

# Energy rate prediction using an equivalent thrust setting profile

Richard Alligier, David Gianazza, Nicolas Durand

► **To cite this version:**

Richard Alligier, David Gianazza, Nicolas Durand. Energy rate prediction using an equivalent thrust setting profile. ICRAT 2012, 5th International Conference on Research in Air Transportation, May 2012, Berkeley, United States. pp xxxx. hal-00938734

**HAL Id: hal-00938734**

**<https://hal-enac.archives-ouvertes.fr/hal-00938734>**

Submitted on 24 Apr 2014

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Energy Rate Prediction Using an Equivalent Thrust Setting Profile

R. Alligier, D. Gianazza, N. Durand  
*ENAC/MAIAA, IRIT/APO Toulouse, France*

**Abstract**—Ground-based aircraft trajectory prediction is a major concern in air traffic management. A safe and efficient prediction is a prerequisite for the implementation of automated tools that detect and solve conflicts between trajectories. This paper focuses on the climb phase because predictions are less accurate in this phase. The Eurocontrol BADA<sup>1</sup> model, as a total energy model, relies on the prediction of energy rate.

In a kinetic model, this energy rate comes from the power provided by the forces applied to the aircraft. Computing these forces requires knowledge of the aircraft state (mass, airspeed, etc), atmospheric conditions (wind, temperature) and aircraft intent (maximum climb thrust or reduced climb thrust, for example). Some of this information like the mass and thrust setting are not available to ground-based systems.

In this paper, we try to infer an equivalent weight and an equivalent thrust profile. These parameters are not meant to be true, however they are designed to improve the energy rate prediction. One common thrust setting profile for all the trajectories is built. This thrust profile is designed in such a way that the estimated equivalent weight provides a good energy rate prediction. We have compared the energy rate prediction using these equivalent parameters and BADA standard parameters.

**Keywords:** trajectory prediction, energy rate, equivalent weight, thrust setting.

## INTRODUCTION

Predicting aircraft trajectories with great accuracy is central to most operational concepts ([1], [2]) and necessary to the automated tools that are expected to improve the air traffic management (ATM) in the near future. The literature on trajectory prediction is fairly wide, and one may refer to [3] for a literature survey on the subject, or [4], [5], or [6] for the statistical analysis and validation of trajectory predictors. Other works focus on the benefits provided to ground-based trajectory predictors by using additional, more accurate, input data ([7], [8], [9]). A good introduction on the use of parametric and non-parametric regression methods for trajectory prediction can be found in [10]. An interesting model-based stochastic approach is presented in [11], although only validated in a simulation environment.

On-board flight management systems predict the aircraft trajectory using a point-mass model of the forces applied to the center of gravity. This model is formulated as a set of differential algebraic equations that must be integrated over a time interval in order to predict the successive aircraft positions in this interval. The point-mass model requires knowledge of the aircraft state (mass, thrust, etc), atmospheric conditions (wind,

temperature), and aircraft intent (target speed or climb rate, for example).

Many of these information are not available to ground-based systems, and those that are available are not known with great accuracy. As a consequence, ground-based trajectory prediction is currently fairly inaccurate, compared to the on-board prediction. A simple solution would be to downlink the on-board prediction to the ground systems. However, this is not sufficient for all applications: some algorithms ([12]) require the computation of a multitude of alternate trajectories that could not be computed and downlinked fast enough by the on-board predictor. There is a need to compute trajectory predictions in ground systems, for all traffic in a given airspace, with enough speed and accuracy to allow a safe and efficient 4D-trajectory conflict detection and resolution.

Thus, downlinking these missing information might not be a solution. Currently, the atmospheric conditions are estimated through meteorological models. Ground-based trajectory predictors make fairly basic assumptions on the aircraft intent (see the "airlines procedures" that go with the BADA model). These default "airline procedures" may not reflect the reality, where the target speeds are chosen by the pilots according to a cost index that is a ratio between the cost of operation and the fuel cost. These costs are specific to each airline operator, and not available in the public domain. The actual aircraft mass is currently not transmitted to the ATM ground systems, although this is being discussed in the EUROCAE group in charge of elaborating the next standards for air-ground datalinks. However, airline operators are reluctant to do this since the mass is a sensitive data.

In this context, this paper focuses on the equivalent weight concept as a workaround to use the BADA point mass model without knowing the actual aircraft mass. This concept of equivalent weight was first discussed in a study [13] based on synthetic data. Assuming a thrust setting and past vertical rates to be known, the equivalent weight is the mass minimizing the gap between computed vertical rates and observed vertical rates. A second study [14] raises doubts about the reliability of the vertical rate for this purpose. It suggests to use the energy rate instead.

Our study focuses on the energy rate prediction, using Mode C radar data and a weather model as input. We try to improve the energy rate prediction by inferring missing parameters: the mass and the thrust setting. For one given trajectory, inferring altogether these two parameters leads to degeneracy issues. To overcome this difficulty, we assume a common thrust setting

<sup>1</sup>BADA: Base of Aircraft Data

profile for all trajectories. This "equivalent thrust" profile is computed using a least square regression method and a set of recorded trajectories as input. Finally, our system is composed of two complementary elements and has been tested on actual data.

The first element is an equivalent weight estimation process. Considering an aircraft, we use the information contained in its past trajectory. With the available information, only the thrust setting is missing to process the equivalent weight. Assuming a known thrust setting profile, the equivalent weight is estimated by minimizing the gap between the computed energy rate and the past observed values.

The second element is the thrust setting profile used for mass estimation and energy rate prediction. Using a large set of trajectories, we design a thrust setting profile oriented towards minimum prediction error.

Using a large validation set, we compare the performance of prediction to the BADA model on fresh data using different setups.

The rest of this paper is organized as follows: section I introduces a widely used simplified point-mass model, BADA. Section II introduces a weight estimation process and its use. The next section, III, describes a way to build a thrust profile using the weight estimation process. The dataset and experimental setup are detailed in section IV, and results are shown in section V.

## I. THE POINT-MASS MODEL

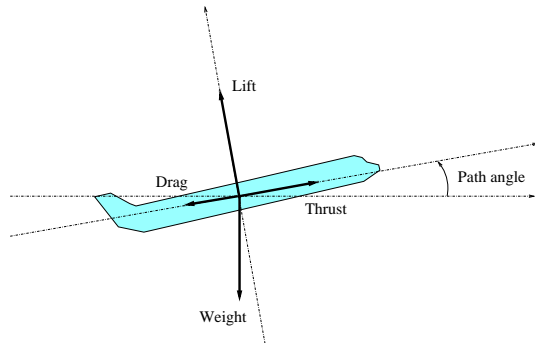


Figure 1. Simplified point-mass model.

### A. Simplified Equations

Most ground systems use a simplified point-mass model, sometimes called *total energy model*, to predict aircraft trajectories. This model, illustrated on figure 1, describes the forces applying to the center of gravity of the aircraft and their influence on the aircraft acceleration, making several simplifying assumptions<sup>2</sup>. It is assumed that the thrust and drag vectors are colinear to the airspeed vector, and that the lift is perpendicular to these vectors. Thus, projecting the forces on the airspeed vector axis, the longitudinal acceleration

<sup>2</sup>Note that more complex point-mass models have been proposed for UAV or fighter airplanes (see [15]), modeling also the side-slip angle.

$a = \frac{dV_{TAS}}{dt}$  along the true airspeed ( $V_{TAS}$ ) axis can be expressed as follows:

$$m.a = T - D - m.g.\sin(\gamma) \quad (1)$$

where  $T$  is the total thrust,  $D$  the aerodynamic drag of the airframe,  $m$  the aircraft mass,  $g$  the gravitational acceleration and  $\gamma$  the path angle (i.e. the angle between the airspeed vector and the horizontal plane tangent to the earth surface).

Introducing the rate of climb/descent  $\frac{dh}{dt} = V_{TAS}.\sin(\gamma)$ , where  $h$  is the altitude in meter, this equation can be rewritten as follows (see [16]):

$$\underbrace{(T - D).V_{TAS}}_{\text{power}} = \underbrace{m.V_{TAS}.\frac{dV_{TAS}}{dt} + m.g.\frac{dh}{dt}}_{\text{total energy variation}} \quad (2)$$

The left member of this equation can be seen as the power of the forces applied to the aircraft, and the right member can be seen as the variation of total energy, the energy rate. Several equivalent forms of this equation can be used (see Eurocontrol BADA User Manual), depending on what unknown variable is being calculated from the other known variables.

### B. The BADA Power Reduction Profile

In the 3.9 BADA model, the forces involved in power calculation and fuel consumption are described as parameterized functions. A Matlab toolbox developed for identification purpose, BEAM, is then used to estimate the value of these parameters [18]. This process relies on a set of 17 well chosen reference trajectories. The parameters are estimated in order to fit the fuel consumption and the rate of climb. Finally, 7 thrust settings are obtained. Each thrust setting is associated to a flight phase. Only one thrust setting is associated to the climb phase.

However, according to [16], many aircraft use a reduced setting during climb in order to extend engine life and save cost. A correction factor  $c_{red}$  is applied to the power computed with the climb thrust setting. This correcting factor has been obtained in an empirical way and has been validated with the help of air traffic controllers.

$$\frac{\text{power}}{m} = c_{red}.\frac{(T - D)}{m}.V_{TAS} \quad (3)$$

With

$$c_{red} = \begin{cases} 1 - 0.15\frac{m_{max} - m}{m_{max} - m_{min}} & \text{if } H_p \leq 0.8H_{max} \\ 1 & \text{otherwise} \end{cases} \quad (4)$$

### C. Discussion

Actually, using equation 2 to predict a trajectory requires a model of the aerodynamic drag for any airframe flying at a given speed through the air. In addition, we need the standard climb thrust, which depends on what engines the aircraft is equipped with. In the experiments presented here, the Eurocontrol BADA model was used to that purpose.

In addition, one cannot use equation 2 without prior knowledge of the state (mass, position, speed,...) of the aircraft, and also of the pilot's intents as to how the aircraft is operated (actual thrust setting). When they are not downlinked from the aircraft, some state variables like the true air speed (TAS) require knowledge of the atmospheric conditions (the air temperature, the wind and pressure) in order to be computed.

One is usually interested in computing the variation of state variables like  $\frac{dV_{TAS}}{dt}$  and  $\frac{dh}{dt}$ . If we assume the current state, the thrust law and the mass to be known, equation 2 is useful since it provides the equation 5.

$$V_{TAS} \cdot \frac{dV_{TAS}}{dt} + g \cdot \frac{dh}{dt} = \frac{power_{computed}}{m_{known}} \quad (5)$$

Given the equation 5 and knowing the repartition<sup>3</sup> of the specific power  $\frac{power_{computed}}{m_{known}}$  between  $\frac{dV_{TAS}}{dt}$  and  $\frac{dh}{dt}$ , it is easy to compute the acceleration  $\frac{dV_{TAS}}{dt}$  and the rate of climb  $\frac{dh}{dt}$ . This is not the purpose of this paper, however. In this study, we are only interested in computing the specific power as precisely as possible.

To go deeper in our analysis, we have to study the forces applied to the aircraft. The standard climb thrust  $T_{std}$  is modeled as a function of the true air speed  $V_{TAS}$ , the geopotential pressure altitude  $H_p$  and the temperature differential  $\Delta T$  (see [17]). However, this standard climb thrust is not always actually used, so we introduce a thrust coefficient  $c$ . Therefore, the effective thrust is  $c \cdot T_{std}$ . The drag  $D$  is a function of the mass  $m$ , the true air speed  $V_{TAS}$ , the geopotential pressure altitude  $H_p$  and the temperature differential  $\Delta T$ . Finally, the specific power is given by the equation 6 below.

$$\frac{power}{m} = \frac{c \cdot T_{std}(V_{TAS}, H_p, \Delta T) - D(V_{TAS}, H_p, \Delta T, m)}{m} \cdot V_{TAS} \quad (6)$$

In a ground-based context, having radar data augmented with a weather model, there are only two missing variables to use this formula, the mass  $m$  and the thrust coefficient  $c$ . These variables have a great impact on the specific power.

Without any additional knowledge, we can use this formula with the BADA reference mass and the BADA reduced climb power<sup>4</sup>. According to [16], the BADA reduced climb power was obtained in an empirical way. In this paper, we extract a common thrust coefficients profile from a large set of recorded trajectories. Using this thrust profile, we infer the mass of each new aircraft using its past trajectory in order to improve the accuracy of the computed specific power.

## II. EQUIVALENT WEIGHT ESTIMATION

This concept was developed in [13]. In his paper, Warren wants to find an "equivalent weight" such that the predicted vertical rate matches the measured vertical rate. Thus, he has to set the share factor which rules the repartition of the available power between kinetic energy and potential energy. Here we directly compute the specific power, and we want it to be equal to the observed energy rate  $V_{TAS} \cdot \frac{dV_{TAS}}{dt} + g \cdot \frac{dh}{dt}$ . We do not have

<sup>3</sup>In BADA, the energy share factor rules this repartition.

<sup>4</sup>In BADA, instead of having a  $c$  coefficient, there is a  $c_{red}$  which is not applied the same way as ours, the BADA formula is  $power = c_{red}(T_{std} - D) \cdot V_{TAS}$

to make any assumption on the airspeed law or a climbing rate or a share factor. However, we have to make an hypothesis on the chosen thrust coefficient  $c$ .

### A. Equivalent Mass at a Given Point

At a given point  $i$ , knowing  $V_{TAS}$ ,  $H_p$ ,  $\Delta T$  and  $c$ , we have :

$$\frac{Power(V_{TAS}^{(i)}, H_p^{(i)}, \Delta T^{(i)}, c^{(i)}, m)}{m} = V_{TAS}^{(i)} \frac{dV_{TAS}^{(i)}}{dt} + g_0 \frac{dh^{(i)}}{dt} \quad (7)$$

The equivalent mass  $m$  is obtained by solving the above equation 7 which can be reduced in a polynomial equation of the second degree in  $m$ , giving us the equation 8.

$$\frac{P_i(m)}{m} = 0 \quad (8)$$

Then, equation 8 can be solved analytically, giving us two possible solutions. In our experimentations, only one solution was positive. Thus, when applying this method independently at each point  $i$  of a trajectory, we observed great variations in the weight estimation, that cannot be explained solely by the fuel consumption. We think that these variations mostly come from bad hypotheses on  $c$  and poor quality of radar data. However, we still need to investigate on this issue.

### B. Equivalent Mass Using a Set of Points

In order to reduce the error due to the lack of accuracy on  $\frac{dh}{dt}$ ,  $\frac{dV_{TAS}}{dt}$ ,  $H_p$ ,  $V_{TAS}$  and  $\Delta T$ , we now consider a set of  $n$  points, knowing the chosen thrust coefficients  $C = (c_1, \dots, c_n)$ .

If we assume that the mass is the same for all the points<sup>5</sup>, finding  $m$  minimizing the difference between the observed energy rate and the computed specific power can be done by minimizing 9, with  $m \in [m_{min}; m_{max}]$ :

$$Traj_{(c_1, \dots, c_n)}(m) = \sum_{i=1}^n \left( \frac{P_i(m)}{m} \right)^2 \quad (9)$$

We search  $m$  in  $[m_{min}; m_{max}]$ . If  $m$  minimizing  $Traj_{(c_1, \dots, c_n)}(m)$  is in  $]m_{min}; m_{max}[$ , then it satisfies the equation 10.

$$\sum_{i=1}^n P_i(m) [m \cdot P_i'(m) - P_i(m)] = 0 \quad (10)$$

One can solve analytically the fourth degree polynomial equation 10 using Ferrari's method. Then, in addition to  $m_{min}$  and  $m_{max}$ , we may have to consider four more potential solutions. Among these six potential solutions, we select the solution in  $[m_{min}; m_{max}]$  minimizing  $Traj_{(c_1, \dots, c_n)}(m)$ .

<sup>5</sup>According BADA simulation, a climb from FL130 to FL300, in ISA+20 atmospheric condition with the nominal mass, consumes 1075kg that is to say 1.68% of the initial mass.

### III. A COMMON EQUIVALENT THRUST PROFILE

As seen before, in a ground-based context, having radar data, a weather model and the thrust coefficient  $c$ , we can compute an equivalent mass. In this section we build a common equivalent thrust profile for all the trajectories, using a set of recorded trajectories and a least square regression method.

#### A. Motivations

1) *Mass Inference Issues:* Inferring altogether the mass  $m$  and the thrust coefficients  $C = (c_1, \dots, c_n)$  of one given aircraft is a difficult task. If you consider a set of  $n$  points like in the previous section II-B, at any mass  $m$ , you can find a  $C$  thrust profile which will perfectly fit the observed energy rate. There are an infinity of couples  $(m, C)$  that fit the observed energy rate perfectly. Intuitively, a large  $C$  with a large mass  $m$  barely produce the same amount of specific power than a small  $C$  with a small mass  $m$ .

One can select one couple by injecting some constraints<sup>6</sup> on the set of coefficients  $C$  for instance. However, considering a given aircraft, the mass  $m$  of the inferred couple  $(m, C)$  depends strongly on the chosen constraint. However, we have no idea on how to design this constraint, that is to say, how to choose one couple  $(m, C)$  among an infinity of potential solutions. So, as we do not know how to infer an individual thrust profile  $C$ , we use a thrust profile  $C$ , common to all aircraft, for inferring the equivalent mass of each individual aircraft.

2) *Energy Rate Prediction Issues:* When computing the energy rate prediction, we need to know a future  $C = (c_1, \dots, c_n)$  thrust profile. The chosen future thrust coefficients  $C$  are likely to be different from one aircraft to another. However, predicting an individual future  $C$  profile seems hard to us without knowledge of the future aircraft intent. Thus, as for mass inference, a predefined thrust profile is used for specific power predictions.

In order to make consistent mass estimation and specific power prediction, we use the same predefined thrust profile for these two purposes.

#### B. Building a Thrust Profile from Data

Considering a set of  $K$  trajectories, we build the thrust profile  $(c_1, \dots, c_n)$  minimizing the sum over the trajectories of the squared error  $\min_{m \in [m_{min}; m_{max}]} Traj_{(c_1, \dots, c_n)}(m)$ . That is to say, we build  $(c_1, \dots, c_n)$  minimizing the overall mean square error. The function to minimize,  $AllTraj$ , is defined in the equation 11.

$$AllTraj(c_1, \dots, c_n) = \sum_{k=1}^K \min_{m \in [m_{min}; m_{max}]} Traj_{(c_1, \dots, c_n)}^{(k)}(m) \quad (11)$$

The resulting thrust profile is an equivalent profile, it might be different from the true mean thrust profile, but this

<sup>6</sup>Continuity, range, etc.

profile is likely to have good predicting performance using the equivalent weight estimation process described in subsection II-B. To minimize  $AllTraj$  we use a quasi-Newton method, BFGS [19], with  $(1, \dots, 1)$  as the initial vector.

### IV. DATA AND EXPERIMENTAL SETUP

#### A. Data Pre-processing

Recorded radar tracks from Paris Air Traffic Control Center were used in this study. This raw data is made of one position report every 1 to 3 seconds, over two months (july 2006, and january 2007). In addition, the wind and temperature data from Meteo France are available at various isobar altitudes over the same two months.

The raw Mode C altitude<sup>7</sup> has a granularity of 100 feet. Trajectories were smoothed, using a local quadratic model, in order to obtain: the aircraft position  $(X, Y)$  in a projection plan, or latitude and longitude in WGS84), the ground velocity vector  $(V_x, V_y)$ , the smoothed altitude  $(z)$ , in feet above isobar 1013,25 hPa, the rate of climb or descent (ROCD). The wind  $(W_x, W_y)$  and temperature  $(T)$  at every trajectory point were interpolated from the meteo datagrid. The temperature at isobar 1000 hPa was also extracted for each point, in order to compute a close approximation of  $\Delta T$ , the difference between the actual temperature and the ISA model temperature at isobar 1013,25 hPa (mean sea level in the ISA atmospheric model). This  $\Delta T$  is one of the key parameters in the BADA model equations.

Using the position, velocity and wind data, we computed the true air speed (TAS), the distance flown in the air (dAIR), the drift angle, the along-track and cross-track winds ( $W_{along}$  and  $W_{cross}$ ). The successive velocity vectors allowed us to compute the trajectory curvature at each point. The actual aircraft bank angle was then derived from true airspeed and the curvature of the air trajectory. The climb, cruise, and descent segments were identified, using triggers on the rate of climb or descent to detect the transitions between two segments.

#### B. Filtering and Sampling Climb Segments

As our aim is to evaluate the performance of the energy rate prediction, we focused on a single aircraft type (Airbus A320), and selected all flights of this type departing from Paris Orly (LFPO). Needless to say, this approach can be replicated to other aircraft types.

We have only kept aircraft trajectories above 13000ft. The trajectories were then filtered so as to keep only the climb segments. An additionnal 80 seconds were clipped from the beginning and end of each segment, so as to remove climb/cruise or cruise/climb transitions.

The trajectories were then sampled every 500 ft. One  $c$  coefficient will be associated to each sampling altitude. When adjusting the coefficients  $c$ , there must be enough climbing trajectories at each altitude. In order to have a good estimation of  $c$ , sampling altitudes with at least 2400 trajectories were

<sup>7</sup>This altitude is directly derived from the air pressure measured by the aircraft. It is the height in feet using the pressure setting selected by the aircraft.

kept. In the prediction phase, we want to estimate the mass on the first ten points and then, using the next points to evaluate the prediction with the estimated mass. Thus, trajectories with less than 30 points were discarded.

At last, our data set contains 3 945 trajectories as shown in figure 2.

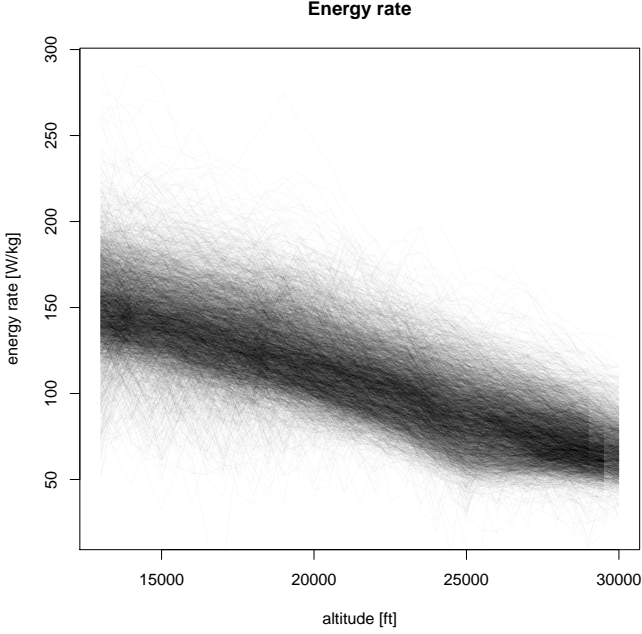


Figure 2. Energy rate of the 3945 trajectories.

### C. Experimental Setup

In our experiments, four different settings are compared. These four settings come from the combination of two mass settings ( $m_{BADA}$  and  $m_{estimated}$ ) and two profile settings (BADA  $C_{red}$  and thrust profile  $C$ ). When estimating an equivalent mass using the standard BADA power reduction profile in an altitude range from 13 000 ft to 18 000 ft, it can be assumed that the inequality  $H_p \leq 0.8 H_{max}(\Delta T, m)$  is true<sup>8</sup>. Then  $c_{red}(m)$  is a first degree polynomial. The equivalent mass can be estimated the same way as in subsection II-B, except that the  $P_i$  in the equation 10 will be third degree polynomials.

The different settings used are summarized by the table I below, where  $error^{(i)}$  denotes the following quantity at the  $i$ -th point of the trajectory:

$$error^{(i)} = \frac{power^{(i)}(m)}{m} - \left( V_{TAS}^{(i)} \frac{dV_{TAS}^{(i)}}{dt} + g_0 \frac{dh^{(i)}}{dt} \right)$$

The set of trajectories was split in two subsets of equal size, one used to learn the thrust profile and one used to evaluate predictions with it, the validation set.

For each trajectory of the validation set, we compute the energy rate predicted from the eleventh point to the last point

<sup>8</sup>Numerical application sets  $0.8.H_{max}$  at 19 144 ft for  $\Delta T$  at 40K and  $m$  at 77 000kg.

setting	description
$m_{BADA}$	$m = m_{BADA}(A320) = 64\,000\text{kg}$
$m_{estimated}$	$m = \underset{m \in [39\,000\text{kg}; 77\,000\text{kg}]}{\operatorname{argmin}} \sum_{i=1}^{10} error^{(i)2}$
BADA $C_{red}$	$power = c_{red} \cdot (T_{std} - D) \cdot V_{TAS}$ $C_{red}$ is the BADA power profile given by the equation 4
thrust profile $C$	$power = (c \cdot T_{std} - D) \cdot V_{TAS}$ $C$ is the thrust profile built according the section III-B

Table I

THE TWO MASS SETTINGS AND THE TWO PROFILE SETTINGS. ONE HAVE TO CHOOSE ONE OF EACH TO COMPUTE THE SPECIFIC POWER.

of the trajectory. The quality of these predictions made with the different settings are compared. If an equivalent mass estimation is processed, the mass estimation will be done using the first ten points. If the BADA reference mass is taken, the first ten points will have no use. All the statistics on energy rate prediction presented in section V are computed using trajectories from the eleventh point to the last point.

## V. RESULTS

The equivalent thrust profile was adjusted using a set of 1 972 trajectories. All the results are computed using a large validation set containing 1 973 fresh trajectories.

### A. Equivalent Parameters

The figure 3 plots the distribution of the estimated masses. It is difficult to interpret. However, the range of the estimated masses seems quite large. The actual masses are likely to be more centred.

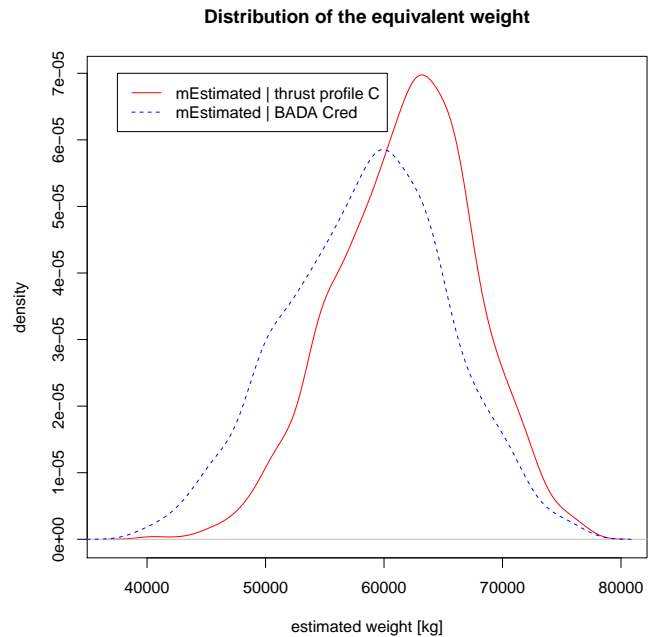


Figure 3. Density of the mass estimated using the thrust profile  $C$  and the BADA profile  $C_{red}$ .

With the figure 4, we can see that the thrust coefficient rapidly crosses the unit value. There are slope variations in the thrust profile. It would be interesting to understand the reason of these changes. It may come from inaccuracies in the BADA modeled power or true changes in the thrust setting. We hit here one possible limitation of this equivalent thrust profile concept. The mean thrust setting may be different from an airport to an other or even from a departure procedure to an other. Some work has to be done to identify if this concern is justified. If it is, we might be able to predict these thrust profile variations and consequently, to further improve the energy rate prediction.

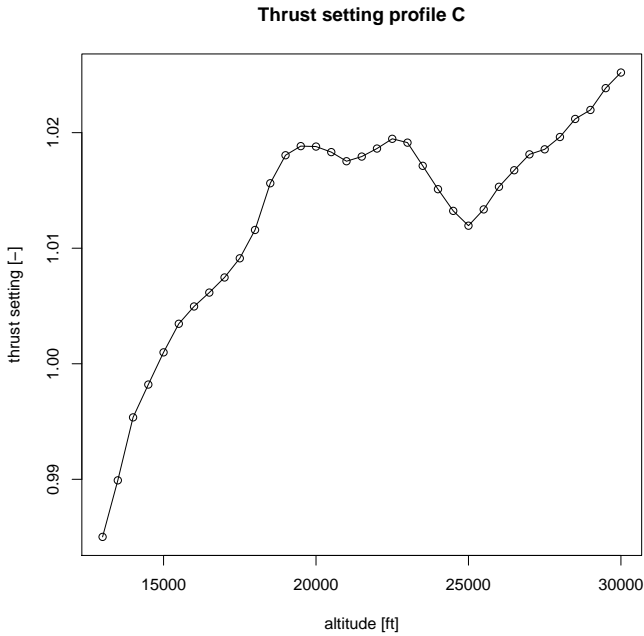


Figure 4. The built thrust coefficients profile.

### B. Prediction Performance as a Function of Altitude

According to the figure 5, the root mean square error decrease with the altitude. This decrease can be explained by the decrease of the standard deviation with the altitude. The four setups have barely the same slope. Setups using the estimated equivalent mass are significantly better than the two other setups.

### C. Overall Prediction Performance

Table II summarizes the root mean square error on the energy rate prediction.

Results table II show prediction improvement using the built thrust profile and the weight estimation. The weight estimation process reduces the root mean square error by approximately 45%. The thrust profile, extracted from the reference data set, reduces the root mean square error on the validation set by 5% which might be not significant. Used together, the root mean square error is reduced by nearly 50%.

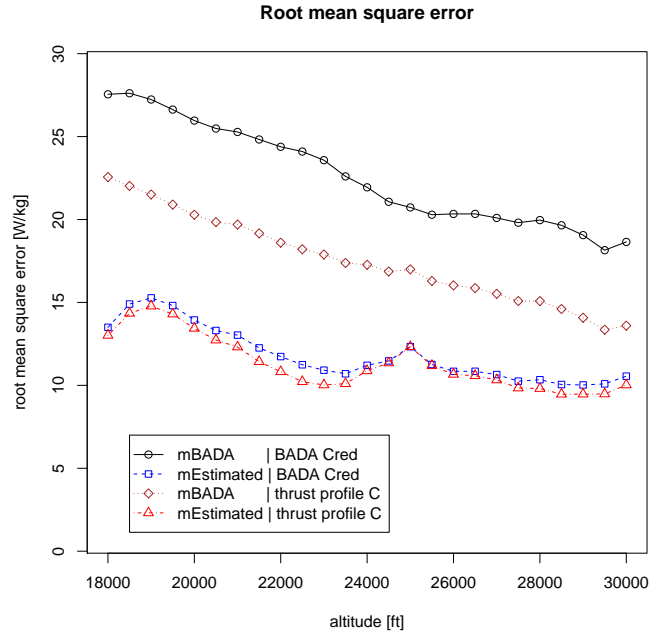


Figure 5. Root mean square error on the predicted energy rate (in W/kg) according to the prediction altitude.

	BADA $C_{red}$	thrust profile $C$
$m_{BADA}$	22.9	17.8
$m_{estimated}$	12.0	11.5

Table II

ROOT MEAN SQUARE ERROR ON THE PREDICTED ENERGY RATE (IN W/KG) FOR AIRBUS A320 AIRCRAFT, USING 10 PAST POINTS FOR MASS ESTIMATION PURPOSE.

## CONCLUSION AND FUTURE WORK

In this paper, we present a practical method to improve the energy rate prediction when the mass and the thrust setting are unknown. The aim was to extract these missing knowledge from the aircraft past trajectory. Thrust coefficients are extracted from a set of trajectories and an equivalent mass is estimated using 10 past points of the trajectory. Focusing on a single aircraft type (A320), this method has been tested using actual radar data and a weather model. On a fresh set of trajectories, the prediction performance of this method was compared to the standard BADA prediction performance. It should be noted that this method can be replicated to other aircraft types.

Our results show that the thrust profile  $C$  combined with the equivalent weight estimation performs better than the standard BADA profile with the reference mass. The thrust profile and the equivalent weight estimation process reduces the root mean square error by 50%.

From an operational point of view, the proposed methods could improve aircraft trajectory predictions. Improving energy rate predictions eventually improves rate of climb and acceleration predictions. It could also be used for simulation

purpose. From a set of trajectories, we can extract a thrust profile and a distribution of equivalent masses. Then using these two elements, we can synthesize aircraft trajectories close to the original set of trajectories.

In future works, we shall implement this thrust setting profile in our BADA 3.9 simulator to quantify improvements in altitude prediction when combined to the equivalent weight estimation. We shall take into account the effect of the wind on the energy rate computation. This should improve the thrust setting extraction, the weight estimation and the prediction. We will test this process on Mode S radar data which are more accurate than Mode C radar data. This concept will be also tested to lower altitude.

Some study on the thrust setting profile has to be done. This thrust setting profile is extracted from climbing segments. The horizontal segments were discarded. However, among climbing segments, the chosen thrust setting may vary according the departure procedure, for instance. If this concern is justified, we might be able to improve even more the energy rate prediction by extracting from the data one specific thrust setting profile for each departure procedure. All these concerns are also related to the robustness of the extracted thrust setting profile.

## REFERENCES

- [1] SESAR Consortium. Milestone Deliverable D3: The ATM Target Concept. Technical report, 2007.
- [2] H. Swenson, R. Barhydt, and M. Landis. Next Generation Air Transportation System (NGATS) Air Traffic Management (ATM)-Airspace Project. Technical report, National Aeronautics and Space Administration, 2006.
- [3] B. Musialek, C. F. Munafo, H. Ryan, and M. Paglione. Literature survey of trajectory predictor technology. Technical report DOT/FAA/TC-TN11/1, Federal Aviation Administration William J. Hughes Technical Center, Atlantic City, November 2010.
- [4] C. Gong and D. McNally. A methodology for automated trajectory prediction analysis. In *AIAA Guidance, Navigation, and Control Conference and Exhibit*, 2004.
- [5] R. A. Vivona, M. M. Paglione, K. T. Cate, and G. Enea. Definition and demonstration of a methodology for validating aircraft trajectory predictors. In *AIAA Guidance, Navigation, and Control Conference*, 2010.
- [6] J. Romanelli, C. Santiago, M. Paglione, and A. Schwartz. Climb trajectory prediction software validation for decision support tools and simulation models. *International Test and Evaluation Association*, 2009.
- [7] Study of the acquisition of data from aircraft operators to aid trajectory prediction calculation. Technical report, EUROCONTROL Experimental Center, 1998.
- [8] Adapt2. aircraft data aiming at predicting the trajectory. data analysis report. Technical report, EUROCONTROL Experimental Center, 2009.
- [9] R. A. Coppenbarger. Climb trajectory prediction enhancement using airline flight-planning information. In *AIAA Guidance, Navigation, and Control Conference*, 1999.
- [10] Christophe Bontemps. Prévision stochastique de trajectoires : procédures paramétriques et non-paramétriques. Master's thesis, Rapport de DEA IFP, 1997.
- [11] I. Lymperopoulos, J. Lygeros, and A. Lecchini Visintini. Model Based Aircraft Trajectory Prediction during Takeoff. In *AIAA Guidance, Navigation and Control Conference and Exhibit*, Keystone, Colorado, August 2006.
- [12] N. Durand, J.M. Alliot, and J. Noailles. Automatic aircraft conflict resolution using genetic algorithms. In *Proceedings of the Symposium on Applied Computing, Philadelphia*. ACM, 1996.
- [13] A.W. Warren and Y.S. Ebrahimi. Vertical path trajectory prediction for next generation atm. In *Digital Avionics Systems Conference, 1998. Proceedings., 17th DASC. The AIAA/IEEE/SAE*, volume 2, pages F11/1–F11/8 vol.2, oct-7 nov 1998.
- [14] A.W. Warren. Trajectory prediction concepts for next generation air traffic management. In *3rd USA/Europe ATM R&D Seminar*, June 2000.
- [15] F. Imado T. Kinoshita. The application of an uav flight simulator - the development of a new point mass model for an aircraft. In *SICE-ICASE International Joint Conference Conference*, 2006.
- [16] A. Nuic. User manual for base of aircraft data (bada) rev.3.9. Technical report, EUROCONTROL, 2011.
- [17] D. Poles. Revision of atmosphere model in bada aircraft performance model. Technical report, EUROCONTROL, 2010.
- [18] D. Poles. Aircraft performance modelling report. Technical report, EUROCONTROL, 2009.
- [19] J. C. Nash. *Compact numerical methods for computers*. Adam Hilger Ltd., Bristol, second edition, 1990.