrics. Relative speed and relative distance between aircraft are used to build such metrics.

The second class is based on topological entropy. In this approach, traffic in the sector is considered as a dynamical system in which the evolution through time of trajectories respects fluid mechanics criteria. Topological makes it possible to compare current situation (on a given time scale) with a completely organized situation (parallel flows with relative speeds equal to zero). the obtained measure can quantify the level of traffic intrinsic interaction in a control sector. The a priori knowledge of future evolution of plane trajectories (flight plans knowledge, prediction of radar positions) is intrinsically taken into account by the dynamical system. The first part of this paper presents the previous related works. The second one gives an extension propose some extension of the geometric approach and the third one resents a new air traffic complexity concept based on the dynamical systems. The results produce by such metrics are then presented in the fourth part.

Previous Related Works

The airspace complexity is related with both the structure of the traffic and the geometry of the airspace. Different efforts are underway to measure the whole complexity of the airspace.

Significant research interest in the concept of ATC complexity was generated by the "Free Flight" operational concept. Integral to Free Flight was the notion of dynamic density. Conceptually, dynamic density is a measure of ATC complexity that would be used to define situations that were so complex that centralized control was required [4].

Windemere inc [5] proposed a measure of the perceived complexity of an air traffic situation. This measure is related with the cognitive workload of the controller with or without the knowledge of the intents of the aircraft. The metric is human oriented and is then very subjective.

Laudeman et al from NASA [5] have developed a metric called « Dynamic Density » which is more quantitative than the previous one and is based on the flow characteristics of the airspace. The « Dynamic Density » is a weighted sum of the traffic density (number of aircraft), the number of heading changes (> 15°), the number of speed change (> 0.02 Mach), the number of altitude changes (> 750 ft), the number of aircraft with 3-D Euclidean distance between 0-5, the number of altitude changes (> 750 ft), the number of aircraft with 3-D Euclidean distance between 0-25, the number of conflicts predicted in 0-25 nautical miles, the number of conflicts predicted in 25-40 nautical miles and the number of conflicts predicted in 40-70 nautical miles. The parameters of the sums have been adjusted by showing different situation of traffic to several controllers. Finally, B.Sridhar (NASA 1998) [7] developed a model to predict the evolution of this metric in the near future. Efforts to define "dynamic density" have identified the importance of a wide range of potential complexity factors, including structural considerations.

A few previous studies have attempted to include structural considerations in complexity metrics, but have done so only to a restricted degree. For example, the Wyndemere Corporation proposed a metric that included a term based on the relationship between aircraft headings and a dominant geometric axis in a sector [5]. The

importance of including structural considerations has been explicitly identified in recent work at Eurocontrol. In a study to identify complexity factors using expert judgment analysis, "Airspace Design" was identified as the second most important factor behind traffic volume [8].

The previous models do not take into account the intrinsic traffic disorder which is related to the complexity. The first efforts related with disorder can be found in [9]. In this paper two new metrics related with disorder in space and speed are developed and are shown to be very adapted to compare situations of traffic. When a set of aircraft is considered in a sector, it is possible to identify different areas for which the structure of traffic is different. For example, it is possible to identify some high density zones and clusters of traffic with strong disorder. This identification is done by our brain which investigate the different structure and is able to recognize structure symmetries. When two aircraft are considered, it is possible to define their relative distance and relative speed (see Figure 1).

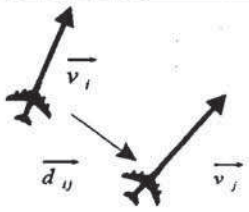


Figure 1. Relative Distance And Relative Speed

The relative distance is given by:

$$\overrightarrow{d_{ij}} = \overrightarrow{P_j} - \overrightarrow{P_i}$$

In the same way, the relative speed is define as follow:

$$\overrightarrow{v_{ij}} = \overrightarrow{v_j} - \overrightarrow{v_i}$$

In the following $\left\|\overrightarrow{d_{ij}}\right\|$ will represent the relative distance vector $\overrightarrow{d_{ij}}$.

It can be easily shown that the derivative of this norm is given by:

$$\frac{d\left\|\overrightarrow{d_{ij}}\right\|}{dt} = \frac{\overrightarrow{d_{ij}}.\overrightarrow{v_{ij}}}{\left\|\overrightarrow{d_{ij}}\right\|}$$

Where "." is the scalar product.

This norm is used to define aircraft proximity measure:

$$\Pr ox(i) = 1 + \sum_{\substack{j=1 \\ j\neq i}}^{N} e^{-\alpha\frac{\left\|\overrightarrow{d_{ij}}\right\|}{R}}$$

where

i is the current aircraft for which the local density is computed

N is the whole number of aircraft

α is a weighted coefficient

R is a neighborhood distance

The exponential function is here to give much more weight to the aircraft, which are close to the current aircraft. Then, each aircraft is located on the density axe on a position between 1 to N.

This paper ([9]) presents also a metric of disorder related to the speed vectors of aircraft: The global convergence of an aircraft "i" is the a weighted sum of all the convergence between pairs of aircraft:

$$Conv(i) = \sum_{\substack{j=1 \\ j\neq i}}^{N} \left| \frac{d\left\|\overrightarrow{d_{ij}}\right\|}{dt} \right| . 1_{R^+} \left\{ \frac{d\left\|\overrightarrow{d_{ij}}\right\|}{dt} \right\} . e^{-\alpha\frac{\left\|\overrightarrow{d_{ij}}\right\|}{R}}$$

Where 1_{R^-} is the set function function on R^-

Those indicators $\Pr ox$ and $Conv$ are very adapted to describe the level of contraction or divergence of the local speed vector field associated to an aircraft.

G. Aigoin has extended and refined these concepts using a cluster-based analysis [10].

Two aircraft are said to be in the same cluster if the product of their relative speed and their proximity (a function of the inverse of the relative distance) is above a threshold.

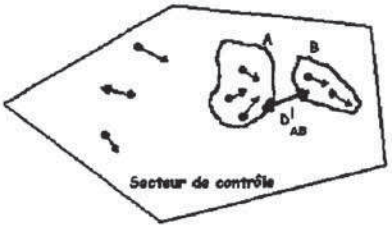


Figure 2. A Cluster Based Analysis Considers Both *Intra* And *Inter* Cluster Complexities

An example of cluster is given in Figure 2. For each cluster, a matrix of relative dependence between aircraft is computed and the whole complexity of the cluster is then given by a weighted sum of some matrix norm. Those norms give an aggregated measure of the level of proximity of aircraft in clusters and the associated convergence with the relative speed. From the cluster matrix it is also possible to compute the difficulty of the cluster. The difficulty captures how hard it is to solve this cluster.

Multiple clusters can exist within a sector, and their interactions must also be taken into account. A measure of this interaction has been proposed by G. Aigoin [10]. This technique allows multiple metrics of complexity to be developed such as average cluster complexity, maximum and minimum cluster complexities, and complexity speeds.

The traffic itself is not enough to describe the complexity associated with an airspace. The impact of the structure on the controller workload can be found on the paper []. This paper shows how strong the structure of the traffic (airways) , sectors etc ... is related with the control workload.

Another approach is based on the fractal dimension of the airspace (see []). Fractal dimension is a metric comparing traffic configurations resulting from various operational concepts. It allows in particular to separate the complexity due to sectorization from the complexity due to traffic flow characteristics. A complete description may be found in "Airspace fractal dimensions and applications". The dimension of geometrical figures is well-known : a line is of dimension 1, a rectangle of dimension 2, etc. Fractal dimension is simply the extension of this concept to more complicated figures, whose dimension may not be an integer. The block count approach is a practical way of computing fractal dimensions : it consists in describing a given geometrical entity in a volume divided in blocks of linear dimension d and counting the number of blocks contained in the entity (N). The fractal dimension (D_0) of the entity is thus :

$$D_0 = \lim_{d \to 0} \frac{\log(N)}{\log(d)}$$

The application of this concept to air route analysis consists in computing the fractal dimension of the geometrical figure composed of existing air routes. An analogy of air traffic with gas dynamics then shows a relation between fractal dimension and conflict rate (number of conflicts per hour for a given aircraft) :

- at constant fractal dimension, the conflict rate is linear with the number of aircraft.
- the log of conflict slope is linear with fractal dimension.

Fractal dimension also provides information on the number of degrees of freedom used in the airspace : a higher fractal dimension indicates more degrees-of-freedom. This information is independent of sectorization and does not scale with traffic volume. Therefore, fractal dimension is a measure of the geometrical complexity of the traffic pattern.

Geometric Disorder Metrics

Covariance Metric

When a set of vector speeds is observed, it seems to be relevant to compute a disorder metric based on the covariance matrix associated to such a set. This matrix is given by:

$$COV = V^T . V$$

$$\text{where } V = \left[\vec{v}_1^{\,T} \quad \vec{v}_2^{\,T} \quad \vec{v}_3^{\,T} \quad \dots \quad \vec{v}_N^{\,T} \right]^T$$

The straight computation of the matrix do not take into account the distance between aircraft and could produce a high level of disorder for aircraft which are away from each other which is not relevant for the ATC purposes. The proposed metric compute local disorder factor which are aggregated by taking into account the relative distance between aircraft. The first step consist in the computation of relative speed between pairs of aircraft.

$$\overrightarrow{V_{ij}} = \overrightarrow{V_j} - \overrightarrow{V_i} = \begin{bmatrix} dv_x \\ dv_y \\ dv_z \end{bmatrix}$$

The associated covariance matrices are then computed:

$$COV_{ij} = \overrightarrow{V_{ij}} . \overrightarrow{V_{ij}}^T$$

Those pair wise covariance matrices are then weighted by a factor depending of the relative distance:

$$\alpha_{ij} = e^{\left(\alpha_p \left\| \overrightarrow{d_{ij}} \right\|\right)}$$

The local level of disorder is then related with the determinants of those new weighted matrices. In order to avoid condition problems, those weighted matrices are first written as singular value decomposition:

$$\alpha_{ij} . COV_{ij} = L_{ij} . S_{ij} . U_{ij}^T$$

The local disorder factor associated with the aircraft pair ij is then given by:

$$c_{ij} = \prod_{k, S_{kk} > 1} S_{kk}$$

The covariance disorder metric is just the summation over all aircraft pair of the factor c_{ij} :

$$C_{cov} = \sum_i \sum_{j, j \neq i} c_{ij}$$

As it can be seen in the result section, such a metric is able to identify disorder of speed vector but is not adapted in a situation where aircraft are organized in a curl moving. As a mater of fact, the metric "sees" the speed vectors of aircraft in disorder even if the situation is completely organized and very easy to control from the ATC point of view. So, speed vector disorder metric is necessary to characterized the ATC system but is not enough to address all situations. The Koenig metric has then been developed.

Koenig Metric

This metric is computed for a set of aircraft which can be determined by a neighborhood . The position of the space barycenter is first computed by using the positions of the aircraft:

$$\overrightarrow{G} = \frac{1}{N} \sum_{i=1}^{i=N} \overrightarrow{X_i}$$

The normalized kinetic moment is then given by:

$$\overrightarrow{m_i} = \frac{1}{\left\| \overrightarrow{d_{ij}} \right\|} \left(\overrightarrow{V_i} \Lambda \overrightarrow{d_{iG}} \right)$$

where Λ is the vector product.

The normalization by $\left\| \overrightarrow{d_{iG}} \right\|$ avoid that aircraft which away from the barycenter have more weight than the ones which are closer. The average kinetic moment located on the barycenter is then given by:

$$\overrightarrow{M_G} = \frac{1}{N} \sum_{i=1}^{i=N} \overrightarrow{m_i}$$

The dispersion of the kinetic moment is given by:

$$COV_M = \frac{1}{N} \sum_{i=1}^{i=N} \left[\overrightarrow{m_i} - \overrightarrow{M_G} \right] \left[\overrightarrow{m_i} - \overrightarrow{M_G} \right]^T$$

The normalization by the factor $\frac{1}{N}$ avoid that the induced metric depends of the number of aircraft. The disorder associated to the kinetic moment D_m is the square root of the trace norm of the matrix COV_M :

$$D_m = \sqrt{\left\| COV_M \right\|_T}$$

Then, when D_m is very small compare to $\left\|\overrightarrow{M_G}\right\|$, it means the aircraft are organize in curl moving.

Based on those factors, a normalized Koenig metric can be build:

$$Koenig = \frac{D_m}{D_m + \left\|\overrightarrow{M_G}\right\|}$$

This metric can evolve from 0 to 1 (0 means that the set of aircraft is organized in a curl moving and 1 means that no curl tendency can be identify).

A computation of such a metric on some traffic situation is given in the results section.

The geometrical metrics previously developed are very well adapted to capture a feature of the complexity but have to aggregated together to produce a relevant metric for the ATC system.

If a more complex organization structure (out of translation or curl) has to be captured, those geometric metrics will fail and a more powerful model is needed and dynamical system are very adapted to address this problem.

Dynamical System

Introduction

This metric is based on the modeling of the set of trajectories by a dynamical system. This enable to identified different structures of organization of the set of speed vector organization. The first approach presented here is limited to the real time analysis of a set of speed vector. The next step which is in progress will take into account a set of trajectories in a 4D space.

Principle

The key idea of this metric is to model the set of aircraft trajectories by a dynamical system. A linear dynamical system is control by the following equation.

$$\dot{\overrightarrow{X}} = A.\overrightarrow{X} + \overrightarrow{B}$$

where $\overrightarrow{X}$ is the state vector of the system:

$$\overrightarrow{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The eigen values of the matrix A control the evolution of the system. The real part of those eigen values is related with the convergence or divergence property of the system in the direction of the eigen vector. When such a eigen value has a positive real part the system is in expansion mode and when it is negative the system is in contraction mode. On the other end , the imaginary parts of eigen values are related with the level of rotation organization of the system. Depending of those eigen values, a dynamical system can evolve in contraction, expansion, rotation or a combination of those three modes. Figure 3 shows examples of evolution with different eigen values. From a situation with two aircraft moving in parallel at the same speed, it shows the position and the speed vectors after a small period of time "d".

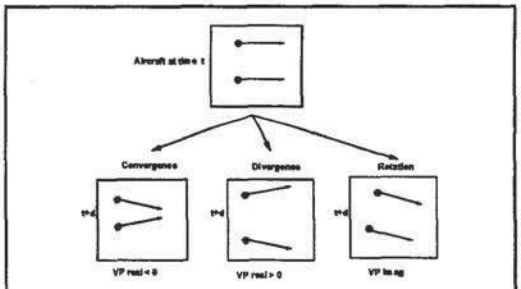


Figure 3. Evolution Examples

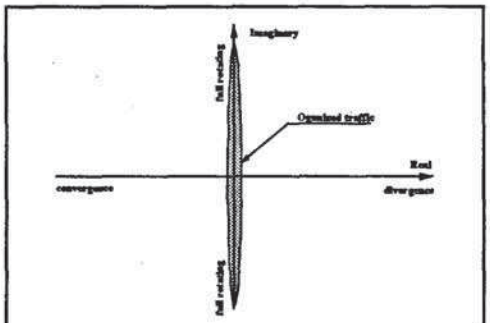


Figure 4. Eigen Values Properties

The eigen value properties can be summarized in the complex coordinates (see Figure 4). In this coordinate system, the vertical ellipse identifies

different organized situation (in translation, in rotation or both).

A dynamical system can then be considered as a map T from the state space X to itself. Based on this mapping T, one can define the topological entropy (or kolmogorov entropy) of dynamical system which measure the level of mixing of X by T. This entropy is associated with the changes of the relative distances between points from X by T. The topological entropy of a dynamical system is then a disorder indicator of the distribution of aircraft. This entropy is not a statistical metric and is very adapted to the air traffic control system for which there are few aircraft in the sectors.

Kolmogorov Entropy Computation

The kolmogorov entropy is computed with the help of the eigen value decomposition of the matrix A:

$$A = L.D.U^T$$

where S is the diagonal matrix of the eigen values. Based on the observations of aircraft (positions and speed vectors), the associated dynamical system has to be adjusted with the minimum error. This fitting has been done with a Least Square Minimization method.

For each aircraft i considered, it is supposed that position X_i and the speed vector V_i are given.

An error criterium between the dynamical system model and the observation is computed:

$$E = \sum_{i=1}^{i=N} \left\| \overrightarrow{V_i} - \left(A.\overrightarrow{X_i} + \vec{B} \right) \right\|$$

In order to use matrix forms the following matrices are introduced.

$$X = \begin{bmatrix} x_1 & x_2 & x_3 & \dots & x_N \\ y_1 & y_2 & y_3 & \dots & y_N \\ z_1 & z_2 & z_3 & \dots & z_N \\ 1 & 1 & 1 & \dots & 1 \end{bmatrix}$$

$$V = \begin{bmatrix} vx_1 & vx_2 & vx_3 & \dots & vx_N \\ vy_1 & vy_2 & vy_3 & \dots & vy_N \\ vz_1 & vz_2 & vz_3 & \dots & vz_N \end{bmatrix}$$

$$C = \begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{bmatrix}$$

With such matrices the error criterium E can be written:

$$E = \|V - C.X\|$$

To minimize E is the same as to minimize:

$$E^2 = \|V - CX\|^2 .$$

The problem is to find the matrix C which minimize such a criterium. By using the matrix derivation properties, the gradient of E^2 is given by:

$$\nabla_C E^2 = -2.(V - C.X).X^T$$

The optimum is given by $\nabla_C E^2 = 0$.

$$\Leftrightarrow C.X.X^T = V.X^T$$

The optimum matrix C_{opt} is then given by:

$$C_{opt} = V.X^T.\left(X.X^T\right)^{-1}$$

The expression $X^T.\left(X.X^T\right)^{-1}$ is the pseudo inverse of the matrix X^T and can be written as:

$$X^T.\left(X.X^T\right)^{-1} = L^T.S^{-1}.R$$

where S is the diagonal matrix of the singular values.

This singular value decomposition is very helpful in order to avoid condition troubles. The matrix C is finally given by

$$C = V.L^T.S^{-1}.R$$

The matrix A is then extracted from the matrix C, and the associated eigen value decomposition is given by:

$$A = L.D.U^T$$

The eigen value of the matrix D are complex number. In order to produce a scalar metric, the larger absolute value of the real part of those complex eigen values is computed. The sign of this larger real part is related with the mode of the system (contraction or expansion). If those eigen values are drawn in the complex coordinates, it gives information about the level of contraction/expansion of the system and also it rotation tendency.

Results

Different traffic samples have been generated in order to compare the previous metrics. The first one consist in a full symmetric convergence of eight aircraft (see Figure 5). The full trajectories of aircraft are shown on this figure with the initial positions symbolized by the arrows.



Figure 5. Eight Aircraft Converging At The Same Point

The second test is given by a random convergence of 41 aircraft (see Figure 6).

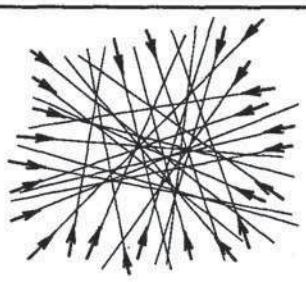


Figure 6. Aicraft (41) Converging In The Same Area

The computation of the covariance metric on the first sample is given in Figure 7.

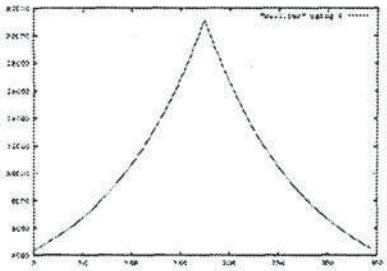


Figure 7. Covariance Metric For 8 Aircraft Converging At The Same Point

When aircraft are away from each other the metric is very small and increase with the reduction of the relative distance. The speed vectors being the same during this simulation, the change of the metric is only due to the relative distance evolution. When the convergence is more fuzzy like in the situation of the Figure 6, the evolution of the covariance metric is more soft but identifies clearly the maximum of interaction between trajectories (see Figure 8). When such a metric is applied on full curl moving (see Figure 9), this metric identify disorder but the Koenig metric stay at zero meaning that the situation is full organized in a curl.

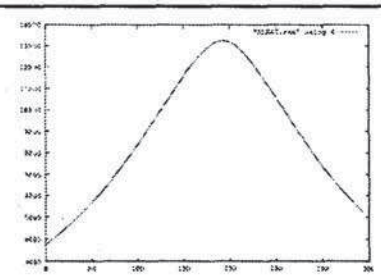


Figure 8. Covariance Metric For A Fuzzy Convergence With 41 Aircraft

The evolution of the larger eigen value of the matrix A is given on the Figure 10. The metric begin to be negative because aircraft are first converging and is positive after the crossing (divergence). The discontinuity identify clearly the conflict. As it can be seen on the complex coordinates system (see Figure 11) , no curl moving is detected by the metric. This graph presents the

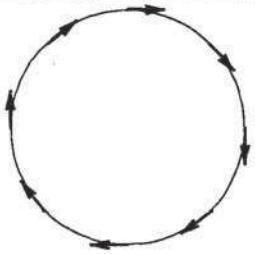


Figure 9. Eight Aircraft In Curl Moving

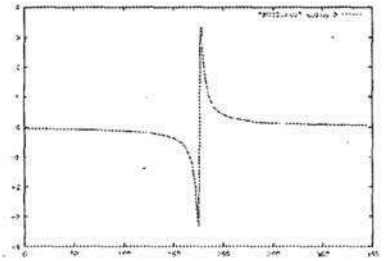


Figure 10. Larger Real Part For 8 Aircraft Converging At The Same Point

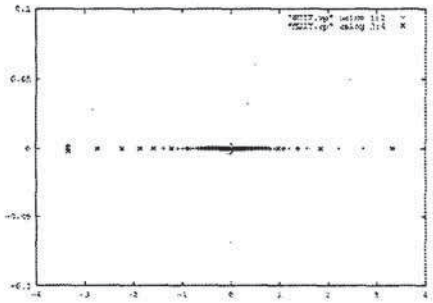


Figure 11. Loci Of Eigen Values For Eight Aircraft Converging At The Same Point

loci of the complex eigen values. For this experiment, the locus is located only on the real axe. Each cross represents the position of an eigen value.

In the case of the fuzzy convergence, the metric first identifies a soft converging situation, reaches a maximum when aircraft are in the central area of crossing and begins to be positive when divergences are more important than divergences (see Figure 12). The associated loci of the eigen values is given in Figure 13. The two eigen values are conjugate. The loci start from left part (negative real part) and finish on the right (positive real part); between the metric identify some rotation tendencies. The full curl moving can be very well identifies by such as it can be seen on the Figure 14. The graph represents the eigen values loci for eight aircraft organized in a full curl moving. The loci degenerate into two single point on the imaginary axes which mean that the associated dynamical system is organize in a full curl.

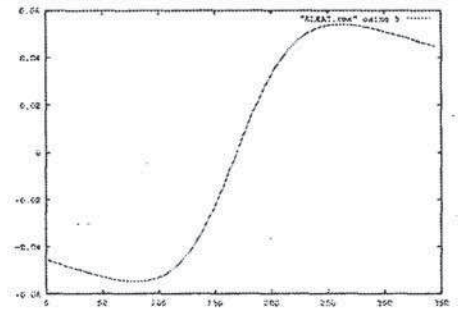


Figure 12. Eigen Values Properties

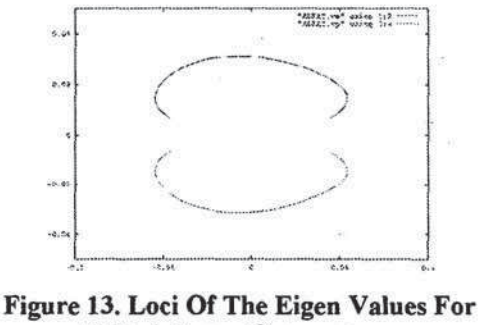


Figure 13. Loci Of The Eigen Values For 41 In A Fuzzy Convergence

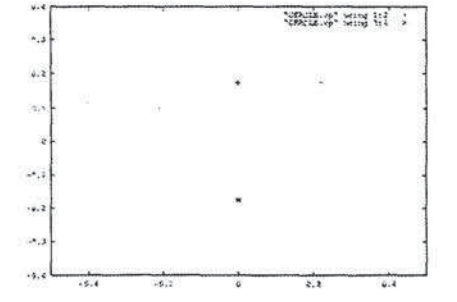


Figure 14. Loci Of The Eigen Values For 8 Aircraft In A Full Curl Moving

Finally, the Kolmogorov entropy has been computed in a unrealistic situation for which eight aircraft are first converging in a spiral moving and go on by a divergence on the same spiral. The moving is the a mix of convergence/divergence and curl (see Figure 15). The associated loci is given in Figure 16 and shows two conjugate complex number moving from the left part (convergence (negative real part)) to the right part (divergence (positive real part)).

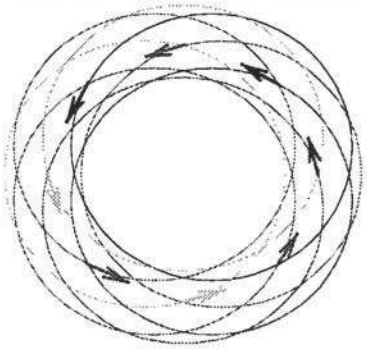


Figure 15. Eight Aircraft In Spiral Moving

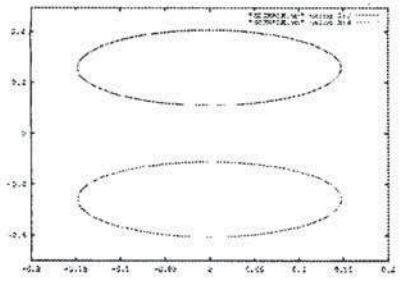


Figure 16. Loci Of The Eigen Value For 8 Aircraft In A Spiral Moving

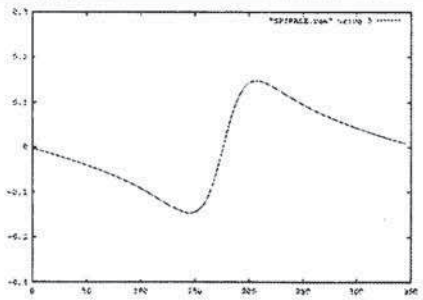


Figure 17. Larger Real Part Of The Eigen Values For 8 Aircraft In A Spiral Moving

The associated real part of those complex number, which is in this case the Kolmogorov entropy is given in Figure 17.

Those initial results have shows that dynamical system modeling is a very good candidate to measure the complexity of a traffic situation on control sector. This metric is now computed for En route sectors in France and is compared with the real controller workload.

Further Works

The previous dynamic system is controlled by a linear equation and is then adapted for situations where such a modeling is relevant. Many situations belong to this class but some others are better modeled by a non-linear dynamical system. The extension of the Kolmogorov entropy is based on the construction of a vector field.

The basic idea of a vector field is to define a non-linear function $\mathbf{f}: R^3 \rightarrow R^3$ describing the spatial evolution of speed in the considered sector : $\mathbf{f} = (f_x, f_y, f_z)^T$ associates a 3-dimensional speed vector to each point of the considered space.

The constraints on **f** are the following

$$\mathbf{f}(x_i) = v_i, \quad i \in [1, N] \tag{1}$$

where N is the number of aircraft in the sector, $\{v_i\}_{i\in[1,N]}$ their observed speeds. These conditions ensure that the resulting dynamical system will give realistic speed values aircraft are located. But these constraints alone cannot define a *unique* dynamical system. In fact, there is an *infinite* number of dynamical systems that satisfy the conditions given in equation (1). Therefore, another criterium must be added to obtain only one solution. Following Chen and Suter (see [11]), we force the dynamical system to minimize the following integral~:

$$\int_{R^3} \alpha \|\nabla div f\|^2 + \beta \|\nabla curl\|^2 \tag{2}$$

The reason to choose this particular integral is based on the shape of the observed trajectories. Indeed, the integral curves of the vector field show

the trajectories that can be inferred from the values of **f**, values which only take into account the current positions and speeds of the planes. If the information given by the field is to be valuable, the resulting trajectories must be realistic (that is, close to the observation), even though past and future positions are not considered. As the planes we deal with are *airliners*, their observed trajectories are *regular~*: the direction and module of the speed vector of a given plane vary slowly. Therefore, to be realistic, the vector field must have limited local variation, hence the choice of criterium [13], which minimizes the divergence and rotational of the field.

α and β are parameters used to tune the criterium~: their values determine which of the divergence and rotational is the more constrained. For instance, setting α at a much higher value than β will lead to a fully rotational field, the divergence being constant.

The solution to the minimization problem, respecting the interpolation conditions, is known~: Amodei and Benbourhim have proved in [14] that it is the sum of a vector kernel and a linear system~:

$$f(\vec{X}) = \sum_{i=1}^{N} a_i^T \Phi\left(\left\|\vec{X} - \vec{x_i}\right\|\right) + A.\vec{X} + \vec{B}$$

Φ is a known spline function which describes the finer variations of the vector field.

The resulting **f** is a 3-dimensional vector field, able to give a value of speed in any point of the sector.

An example of global field construction is given in Figure 18, for which eight aircraft are organized into two curl moving. The aircraft on the left are moving counter clockwise and the ones on the right circle are moving clockwise. The global field is the summation of the linear part and the spline part.

The evolution of the field is analyzed by calculating the eigen values of the gradient matrix in every grid point. Indeed, the real part of the eigen values with the greatest module is representative of the local variation of the field~: if it is negative, the vector field is locally contracting; if it is positive, the vector field is locally diverging. Because of the minimization of criterium (2) , such local variations are due to an interaction between aircraft. It is thus possible to obtain a *complexity map* of the sector. As the positions and speed vectors that determine the vector field are radar observations made at one given instant, the complexity map is time-dependent and thus gives instantaneous information. This work is still in progress and is completed by the time extension in order to address trajectories set complexity.

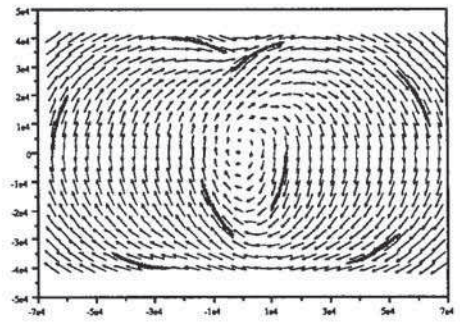


Figure 18. Global Field

Conclusion

This paper has presented new concepts to address the air traffic complexity modeling problem. Two new geometrical metrics have been introduced and have been found very useful to capture typical features of traffic complexity. The covariance metric is very adapted to identify disorder in a set of speed vectors and can be applied for En route airspace (En route airspace is the airspace between airports). Similarly, the Koenig metric identifies easily the curl movement organizations and can be applied to areas around airports where Air Traffic Control procedures impose turns on aircraft trajectories.

In order to have a more aggregated metric, gathering many features of the geometrical metrics, dynamical system modeling of the air traffic system has been proposed. Based on a linear modeling, the Kolmogorov entropy associated to air traffic distributions, has been shown to be relevant to measure the intrinsic complexity of the speed vectors. This metric is able to capture any speed vector organization of the air traffic.

Afterwards, a non-linear dynamical system has been proposed and will be used to develop complexity maps for any airspace. The next step, which is in progress, will extend this last non-linear model into the time dimension in order to compute the complexity of a set of trajectories in a 4D space.

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