Aircraft flight management with actuator major failure

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Abstract— **This communication considers the case of total loss of hydraulic power by a transportation aircraft. The aim of this study is to propose control solutions to the design of new guidance function for the flight director system, or an emergency mode for the auto-throttle to control the thrust of a multi-engine transportation aircraft to make it return safely near ground level with acceptable landing conditions. The approach is based on a model of the flight dynamics involving all its major nonlinearities and couplings. Since after this major failure the aircraft is no more fully controllable, non linear inverse control technique is used to design control laws allowing the realization of a sequence of basic manoeuvres sufficient to turn achievable emergency landing. Some simulation results are displayed.**
Key words: aircraft flight safety, actuator failure, emergency management, non linear inverse control.

I. Introduction

THIS communication considers the case of a major failure for the aircraft flight control system: the total loss of hydraulic power used by the main aerodynamic actuators. This situation is extremely rare but the possibility of a crash involving the failure of all aerodynamic actuators cannot be excluded. In this paper we are interested in proposing control considerations that can either lead to the synthesis of new functions for the flight director system, or an emergency mode for the auto-throttle to control the thrust of a multi-engine aircraft so that it is able to return near ground level with acceptable landing conditions. The approach is based on a model of the flight dynamics of a commercial aircraft involving all their major nonlinearities and couplings. Since after this major failure the aircraft is no more fully controllable, non linear inverse control technique is used to design control laws allowing the realization of a sequence of basic manoeuvres. Some simulation results involving A300 are displayed.

F. Mora-Camino is Professor at the French Civil Aviation Institute – ENAC, Automation and Operations Research Laboratory-LARA, Air Transportation Department, 31055, Toulouse, France. Phone: 0033562174358, fax: 0033562174403, e-mail: felix.mora@enac.fr.

S. Simões Cunha is Associate Professor at Federal University of Itajuba, Brazil, Mechanical engineering department, he has a post-doctoral position at ENAC, Automation and Operations research Laboratory, funded by CAPES. Phone: 00553536291166, e-mail: sebas@unifei.edu.br.

A. Doncescu is Professor at Toulouse University and senior researcher at LAAS du CNRS, 31077, Toulouse, France. Phone: 0033561336200, e-mail: adoncesc@laas.fr.

II. The A300 Baghdad Accident

This study takes as reference the accident of November 22, 2003 in Baghdad [48] when shortly after takeoff from Baghdad International Airport an A300 freighter aircraft belonging to European Air Transport had its left wing hit by a ground-to-air missile. The damage on the wing led to the complete loss of hydraulics for the aircraft flight control *systems*. Just before the impact, the aircraft was climbing at about 9 000ft with a 10° path angle and a speed V= 210kts. The strike pierced the left wing tanks, a fuel leak began and fire started at the tip of the left wing. The missile also caused a 5-metre long crack in the wing and cut its three hydraulic lines in the wing. The hydraulic fluid bled away, turning ineffective all the aerodynamic actuator control channels.

Fig. 1 The damaged left wing after landing

The airplane started to oscillate up and down (phugoïd cycle), this motion being characteristic of a loss of elevator authority. This motion faded away slowly and the pilot tried to control the flight by acting on the thrust levers controlling its two engines which looked to behave normally: a differential setting of the thrust levers was able to produce a yaw moment allowing to control somehow the aircraft heading through slowly damping yaw and roll oscillations, while an equal reduction of the thrust of the engines led to a reduction of the path angle to 0° , but also a significant increase of the speed of the aircraft to 280kts at which no landing can be safely performed, inducing total destruction of the aircraft (maximum structural landing speed for this aircraft being 220kts). As they flown over the area, the flight engineer extended the landing gear by gravity. This maneuver altered the balance of the aircraft, which began again to oscillate upwards. The captain reduced the power on both engines with the aim to pitch the nose down while the aircraft was approaching stall speed and he achieved to level the flight out. With the landing gear extended, the airplane became easier to control and he put the plane into a turn to line up with runway 33R at Baghdad International Airport. As they approached this runway, it became clear that they were too high to perform safely a landing. After abandoning this landing attempt, they flew around to give way to a longer final approach to Runway 33L, while this increased the danger of losing the left wing which was in fire. To compensate for the left wing, the pilot applied more power to the left engine to bank right, and managed to line up with runway 33L. However at the height of 400 feet, crosswinds disturbed the flight path and attitude of the airplane. At landing, with a path angle of -2°, a large bank angle and a speed of 210kt, the right main landing gear hit violently the runway followed by the left one and then the nose gear. Then the aircraft deviated from the runway axis, and with its belly rubbing the sandy ground, slowed down before coming to a stop between the runway and taxiway. The crew was then able to deploy the right emergency slide and made it out safely from the wrecked aircraft.

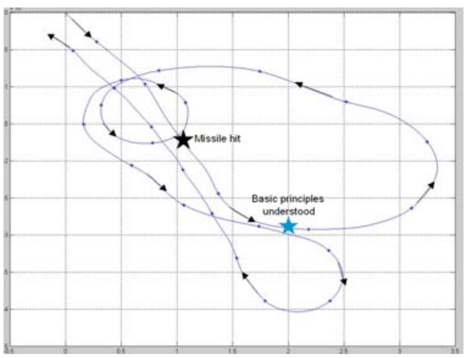

Fig. 2 The aircraft trajectory around Baghdad airport

All this displays the extreme difficulty to manually control the aircraft, through engine thrusts, in such a fault situation. So it appears of utmost interest to design new control approaches to cope as safely as possible with this case.

III. Flight Dynamics of the Damaged Aircraft

Here we consider the flight dynamics of an aircraft with uncontrolled aerodynamic actuators (they are either stuck to a fixed position, or free to rotate creating then no significant moment). The aircraft is assumed to be a rigid body flying a no wind standard atmosphere over a flat Earth. Here the thrust produced by each turbofan engine is particularized.

A. Rigid body flight equations

In this case the flight equations [1] are composed of:

a) Force equations :

$$m(\dot{u} + q\,w - r\,v) = -mg\sin\theta - 1/2\,\rho V^2 S\,C_x + (P_L + P_R)\cos\sigma$$

$$m(\dot{v} + u\,r - p\,w) = mg\,\cos\theta\,\sin\phi + 1/2\,\rho V^2 S\,C_y$$

$$m(\dot{w} + p\,v - q\,u) = mg\cos\theta\,\cos\phi + 1/2\,\rho V^2 S\,C_z + (P_L + P_R)\sin\sigma$$

where the coefficients for aerodynamic forces are given by :

$$C_x = C_{x0l} + k\,C_z^2 \quad (2.1)$$

$$C_y = C_{y\beta}\,.\,\beta + C_{y\delta p}\,.\,\delta_p + C_{y\delta r}\,.\,\delta_r + C_{yp}\,.\,p\,.\,l_A/V_0 + C_{yr}\,.\,r\,.\,l_A/V_0 \quad (2.2)$$

$$C_z = C_{z0} + C_{z\alpha}.\alpha + C_{zphr}\,.\,\delta_{phr} + C_{zq}\,.\,\delta_q \quad \text{with } \alpha \approx w/V \quad (2.3)$$

b) Moment equations: (3.1)

$$A\,\dot{p} - E\,\dot{r} + (C - B)r\,q - E\,p\,q = 1/2\,\rho V^2 S\,l\,C_l - a(P_L - P_R)\sin\sigma \quad (3.2)$$

$$B\,\dot{q} + (A - C)r\,p - E(p^2 - r^2) = 1/2\,\rho V^2 S\,l\,C_m + b\,(P_L + P_R) \quad (3.3)$$

$$C\,\dot{r} - E\,\dot{p} + (B - A)q\,p = 1/2\,\rho V^2 S\,l\,C_n + a\,(P_L - P_R)\cos\sigma$$

with the inertia matrix :

$$I = \begin{bmatrix} A & 0 & -E \\ 0 & B & 0 \\ -E & 0 & C \end{bmatrix} \quad (4)$$

where $\underline{V} = (u, v, w)$' and $\underline{\Omega} = (p, q, r)$' are respectively the instant translation speed and the rotation speed vectors expressed in the body frame. Here C_x, C_y, C_z, C_l, C_m and C_n are the aerodynamic coefficients , m is the mass of the aircraft and A, B, C and E are the aircraft main axis inertia moments. Here θ and ϕ are respectively the pitch angle and the bank angle while ψ is the heading angle. The non dimensional coefficients of the different aerodynamic moments are supposed to follow :

$$C_m = C_{m\alpha}\,.\,\alpha + C_{m\delta phr}\,.\,\delta_{phr} + C_{m\delta m}\,.\,\delta_m + C_{mq}\,.\,q\,.\,l_A/V_0 \quad (5.1)$$

$$C_l = C_{l\beta}\,.\,\beta + C_{lp}\,.\,p\,.\,l_A/V_0 + C_{lr}\,.\,r\,.\,l_A/V_0 + C_{l\delta p}\,.\,\delta_p + C_{l\delta r}\,.\,\delta_r \quad (5.2)$$

$$C_n = C_{n\beta}\,.\,\beta + C_{np}\,.\,p\,.\,l_A/V_0 + C_{nr}\,.\,r\,.\,l_A/V_0 + C_{n\delta p}\,.\,\delta_p + C_{n\delta r}\,.\,\delta_r \quad (5.3)$$

c) The Euler equations

They provide the relation between the rates of the attitude and heading angles (ϕ, θ and ψ) and the body components (p,q,r) of the rotational speed :

$$\begin{cases} \dot{\theta} = -r\sin\phi + q\cos\phi & (6.1) \\ \dot{\psi} = \dfrac{q\sin\phi + r\cos\phi}{\cos\theta} & (6.2) \\ \dot{\phi} = p + \mathrm{tg}\theta(q\sin\phi + r\cos\phi) & (6.3) \end{cases}$$

V is the modulus of the translation speed, α is the angle of attack, β is the sideslip angle, where:

$$u = V\cos\beta\,\cos\alpha\ ,\quad v = V\sin\beta\ ,\quad w = V\cos\beta\,\sin\alpha \quad (7)$$

The thrust of the left and right engines P_L and P_R are supposed to follow first order linear dynamics such as :

$$\tau_M\,\dot{P}_L + P_L = P_{LC} \quad \text{and} \quad \tau_M\,\dot{P}_R + P_R = P_{RC} \quad (8)$$

where τ_M is a time constant. However, for simplicity, in the subsequent study these dynamics will be considered to be fast with respect to the aircraft flight dynamics.

B. Minimal equilibrium conditions

The minimal equilibrium state corresponds to the situation in which the rotation speed is null while the aircraft is level flight at altitude z_0 with a constant translation speed V_0. Regulations with respect to transport aircraft flight qualities require that such equilibrium can be reached for adequate setting of thrust while aerodynamic actuators are stuck to a given position. Furthermore this equilibrium state should be stable. The equilibrium equations can then be written:

$$-mg\sin\theta - 1/2\ \rho(z_0)V_0^2 S\ C_x(\alpha,\beta,p,q,r,\delta_{phr},\delta_p,\delta_q,\delta_r) + (P_L+P_R) = 0 \quad (9.1)$$

$$mg\ \cos\theta\ \sin\phi + 1/2\ \rho(z_0)V_0^2 S\ C_y(\alpha,\beta,p,q,r,\delta_{phr},\delta_p,\delta_q,\delta_r) = 0 \quad (9.2)$$

$$mg\cos\theta\ \cos\phi + 1/2\ \rho(z_0)V_0^2 S\ C_z(\alpha,\beta,p,q,r,\delta_{phr},\delta_p,\delta_q,\delta_r) = 0 \quad (9.3)$$

and

$$1/2\ \rho(z_0)V_0^2 S\,l\,C_l(\beta,p,r,\delta_p,\delta_r) = 0 \quad (10.1)$$

$$1/2\ \rho(z_0)V_0^2 S\,l\,C_m(\alpha,q,\delta_{phr},\delta_m) + b\ (P_L+P_R) = 0 \quad (10.2)$$

$$1/2\ \rho(z_0)V_0^2 S\,l\,C_n(\beta,p,r,\delta_p,\delta_r) + a\ (P_L-P_R) = 0 \quad (10.3)$$

with

$$\gamma = \theta - \alpha = 0 \quad (11)$$

and $\dot\theta = q = 0, \dot\psi = r = 0, \dot\phi = p = 0$ (12)

Assuming that

$$\delta_p = \delta_q = \delta_r = 0 \text{ and } \delta_{phr} = \delta_{phr0} \quad (13)$$

we get

$$p = q = r = 0,\quad \beta = \phi = 0 \quad (14)$$

and the two conditions :

$$-mg\sin\theta - 1/2\ \rho(z_0)V_0^2 S\ C_x(\theta,\delta_{phr0}) + (P_L+P_R) = 0 \quad (15.1)$$

$$1/2\ \rho(z_0)V_0^2 S\,l\,C_m(\theta,\delta_{phr}) + b\ (P_L+P_R) = 0 \quad (15.2)$$

with $P_L = P_R$

Then the equilibrium pitch angle is solution of equation:

$$mg\sin\theta_{\dot a} + 1/2\ \rho(z_0)V_0^2 S\ (C_x(\theta_{\dot a},\delta_{phr0}) + l\,C_m(\theta_{\dot a},\delta_{phr})) = 0 \quad (16)$$

Where the equilibrium thrust is such that :

$$P_{L0} = P_{R0} = 1/2 \quad (mg\sin\theta + 1/2\ \rho(z_0)V_0^2 S\ C_x(\theta,\delta_{phr0})) \quad (17)$$

We suppose here that the stability region around this equilibrium state is large enough so that starting from a normal transition state as initial flight conditions and after natural oscillations (short period oscillation, Dutch roll and phugoïd) , the aircraft returns asymptotically towards such an equilibrium state. However in the case of turbulent atmosphere will remain only the natural tendency to reach such a state.

C. Engine actuator limitations

After occurrence of the failure only two actuators remain: the left and right turbofan engines. These actuators present different limitations :

A limited in flight range of values :

$$P_{\min}(z,V) \le P_L, P_R \le P_{\max}(z,V) \quad (18)$$

A varying response time (with flight level and airspeed) to throttle lever settings.

Their effectiveness depends on the availability of carburant and the operational state of engines.

- limited effects: a traction force P_L+P_R, a pitch moment $b(P_L+P_R)$ and a yaw moment $a\ (P_L\text{-}P_R)$ where a and b are lever lengths.

Thus the objectives of flight guidance objectives can only be very limited and should cover at least the maneuvers that contribute directly to the success of a descent toward an emergency landing. With these actuators which provide directly no roll moment it will be quite impossible to master efficiently the aircraft fast dynamics, however they will be able to alter the energy balance of the aircraft between kinetic energy and to create a yaw moment affecting the lateral guidance of the aircraft. It will be also necessary to control the speed of the airplane during descent (to avoid as well excessive vertical and longitudinal speeds as too low airspeed leading to stall conditions) and near the ground while steering it towards the axis of a landing site (a runway, a road, any open space, etc..).

IV. Non linear Inverse control Approach

While twenty years ago only linear control law design techniques [2] where available, today many non linear control law design techniques are available to master aircraft dynamics and perform safe and accurate flights: sliding mode and robust control [3], [4], nonlinear inverse control [5], backstepping control [6], differentially flat control [7] and neural control [8] as well as combinations of these techniques [9]. One of these techniques, non linear inverse control, formalized by the theoretical work of *Isidori* [10], has been of particular interest in the field of flight control [11]. In this study we adopt this technique to a develop solution path to the considered critical situation.

A. Non linear inverse control basics

Consider now a non-linear dynamic system given by:

$$\dot{\underline{X}} = f(\underline{X}) + g(\underline{X}).\underline{U} \quad (19\text{-}1)$$

$$\underline{Y} = h(\underline{X}) \quad (19\text{-}2)$$

where $\underline{X} \in R^n$, $\underline{U} \in R^m$, $\underline{Y} \in R^m$, f and g are smooth vector fields of $\underline{X}$ and h is a smooth vector field of $\underline{X}$. The system has, with respect to each independent output Y_i , a relative degree r_i ($\sum_{i=1}^{m}(r_i+1) \le n$, $i = 1, \dots, m$) around the state $\underline{X_0}$ if the output dynamics can be written as:

$$\begin{pmatrix} Y_1^{(r_1+1)} \\ \vdots \\ Y_m^{(r_m+1)} \end{pmatrix} = \underline{A}(\underline{X}) + B(\underline{X}).\underline{U} \quad (20)$$

If $B(\underline{X})$ is invertible, a feedback control law such as:

$$\underline{U} = B^{-1}(\underline{X})(\underline{v} - \underline{A}(\underline{X})) \quad (21)$$

can be obtained. Here the new control input $\underline{v} = [v_1, \dots, v_m]'$ is chosen such as:

$$v_i = Y_{di}^{(r_i)} - \sum_{k=0}^{r_i-1} c_{ik}\left(Y_i^{(k)} - Y_{di}^{(k)}\right) \quad i=1 \text{ to } m \quad (22)$$

where Y_{di} is the reference control input for the output dynamics. Then the dynamics of the tracking error given by $e_i = Y_i - Y_{di}$ i=1 to m , are such as:

$$e_i^{(r_i)} + c_{ir_i-1}e_i^{(r_i-1)} + \cdots + c_{i1}e_i^{(1)} + c_{i0}e_i = 0 \quad (23)$$

where the coefficients c_{ik} can be chosen to make the output dynamics asymptotically stable and ensure the tracking of output y_i towards the reference output y_{di}. However the derived feedback control law works only if either no internal dynamics ($\sum_{i=1}^{m}(r_i+1)=n$) are present or if the internal dynamics ($\sum_{i=1}^{m}(r_i+1)<n$) are stable. To cope with the saturation of the actuators, the choice of the coefficients c_{ik} should be the result of a trade-off between the characteristics of the transient dynamics of the different outputs and the solicitations of the inputs.

B. Guidance through non linear inverse control

Here we consider the flight dynamics of an aircraft with uncontrolled aerodynamic actuators (they are either stuck to a fixed position, or free to rotate and generating no significant aerodynamic moment). The aircraft is assumed to be a rigid body flying within a no wind standard atmosphere over a flat Earth. The thrust produced by each of its turbofan engine is particularized and each engine will be supposed to work nominally.

It will be of interest here, as quoted before, to master the speed and the heading while the aircraft is flying at constant level or is engaged in a descent maneuver. However, since we have only two independent control inputs (the total thrust and the difference of thrusts) we can only control two independent outputs during a given maneuver. If we choose the vertical speed w and the heading ψ as controlled outputs, we can determine their relative degrees through the computation of their successive time derivatives:

$$\dot{w} = -pv + qu + g\cos\theta\cos\phi + (1/2)\rho V^2 (S/m)\, C_z$$
$$\dot{\psi} = \frac{q\sin\phi}{\cos\theta} + \frac{\cos\phi}{\cos\theta} r \quad (24)$$

$$\ddot{w} = W(p,q,r,u,v,w,\theta,\phi) + (b/B)\, u\, (P_L + P_R)$$
$$\ddot{\psi} = P(p,q,r,u,v,w,\theta,\phi) + \frac{\sin\phi}{\cos\theta}\frac{b}{B}(P_L + P_R) + \frac{\cos\phi}{\cos\theta}\frac{a}{C}(P_L - P_R) \quad (25)$$

where

$$\begin{aligned} &W(p,q,r,u,v,w,\theta,\phi) = \\ &-(Epq + (B-C)rq + 1/2\,\rho V^2 SlC_l)v/A \\ &+ p(pw - ur + g\cos\theta\sin\phi + 1/2\rho V^2 C_y/m) \\ &+ (E(p^2 - r^2) - (A-C)pr + 1/2\,\rho V^2 SlC_m)\,u/B \\ &- g\,(\sin\theta\cos\phi\,\dot{\theta} + \cos\theta\sin\phi\,\dot{\phi}) + 1/2\,\rho V^2 (S/m)\, C_{z\alpha}\dot{w}/V \end{aligned} \quad (26)$$

$$\begin{aligned} &P(p,q,r,u,v,w,\theta,\phi) = \\ &q\Big(\frac{\cos\phi}{\cos\theta}(p + \mathrm{tg}\theta(q\sin\phi + r\cos\phi)) - \frac{\sin\phi\sin\theta}{\cos^2\theta}(-r\sin\phi + q\cos\phi)\Big) \\ &+ r\Big(\frac{\sin\phi}{\cos\theta}(p + \mathrm{tg}\theta(q\sin\phi + r\cos\phi) - \frac{\sin\phi\sin\theta}{\cos^2\theta}(-r\sin\phi + q\cos\phi)\Big) \\ &+ \frac{\sin\phi}{B\cos\theta}(E(p^2 - r^2) + (C-A)rp + 1/2\,\rho V^2 S\, lC_m) \\ &+ \frac{\cos\phi}{\cos\theta}\Big(\frac{(A-B)A}{AC-E^2}pq + 1/2\,\rho V^2 S\, l\,(\frac{A}{AC-E^2}C_n + \frac{E}{AC-E^2}C_l)\Big) \end{aligned} \quad (27)$$

Then the relative degree of each of these outputs is equal to 1 and there is an inner dynamics of order 6 involving flight variables such as: p, q, r, θ, ϕ and v.

If the pitch angle θ remains within the set $[-\pi/2, +\pi/2]$, which is likely, system (25) is invertible with respect to the two independent inputs (P_L+P_R) and (P_L-P_R). Then adopting second order dynamics for w and ψ such as :

$$\ddot{w} = -2\,Z_w\omega_w\,\dot{w} - \omega_w^{\,2}(w - w_c)$$
$$\ddot{\psi} = -2\,Z_\psi\omega_\psi\,\dot{\psi} - \omega_\psi^{\,2}(\psi - \psi_c) \quad (28)$$

where w_c and ψ_c are target values for w and ψ, where Z_w and Z_ψ are damping (positive) coefficients and where ω_w and ω_ψ are natural frequencies. Then we get the following control laws:

$$P_L = \frac{BC}{2ab\cos\phi}((\sin\phi\, b/B - \cos\phi\, a/C)((2\,Z_w\omega_w\,\dot{w} + \omega_w^{\,2}(w - w_c) + W) - b/B\cos\theta(2\,Z_\psi\omega_\psi\,\dot{\psi} + \omega_\psi^{\,2}(\psi - \psi_c) + P)) \quad (29.1)$$

$$P_R = -\frac{BC}{2ab\cos\phi}((\sin\phi\, b/B + \cos\phi\, a/C)((2\,Z_w\omega_w\,\dot{w} + \omega_w^{\,2}(w - w_c) + W) + b/B\cos\theta(2\,Z_\psi\omega_\psi\,\dot{\psi} + \omega_\psi^{\,2}(\psi - \psi_c) + P)) \quad (29.2)$$

V. PHASED GUIDANCE USING ENGINE CONTROL

Here we consider that the two basic guidance maneuvers that should be performed by an emergency guidance system in the failure case are engine controlled symmetrical descent at a time and engine controlled level turn. In both cases we make use of the non linear inverse control law design technique to propose and then analyse a solution.

A. Engine controlled symmetrical descent

In this case the aircraft lateral dynamics are supposed to remain unchanged at equilibrium (no lateral wind). Then we have :

$$P_L = P_R = -\frac{B}{2b}(2\,Z_w\omega_w\,\dot{w} + \omega_w^{\,2}(w - w_c) + W) \quad (30)$$

with

$$W(p,q,r,u,v,w,\theta,\phi) = (1/2\,\rho V^2 SlC_m)\,u/B - g\sin\theta\, q + 1/2\,\rho V^2 S\, C_{z\alpha}\dot{w}/V \quad (31)$$

The longitudinal inner dynamics are then given by the equations :

$$\dot{u} = -qw - g\sin\theta - 1/2\,\rho V^2 (S/m)\, C_x - \frac{B}{2bm}(2\,Z_w\omega_w\,\dot{w} + \omega_w^{\,2}(w - w_c) + W) \quad (32.1)$$

$$\dot{q} = 1/2\,\rho V^2 (S\,l/B)\, C_m - (2\,Z_w\omega_w\,\dot{w} + \omega_w^{\,2}(w - w_c) + W)/u \quad (32.2)$$

$$\dot{\theta} = q \quad (32.3)$$

Then

$$\dot{q} = -(2\,Z_w\omega_w\,\dot{w} + \omega_w^{\,2}(w - w_c) - g\sin\theta\, q + 1/2\,\rho V^2 S\, C_{z\alpha}\dot{w}/V)/u \quad (33)$$

Then, when $w = w_c$, the pitch dynamics becomes:

$$\dot{\theta} = q$$
$$\dot{q} = -(g/u)\, q\ \sin\theta \quad (34)$$

with

$$W(p,q,r,u,v,w,\theta,\phi) = (1/2\,\rho V^2 SlC_m)\,u/B - g\sin\theta\, q \quad (35)$$

To study the stability of the pitch dynamics, consider the evolution of q in the two situations:
If $\theta \geq 0$ then:

$$d(q^2)/dt = q\dot{q} = -(g/u)\sin\theta\, q^2 \leq 0 \qquad (36.1)$$

then q goes to 0 and θ reaches a constant value θ_c.
if $\theta < 0$ then :

$$d(q^2)/dt = q\dot{q} = -(g/u)\sin\theta\, q^2 \geq 0 \qquad (36.2)$$

and in this second case there is pitch instability.
In general at cruise and furthermore at the minimal equilibrium state, the aircraft has a slightly positive pitch angle. Then it appears important to maintain this pitch angle positive during the various maneuvers which can be attempted to reach safely the ground. In particular, to perform a descent, no pitch down motion should be created using total thrust, but instead a lift deficit should be established so that the aircraft sinks progressively in the air towards the ground.
In the stable situation, the longitudinal speed obeys to the equation :

$$\dot{u} = -qw_c - g\sin\theta\,(1 - \frac{B}{2bm}q) - 1/2\,\rho V^2 (S/m)\,(C_x(w_c,u) + \frac{u}{2b} l C_m(w,u,q)) \qquad (37)$$

And when the pitch angle stabilizes to θ_c, it will be such as :

$$\dot{u} = -g\sin\theta_c - 1/2\,\rho(u^2 + w_c^2)(S/m)\,(C_x(w_c,u) + \frac{u}{2b} l C_m(w_c,u)) \qquad (38)$$

During descent the thrust should follow the following relation:

$$P_L = P_R = -(1/4\,\rho V^2 S(l/b)C_m)\,u \qquad (39)$$

The choice of w_c will determine the rate of acceleration or deceleration in the above equation during descent. Also, the instant and position of the start of this maneuver will be critical to define the descent trajectory towards some safe landing place. Figure 3 displays an example of evolution of speed, heading, pitch angle and thrusts during such type of descent.

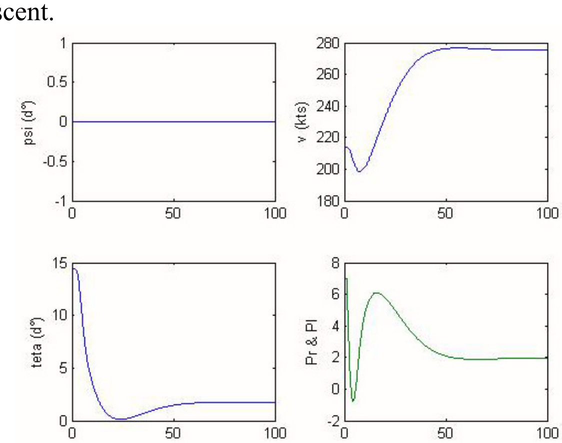

Fig. 3 Simulated evolution of the speed, the pitch angle and thrust during engine controlled symmetrical descent for an A300 aircraft.

B. *Engine controlled turn*

Here it is proposed to perform this maneuver through the direct application of the non linear inverse control technique to the lateral dynamics while the evolution of the pitch angle is checked to stay within a safe range value.
We get the following control laws when a linear second order dynamics is pursued for the heading:

$$P_L = \frac{BC}{2ab\cos\phi}((\sin\phi\, b/B - \cos\phi\, a/C)(W) - b/B\cos\theta(2\,Z_\psi\omega_\psi\,\dot{\psi} + \omega_\psi^{\,2}(\psi - \psi_c) + P)) \qquad (40.1)$$

$$P_R = -\frac{BC}{2ab\cos\phi}((\sin\phi\, b/B + \cos\phi\, a/C)(W) + b/B\cos\theta(2\,Z_\psi\omega_\psi\,\dot{\psi} + \omega_\psi^{\,2}(\psi - \psi_c) + P)) \qquad (40.2)$$

The total thrust is given by :

$$(P_L + P_R) = -BW/b \qquad (41)$$

And the difference thrust is such as :

$$(P_L - P_R) = \frac{BC}{ab\cos\phi}((\sin\phi\, b/B)W - b/B\cos\theta\,(2\,Z_\psi\omega_\psi\,\dot{\psi} + \omega_\psi^{\,2}(\psi - \psi_c) + P)) \qquad (42)$$

This difference of thrust generates a lateral motion characterized first by a yaw acceleration:

$$\dot{r} = \frac{(A-B)A}{AC-E^2}pq + 1/2\,\rho V^2 S\, l\,(\frac{A}{AC-E^2}C_n + \frac{E}{AC-E^2}C_l) - \frac{Aa}{AC-E^2}(P_L - P_R) \qquad (43)$$

and then a roll acceleration :

$$\dot{p} = (E\,pq + (B-C)\,rq + 1/2\,\rho V^2 Sl\,C_l)/A + (E/A)\,\dot{r} \qquad (44)$$

Then this lateral motion induces a small pitch acceleration given by :

$$\dot{q} = (-(A-C)r\,p + E(p^2 - r^2) + 1/2\,\rho V^2 S l C_m + b\,(P_L + P_R))/B \qquad (45)$$

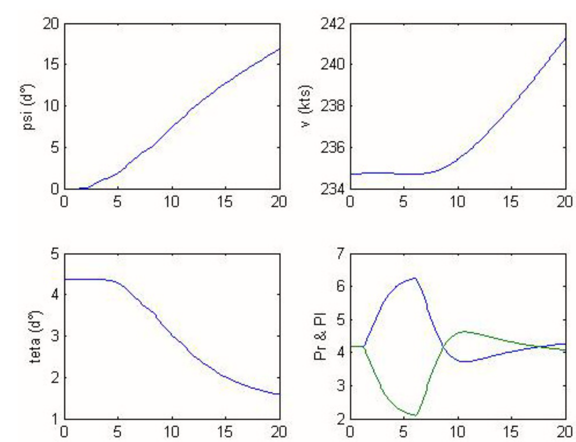

Fig. 4 Simulated evolution of heading, speed, pitch angle and thrusts during an engine controlled turn manoeuvre for an A300 aircraft.

In figure 4 it appears that while the heading seems to be adequately controlled, the adopted total thrust is unable to maintain the level flight and the turn maneuver leads to a loss of altitude. In fact, it should be necessary to increase total thrust at the start of the turning maneuver. In this case also, the non linear inverse technique can be applied to complete the lateral and longitudinal basic maneuver.

VI. CONCLUSION

In this communication we considered the problem of controlling the trajectory of an aircraft which has been subject to a major actuator failure. In that case the only admissible objective is to achieve the guidance of the aircraft towards a safe landing place with acceptable flight conditions (speed, heading, path angle and pitch angle). However the whole flight dynamics cannot be any more fully controlled and this implies that the emergency trajectory must be composed of basic maneuvers with partial objectives compatible with the effectiveness of the remaining actuators.

In this paper it has been shown how the non linear inverse control law design technique can be a basis to perform such basic maneuvers.

This does not allow to get rid of all danger since this results in poor guidance performances while the whole attempt can be jeopardized by any perturbation (lateral wind) or bad timing in phasing of the basic maneuvers.

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