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▶ **To cite this version:**

Hakim Bouadi, Hongying Wu, Felix Mora-Camino. Flight path tracking based-on direct adaptive sliding mode control. IV 2011, IEEE Intelligent Vehicles Symposium, Jun 2011, Baden-Baden, Germany. pp 25-30, 10.1109/IVS.2011.5940395 . hal-00938420

HAL Id: hal-00938420

https://enac.hal.science/hal-00938420

Submitted on 20 Jun 2014

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2011 IEEE Intelligent Vehicles Symposium (IV)
Baden-Baden, Germany, June 5-9, 2011

Flight Path Tracking Based-on Direct Adaptive Sliding Mode Control

Hakim Bouadi, H.Wu and F.Mora-Camino

***Abstract*— Adaptive control algorithms are of interest in the flight control systems design not only for their capability to improve performance and reliability but also for handling aerodynamic parameter uncertainties, external disturbances and modeling inaccuracies. This paper addresses a nonlinear direct adaptive sliding mode controller to perform a flight path tracking for a transportation aircraft. The synthesis of the adaptation laws is based on the positivity and Lyapunov design principle. Simulation results for different flight conditions for transportation aircraft are provided to illustrate the effectiveness of the proposed controller.**

I. INTRODUCTION

The application of modern flight control design methodologies to transportation aircraft is currently limited by the complexity of the problem and the need of extensive testing to validate the proposed solutions mainly when conventional gain scheduling is involved. Also, any control methodology proposed for transportation aircraft should enable people to safely fly without requiring increased workload from the pilots. In this context, nonlinear adaptive control could be a promising way to design improved solutions to this problem since these techniques try to compensate for modeling inaccuracies and external disturbances; this is why many studies [1], [2], [4], [5], [6], [7], [8], [9], [10], [11]-[12] have been performed in this field.

In this paper, is proposed a new nonlinear direct adaptive sliding mode controller to ensure flight path tracking by a transportation aircraft.

The proposed controller is a combination of nonlinear dynamic inversion and sliding mode approach and it is synthesized in two steps; the first one consists in performing consecutive time derivatives of the considered output until the control input appears and the resulting equation is put under a linearly parameterized form while the second step consists in defining a sliding surface, a function of the tracking error and its time derivatives; then a Lyapunov function is chosen in order to extract the control law which verifies both asymptotic Lyapunov stability and sliding conditions.

Manuscript received December 23; 2010.This work was supported by Automation and Operational Research Laboratory of Air Transportation Department in the National Civil Aviation Institute in Toulouse-France.

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Developed controller guarantees a good robustness against modelling parameter uncertainties while keeping the angle of attack α within an acceptable range. The synthesis of the adaptation mechanism is based on the positivity and Lyapunov design principle in order to estimate the controller parameters directly while the exponentially asymptotic convergence of the chosen sliding surface σ is shown.

First are presented the vertical flight-path angle dynamics for a transportation aircraft while the controllability condition is checked.

A sliding surface σ is chosen with respect to the relative degree of the considered output, flight path angle, and a control law is synthesized based on the sliding mode approach. Subsequently, the positivity and Lyapunov design principle is used to determine adaptation laws which ensure an exponentially asymptotic convergence of the closed-loop dynamics.

Finally, through simulation the performance of the proposed adaptive controller are studied with several flight conditions for a transportation aircraft.

II. FLIGHT DYNAMICS MODELING

Considering classical simplifying assumptions, the vertical flight equations adopted here are given by:

$$\dot{V} = \frac{1}{m}\left[T\cos(\alpha+i) - D - mg\sin(\gamma)\right] \tag{1a}$$

$$\dot{\gamma} = \frac{1}{mV}\left[T\sin(\alpha+i) + L - mg\cos(\gamma)\right] \tag{1b}$$

$$\dot{q} = \frac{M}{I_{yy}} \tag{1c}$$

$$\dot{\theta} = q \tag{1d}$$

where γ, V, α and i are respectively the flight path angle, the true airspeed, the angle of attack and the incidence angle of the thrust vector with respect to the centerline. θ, q and M represent the pitch angle, the pitch rate and the pitch moment, respectively. g is the gravity acceleration and I_{yy} is the inertial moment around the pitch axis. L and D denote lift and drag forces, respectively.

$$M = \frac{1}{2}\rho V^2 S\bar{c} C_M \tag{2a}$$

$$C_M = C_{m_\alpha}\alpha + \frac{q\bar{c}}{2V}C_{m_q} + C_{m_{\delta_e}}\delta_e \tag{2b}$$

where C_M, ρ, S, $\bar{c}$ and δ_e represent the pitching moment coefficient, the air density, the wing reference surface area, the mean chord line and the elevator deflection, respectively.

C_{m_α}, C_{m_q} and $C_{m_{\delta_e}}$ are non-dimensional stability derivatives and they denote respectively the change in the pitching

moment coefficient with respect to the angle of attack, the change in the pitching moment coefficient with respect to the pitch rate and the elevator control effectiveness.

Lift force is expressed as follows:

$$L = \frac{1}{2}\rho V^2 S C_L \tag{3a}$$

$$C_L = C_{L_0} + C_{L_\alpha}\alpha \tag{3b}$$

where C_L, C_{L_0} and C_{L_α} represent the lift coefficient, the reference lift coefficient and the lift curve slope respectively.

Since $\alpha = \theta - \gamma$ then:

$$\dot{\alpha} = q - \frac{1}{mV}\left[T\sin(\alpha + i) + L - mg\cos(\gamma)\right] \tag{4}$$

Here it is considered that at cruise, the true airspeed (TAS) is taken as a constant for a short period and the thrust term $T\sin(\alpha + i)$, is neglected as it is generally much smaller than lift [4]. Then, the longitudinal motion dynamics can be written under the nonlinear affine state representation as follows:

$$\dot{x} = f(x) + g(x)u \tag{5}$$

with:

$$f(x) = \begin{pmatrix} a_1\cos(\gamma) + a_2(\theta - \gamma) + \varepsilon_\gamma \\ q \\ a_3 q + a_4(\theta - \gamma) \end{pmatrix}, g(x) = \begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \tag{6}$$

where: x=$[\gamma, \theta, q]^T$, $u = \delta_e$ and:

$$\begin{aligned} a_1 &= -\frac{g}{V} & a_2 &= \frac{1}{2m}\rho V S C_{L_\alpha} \\ a_3 &= \frac{1}{4I_{yy}}\rho V S \bar{c}^2 C_{m_q} & a_4 &= \frac{1}{2I_{yy}}\rho V^2 S \bar{c} C_{m_\alpha} \\ b &= \frac{1}{2I_{yy}}\rho V^2 S \bar{c} C_{m_{\delta_e}} & \varepsilon_\gamma &= \frac{1}{2m}\rho V S C_{L_0} \end{aligned} \tag{7}$$

The controllability matrix Υ given by:

$$\Upsilon = \left[g, ad_f g, ad_f^2 g\right] \tag{8}$$

is such that:

$$\Upsilon = \begin{pmatrix} 0 & 0 & a_2 b \\ 0 & -b & a_3 b \\ b & -a_3 b & b(a_3^2 + a_4) \end{pmatrix} \tag{9}$$

Since, $det(\Upsilon) = a_2 b^3$, the above system is locally controllable over $\mathbb{R}^3$ if and only if $a_2 \neq 0$ and $b \neq 0$.

III. CONTROL DESIGN WITH PARAMETER UNCERTAINTY

Here it is assumed that some uncertainties remain with respect to the main aerodynamic coefficients of the above model. The main control objective is here the tracking of a given flight path angle which changes according to the guidance needs. In order to safely achieve this control objective, a nonlinear adaptive sliding mode control is developed.

The relative degree of the considered output $y = \gamma$ is equal to three:

$$\begin{aligned} \dot{y} &= \dot{\gamma} \\ &= a_1\cos(\gamma) + a_2(\theta - \gamma) + \varepsilon_\gamma \end{aligned} \tag{10a}$$

$$\begin{aligned} y^{(2)} &= \gamma^{(2)} \\ &= -\frac{a_1^2}{2}\sin(2\gamma) - a_1\sin(\gamma)\left[a_2(\theta - \gamma) + \varepsilon_\gamma\right] \\ &\quad + a_2\left[q - a_1\cos(\gamma) - a_2(\theta - \gamma) - \varepsilon_\gamma\right] \end{aligned} \tag{10b}$$

$$\begin{aligned} y^{(3)} &= \gamma^{(3)} \\ &= f_0 + a_2 f_1 + a_2^2 f_2 + (a_2^3 + a_2 a_4) f_3 + a_2 a_3 f_4 \\ &\quad + a_2 b\delta_e \end{aligned} \tag{10c}$$

where:

$$\begin{aligned} f_0 &= -a_1^2\varepsilon_\gamma\left[\cos^2(\gamma) + \cos(2\gamma)\right] - a_1\varepsilon_\gamma^2\cos(\gamma) \\ &\quad - a_1^3\cos(\gamma)\cos(2\gamma) \end{aligned} \tag{11a}$$

$$\begin{aligned} f_1 &= -a_1^2(\theta - \gamma)\left[\cos^2(\gamma) + \cos(2\gamma)\right] + a_1^2\sin(2\gamma) \\ &\quad - 2a_1\varepsilon_\gamma(\theta - \gamma)\cos(\gamma) + a_1(2\varepsilon_\gamma - q)\sin(\gamma) \end{aligned} \tag{11b}$$

$$\begin{aligned} f_2 &= 2a_1(\theta - \gamma)\sin(\gamma) - a_1(\theta - \gamma)^2\cos(\gamma) \\ &\quad + a_1\cos(\gamma) - q + \varepsilon_\gamma \end{aligned} \tag{11c}$$

$$f_3 = \theta - \gamma \tag{11d}$$

$$f_4 = q \tag{11e}$$

Then, no internal dynamics is associated with this output and (10c) can be rewritten under a linearly parameterized form such as:

$$h[\gamma^{(3)} - f_0] + \sum_{i=1}^{4}\lambda_i f_i = \delta_e \tag{12}$$

where:

$$\begin{aligned} h &= \frac{1}{a_2 b} & \lambda_1 &= -\frac{1}{b} & \lambda_2 &= -\frac{a_2}{b} \\ \lambda_3 &= -\frac{a_2^2 + a_4}{b} & \lambda_4 &= -\frac{a_3}{b} \end{aligned} \tag{13}$$

To synthesize the adaptive control law, it is assumed that f_0 and f_i are known nonlinear functions of the state and time; the parameters h and λ_i are unknown constants. We assume also that the full state vector components are available through the measure, and that the sign of h is known [3].

Now, let us choose a sliding surface σ such as:

$$\sigma = y^{(2)} - v \tag{14a}$$

$$v = \gamma_d^{(2)} - k_1\dot{z} - k_2 z \tag{14b}$$

with:

$$z = \gamma - \gamma_d \tag{15}$$

where z represents the tracking error, γ_d is the desired flight path angle and k_1, k_2 are real positive parameters.

Now a candidate Lyapunov positive definite function $V_1(\sigma)$ can be defined:

$$V_1(\sigma) = \frac{1}{2}\sigma^2 \tag{16a}$$

$$\dot{V}_1(\sigma) = \sigma\dot{\sigma} = \sigma\left[\frac{1}{h}\left(\delta_e + hf_0 - \sum_{i=1}^{4}\lambda_i f_i\right) - \dot{v}\right] \tag{16b}$$

asymptotic Lyapunov stability will be guaranteed if $\dot{V}_1(\sigma) = \sigma\dot{\sigma} < 0$, for which the control law could be chosen as follows:

$$\delta_e = h\left[\gamma_d^{(3)} - k_1 z^{(2)} - k_2\dot{z} - f_0\right] + \sum_{i=1}^{4}\lambda_i f_i - k\sigma \tag{17}$$

where k is a real positive parameter.

Parameters h and λ_i are unknown, and they are replaced by their estimates $\hat{h}$ and $\hat{\lambda}_i$ respectively and the proposed control law is expressed such as:

$$\delta_e = \hat{h}\left[\gamma_d^{(3)} - k_1 z^{(2)} - k_2\dot{z} - f_0\right] + \sum_{i=1}^{4}\hat{\lambda}_i f_i - k\sigma \tag{18}$$

To get the closed-loop dynamics, let us replace the synthesized control law expression (18) in the time derivative of the sliding surface σ:

$$h\dot{\sigma} + k\sigma = \sum_{i=1}^{4}\tilde{\lambda}_i f_i + \tilde{h}(\dot{v} - f_0) \tag{19}$$

with:

$$\tilde{h} = \hat{h} - h \tag{20a}$$

$$\tilde{\lambda}_i = \hat{\lambda}_i - \lambda_i \tag{20b}$$

and these represent the estimation errors related to the controller parameters. The closed-loop dynamics (19) shows that if the estimation errors related to the controller parameters converge to zero, then the tracking error dynamics tend to zero exponentially as shown:

$$h\dot{\sigma} + k\sigma = 0 \tag{21}$$

To synthesize an adaptation mechanism, the positivity and Lyapunov design principle is applied in order to determine the adaptation laws which allow the on-line estimation of the unknown controller parameters $\hat{h}$ and $\hat{\lambda}_i$. Consequently another Lyapunov positive definite function $V_2(\sigma, \tilde{\lambda}_i, \tilde{h})$ is introduced:

$$V_2(\sigma, \tilde{\lambda}_i, \tilde{h}) = \frac{1}{2}|h|\sigma^2 + \frac{1}{2\eta}\left[\tilde{h}^2 + \sum_{i=1}^{4}\tilde{\lambda}_i^2\right] \tag{22}$$

where η denotes the gain adaptation. The idea is to choose $\dot{\hat{h}}$ and $\dot{\hat{\lambda}}_i$ such that $\dot{V}_2(\sigma, \tilde{\lambda}_i, \tilde{h}) \leqslant 0$. Since:

$$\dot{V}_2(\sigma, \tilde{\lambda}_i, \tilde{h}) = \frac{|h|}{h}\sigma\left[\sum_{i=1}^{4}\tilde{\lambda}_i f_i + \tilde{h}(\dot{v} - f_0) - k\sigma\right] + \frac{1}{\eta}\left[\tilde{h}\dot{\tilde{h}} + \sum_{i=1}^{4}\tilde{\lambda}_i\dot{\tilde{\lambda}}_i\right] \tag{23}$$

choosing $\dot{\hat{h}}$ and $\dot{\hat{\lambda}}_i$ such as:

$$\dot{\hat{\lambda}}_i = -\eta\, sgn(h)\sigma f_i \tag{24a}$$

$$\dot{\hat{h}} = -\eta\, sgn(h)\sigma(\dot{v} - f_0) \tag{24b}$$

then:

$$\dot{V}_2(\sigma, \tilde{\lambda}_i, \tilde{h}) = -|k|\sigma^2 \tag{25}$$

and the global tracking convergence of the adaptive control system is guaranteed.

IV. SIMULATION STUDY

To test the effectiveness of the developed controller, it has been applied to the path angle tracking of a very large, four-engine, passenger jet aircraft for several flight conditions as it is shown in Table 3 below.

The considered aircraft has the following general parameters as it is shown in Table 1:

TABLE I
AIRCRAFT GENERAL PARAMETERS

Parameter	Value
Wing area (m^2)	510
Aspect ratio (AR)	7.0
Mean chord line $\bar{c}$ (m)	8.3
Gravity centre	$0.25\bar{c}$
Total related thrust (KN)	900

In Table 2. the weight and the inertias of the aircraft during the approach segment and in all other considered flight conditions are shown.

TABLE II
WEIGHT AND INERTIAS

	Approach	All other flight conditions
Weight (Kg)	250000	290000
$I_{xx}(Kg.m^2)$	18.6×10^6	24.6×10^6
$I_{yy}(Kg.m^2)$	41.35×10^6	45×10^6
$I_{zz}(Kg.m^2)$	58×10^6	67.5×10^6
$I_{xz}(Kg.m^2)$	1.2×10^6	1.32×10^6

TABLE III

FLIGHT CONDITIONS

parameter	FC1	FC2	FC3	FC4
Height (m)	S.L	6100	6100	12200
Mach	0.198	0.5	0.5	0.8
$V(m.s^{-1})$	67	158	250	250
$\overline{q}(N.m^{-2})$	2810	8667	24420	9911
$\alpha_0(degree)$	8.5	6.8	0	4.6

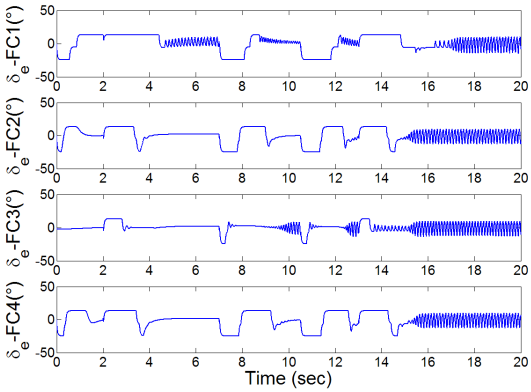

Fig. 1. Control Input

Fig. 1 shows the response due to the elevator deflection since it is considered that it follows saturated first order dynamics in order to respect the physical limitations of the considered actuator. Obtained results show chattering phenomenon often present while sliding mode control is applied.

This phenomenon can create hazardous consequences both on the considered actuator and on the flight safety.

Fig. 2 displays desired flight path tracking of the aircraft for different flight conditions, desired flight path is chosen as a set of several straight lines with different slopes; this choice can be considered a hard test for the robustness of the proposed controller in real life. The obtained tracking results show that a good path angle tracking performance is achieved.

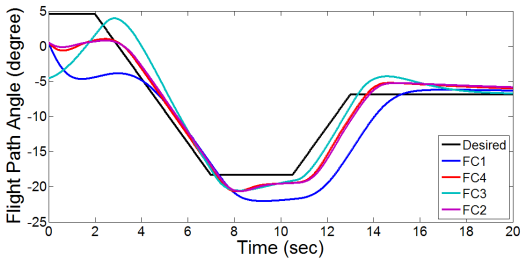

Fig. 2. Desired Flight Path Tracking

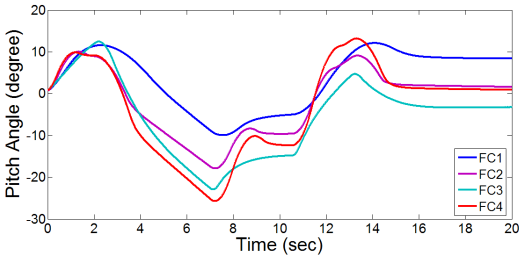

Fig. 3. Pitch Angle Evolution

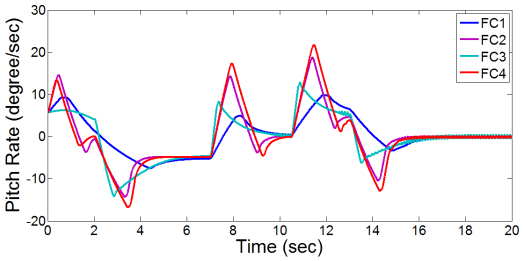

Fig. 4. Pitch Rate Evolution

Fig. 3 and Fig. 4 show the evolution of the pitch angle and the pitch rate according to the considered flight conditions.

The angle of attack remains among the main variables affecting the flight safety for civil aviation. It can be seen in Fig. 5 that the behavior of the angle of attack remains within an acceptable range for considered flight conditions.

To deal with chattering phenomenon, we have added to the desired flight path angle defined above a second order model reference to get a smooth reference input and to considerably decrease the chattering phenomenon since this model reference also has the role of a filter as it is shown in Fig. 6 and Fig. 10 below.

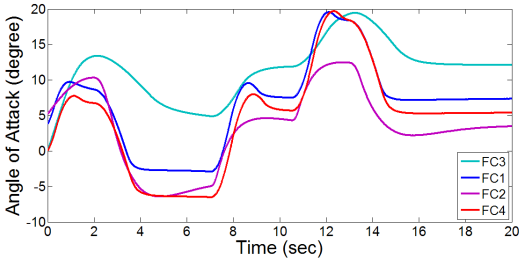

Fig. 5. Angle of Attack Evolution

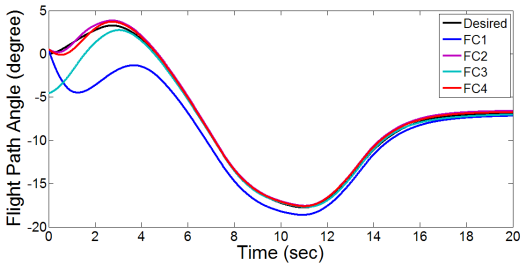

Fig. 6. Desired Flight Path Tracking with Model-Reference

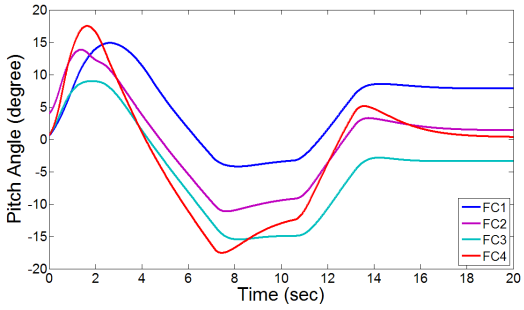

Fig. 7. Pitch Angle Evolution with Model-Reference

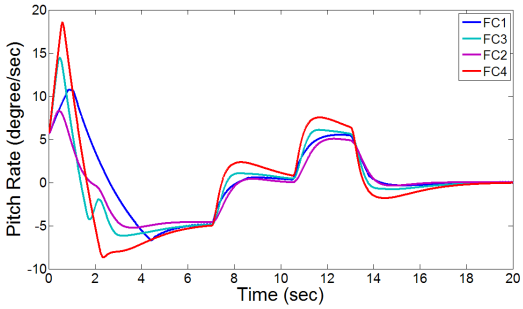

Fig. 8. Pitch Rate Evolution with Model-Reference

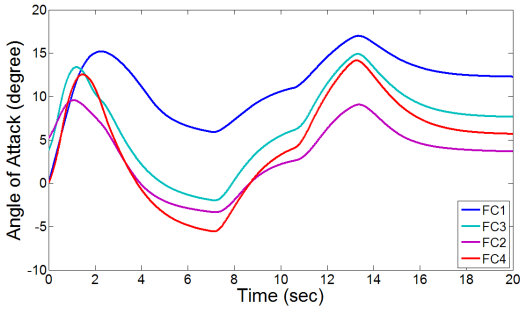

Fig. 9. Angle of Attack Evolution with Model-Reference

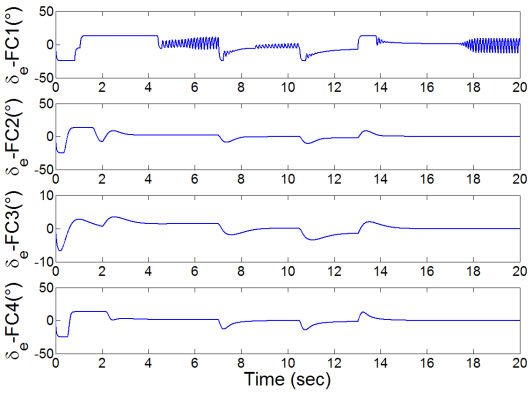

Fig. 10. Control Input with Model-Reference

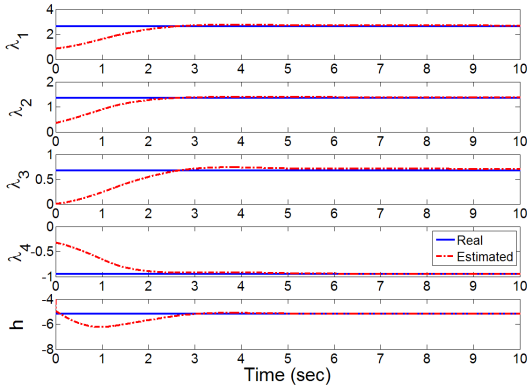

Fig. 11. Controller Parameters Estimation Related to FC1

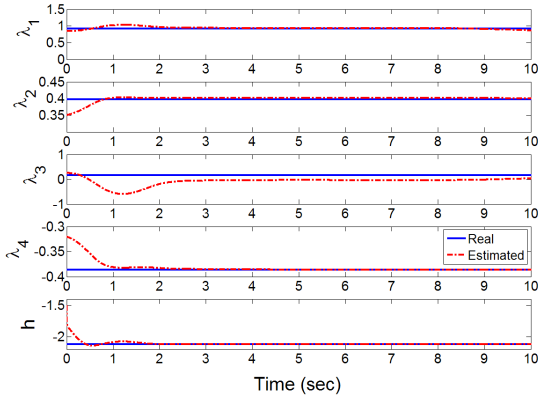

Fig. 12. Controller Parameters Estimation Related to FC2

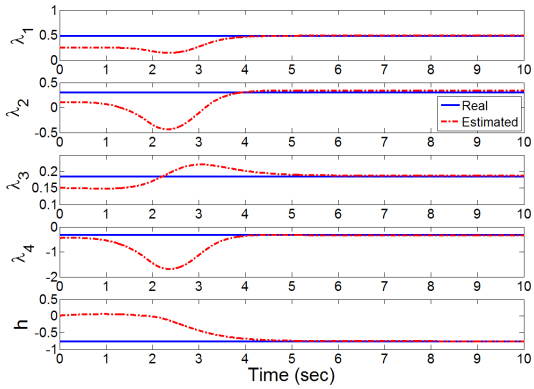

Fig. 13. Controller Parameters Estimation Related to FC3

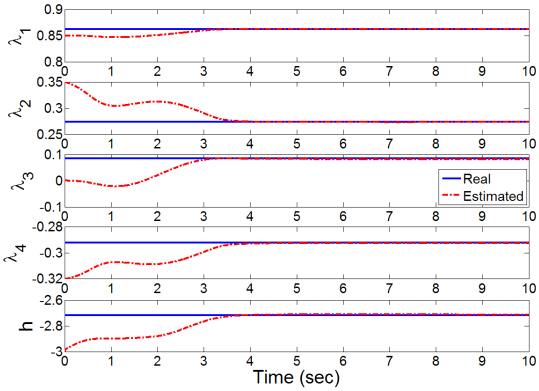

Fig. 14. Controller Parameters Estimation Related to FC4

Fig. 11, Fig. 12, Fig. 13 and Fig. 14 display the estimation of the considered controller parameters $\hat{h}$ and $\hat{\lambda}_i$ according to the applied flight conditions FC1, FC2, FC3 and FC4.

The obtained simulation results show acceptable estimation performances.

V. CONCLUSION

In this paper, an adaptive flight path control for transportation aircraft is considered while the controllability condition is checked.

Control design is a hybridization between a nonlinear dynamic inversion method and adaptive sliding mode control approach; developed controller had ensured asymptotic Lyapunov stability and guaranteed robustness against the system parameters uncertainties.

Since direct adaptive control design approach is used, an adaptation mechanism is developed based on the positivity and Lyapunov principle design in order to synthesize the adaptation laws which allow the estimation of the controller parameters and to guarantee the exponential convergence of the tracking error dynamics.

Despite the use of saturated first order dynamics for the control input in order to respect the physical limitations of the considered actuator. Desired flight path tracking is performed, chattering phenomenon is considerably decreased and angle of attack is maintained within an acceptable range.

To test the robustness of the proposed control design we have applied synthesized control law to several flight conditions. Simulation results show satisfactory performances.

Adaptive control can provide even some fault tolerance against unforeseen failures and improve the aircraft chances of survivability under adverse conditions. In perspective, we would like to develop adaptive controllers which can deal with failures and also with dynamic parameters.

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