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Adaptive Sliding Mode Control for Quadrotor Attitude Stabilization and Altitude Tracking

Hakim Bouadi, S. Simoes Cunha, A. Drouin and F. Mora-Camino

***Abstract*—Adaptive control algorithms are of interest in flight control systems design not only for their capability to improve performance and reliability but also for handling aerodynamic parameter uncertainties, external disturbances and modeling inaccuracies. In this paper, a direct adaptive sliding mode control is developed for the quadrotor attitude stabilization and altitude trajectory tracking. First, developed controller is applied without considering disturbances and parameter uncertainties. After, a centered white gaussian noise with some parameter uncertainties are added to the considered output vector, mass and inertia matrix, respectively. The synthesis of the adaptation laws is based on the positivity and Lyapunov design principle. Numerical simulations are performed showing the robustness of the proposed control technique.**

***Index Terms*—feedback linearization, adaptive control, adaptation mechanism, positivity principle, sliding mode.**

I. Introduction

UNMANNED aerial vehicles (UAV) have known a growing interest thanks to recent technological advances, especially those related to instrumentation which turn possible the design of powerful systems (mini drones) endowed with real capacities of autonomous navigation at reasonable cost [1].

These flying devices have been used in many civilian and military applications while new configurations and control laws should be developed to insure their applicability to new mission under more adverse environments. A special class of UAVs, called quadrotors, has been studied recently by some authors [1], [2], [3], [4], [5], [6]-[7]. These systems as many other dynamic systems, present constant or slowly-varying uncertain parameters.

Adaptive control provides many tools to tackle such systems. The basic idea in adaptive control is to estimate the uncertain plant parameters or, equivalently, is to perform an on-line estimation of the corresponding controller parameters based on the measured system signals, and to use the estimated parameters in the control input computation [8], [9], [10]-[11].

In this paper, The control of a quadrotor flying in an environment subject to atmospheric perturbation is considered where the controllability condition is checked. First, a dynamic modeling of a quadrotor is considered taking into account various parameters which can affect the dynamics of a flying structure such as frictions due to the aerodynamic torques, drag forces along (x, y, z) axis and gyroscopic effects. Then, an adaptive sliding mode controller is synthesized in order to stabilize both bank and pitch angles while tracking heading and altitude trajectories and to compensate additive perturbation and parameter uncertainties related to the mass and inertia matrix of the quadrotor. Adaptation laws are developed based-on Lyapunov approach where the asymptotic stability is guaranteed. Finally, numerical simulations are displayed, showing acceptable results for the proposed control solution.

II. Flight Dynamics Modeling

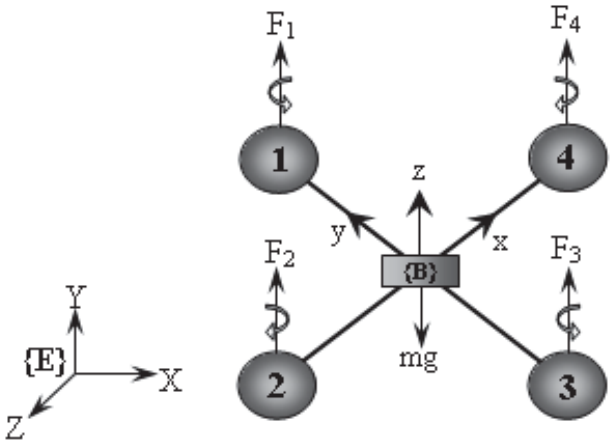


Fig. 1. Quadrotor Configuration

The quadrotor has four propellers in cross configuration where two pairs of propellers turn in opposite directions. By varying the rotor speeds, one can change the lift forces and create motion. Thus, increasing or decreasing evenly the four propellers speeds together generates vertical motion.

Changing evenly the 2 and 4 propellers speed conversely produces roll rotation coupled with lateral motion.

Pitch rotation and the corresponding lateral motion result from 1 and 3 propellers speed conversely modified. Yaw rotation is more subtle, as it results from the difference in the counter-torque between each pair of propellers.

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To limit the complexity of the dynamics modeling, the following assumptions are adopted [1]:

1. The quadrotor structure is rigid and symmetrical
2. The propellers are rigid
3. Thrust and drag forces are proportional to the square of propellers speed rotation.

Under these assumptions, it is possible to describe its flight dynamics as that of a rigid body under aerodynamic forces and moments caused by the propellers rotation. Using the formalism of Newton-Euler, the dynamics equations are written in the following way [1]:

$$m\ddot{\zeta} = F_f + F_d + F_g \tag{1a}$$

$$J\dot{\Omega} = -\Omega \wedge J\Omega + \Gamma_f - \Gamma_a - \Gamma_g \tag{1b}$$

where ζ is the position of the quadrotor centre of mass with respect to the inertial frame E, m is the total mass of the structure and $J \in \mathbb{R}^{3\times3}$ is a symmetric positive definite constant inertia matrix of the quadrotor with respect to the body fixed frame.

$$J = \begin{pmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{pmatrix} \tag{2}$$

Ω represents the angular velocity of the quadrotor expressed in the body fixed frame such as:

$$\Omega = \begin{pmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{pmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} \tag{3}$$

where ϕ, θ and ψ are bank, pitch and heading angles, respectively.

In the case when the quadrotor performs many angular motions of low amplitude, Ω can be assimilated to $[\dot{\phi}, \dot{\theta}, \dot{\psi}]^T$. F_f denotes the resultant of the forces generated by the four rotors such as:

$$F_f = \begin{pmatrix} \cos\phi\cos\psi\sin\theta + \sin\phi\sin\psi \\ \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ \cos\phi\cos\theta \end{pmatrix} \sum_{\mathbf{i=1}}^{\mathbf{4}} F_i \tag{4a}$$

$$F_i = K_p\omega_i^2 \tag{4b}$$

where K_p is a coefficient related to the lift force and ω_i is the angular rotor speed. F_d denotes the resultant of the drag forces along (X, Y, Z) axis such as:

$$F_d = \begin{pmatrix} -K_{fdx} & 0 & 0 \\ 0 & -K_{fdy} & 0 \\ 0 & 0 & -K_{fdz} \end{pmatrix} \dot{\zeta} \tag{5}$$

where K_{fdx}, K_{fdy} and K_{fdz} are positive translation drag coefficients. F_g is the gravity force and it is expressed as follows:

$$F_g = \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix} \tag{6}$$

Γ_f is the moment developed by the quadrotor according to the body fixed frame such as:

$$\Gamma_f = \begin{pmatrix} d(F_3 - F_1) \\ d(F_4 - F_2) \\ C_D(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \end{pmatrix} \tag{7}$$

d is the distance between the quadrotor centre of mass and the rotation axis of propeller and C_D is the drag coefficient. Γ_a is the resultant of aerodynamic frictions torques such as:

$$\Gamma_a = \begin{pmatrix} K_{fax} & 0 & 0 \\ 0 & K_{fay} & 0 \\ 0 & 0 & K_{faz} \end{pmatrix} \|\Omega\|^2 \tag{8}$$

where K_{fax}, K_{fay} and K_{faz} are the coefficients of aerodynamic frictions. Γ_g is the resultant of torques due to the gyroscopic effects:

$$\Gamma_g = \sum_{i=1}^{4} \Omega \wedge J_r \begin{pmatrix} 0 \\ 0 \\ (-1)^{i+1}\omega_i \end{pmatrix} \tag{9}$$

where J_r represents the rotor inertia.

Consequently the complete dynamic model which governs the quadrotor is as follows:

$$\ddot{\phi} = \frac{1}{I_x}\left[(I_y - I_z)\dot{\psi}\dot{\theta} - K_{fax}\dot{\phi}^2 - J_r\overline{\Omega}\dot{\theta} + dU_2\right] \tag{10a}$$

$$\ddot{\theta} = \frac{1}{I_y}\left[(I_z - I_x)\dot{\psi}\dot{\phi} - K_{fay}\dot{\theta}^2 + J_r\overline{\Omega}\dot{\phi} + dU_3\right] \tag{10b}$$

$$\ddot{\psi} = \frac{1}{I_z}\left[(I_x - I_y)\dot{\phi}\dot{\theta} - K_{faz}\dot{\psi}^2 + C_DU_4\right] \tag{10c}$$

$$\ddot{x} = \frac{1}{m}\left[-K_{fdx}\dot{x} + (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)U_1\right] \tag{10d}$$

$$\ddot{y} = \frac{1}{m}\left[-K_{fdy}\dot{y} + (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)U_1\right] \tag{10e}$$

$$\ddot{z} = \frac{1}{m}\left[-K_{fdz}\dot{z} + (\cos\phi\cos\theta)U_1\right] - g \tag{10f}$$

where U_1, U_2, U_3 and U_4 are the control inputs of the system which are written according to the angular velocities of the four rotors as follows:

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} = \begin{pmatrix} K_p & K_p & K_p & K_p \\ -K_p & 0 & K_p & 0 \\ 0 & -K_p & 0 & K_p \\ C_D & -C_D & C_D & -C_D \end{pmatrix} \begin{pmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{pmatrix} \tag{11a}$$

$$\overline{\Omega} = \omega_1 - \omega_2 + \omega_3 - \omega_4 \tag{11b}$$

The dynamic modeling developed above (10) is completed by the following control inputs constraints:

$$0 \leqslant U_1 \leqslant 4K_p\omega_{max}^2 \tag{12a}$$

$$-K_p\omega_{max}^2 \leqslant U_2 \leqslant K_p\omega_{max}^2 \tag{12b}$$

$$-K_p\omega_{max}^2 \leqslant U_3 \leqslant K_p\omega_{max}^2 \tag{12c}$$

$$-2C_D\omega_{max}^2 \leqslant U_4 \leqslant 2C_D\omega_{max}^2 \tag{12d}$$

The rotor is a unit constituted by D.C-motor actuating a propeller via a reducer. The D.C-motor is governed by the following dynamics:

$$V = RI + L\frac{dI}{dt} + K_e\omega \tag{13a}$$

$$K_mI = J_r\frac{d\omega}{dt} + K_r\omega^2 + C_s \tag{13b}$$

where V is the motor input tension. K_e and K_m represent the electrical and mechanical torque constant, respectively. K_r is the load constant torque. R and J_r are the motor internal resistance and the rotor inertia, respectively. C_s denotes the solid friction.

Then the model which characterizes the rotor dynamics is:

$$\dot{\omega}_i = bV + \beta_0 + \beta_1\omega_i + \beta_2\omega_i^2 \tag{14}$$

with:

$$\beta_0 = \frac{-C_s}{J_r} \quad \beta_1 = \frac{-K_eK_m}{RJ_r} \quad \beta_2 = \frac{-K_r}{J_r} \quad b = \frac{K_m}{RJ_r}$$

III. Control Design With Parameter Uncertainty

To synthesize adaptive laws, it is assumed that some uncertainty remains with respect to the main coefficients related to the aerodynamic torques, inertia matrix and mass of the system. The main control objective is here the stabilization of bank and pitch angles while tracking heading and altitude trajectories. In order to achieve safely this control objective a nonlinear adaptive sliding mode control is developed.

Let $X = [\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, z, \dot{z}]^T$ and $U = [U_2, U_3, U_4, U_1]^T$ be the state and the control input vectors, respectively. The equations (10) can be written under the following affine state representation such as:

$$\dot{X} = f(X) + g(X)U \tag{15a}$$

$$f(X) = \begin{pmatrix} x_2 \\ a_1x_4x_6 + a_2x_2^2 + a_3\overline{\Omega}x_4 \\ x_4 \\ a_4x_2x_6 + a_5x_4^2 + a_6\overline{\Omega}x_2 \\ x_6 \\ a_7x_2x_4 + a_8x_6^2 \\ x_8 \\ a_9x_8 - g \end{pmatrix} \tag{15b}$$

and $g(X) = [g_1, g_2, g_3, g_4]$ represents the control matrix:

$$g(X) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ b_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & b_3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\cos x_1 \cos x_3}{m} \end{pmatrix} \tag{16}$$

with:

$$a_1 = \frac{I_y - I_z}{I_x} \quad a_2 = \frac{-K_{fax}}{I_x} \quad a_3 = \frac{-J_r}{I_x} \quad a_4 = \frac{I_z - I_x}{I_y}$$
$$a_5 = \frac{-K_{fay}}{I_y} \quad a_6 = \frac{J_r}{I_y} \quad a_7 = \frac{I_x - I_y}{I_z} \quad a_8 = \frac{-K_{faz}}{I_z}$$
$$a_9 = \frac{-K_{fdz}}{m} \quad b_1 = \frac{d}{I_x} \quad b_2 = \frac{d}{I_y} \quad b_3 = \frac{C_D}{I_z}$$

The controllability matrix Υ for the system (15a) is given by:

$$\Upsilon = \Big[g_1, g_2, g_3, g_4, ad_fg_1, ad_fg_2, ad_fg_3, ad_fg_4\Big] \tag{17}$$

where ad_fg_i represents the Lie brackets such as:

$$ad_fg_i = \bigtriangledown g_i f - \bigtriangledown f g_i \tag{18}$$

$$det(\Upsilon) = \frac{b_1^2b_2^2b_3^2\cos x_1^2 \cos x_3^2}{m^2} \tag{19}$$

It can be concluded that $\forall(x_1, x_3) \in \mathbb{R}^2$ such that $(x_1, x_3 \neq \frac{\sqrt{2\pi}}{2})$ and $\forall(b_1, b_2, b_3) \in \mathbb{R}^{*3}$, the considered above system (15a) is locally controllable over $\mathbb{R}^8$.

Let $y = [\phi, \theta, \psi, z]^T$ be the output vector. The relative degree of the considered system is equal to 8 since:

$$\ddot{y}_1 = a_1x_4x_6 + a_2x_2^2 + a_3\overline{\Omega}x_4 + b_1U_2 \tag{20a}$$
$$\ddot{y}_2 = a_4x_2x_6 + a_5x_4^2 + a_6\overline{\Omega}x_2 + b_2U_3 \tag{20b}$$
$$\ddot{y}_3 = a_7x_2x_4 + a_8x_6^2 + b_3U_4 \tag{20c}$$
$$\ddot{y}_4 = a_9x_8 + \frac{\cos x_1 \cos x_3}{m}U_1 - g \tag{20d}$$

then, no internal dynamics is associated with this output vector. The equation (20a) can be rewritten under a linearly parameterized form such as:

$$U_2 = h_1\ddot{y}_1 - \lambda_1x_4x_6 - \lambda_2x_2^2 - \lambda_3\overline{\Omega}x_4 \tag{21}$$

with:

$$h_1 = \frac{1}{b_1} \quad \lambda_1 = \frac{a_1}{b_1} \quad \lambda_2 = \frac{a_2}{b_1} \quad \lambda_3 = \frac{a_3}{b_1}$$

To synthesize the adaptive control law U_2, it is assumed that the full state vector components are available through the measure and that the sign of h_1 is known [12]. We assume also that the parameters h_1 and λ_i are unknown constants.

Now, let us choose a sliding surface σ_1 such as:

$$\sigma_1 = \dot{y}_1 - v_1 \tag{22a}$$
$$v_1 = \dot{\phi}_d - k_1\epsilon_\phi \tag{22b}$$
$$\epsilon_\phi = \phi - \phi_d \tag{22c}$$

where ϵ_ϕ represents the tracking error, ϕ_d is the desired bank angle and k_1 denotes a real positive parameter. Now a candidate Lyapunov positive definite function $V_1(\sigma_1)$ is defined:

$$V_1(\sigma_1) = \frac{1}{2}\sigma_1^2 \tag{23a}$$
$$\dot{V}_1(\sigma_1) = \sigma_1\left[\frac{1}{h_1}(U_2 + \lambda_1x_4x_6 + \lambda_2x_2^2 + \lambda_3\overline{\Omega}x_4) - \dot{v}_1\right] \tag{23b}$$

then, if $\dot{V}_1(\sigma_1) = \sigma_1\dot{\sigma}_1 \leqslant 0$, asymptotic Lyapunov stability will be guaranteed. For that the control law could be chosen as:

$$U_2 = \hat{h}_1[\ddot{\phi}_d - k_1\dot{\epsilon}_\phi - k\sigma_1] - \hat{\lambda}_1x_4x_6 - \hat{\lambda}_2x_2^2 - \hat{\lambda}_3\overline{\Omega}x_4 \tag{24}$$

since the parameters λ_1, λ_2, λ_3 and h_1 are unknown, so they are replaced by their estimates in the control law (24) and k is a real positive parameter.

By replacing the synthesized control law expression (24) in the time derivative equation of the sliding surface σ_1, the closed loop dynamics can be written such as:

$$h_1\dot{\sigma}_1 + k\sigma_1 = \tilde{\lambda}_1x_4x_6 + \tilde{\lambda}_2x_2^2 + \tilde{\lambda}_3\overline{\Omega}x_4 - \tilde{h}_1[\ddot{\phi}_d - k_1\dot{\epsilon}_\phi] \tag{25}$$

where $\tilde{\lambda}_i$ and $\tilde{h}_1$ represent the estimation errors related to the controller parameters:

$$\tilde{\lambda}_i = \lambda_i - \hat{\lambda}_i \tag{26a}$$

$$\tilde{h}_1 = h_1 - \hat{h}_1 \tag{26b}$$

The closed-loop dynamics (25) shows that if the estimation errors related to the controller parameters converge to zero, then the tracking error dynamics tend to zero exponentially as shown:

$$h_1\dot{\sigma}_1 + k\sigma_1 = 0 \tag{27}$$

To synthesize an adaptation mechanism, the positivity and Lyapunov design principle is applied in order to determine the adaptation laws which allow the on-line estimation of the unknown controller parameters $\hat{h}_1$ and $\hat{\lambda}_i$. Consequently another Lyapunov positive definite function $V_2(\sigma_1, \tilde{\lambda}_i, \tilde{h}_1)$ is introduced:

$$V_2(\sigma_1, \tilde{\lambda}_i, \tilde{h}_1) = \frac{1}{2}|h_1|\sigma_1^2 + \frac{1}{2\gamma_1}\left[\tilde{h}_1^2 + \sum_{i=1}^{3}\tilde{\lambda}_i^2\right] \tag{28}$$

where γ_1 denotes the gain adaptation. The idea is to choose $\dot{\hat{h}}_1$ and $\dot{\hat{\lambda}}_i$ such that $\dot{V}_2(\sigma_1, \tilde{\lambda}_i, \tilde{h}_1) \leqslant 0$. Since:

$$\begin{aligned}\dot{V}_2(\sigma_1, \tilde{h}_1, \tilde{\lambda}_i) = \sigma_1\frac{|h_1|}{h_1}\Big[&\tilde{\lambda}_1 x_4 x_6 + \tilde{\lambda}_2 x_2^2 + \tilde{\lambda}_3\overline{\Omega}x_4 \\ &- h_1(\ddot{\phi}_d - k_1\dot{\epsilon}_\phi) - k\sigma_1\Big] \\ &- \frac{1}{\gamma_1}\left[\tilde{h}_1\dot{\tilde{h}}_1 + \sum_{i=1}^{3}\tilde{\lambda}_i\dot{\tilde{\lambda}}_i\right]\end{aligned} \tag{29}$$

choosing the adaptation laws $\dot{\tilde{h}}_1$ and $\dot{\tilde{\lambda}}_i$ such as:

$$\dot{\tilde{\lambda}}_1 = \gamma_1 sgn(h_1)\sigma_1 x_4 x_6 \tag{30a}$$

$$\dot{\tilde{\lambda}}_2 = \gamma_1 sgn(h_1)\sigma_1 x_2^2 \tag{30b}$$

$$\dot{\tilde{\lambda}}_3 = \gamma_1 sgn(h_1)\sigma_1\overline{\Omega}x_4 \tag{30c}$$

$$\dot{\tilde{h}}_1 = -\gamma_1 sgn(h_1)\sigma_1(\ddot{\phi}_d - k_1\dot{\epsilon}_\phi) \tag{30d}$$

implies that:

$$\dot{V}_2(\sigma_1, \tilde{h}_1, \tilde{\lambda}_i) = -|k|{\sigma_1}^2 \tag{31}$$

and the global tracking convergence of the adaptive control system is guaranteed.

The next control and adaptation laws are obtained by following the same steps presented above, therefore:

$$U_3 = \hat{h}_2[\ddot{\theta}_d - k_2\dot{\epsilon}_\theta - k'\sigma_2] - \hat{\lambda}_4 x_2 x_6 - \hat{\lambda}_5 x_4^2 - \hat{\lambda}_6\overline{\Omega}x_2 \tag{32}$$

with:

$$\dot{\hat{\lambda}}_4 = \gamma_2 sgn(h_2)\sigma_2 x_2 x_6 \tag{33a}$$

$$\dot{\hat{\lambda}}_5 = \gamma_2 sgn(h_2)\sigma_2 x_4^2 \tag{33b}$$

$$\dot{\hat{\lambda}}_6 = \gamma_2 sgn(h_2)\sigma_2\overline{\Omega}x_2 \tag{33c}$$

$$\dot{\hat{h}}_2 = -\gamma_2 sgn(h_2)\sigma_2(\ddot{\theta}_d - k_2\dot{\epsilon}_\theta) \tag{33d}$$

and

$$U_4 = \hat{h}_3[\ddot{\psi}_d - k_3\dot{\epsilon}_\psi - k''\sigma_3] - \hat{\lambda}_7 x_2 x_4 - \hat{\lambda}_8 x_6^2 \tag{34}$$

with:

$$\dot{\hat{\lambda}}_7 = \gamma_3 sgn(h_3)\sigma_3 x_2 x_4 \tag{35a}$$

$$\dot{\hat{\lambda}}_8 = \gamma_3 sgn(h_3)\sigma_3 x_6^2 \tag{35b}$$

$$\dot{\hat{h}}_3 = -\gamma_3 sgn(h_3)\sigma_3(\ddot{\psi}_d - k_3\dot{\epsilon}_\psi) \tag{35c}$$

and

$$U_1 = \frac{1}{\cos x_1 \cos x_3}\left[\hat{\lambda}_9 x_8 + \hat{m}(g + \ddot{z}_d - k_4\dot{\epsilon}_z - k'''\sigma_4)\right] \tag{36}$$

with

$$\dot{\hat{\lambda}}_9 = -\gamma_4\sigma_4 x_8 \tag{37a}$$

$$\dot{\hat{m}} = -\gamma_4\sigma_4(\ddot{z}_d - k_4\dot{\epsilon}_z + g - k'''\sigma_4) \tag{37b}$$

where the sliding surfaces σ_2, σ_3 and σ_4 are such as:

$$\sigma_2 = \dot{y}_2 - \dot{\theta}_d - k_2\epsilon_\theta \tag{38a}$$

$$\sigma_3 = \dot{y}_3 - \dot{\psi}_d - k_3\epsilon_\psi \tag{38b}$$

$$\sigma_4 = \dot{y}_4 - \dot{z}_d - k_4\epsilon_z \tag{38c}$$

and ϵ_θ, ϵ_ψ and ϵ_z denote the tracking errors related to the pitch, heading and altitude, respectively:

$$\epsilon_\theta = \theta_d - \theta \tag{39a}$$

$$\epsilon_\psi = \psi_d - \psi \tag{39b}$$

$$\epsilon_z = z_d - z \tag{39c}$$

Adopted control strategy is shown in the Fig. 2 below:

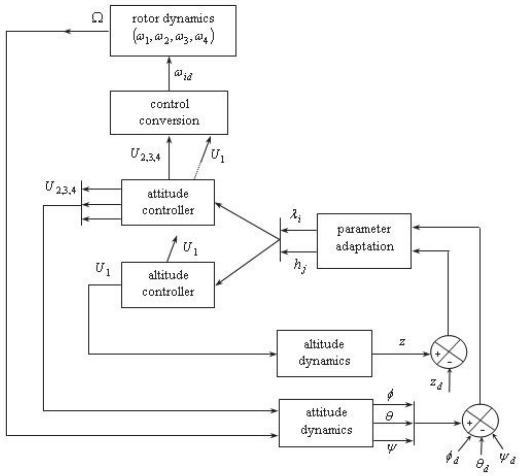


Fig. 2. Adopted Control Strategy

IV. Numerical Simulation

The simulation results are obtained based on the real parameters in Table. 1 shown below.

1. Results without perturbation are shown in Fig. 3 to Fig. 10.
2. Flight with parametric uncertainties:
   Two cases are treated. First, for an uncertainty of 20% on m and 10% on (I_x, I_y, I_z). After, for an uncertainty of 35% on m and 15% on (I_x, I_y, I_z). Desired trajectories tracking and parameters estimation are shown below (Fig. 11 to Fig. 14):

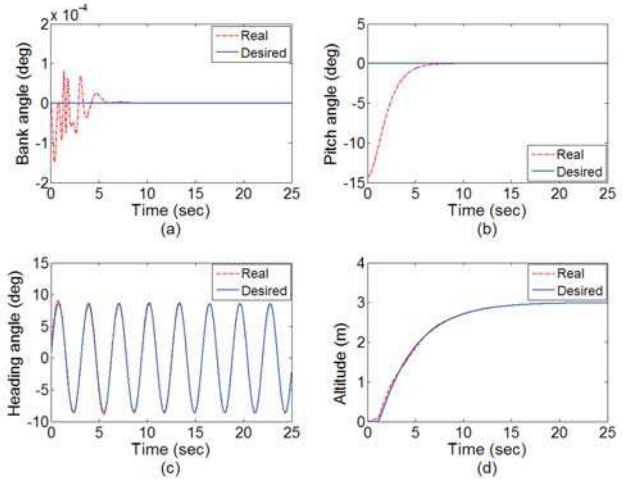


Fig. 3. Desired trajectories tracking related to the bank, pitch, heading angles and altitude, respectively

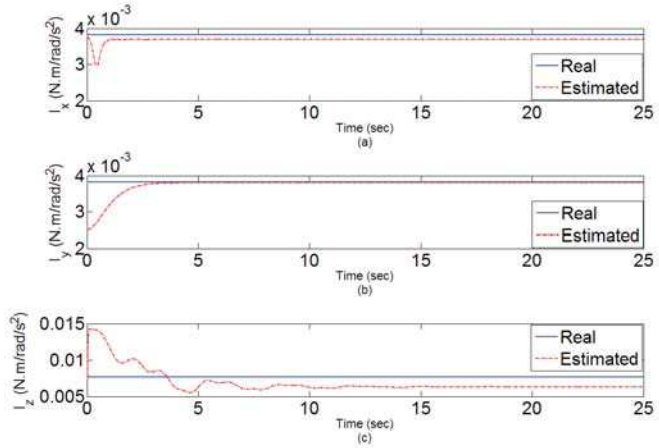


Fig. 4. Simulation results of inertial moments estimation

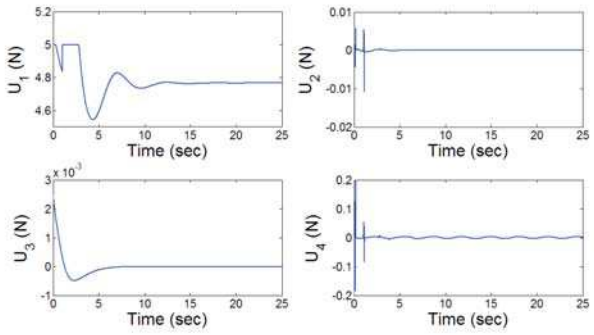


Fig. 5. Control inputs

V. Conclusion

In this paper, a direct adaptive sliding mode control have been developed in order to stabilize roll and pitch motions while tracking heading and altitude trajectories of a quadrotor.

During simulations, two cases are treated. the First one is considered for an uncertainty of 20% on the quadrotor mass

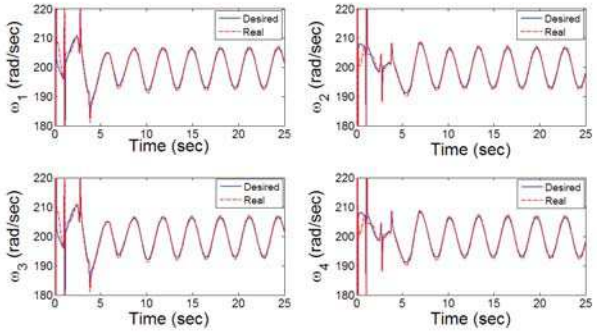


Fig. 6. Desired rotors velocities tracking

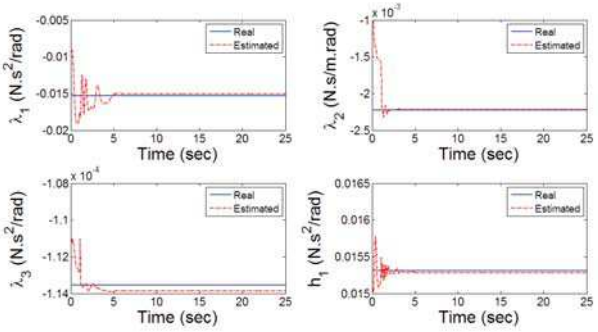


Fig. 7. Bank controller parameters estimation

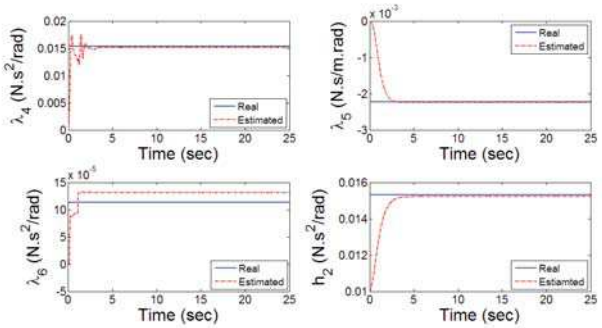


Fig. 8. Pitch controller parameters estimation

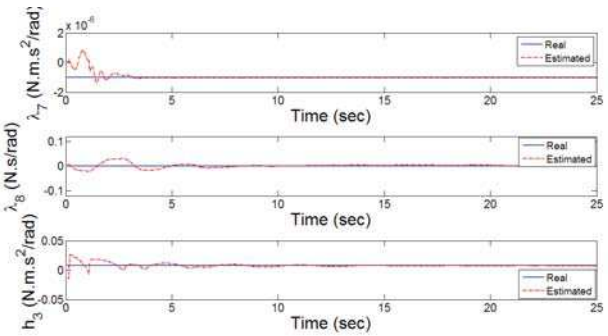


Fig. 9. Heading controller parameters estimation

m and 10% on the inertia matrix $diag(I_x, I_y, I_z)$ while the second case is for an uncertainty of 35% on m and 15% on inertia matrix. Both of them are subject to an additional

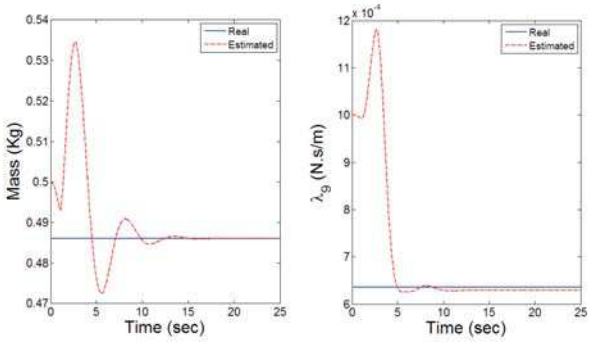


Fig. 10. Altitude controller parameters estimation

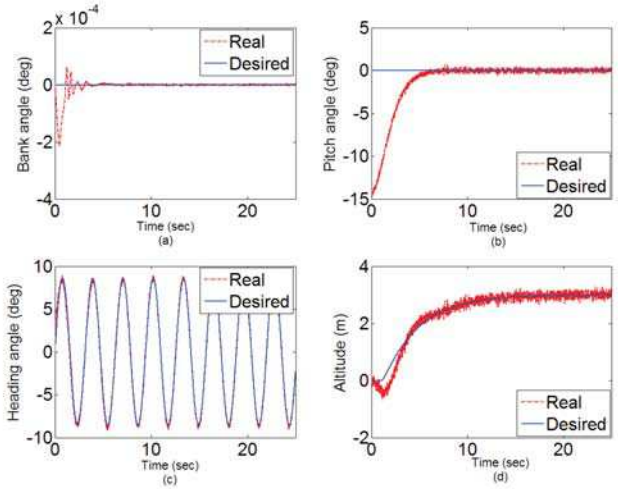


Fig. 11. Desired trajectories tracking for uncertainties of 20% on m and 10% on (I_x, I_y, I_z)

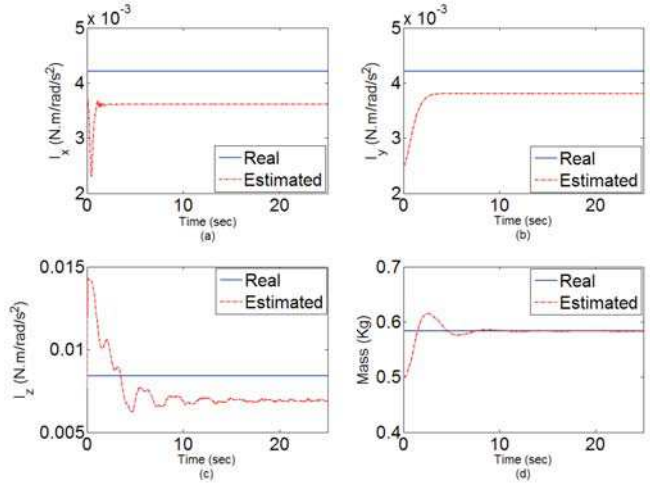


Fig. 12. Inertia matrix and mass estimation for uncertainties of 20% on m and 10% on (I_x, I_y, I_z)

centered white gaussian noise.

Controller parameters estimation is based on the positivity and Lyapunov design principle. Considered unknown coefficients are related to the aerodynamic torques, inertia matrix and mass, respectively.

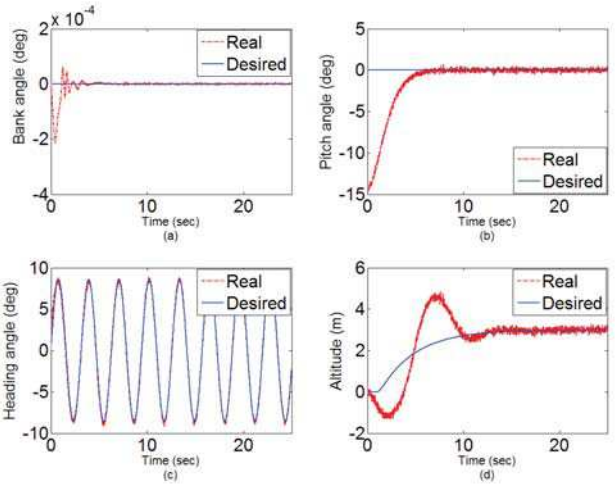


Fig. 13. Desired trajectories tracking for uncertainties of 35% on m and 15% on (I_x, I_y, I_z)

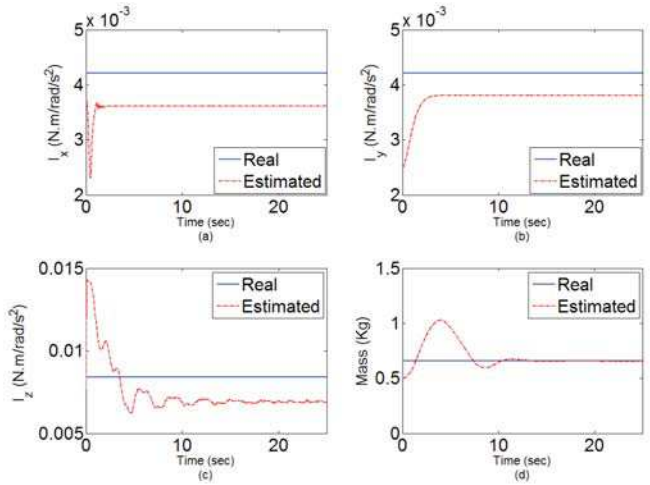


Fig. 14. Inertia matrix and mass estimation for uncertainties of 35% on m and 15% on (I_x, I_y, I_z)

The effectiveness of the adaptive control laws have been evaluated by numerical simulations, showing good convergence results.

Further study will focus in a full state control of the quadrotor engaged in complex trajectory tracking.

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TABLE I
QUADROTOR GENERAL PARAMETERS

Parameter	Value
Mass (Kg)	0.486
Distance d (m)	0.25
I_x $(N.m/rad/s^2)$	3.8278×10^{-3}
I_y $(N.m/rad/s^2)$	3.8278×10^{-3}
I_z $(N.m/rad/s^2)$	7.6566×10^{-3}
K_{fax} $(N/rad/s)$	5.5670×10^{-4}
K_{fay} $(N/rad/s)$	5.5670×10^{-4}
K_{faz} $(N/rad/s)$	6.3540×10^{-4}
K_{fdx} $(N/m/s)$	5.5670×10^{-4}
K_{fdy} $(N/m/s)$	5.5670×10^{-4}
K_{fdz} $(N/m/s)$	6.3540×10^{-4}
Lift force coefficient K_p $(N.m/rad/s)$	2.9842×10^{-5}
Drag force coefficient C_D $(N.m/rad/s)$	3.2320×10^{-7}
Rotor inertia J_r $(N.m/rad/s^2)$	2.8385×10^{-5}
β_0	189.63
β_1	6.0612
β_2	0.0122
b	280.19

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