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Abstract— **The purpose of this communication is to display a non-linear control approach based on backstepping for the positioning and orientation for a four-rotor aircraft. Realistic rotorcraft flight dynamics are introduced and the effectiveness of its control channels is analyzed. Then two complementary implementations of the backstepping control approach are considered. The compatibility of these control approaches with a two-layer control structure devoted to the guidance of the rotorcraft is displayed. The resulting control laws are detailed and their expected performances are discussed. A simulation study is performed where the performances of the proposed control structure are compared with those of a classical non linear inverse control solution.**

I. Introduction

IN the last years a large interest has risen for the four-rotor concept since it appears to present simultaneously hovering, orientation and trajectory tracking capabilities of interest for many practical applications [1]. The flight mechanics of this rotorcraft are highly non-linear and different control approaches [2], [3], have been considered with little success to achieve either only autonomous hovering and orientation or also trajectory tracking.

In this paper we consider the flight dynamics of a four-rotor aircraft with fixed pitch blades. The control problem of interest is the design of flight control laws enabling autonomous positioning and orientation for this class of rotorcraft. This study investigates the solution of this problem using a backstepping control approach. Here it is required that a single continuous control law performs the whole maneuver while to achieve it by manual control it appears necessary to go through a succession of elementary maneuvers since the system is underactuated.

The backstepping control approach, which has also been applied to airships flight control law design [4], is introduced. Two different design techniques are developed following the main guidelines of this approach. These two implementations appear of direct interest for the design of a new two-layer control structure based on backstepping control laws. The proposed control approach is compared with a nonlinear inverse control approach introduced in [10].

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II. Rotorcraft Flight Dynamics

The considered system is shown in figure 1 where rotors one and three are clockwise while rotors two and four are counter clockwise. The main simplifying assumptions adopted with respect to flight dynamics in this study are a rigid cross structure, constant wind, negligible aerodynamic contributions resulting from translational speed, no ground effect as well as small air density effects and negligible response times for the rotors. It is then possible to write the rotorcraft flight equations as follows [7].

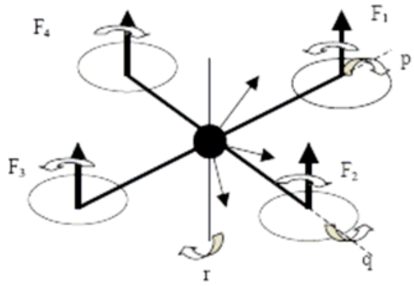

Fig. 1. Four rotor aircraft

A. *Rotorcraft Flight Equations*

The rotor forces and moments are given by:

$$F_i = f\ \omega_i^2 \qquad i \in \{1, 2, 3, 4\} \quad (1\text{-}1)$$

$$M_i = k\ F_i = k\ f\ \omega_i^2 \qquad i \in \{1, 2, 3, 4\} \quad (1\text{-}2)$$

Where f and k are positive constants and ω_i is the rotational speed of rotor i. These speeds and forces satisfy the constraints:

$$0 \le \omega_{i_i} \le \omega_{\max} \qquad i \in \{1, 2, 3, 4\} \quad (2\text{-}1)$$

$$0 \le F_{i_i} \le F_{i\max} = f\ \omega_{\max} \qquad i \in \{1, 2, 3, 4\} \quad (2\text{-}2)$$

Since the inertia matrix of the rotorcraft can be considered diagonal with $I_{xx} = I_{yy}$, the roll, pitch and yaw moment equations may be written as:

$$\dot{p} = (l\ (F_4 - F_2) + k_2\ q\ r) / I_{xx} \quad (3\text{-}1)$$

$$\dot{q} = (l\ (F_1 - F_3) + k_4\, p\ r) / I_{yy} \quad (3\text{-}2)$$

$$\dot{r} = (k\,(F_2 - F_1 + F_4 - F_3))\,/\,I_{zz} \qquad (3\text{-}3)$$

Where p, q and r are the roll, pitch and yaw body angular rates. Here $k_2 = (I_{zz} - I_{yy})$ and $k_4 = (I_{xx} - I_{zz})$, where I_{xx}, I_{yy} and I_{zz} are the inertia moments in body-axis, and l is the length of the four arms of the rotorcraft.

Let ϕ, θ and ψ be respectively the bank, pitch and heading angles, then the Euler equations relating the derivatives of the attitude angles to the body angular rates, are given by:

$$\dot{\phi} = p + tg(\theta)(\sin\phi\ q + \cos\phi\ r) \qquad (4\text{-}1)$$

$$\dot{\theta} = \cos\phi\ q - \sin\phi\ r \qquad (4\text{-}2)$$

$$\dot{\psi} = (\sin\phi\, q + \cos\phi\, r)\,/\,\cos\theta \qquad (4\text{-}3)$$

In this study the wind is given in the local Earth reference frame by $\underline{w} = (w_x \quad w_y \quad w_z)$. The wind is supposed constant while the ground effect is neglected. The acceleration $\underline{a} = (a_x \quad a_y \quad a_z)'$ of the centre of gravity, taken directly in the local Earth reference frame, is such as:

$$a_x = (1/m)((\cos(\psi)\sin(\theta)\cos(\phi) + \sin(\psi)\sin(\phi))\ F - d_x) \qquad (5\text{-}1)$$

$$a_y = (1/m)((\sin(\psi)\sin(\theta)\cos(\phi) - \cos(\psi)\sin(\phi))\ F - d_y) \qquad (5\text{-}2)$$

$$a_z = g - (1/m)(\cos(\theta)\cos(\phi)\ F + d_z) \qquad (5\text{-}3)$$

where x, y and z are the centre of gravity coordinates, m is the total mass of the rotorcraft and:

$$F = F_1 + F_2 + F_3 + F_4 \qquad (6)$$

Here the drag force $\underline{d} = (d_x \quad d_y \quad d_z)'$ is given by:

$$\underline{d} = c\,\sqrt{(\dot{x} - w_x)^2 + (\dot{y} - w_y)^2 + (\dot{z} - w_z)^2}\ \ (\dot{x} - w_x \quad \dot{y} - w_y \quad \dot{z} - w_z)' \qquad (7)$$

with $c = 1/2\ \rho\ S\ C_d$ where ρ is the volumetric mass of the air, S is the reference surface for the aerodynamics of the rotorcraft and C_d is its dimensionless drag factor. The components of the rotorcraft airspeed are:

$$u_a = \dot{x} - w_x\,,\quad v_a = \dot{y} - w_y \quad \text{and} \quad w_a = \dot{z} - w_z \qquad (8)$$

The equilibrium conditions (hovering) in an horizontal wind (w_z=0) with a given heading ψ are such that:

$$p = 0,\quad q = 0,\quad r = 0,\ \dot{x} = 0,\quad \dot{y} = 0,\quad \dot{z} = 0 \qquad (9\text{-}1)$$

$$\theta_e = arctg((c\,\|\underline{w}\|\,/\,mg)(\cos\psi\ \ w_x + \sin\psi\ \ w_y)) \qquad (9\text{-}2)$$

$$\phi_e = arctg((c\,\|\underline{w}\|\,/\,mg)(\sin\psi\ \ w_x - \cos\psi\ \ w_y)\,\cos\theta_e) \qquad (9\text{-}3)$$

with

$$F_1 = F_2 = F_3 = F_4 = mg\,/(4\cos\theta_e\ \cos\phi_e) \qquad (9\text{-}4)$$

B. Analysis of Rotorcraft Flight Dynamics

Here we are interested in controlling the four-rotor aircraft so that its centre of gravity reaches and stays hovering at a predefined position while its heading acquires and maintains a given orientation. Many potential applications require this capability to be available in UAVs' while this problem can be also considered as a first step towards the design of more efficient trajectory tracking systems.

The manoeuvre under study is, when performed manually through direct radio control of the four engine thrusts (see picture 1), quite difficult to be achieved in one step. Experimentally it appears that no direct approach is feasible and that much depends on the rotorcraft attitude angles ϕ and θ specially when considering the control of its horizontal position error (x-x_c, y-y_c).

Equations (5-1) and (5-2) show that to get any horizontal acceleration, it is necessary to have a non zero attitude ($\phi \neq 0$ or $\theta \neq 0$), they show also that the orientation of the acceleration is dependent of the heading angle ψ. Equations (3-3) with (4-3) and (5-3) show that given the attitude angles ϕ and θ, it is easy to master the heading angle error (ψ-ψ_c) and the vertical position error (z-z_c). From equations (3-1), (3-2) and (3-3), it appears that the effectiveness of the rotor actuators is much larger with respect to the roll and pitch axis than with respect to the yaw axis. Then we consider that attitude piloting is involved with controlling the angles θ and ϕ. In equations (3-1) and (3-2), the effect of the rotor forces appears as differences so, we define new attitude inputs u_q and u_p as:

$$u_q = F_1 - F_3 \qquad u_p = F_4 - F_2 \qquad (10.1)$$

In the heading and position dynamics, the effects of rotor forces and moments appear as sums, so we define new guidance inputs u_ψ and u_z as:

$$u_\psi = (F_2 + F_4) - (F_1 + F_3) \quad u_z = F = F_1 + F_2 + F_3 + F_4 \qquad (10.2)$$

$$\underline{F} = [F_1\ F_2\ F_3\ F_4]' \quad \underline{u} = [u_p\ u_q\ u_\psi\ u_z]' \qquad (11\text{-}3)$$

Equations (3-1), (3-2) and (3-3) are rewritten:

$$\dot{p} = (l\ u_p + k_2\ q\ r)/I_{xx} \qquad (12\text{-}1)$$

$$\dot{q} = (l\ u_q + k_4 p\ r)/I_{yy} \qquad (12\text{-}2)$$

$$\dot{r} = k\ u_\psi\,/\,I_{zz} \qquad (12\text{-}3)$$

It appears that u_q and u_p can be made to vary significantly with u_ψ and u_z remaining constant. Attitude angles ϕ and θ can be seen as virtual controls for the horizontal position of the rotorcraft. Here the attitude dynamics are considered to be the fast dynamics , they are at the heart of the control system. The heading and height dynamics are intermediate while the dynamics of the horizontal position coordinates are the slower. This can lead to multilevel closed-loop control structures.

III. BACKSTEPPING CONTROL

A. The Backstepping Control Approach

The backstepping technique is a rather recent non-linear control technique, which applies to cascaded systems. The main idea is to use intermediate state variables as virtual inputs to take advantage of the causality relations displayed by the cascaded state representation. The convergence of the output variables towards their target values is obtained by the construction, step by step, of an auxiliary Lyapunov function. This general idea can be developed in different ways, as it will be shown in the next sub-section.

The main interest of the backstepping approach is that the stability of the controlled system as well as the convergence of the outputs towards their reference values can be guaranteed without inducing, like in the case of the non-linear control approach, the decoupling of the outputs dynamics. Indeed, it can be considered that the decoupling of the outputs dynamics demands an additional effort from the control channels with then a higher possibility of saturation for the actuators, either in position or speed, resulting in downgraded performances. Finally, another advantage of this approach is that several matrices of parameters are introduced while constructing the control law, providing a large variety of possibilities to shape conveniently the outputs dynamics as well as the control signals.

B. Direct Implementation of the Backstepping

Consider a cascaded system whose state representation is given by:

$$\dot{\underline{x}}_1 = \underline{x}_2 \quad \text{and} \quad \dot{\underline{x}}_2 = g(\underline{x}_2, \underline{U}) \tag{13}$$

where $\underline{x}_1 \in R^n$, $\underline{x}_2 \in R^n$ are state variables and $\underline{u} \in R^n$ is the control input and *g* is a smooth diffeomorphism with respect to $\underline{u}$. The control objective here is to design a control law such that the state $\underline{x}_1$ can be stabilized at $\underline{x}_{1c}$. Here also, $\underline{x}_2$ can be regarded as a virtual control input for the dynamics of $\underline{x}_1$ while the dynamics of $\underline{x}_2$ are controlled by the real control input $\underline{U}$. Now, suppose that there exists a control law $\underline{x}_2 = G(\underline{x}_1, \underline{x}_{1c})$ such that the dynamics of $\underline{x}_1$ can be stabilized at $\underline{x}_{1c}$ while we can find a Lyapunov function $V_1(\underline{x}_1 - \underline{x}_{1c})$, which satisfies the condition:

$$\dot{V}_1(\underline{x}_1 - \underline{x}_{1c}) = (\partial V_1 / \partial \underline{x}_1)' G(\underline{x}_1, \underline{x}_{1c}) \le -W(\underline{x}_1 - \underline{x}_{1c}) \tag{14}$$

where $W(\underline{x}_1 - x_{1c})$ is a positive definite function of $\underline{x}_1$. A possible choice is:

$$G(\underline{x}_1, \underline{x}_{1c}) = -\Lambda(\underline{x}_1 - \underline{x}_{1c}) \tag{15}$$

where Λ is a positive definite symmetric matrix. Then in this case:

$$V_1(\underline{x}_1 - \underline{x}_{1c}) = W(\underline{x}_1, \underline{x}_{1c}) = \frac{1}{2}(\underline{x}_1 - \underline{x}_{1c})'(\underline{x}_1 - \underline{x}_{1c}) \tag{16}$$

The whole dynamics can be expressed as:

$$\dot{\underline{x}}_1 = G(\underline{x}_1, \underline{x}_{1c}) + \underline{z} \quad \text{and} \quad \dot{\underline{z}} = \underline{w} \tag{17-1}$$

where $$\underline{z} = \underline{x}_2 - G(\underline{x}_1, \underline{x}_{1c}) \tag{17-2}$$

and

$$\underline{w} = g(\underline{x}_2, \underline{u}) - (\partial G / \partial \underline{x}_1)\, \underline{x}_2 \tag{17-3}$$

Then a candidate Lyapunov function of the full system is given by:

$$V(\underline{x}_1 - \underline{x}_{1c}, \underline{z}) = V_1(\underline{x}_1 - \underline{x}_{1c}) + 1/2\, \underline{z}'\, \underline{z} \tag{18}$$

The time derivative of $V(\underline{x}_1 - \underline{x}_{1c}, \underline{z})$ is given by:

$$\dot{V}(\underline{x}_1 - \underline{x}_{1c}, \underline{z}) = (\partial V_1 / \partial \underline{x}_1)'(G(\underline{x}_1, \underline{x}_{1c}) + \underline{z}) + \underline{z}'\underline{w} \tag{19}$$

then:

$$\dot{V}(\underline{x}_1 - \underline{x}_{1c}, \underline{z}) \le -W(\underline{x}_1 - \underline{x}_{1c}) + (\partial V_1 / \partial \underline{x}_1)'\, \underline{z} + \underline{z}'\underline{w} \tag{20}$$

and by an adequate choice of $\underline{w}$, such as :

$$\underline{w} = -(\partial V_1 / \partial \underline{x}_1) - \Omega\, \underline{z} \tag{21}$$

where Ω is a symmetric positive definite matrix, the full system is globally asymptotically stable since it satisfies the following condition:

$$\dot{V}(\underline{x}_1 - \underline{x}_{1c}, \underline{z}) \le -W(\underline{x}_1 - \underline{x}_{1c}) - \underline{z}'\Omega\, \underline{z} \tag{22}$$

Finally, the effective control input is given by:

$$\underline{U} = -g^{-1}(\underline{x}_2)\left((\partial V_1 / \partial \underline{x}_1) + \Omega\left(\underline{x}_2 - G(\underline{x}_1, \underline{x}_{1c})\right)\right) \tag{23}$$

C. Indirect Implementation of Backstepping

Now we consider the case where the cascaded system cannot be written easily in the form (13), but it obeys to:

$$\dot{\underline{X}} = g(\underline{X}, \underline{U}) \quad \text{with} \quad \underline{Y} = h(\underline{X}) \tag{24}$$

where $\underline{X} \in \mathrm{R}^n$, $\underline{U} \in \mathrm{R}^m$, $\underline{Y} \in \mathrm{R}^m$, *g* is a smooth vector field of $\underline{X}$ and $\underline{U}$ and *h* is a smooth vector field of $\underline{X}$. The system has, with respect to each independent output Y_i, a relative degree r_i ($\sum_{i=1}^{m} (r_i + 1) \le n$, $i = 1, \ldots, m$) around the state $\underline{X}_0$ if the output dynamics can be written as:

$$\begin{pmatrix} Y_1^{(r_1+1)} \\ \vdots \\ Y_m^{(r_m+1)} \end{pmatrix} = \underline{A}(\underline{X}) + B(\underline{X}, \underline{U}) \tag{25}$$

Here we assume that $r_1 = r_2 = \cdots = r_m = 1$, where the jacobian of *B* with respect to the control inputs is invertible. In that case, two auxiliary outputs can be defined:

$$\underline{Z}_1 = L\,(\underline{Y} - \underline{Y}_c) + \dot{\underline{Y}} \quad \text{and} \quad \underline{Z}_2 = \dot{\underline{Y}} \tag{26}$$

where *L* is a positive definite symmetric matrix. A candidate Lyapunov function is then given by:

$$V_2 = \frac{1}{2}(\underline{Z}_1{}'\underline{Z}_1 + \underline{Z}_2{}'\underline{Z}_2) \tag{27}$$

The time derivative of V_2 is such as:

$$\dot{V}_2 = \underline{Z}_1{}'\,\underline{Z}_1 + \underline{Z}_2{}'\,\underline{Z}_2 \tag{28-1}$$

or $\dot{V}_2 = (L\,(\underline{Y}-\underline{Y}_c)+2\,\dot{\underline{Y}})'\,(L\,\dot{\underline{Y}}+\ddot{\underline{Y}}) - L\,\dot{\underline{Y}}'\,\dot{\underline{Y}}$ (28-2)

Choosing a control such as :

$$L\,\dot{\underline{Y}}+\ddot{\underline{Y}} = -\Lambda(L\,\underline{Y}+2\,\dot{\underline{Y}}) \quad (29)$$

where Λ is another symmetric positive definite matrix. We have:

$$\dot{V}_2 = -(L\,(\underline{Y}-\underline{Y}_c)+2\,\dot{\underline{Y}})'\,\Lambda\,(L\,(\underline{Y}-\underline{Y}_c)+\dot{\underline{Y}}) - \dot{\underline{Y}}'L\,\dot{Y} \quad (30)$$

and it is straightforward to show that the system is globally asymptotically stable. The corresponding control law is given by :

$$\underline{U} = -B^{-1}(\underline{X})(\Lambda\,L\,(\underline{Y}-\underline{Y}_c)+(L+2\Lambda)\,\dot{\underline{Y}}+\underline{A}(\underline{X})) \quad (31)$$

IV. APPLICATION OF BACKSTEPPING TO ROTORCRAFT FLIGHT CONTROL

Analyzing relations (3-i), (4-i) and (5-i), i =1 to 3, it appears that their equations can be separated into two sets: one relative to the slower dynamics, the horizontal dynamics, and corresponding to the first case considered in the previous section and one relative to other dynamics and corresponding to the second case. The above approaches of backstepping are now applied to each of the control layer necessary to perform attitude control and guidance of the rotorcraft.

A. Control of Rotorcraft Attitude and Level

The attitude and altitude dynamics can be given by the state equations:

$$\dot{\phi} = p + tg(\theta)(\sin\phi\;\; q + \cos\phi\;\; r) \quad (32\text{-}1)$$

$$\ddot{\phi} = (l\,u_p + l\;tg\theta\,\sin\theta\,u_q + k\;tg\theta\,\cos\phi\,u_\psi)/I_{xx} + P_p(p,q,r,\phi,\theta) \quad (32\text{-}2)$$

$$\dot{\theta} = \cos\phi\;\; q - \sin\phi\;\; r \quad (32\text{-}3)$$

$$\ddot{\theta} = (l\,\cos\phi\,u_q - k\,\sin\phi\,u_\psi)/I_{yy} + P_q(p,q,r,\phi,\theta) \quad (32\text{-}4)$$

$$\dot{\psi} = (\sin\phi\;q + \cos\phi\;r)/\cos\theta \quad (32\text{-}5)$$

$$\ddot{\psi} = k\,\cos\phi\,u_\psi\,/(\cos\theta\,I_{zz}) + l\,\sin\phi\,u_p\,/(\cos\theta\,I_{yy}) + g_\psi(p,q,r,\phi,\theta,\psi) \quad (32\text{-}6)$$

$$\dot{z} = v_z \quad (32\text{-}7)$$

$$\ddot{z} = g - (1/m)(\cos(\theta)\cos(\phi)\,u_z + d_z) \quad (32\text{-}8)$$

where the exact expressions of $P_p(p,q,r,\phi,\theta)$, $P_q(p,q,r,\phi,\theta)$ and $g_\psi(p,q,r,\phi,\theta,\psi)$ can be derived from relations (3-i) and (4-i) .

The outputs dynamics (32-2), (32-4), (32-6) and (32-8) take the form (24) with $\underline{Y}=[\phi\;\;\theta\;\;\psi\;\;z]'$, $\underline{U} = \underline{u}$ with $\underline{u} = (u_p, u_q, u_\psi, u_z)'$. Here $\underline{X} = (p,q,r,\phi,\theta,\psi,\dot{z},z)'$ and $B(\underline{X}, \underline{u})$ is such as:

$$B(\underline{X},\underline{u}) = J(\underline{Y})\;\underline{u} \quad (33\text{-}1)$$

with

$$J(\underline{Y}) = \begin{bmatrix} \dfrac{l}{I_{xx}} & \dfrac{l\,tg\theta\sin\theta}{I_{xx}} & \dfrac{k\,tg\theta\cos\phi}{I_{xx}} & 0 \\ 0 & \dfrac{l\,\cos\phi}{I_{yy}} & \dfrac{-k\sin\phi}{I_{yy}} & 0 \\ \dfrac{l\sin\phi}{\cos\theta\,I_{yy}} & 0 & \dfrac{k\cos\phi}{\cos\theta\,I_{zz}} & 0 \\ 0 & 0 & 0 & \dfrac{-\cos\phi\cos\theta}{m} \end{bmatrix} \quad (33\text{-}2)$$

and the above matrix is invertible if:

$$\frac{\cos^2\phi}{I_{zz}} - \frac{tg\theta\,\sin\phi}{I_{yy}}(\sin\theta\,\sin\phi + \cos^2\phi) \neq 0 \quad (34)$$

which is the case when ϕ and θ remains small with respect to $\pm\pi/2$. We introduce now the two R^{4x4} symmetric positive definite matrices L and Λ and adopt the control law (31) with

$$\underline{A}(\underline{X}) = [P_p \quad P_q \quad g_\psi \quad (g - d_z/m)]' \quad (35)$$

to compute the current input vector $\underline{u}$.

B. Horizontal backstepping control layer

The state representation of the horizontal dynamics is given by:

$$\dot{x} = v_x \quad (36\text{-}1)$$

$$\dot{y} = v_y \quad (36\text{-}2)$$

$$\dot{v}_x = (1/m)((\cos(\psi)\sin(\theta)\cos(\phi) + \sin(\psi)\sin(\phi))\;F - d_x) \quad (36\text{-}3)$$

$$\dot{v}_y = (1/m)((\sin(\psi)\sin(\theta)\cos(\phi) - \cos(\psi)\sin(\phi))\;F - d_y) \quad (36\text{-}4)$$

where ψ and F are defined by the inner control loop.

This state representation corresponds to the one studied in the case of (13-1) and (13-2). Then following the corresponding backstepping approach, we get with V_1 chosen according to relation (23) the following reference values for ϕ and θ:

$$\phi_c = \arcsin((\sin\psi\;(m\,\varepsilon_x + d_x) - \cos\psi\;(m\,\varepsilon_y + d_y)/u_z) \quad (37\text{-}1)$$

$$\theta_c = \arcsin((\cos\psi(m\varepsilon_x + d_x)/u_z + \sin\psi(m\varepsilon_y + d_y)/u_z/\cos\phi_c) \quad (37\text{-}2)$$

where ε_x and ε_y are given by:

$$\begin{bmatrix}\varepsilon_x \\ \varepsilon_y\end{bmatrix} = -(I_2 + \Omega\,\Lambda)\begin{bmatrix}x - x_c \\ y - y_c\end{bmatrix} - \Omega\begin{bmatrix}\dot{x} \\ \dot{y}\end{bmatrix} \quad (37\text{-}3)$$

where Λ and Ω are symmetric positive definite matrices. Then, the horizontal position of the rotorcraft follows the linear dynamics:

$$\begin{bmatrix}\ddot{x} \\ \ddot{y}\end{bmatrix} + \Omega\begin{bmatrix}\dot{x} \\ \dot{y}\end{bmatrix} + (I_2 + \Omega\,\Lambda)\begin{bmatrix}x - x_c \\ y - y_c\end{bmatrix} = \underline{0} \quad (38)$$

Since the actuator settings are determined by the inner control loop, let us have a look at the corresponding dynamics. The outputs ϕ, θ, ψ and z of the inner closed loop follows the dynamics given by:

$$(\Lambda\ L)^{-1}\ddot{\underline{Y}} + (\Lambda\ L)^{-1}\ (L+2\Lambda)\ \dot{\underline{Y}} + (\underline{Y} - \underline{Y}_c) = \underline{0} \quad (39)$$

When matrices Λ and L are diagonal, these dynamics are decoupled and the poles of the decoupled dynamics are the roots of the *m* different characteristic polynomials:

$$s^2 + (\mu_i + 2\,\lambda_i)\,s + \lambda_i\ \mu_i = 0 \quad i = 1 \text{ to } m \qquad (40\text{-}1)$$

where

$$\left.\begin{array}{l} \Lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_m) \\ \text{and} \\ L = diag(\mu_1, \mu_2, \cdots, \mu_m) \end{array}\right\} \qquad (40\text{-}2)$$

and where *s* is the Laplace variable. In this case, since the λ_i and the μ_i are positive real, we get always real negative roots. In the general case, the dynamics modes of the outputs will be characterized by the solutions of the global characteristic polynomial:

$$\det\left(\begin{bmatrix} sI_m & -I_m \\ \Lambda\ L & sI_m + (L+2\Lambda) \end{bmatrix}\right) = 0 \qquad (41)$$

Since this last relation is independent of the application it is possible to study once for all the reachable pole sets within the left half complex plane.

Remark: In the case of an horizontal wind, a necessary condition for final convergence and equilibrium, independent of the choice of matrices Λ and L or even of the control approach is given by:

$$m\ g\ /(4\cos\theta_e\ \cos\phi_e) \le F_{i\max} \qquad (42\text{-}1)$$

with

$$\theta_e = arctg((c\ \|\underline{w}\|/mg)(\cos\psi_c\ \ w_x + \sin\psi_c\ \ w_y)) \qquad (42\text{-}2)$$

$$\phi_e = arctg((c\ \|\underline{w}\|/mg)(\sin\psi_c\ \ w_x - \cos\psi_c\ \ w_y)\cos\theta_e) \qquad (42\text{-}3)$$

where ψ_c is the heading reference value.

V. SIMULATION RESULTS

The selected gains for the backstepping control law are displayed on Table 1 while the selected dynamics for the attitude, the altitude, the heading and the horizontal position are second order linear dynamics characterized by their respective damping coefficients and natural frequencies. These values are reported on Table 2.

The produced figures display different time responses of the rotorcraft under either the backstepping control law or a reference non-linear inverse control law. The comparison is performed in two stages: first, the responses of the rotorcraft to a step in attitude (either ϕ or θ) for each control law, are evaluated. Then, for each control law , the responses of the rotorcraft to a step in position (*x*, *y* or *z*) are evaluated and compared. The evaluation of the inner attitude control loop is important since the guidance capability of the rotorcraft, an under actuated device, is directly dependent of the controllability of its attitude angles.

TABLE I
SELECTED GAINS FOR BACKSTEPPING CONTROL

K_θ = -2	K_q =-0.23
K_ϕ= - 2	K_p = -0.23
K_ψ = -0.02	K_r = -0.025
K_z = 0.12	$K_{\dot{z}}$ =0.15
K_x = 0.137	$K_{\dot{x}}$=0.183
K_y = 0.137	$K_{\dot{y}}$ = 0.183

TABLE II
SELECTED DYNAMIC PARAMETERS

ζ_θ = 0.8	ω_θ= 10 rad/s
ζ_ϕ = 0.8	ω_ϕ = 10 rad/s
ζ_ψ = 0.8	ω_ψ = 2 rad/s
ζ_z = 0.8	ω_z = 1.5 rad/s
ζ_x = 0.8	ω_x = 1.5 rad/s
ζ_y = 0.8	ω_y = 1.5 rad/s

The results (see figures 2 to 5) show that the two control laws, in both levels, present equivalent performances. However, while the non linear inverse control law produces second order linear dynamics for the attitude angles and the position and heading outputs, the backstepping control law produces clearly a non linear behaviour for these variables. In particular (figure 4) , since the final convergences of the backstepping control law is rather slow, the non linear inverse solution can produce, for a same response time, a less input demanding solution. However, as shown in the following figures, other parameters settings may lead to responses where the backstepping approach is slightly superior. Other simulation studies should be performed in particular to show clearly the advantage of using advanced non-linear control law instead of empirical-intuitive ones. Also, the realisation of simulation studies should be of interest to explore the impact of actuator saturations on the flight domain and feasible manoeuvres of the rotorcraft.

VI. CONCLUSION

In this communication the applicability of a non-linear control approach to the positioning and orientation of a rotorcraft has been treated. Since this system is highly nonlinear, naturally unstable and rather under-actuated, the design of a unique control law to perform safely the whole manoeuvre is not straightforward and a multilevel control approach must be considered. So a multilevel control structure has been introduced. It appeared that the direct application of the backstepping control approaches was not desirable and that it was more judicious to realize two different implementations of the backstepping guidelines to insure first the internal stability and then guidance of the controlled system.

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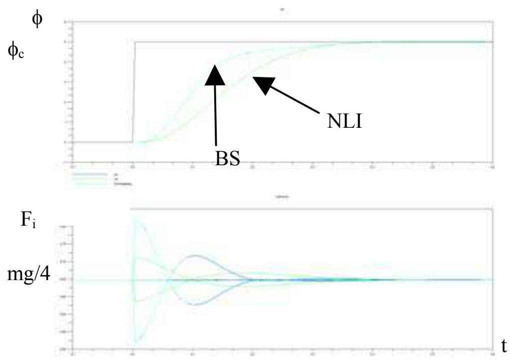
FIGURES



(NLI : non linear inverse, BS : backstepping)

Fig.2 Step response of ϕ (ω_ϕ=10 rad/s)

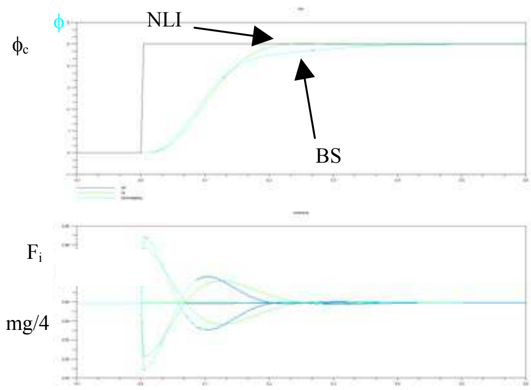


Fig.3. Step response of ϕ (ω_ϕ=15 rad/s)

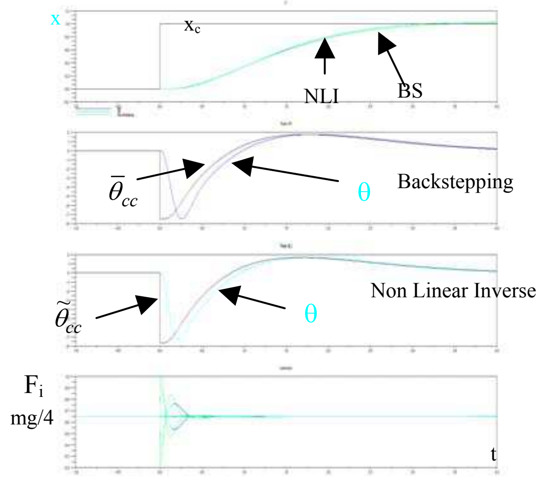


Fig.4. Step response of x
(with θ, θ_c, ω_x=1.5, rad/s, ω_θ=15 rad/s)

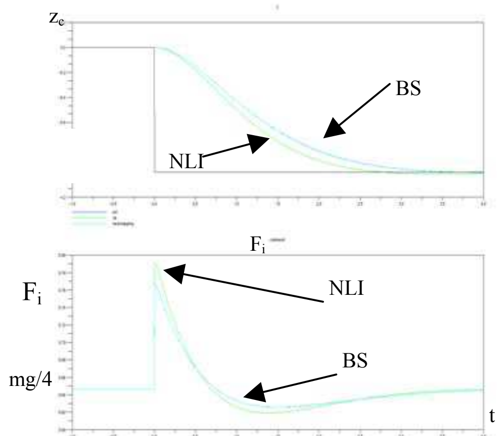


Fig. 5. Step response of z ($\omega_z = 1.5\ rad/s$)