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We dedicate this paper to Pascal Brisset, our friend and colleague who died on May 22th 2010 in a tragic hiking accident. Pascal, we miss you so much!

Abstract A mixed-integer nonlinear optimization model is presented for the resolution of aircraft conflict. Aircraft conflicts occur when aircraft sharing the same airspace are "too close" to each other and represent a crucial problem in Air Traffic Management. We describe the model and show some numerical experiments.

Keywords: aircraft conflict aivoidance, modeling, global optimization, MINLP

1. Introduction

The problem of detecting and solving aircraft conflicts, that occur when the distance between two aircraft sharing the same airspace is less than a given safety distance, is crucial in Air Traffic Management to guarantee air traffic safety. Currently, the resolution of conflicts is still largely performed manually by air traffic controllers watching the movement of traffic on a radar screen. Therefore, a great interest is devoted to the development of automatic tools.

One aims at solving a conflict while deviating as little as possible from the original flight plan. An optimization problem can thus be naturally defined. Notwithstanding the importance of the problem and the urgent need of automatic tools able to integrate human work to face the growing air traffic security requirements, there is still a need for suitable models. Different models have been proposed based on allowing both heading angle deviation and speed change maneuvers, either in a centralized [5][6][7] or in an autonomous [4][3] approach. The advantages of subliminal control using only small speed adjustments were shown in the ERASMUS [2] project. In this paper, we propose a new model for air conflict avoidance based on velocity changes. It is mixed-integer because it requires the use of continuous and discrete variables, in particular 0-1 variables to represent logic choices, and involves nonlinear terms. The model is then in the area of Mixed-Integer Nonlinear Programming. In the following sections we describe the model and we show some computational results obtained using a general-purpose global optimization solver.

2. Modelization

Aircraft are assumed to be flying on a horizontal plane and are identified by points in the plane. We propose a model based on instantaneous velocity changes, while the trajectory is kept unchanged. The main idea is to deal with the different time windows where aircraft fly with their original (known) speed v or with a changed speed $v+q$, q representing a possible positive or negative speed change. Time windows are defined by instant times such that each aircraft changes its original velocity, i.e., it starts or ends flying with speed $v+q$. Because of the assumption of instantaneous velocity changes, we can consider uniform motion laws in each time window, where the velocity to be considered for each aircraft k is v_k or $v_k + q_k$ depending on the time configuration. There are 6 possible time configurations, obtained considering permutations of instant times when aircraft change their speed, and, for each time configuration, 5 time intervals have to be taken into account. Given a pair of aircraft i and j, let t_{1i}, t_{1j} and t_{2i}, t_{2j} be the instant times when i and j start and respectively end flying with changed speed. An order for $t_{1i}, t_{2i}, t_{1j}, t_{2j}$ is not a priori known. By permutations of these instant times, excluding some cases giving rise to inconsistency (i.e., taking into account that $\forall k \ t_{1k} \le t_{2k}$ and so a time sequence always starts with a t_1 instant and ends with a t_2 one), we obtain the following time configurations, where T represents the upper bound on time instants:

$$0 \le t_{1i} \le t_{1j} \le t_{2i} \le t_{2j} \le T \tag{1}$$
$$0 \le t_{1j} \le t_{1i} \le t_{2i} \le t_{2j} \le T \tag{2}$$
$$0 \le t_{1i} \le t_{2i} \le t_{1j} \le t_{2j} \le T \tag{3}$$
$$0 \le t_{1j} \le t_{2j} \le t_{1i} \le t_{2i} \le T \tag{4}$$
$$0 \le t_{1i} \le t_{1j} \le t_{2j} \le t_{2i} \le T \tag{5}$$
$$0 \le t_{1j} \le t_{1i} \le t_{2j} \le t_{2i} \le T. \tag{6}$$

Each of these configurations defines 5 time intervals.

We use as decision variables:

- $\forall k \in A \quad q_k$, where A is the set of aircraft, expressing the speed change of aircraft. Note that q_k can be positive (acceleration), negative (deceleration) and null (if there is no speed change). We impose, as it is done in practice, that the speed change for aircraft k cannot be greater than $+3\%$ and smaller than -6% of its original speed.
- $\forall k \in A \quad t_{1k}, t_{2k}$, representing the instant times such that aircraft k starts and respectively ends flying with changed speed. Instant time are always ≥ 0 and have an upper bound T. They are such that $t_{1k} \le t_{2k}$.

We also employ auxiliary variables to model the problem, both continuous and integer (and in particular binary). Suitable integer variables are in particular used to describe all possible time configurations.

We aim at obtaining conflict avoidance with the minimum speed change for aircraft that should fly with changed speed during a time interval which also has to be minimized. We then use as objective function:

$$\min \sum_{k \in A} q_k^2 (t_{2k} - t_{1k})^2. \tag{7}$$

We impose a number of constraints that are used to handle time configurations and to express aircraft separation conditions in each time interval.

Firstly, the interval time for speed change must be at least equal a certain amount t_{min}:

$$\forall k \in A \quad t_{2k} - t_{1k} \ge t_{min}. \tag{8}$$

Modeling all possible time configurations needs the introduction of binary variables z_ℓ, $\ell \in \{1, \ldots, 6\}$ stating, for each time configuration, what is the order of instant times for that configuration. So, for example, the binary variable z_1 is such that:

$$t_{1i} \le t_{1j} \quad \text{and} \quad t_{1j} \le t_{2i} \quad \text{and} \quad t_{2i} \le t_{2j}. \tag{9}$$

The following constraint imposes that only one configuration must hold:

$$\sum_{\ell \in \{1,\ldots,6\}} z_\ell = 1. \tag{10}$$

Aircraft separation is expressed by the following condition:

$$||\overrightarrow{x}^r(t)|| \geq d, \tag{11}$$

where d is the minimum required separation distance and $\overrightarrow{x}^r(t)$ is given by

$$\overrightarrow{x}^r(t) = \overrightarrow{x}^{rd}_{ij} + \overrightarrow{v}^r_{ij} t, \tag{12}$$

where $\overrightarrow{x}^{rd}_{ij}$ is the relative initial position of aircraft i and j and $\overrightarrow{v}^r_{ij}$ their relative speed. Squaring (11) and deriving with respect to t, one can see that the minimum is attained for $t_m = -\frac{\overrightarrow{v}^r_{ij} \overrightarrow{x}^{rd}_{ij}}{(v^r_{ij})^2}$. We are only interested in the minimum in each interval $[t_s, t_{s'}]$ Substituting, the following separation condition is obtained:

$$(x^{rd}_{ij})^2 - \frac{(\overrightarrow{v}^r_{ij} \overrightarrow{x}^{rd}_{ij})^2}{(v^r_{ij})^2} - d^2 \geq 0. \tag{13}$$

Initial position in each time interval, relative distances and speeds between aircraft are then exploited, and new variables introduced accordingly. Distances covered by aircraft during each time interval are computed exploiting laws of uniform motion because of the aircraft's constant speed in each of such intervals. In the h-th time interval $[t_s, t_{s'}]$, $h \in \{1, \ldots, 5\}$, for all aircraft $k \in A$ the initial position x_{kh} is given by

$$x_{kh} = x_{k(h-1)} + (t_{s'} - t_s)\bar{v}_k, \tag{14}$$

where $\bar{v}_k$ is the original speed v_k or the changed speed $v_k + q_k$, depending on the time configuration holding. So, (continuous) variables x_{kh} $\forall k \in A$ $\forall h \in \{1, \ldots, 5\}$, are introduced and corresponding constraints added to the formulation, expressing for each aircraft the 5 initial positions in the 5 time intervals. Each aircraft k has speed $\bar{v}_k$ equal to its original speed or to the changed speed depending on the time configuration, so that variables z_ℓ are used to identify the configuration holding. Relative distances x^{rd}_{ij} and relative speeds v^r_{ij} between aircraft are also defined, for each time configuration and each time interval, and constraints adjoined accordingly using variables x_{kh} and again z_ℓ.

Further constraints are then adjoined to the model to impose the condition (13) in each of the 5 time intervals, when $t_m \in [t_s, t_{s'}]$, where $[t_s, t_{s'}]$ is the h-th time interval. In order to check if $t_m \in [t_s, t_{s'}]$, binary variables are used. For all $h \in \{1, \ldots, 5\}$ a binary variable y_{lh} is introduced such that $y_{lh} = 1$ if $t_{sh} \leq t_{mh}$ and 0 otherwise, y_{rh} is such that $t_{mh} \leq t_{s'h}$ and 0 otherwise. The following constraints are then imposed:

$$\forall h \in \{1, \ldots, 5\} \qquad t_{sh} \leq t_{mh} + M(1 - y_{lh}), \quad t_{mh} \leq t_{sh} + M y_{lh} \tag{15}$$

$$t_{mh} \leq t_{s'h} + M(1 - y_{rh}), \quad t_{s'h} \leq t_{mh} + M y_{rh} \tag{16}$$

with M sufficiently large. Condition (13) is then imposed for each time configuration $\ell \in \{1, \ldots, 6\}$, $\forall h \in \{1, \ldots, 5\}$ and $i, j \in A$, as follows:

$$\left(y_{lh} y_{rh} \left((x^{rd}_{ijh})^2 - \frac{(v^r_{ijh} x^{rd}_{ijh})^2}{(v^r_{ijh})^2} - d^2 \right) \right) \geq 0 \tag{17}$$

Finally, for each time interval, the following separation condition is also imposed:

$$\forall h \in \{1, \ldots, 5\}, \forall i, j \in A \qquad (x^{rd}_{ijh})^2 \geq d^2. \tag{18}$$

3. Computational experience

We carried out preliminary computational experiments considering a pair of aircraft. The two aircraft are supposed to move from an initial position given, in 2-dimensional space, by $(-100, 0)$ and $(0, -100)$ respectively and with a velocity $v = 400Nm/h$. Separation distance d is equal to $5Nm$. Aircraft k is assumed to change its speed on an instant t_{1k} and keep the new speed. We solved the problem using the `Couenne` [1] software for MINLP, obtaining the following optimal solution:

$$q_1 = -0.05636 \times v,\ q_2 = 0.02492 \times v,\ t_{11} = 0.00611072,\ t_{12} = 0.0115235, \tag{19}$$

corresponding to the objective function value 0.00086678. This solution required 1.99 seconds of CPU time on a 2.4 GHz CPU.

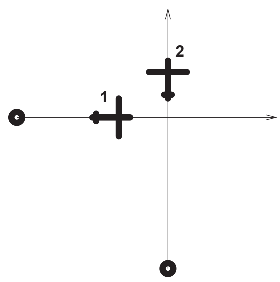

Figure 1. Example of conflict resolution, as described in Sect.3. The conflict in (0,0) is solved by decelerating the first aircraft and accelerating the second one in an optimal way.

4. Summary

We presented a mixed-integer nonlinear model for the problem of aircraft conflict resolution, a challenging problem in Air Traffic Management. In this model, conflicts are avoided allowing aircraft to only accelerate or decelerate in a time window, and speed changes are minimized together with time windows when they occur. Preliminary computational experiments show that the model is promising in air conflict resolution. We plan to extend the proposed model to the case of $n > 2$ aircraft.

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