Aircraft conflict avoidance: a mixed-integer nonlinear optimization approach

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Abstract Detecting and solving aircraft conflicts, which occur when aircraft sharing the same airspace are too close to each other according to their predicted trajectories, is a crucial problem in Air Traffic Management. We focus on mixed-integer optimization models based on speed regulation. We first solve the problem to global optimality by means of an exact solver. The problem being very difficult to solve, we also propose a heuristic procedure where the problem is decomposed and it is locally solved by an exact solver. Computational results show that the proposed approach provides satisfactory results in reasonable time.

Keywords: air traffic management, conflict avoidance, MINLP, modeling, global exact solution, heuristic

1. Introduction

The air traffic level currently attained in Europe is around tens of thousands of flights per day and it is expected to be multiplied by a factor of two during next 20 years. Air traffic is therefore at the core of the social and economic dynamism of our society. The European project SESAR gives the guidelines to go towards an Air Traffic Management (ATM) characterized by more efficiency and more safety, which should essentially result from a higher level of automation of ATM. The need for automatical tools to integrate human work specially arises in the context of aircraft conflicts detection and resolution.

Aircraft potential conflicts can be solved in different ways. The most commonly exploited is based on the idea of achieving separation changing the trajectory (heading angle) or the flight level of the aircraft involved in the conflict. Another way is based on the idea of separating aircraft by slightly changing their speeds but keeping the predicted trajectories. A speed regulation which occurs in a reasonable small range allows a *subliminal control* as suggested by the European ERASMUS project [3]. This project showed the advantage of such a control, which is not even perceived by air traffic controllers. Conflict avoidance is expected to be performed while deviating as little as possible from the original aircraft flight plan, minimizing the impact of the separation maneuvers. Various solution strategies have been proposed. A review is provided in [6]. Solution algorithms are currently mainly based on evolutionary computation [4]. These methods are computationally efficient, but the global optimal solution and even a feasible solution (with no conflicts) is not guaranteed to be achieved in a given time. Recent advances in mixed-integer linear and nonlinear programming open new perspective for modeling and efficiently solving the addressed problem. The first attempt to use mixed-integer optimization is by Pallottino et al. in [8], where, though under very stringent hypothesis, a geometrical construction leads to a mixed-integer linear programming model. More recently, mixed-integer programming has been proposed again for aircraft conflict resolution (see e.g.[7]).

In this paper, the speed regulation strategy is modeled by mixed-integer nonlinear programming, building on [3]. A deterministic global solution is first proposed, using a general-purpose solver for MINLP. Then, to deal with the difficulty of the problem, another strategy is also proposed, where the optimality guarantee is forsaken in exchange for the computational efficiency. This solution strategy is based on hybridizing mathematical programming and a heuristic tailored on the problem.

The paper is organized as follows. In Sect. 2 we review mixed-integer nonlinear modeling of the aircraft conflict avoidance problem. In Sect. 3 we present a global exact solution of a few randomly generated instances and we propose a heuristic tailored on the problem to gain computational efficiency. Sect. 4 concludes the paper.

2. Modeling the aircraft conflict avoidance problem

Aircraft are said to be potentially *in conflict* when their horizontal or altitude distances are less than given standard separation distances (5NM and 1000 ft[1]). So, assuming the aircraft flying on a horizontal plane, the separation between aircraft i and j at the instant time t is expressed by the following condition:

$$||\mathbf{x}^r_{ij}(t)|| \geq d, \tag{1}$$

where d is the minimum required separation distance and $\mathbf{x}^r_{ij}(t)$ is the vector of relative distance between i and j. We assume, as usually done, that speed changes occur instantaneously. We can therefore consider uniform motion laws. Hence, for each t, we have:

$$\mathbf{x}^r_{ij}(t) = \mathbf{x}^{rd}_{ij} + \mathbf{v}^r_{ij}t, \tag{2}$$

where $\mathbf{x}^{rd}_{ij}$ is the relative initial position of aircraft i and j and $\mathbf{v}^r_{ij}$ is their relative speed. Observing that the minimum t is given by $t_m = -\mathbf{v}^r_{ij}\mathbf{x}^{rd}_{ij}/(v^r_{ij})^2$ the separation condition can be rewritten as follows:

$$(x^{rd}_{ij})^2 - \frac{(\mathbf{v}^r_{ij}\mathbf{x}^{rd}_{ij})^2}{(v^r_{ij})^2} - d^2 \geq 0. \tag{3}$$

Note that condition (3) has to be checked only when the inner product $\mathbf{v}^r_{ij}\mathbf{x}^{rd}_{ij}$ is negative. In this case, indeed, aircraft are converging.

In our model, conflict avoidance is achieved by performing a speed change maneuvre. Aircraft which are in conflict accelerate or decelerate in order to cross their conflict zone at different instant times, solving the conflict. Let A be the set of n aircraft. Decision variables are $q_k,\ k \in A$, expressing the percentage of speed change of each aircraft with respect to its original speed. As prescribed by the ERASMUS project [2], we impose bounds on these variables in order to have speed changes ranging between -6% and $+3\%$ of the original speeds. In this way, the so-called *subliminal control* is achieved. We minimize the speed change for each aircraft together with time intervals during which it flies with a modified speed, in order to deviate as less as possible from the original flight plan:

$$\min \sum_{k \in A} q_k^2 (t_{2k} - t_{1k})^2, \tag{4}$$

where t_{1k} and t_{2k} are decision variables representing starting and ending instant times for aircraft k changing its speed. The order of instant times when aircraft change speed being unknown, 6 possible time configurations have to be considered for each pair of aircraft and, for each of them, 5 time intervals $[t_s, t'_s]$. The constraints of the problem impose aircraft separation (3) for each time configuration and time interval. This needs the introduction of new (integer) variables and constraints. See [3] for details. The described model can be relaxed, for example imposing that aircraft speed changes occur at the instant time $t = 0$ and that the new speeds are kept during the trajectories. We consider in the following this relaxed model. First, we have to express aircraft speeds in (3) in terms of their original speed v and speed modification q. We also have to check if t_m is greater than 0. Equation for t_m gives rise to a constraint, for each pair of aircraft, defining the minimum instant time. To check if $t_m \geq 0$,

[1] 1 NM (nautical mile)= 1852 m, 1 ft (feet) = 0.3048 m

a binary variable y_{ij} for each (*i*,*j*) is introduced ($y_{ij} = 1$ if $t_m \geq 0,$ and 0 otherwise), and constraints are adjoined accordingly. The separation condition is then imposed, for each pair of aircraft, only when $t_m \geq 0$:

$$\forall i, j \in A, i \neq j, \quad y_{ij} \left(\mathbf{x}_{ij}^{rd} (\mathbf{v}_{ij}^{r})^2 - (\mathbf{v}_{ij}^{r} \mathbf{x}_{ij}^{rd})^2 - (d^2 (\mathbf{v}_{ij}^{r})^2) \right) \geq 0. \tag{5}$$

The obtained mathematical programming model has as many (nonlinear) separation constraints as pairs of aircraft.

3. Solving the conflict avoidance problem

3.1 Global exact solution

We use as a testbed n aircraft in 2-dimensional space, placed on a circle of a given radius r, with speed v and a heading angle such that their trajectory is toward the center of the circle (or slightly deviated with respect to such direction). The zone of conflict is around the center of the circle where aircraft are placed, and each aircraft is in conflict with each other. It is easy to see that the number of conflicts is $n(n-1)/2$, so a large number of conflicts is generated in the same conflict zone. We solve the problem to global optimality using COUENNE [1], which implements a spatial Branch-and-Bound based on convex relaxations. Results are reported in Table 1a (v=400 NM/h). They show that we are able to obtain global exact solutions up to $n = 6$ (i.e. 15 conflicts). However, an exact solution turns to be high memory and time demanding, even for a small number of aircraft, due to the high number of conflicts and the number of variables and constraints largely increasing with n. Hence, we are not able to solve the problem for $n > 6$ even with the relaxed modeling. Objective function values show that aircraft separation is always achieved with very slight speed changes.

3.2 A heuristic based on local exact solutions

We then propose a heuristic procedure where we solve at global optimality subproblems involving up to 4 aircraft at a time, based on the observation that a solution can be efficiently computed for problems involving a small number of aircraft.

Let a *cluster* be the transitive closing on conflicting pairs of aircraft (see, e.g., [5]). The heuristic is based on the idea of decomposing the problem in subproblems (clusters) and solve the conflict avoidance problem on clusters. Let *ncl* be the number of clusters. At each step, *ncl* problems are sequentially solved by using an exact solver (COUENNE). Combining together all the results, in general the conflicts are not all solved because aircraft inside clusters are typically in conflict with aircraft inside other clusters too. After the resolution step on subproblems, the number of remaining conflicts is computed. If it is greater than 0, a new step is performed. To do so, the initial speed (which together with the initial position represents the data of the problem) of aircraft that are still in conflict is re-initialized taking into account the solution obtained at the last step. That is, if the (optimal) solution obtained for cluster i is such that an aircraft in this cluster has been accelerated with respect to its original speed, then its speed is modified by a random slight further increase. If it has been decelerated, then its speed is modified by a random slight further decrease. In this way, the information obtained at the previous step is preserved and the chances to keep aircraft separated inside clusters increase. To update the speeds, a local search is performed testing a number of candidates and choosing the one that minimizes the sum, over all conflicting aircraft, of the maximum violation of the separation constraints for each considered aircraft, divided by the number of remaning conflicts. When only one conflict is to be solved, this search is intensified to increase the chances to solve the problem. Aircraft speeds have to be bounded in the small range $[-6\%v, +3\%v]$, so when speeds are modified these bounds have to be checked and speeds adjusted to fulfill this requirement. This may eventually lead to change the speed scenario provided by local solutions.

Results are reported in Table 1b. Values are averaged over 10 runs. For all test problems, all conflicts are solved. Comparing with global exact solutions (Table 1a), it appears that decomposing the problem does not significantly affect the quality of the result. In general, increasing n, faster solutions are obtained using a higher number of smaller subproblems. Solutions are obtained in reasonable time on problems involving up to 10 aircraft.

Table 1a. Results obtained with `COUENNE`

ID	n	r	obj	CPU time (sec.)
pb_n2	2	1×10^2	0.002531	0.15
pb_n3	3	2×10^2	0.001667	1.45
pb_n4	4	2×10^2	0.004009	12.87
pb_n5	5	3×10^2	0.003033	841.33
pb_n6	6	3×10^2	0.006033	51863.37

n= number of aircraft
r= radius of the circle (NM)
obj= objective function value
ncl= number of aircraft clusters

Table 1b. Results obtained with the proposed heuristic

ID	n	r	ncl	obj	CPU time (sec.)
pb_n4	4	2×10^2	2	0.005151	26.97
pb_n5	5	3×10^2	2	0.004729	17.98
pb_n6	6	3×10^2	2	0.006402	17.33
pb_n6	6	3×10^2	3	0.007438	341.12
pb_n7	7	3×10^2	2	0.009215	131.34
pb_n7	7	3×10^2	3	0.008144	22.99
pb_n8	8	4×10^2	2	0.008220	759.40
pb_n8	8	4×10^2	3	0.007551	39.66
pb_n8	8	4×10^2	4	0.012034	48.99
pb_n9	9	4×10^2	3	0.009238	97.41
pb_n10	10	4×10^2	3	0.014047	484.49

4. Conclusions

We presented an approach based on mixed-integer nonlinear optimization for the aircraft conflict avoidance problem. We are able to obtain global exact solutions for problems with up to 6 aircraft, while a new heuristic tailored on the problem and based on local exact solutions allow us to obtain good quality results even on problems involving many conflicts at a time.

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