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## Optimal Reallocation of Aircraft Actuators after Failure

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### Abstract

Control allocation compute the commands that are applied to the actuators so that a certain set of forces or moments are generated by the control effectors. In this communication we consider the problem of the design of an airborne system whose function is to solve the problem of allocation of transportation aircraft actuators during manoeuvres. The case of a roll manoeuvre, which may be quite demanding for the wings structure, is more particularly considered. The aim of this supervision function, similar to the load alleviation function, is to control the bending and flexion moments of the wings while performing a manoeuvre which can turn to be extreme. The wings actuators considered are the ailerons, the spoilers and the flaps, but the other aerodynamic actuators, elevators and rudders, must be considered to take into account the existing coupling effects between the three body axis. The proposed approach may be applied specially in the case in which some of the actuators are failed (stuck to a particular deflection or flapping in the wind stream) while their limitations, such as position, speed and response time, are considered explicitly in this study.

**Keywords:** Optimization, Actuator Failure, Control Allocation, Aircraft

### Nomenclature

$p, q, r$  - angular rates in body axis frame;

$I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{yz}, I_{zx}$  - inertia moments in body-axis;

$L, M, N$  - aerodynamics moments in body-axis;

$C_l, C_m, C_n$  - aerodynamics moments coefficients in body-axis;

$S$  - aerodynamic surface;

$V, V_{ail}$  - the air speed, air speed at the aileron level

$\phi$  - sweep angle;

$\eta$  - elastic axis;  $\eta = y / \cos \phi$

$c$  - aerodynamic wing cord;

$c_{am}$  - middle aerodynamic wing cord;

$C_{m0}$  - pitching torque generated by null lift and deflection;

$C_{m\alpha}$  - incidence torque; the torque generated by the horizontal empennage;

$C_{m\delta m}$  - torque due to elevator, tail deflection ( $\delta_{elev}, \delta_{tail}$ );

$C_{mq}$  - pitching torque due to pitch speed ( $q$ );

$C_{n\beta}$  - represent the stability of the way, is the sideslip influence of fuselage and elevator;

$C_{l\delta ail}, C_{n\delta ail}$  - yawing torque and inverse yawing torque due to drag aileron;

$C_{n\delta rud}, C_{l\delta rud}$  - roll torque and inverse roll couple due to rudder deflection ( $\delta_{rud}$ );

$C_{mr}, C_{lr}$  - pitching torque and induct pitching torque due to pitch speed ( $r$ );

$C_{np}, C_{lp}$  - roll torque and induct roll torque due to roll speed ( $p$ );

$C_{l\delta spl}$  - torque due to spoiler deflection;

$C_{l\beta}$  - torque due to dihedral effect, to wing and rudder arrow;

$\delta_{ail}, \delta_{rud}, \delta_{spl}$  - aileron, rudder, spoiler deflection,

### 1. Introduction

Conventional aircraft make use of their main actuators, elevator for pitch control, the ailerons for roll control, and the rudder for yaw control, but many other actuators are available. Then the mixing of these actuators to achieve some desired control objectives lead to formulate a real time allocation problem. Due to the diversity and the coupling of effects of the different control surfaces, it is difficult to translate a flight control command into a single control surface commands. On one side rate and position limits of the control surfaces must be considered in order to achieve a realistic solution and on the other side strain constraints must be taken into account to preserve the structural integrity of the aircraft. The ability to mix or to complement main actuators by secondary ones should also provide capability to cope safely with situations in which some control surface is no more available due to failure.

In this paper we consider the case of a lateral manoeuvre which should be performed according to some prescribed dynamics. This leads to the formulation of the necessary moments to be generated along the body axis of the aircraft, then, taking into account the

current limitations of each actuator as well as flexion and bending moment limitations, an optimal actuator allocation problem is formulated. This leads to the design of a new flight safety supervision function whose objective is to protect the aircraft from structural strain and when actuator failures happen, to maintain as much as possible a safe behaviour for the damaged aircraft.

## 2. Aircraft Flight Dynamics

Adopting a classical approach for the modelling of the aircraft flight dynamics, the moment equations in the body frame are given by:

$$\begin{aligned} L &= I_{xx}\dot{p} - I_{xz}\dot{r} + (I_{zz} - I_{yy})qr - I_{xz}pq \\ M &= I_{yy}\dot{q} + (I_{xx} - I_{zz})rp + I_{xz}(p^2 - r^2) \\ N &= -I_{xz}\dot{p} + I_{zz}\dot{r} + (I_{yy} - I_{xx})qr + I_{xz}qr \end{aligned} \quad (1)$$

According to the work of Lu *et al* in the field of differential flatness, it can be shown that the knowledge of the trajectory of the center of gravity of the aircraft leads to the knowledge of the necessary values for the attitude angles  $\theta$  and  $\phi$ , and the heading angle  $\psi$ , during the same time span. Then considering the angular Euler equations, it is possible to derive from them the necessary history for the body components  $p$ ,  $q$  and  $r$  of the aircraft rotation vector. Then from equations (1) it is possible to get the values of the roll, pitch and yaw moments necessary to perform the desired manoeuvre. The roll, pitch and yaw moments  $M$ ,  $N$  and  $L$  can be related to airspeed and actuator deflections by the classical relations:

$$M = \frac{1}{2} \rho S V^2 \cdot c_{am} \cdot (C_m + C_{mF}) \quad (2)$$

$$N = \frac{1}{2} \rho S V^2 \cdot c_{am} \cdot (C_n + C_{nF}) \quad (3)$$

$$L = \frac{1}{2} \rho S V^2 \cdot c_{am} \cdot (C_l + C_{lF}) \quad (4)$$

where the dimensionless coefficients  $C_m$ ,  $C_n$ ,  $C_l$ , are defined as:

$$C_m = C_{m0} + C_{m\alpha} \cdot (\alpha - \alpha_0) + C_{m\delta m} \cdot \delta_m + C_{m\delta mT} \cdot \delta_{mT} + C_{mq} \cdot \frac{qc_{am}}{V} \quad (5)$$

$$C_n = C_{n\beta} \cdot \beta + C_{n\delta ail} \cdot \delta_{ail} + C_{n\delta rud} \cdot \delta_{rud} + C_{np} \cdot \frac{pc_{am}}{V} + C_{nr} \cdot \frac{rc_{am}}{V} \quad (6)$$

$$C_l = C_{l\beta} \cdot \beta + C_{l\delta ail} \cdot \delta_{ail} + C_{l\delta spl} \cdot \delta_{spl} + C_{l\delta rud} \cdot \delta_{rud} + C_{lp} \cdot \frac{pc_{am}}{V} + C_{lr} \cdot \frac{rc_{am}}{V} \quad (7)$$

In the Figure 1 we consider the body frame  $Oxyz$  and we introduce the wing elastically axis  $O\eta$ .

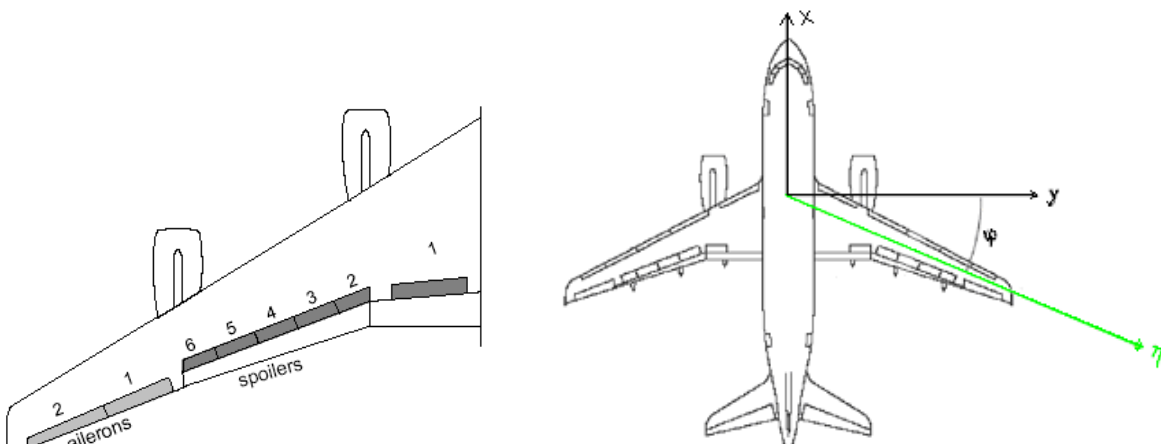


Figure 1. Presentation of left wing on A340-600. Definition of elastic chord  $\eta$

In the next sections detailed models of the aerodynamic effects over the wings of a transportation aircraft are provided, allowing to compute the resulting flexion and bending moments.

### 3. The Roll Moment

For a classical civil transportation aircraft it can be considered that the aerodynamic wing cord varies linearly according to  $\eta$ . This aerodynamic wing cord is parallel to the aircraft axle and represents the length of a wing area plug in the direction of the upstream flow of the air. The aerodynamic wing cord is given by:

$$c = c_{emp} \left( 1 + (\varepsilon - 1) \frac{y - r_{fus}}{b/2 - r_{fus}} \right) \quad (8)$$

The lift forces repartition is considered to be elliptical along the span of the wings and its expression is given by:

$$dP(y) = \frac{1}{2} \rho S V a^2 C_z \sqrt{1 - \left( \frac{y}{b/2} \right)^2} \cdot \frac{4}{\pi b} dy \quad (9)$$

Here  $dP$  is the lift created by the  $dy$  section of the wing at position  $y$ .

The whole roll moment is the result of the effects of the ailerons and the spoilers, it can be broken up into rigid and flexible components.

- Roll moment due to the deflexion of the ailerons ( $\delta_{ail}$ ):

The rigid roll moment  $M_r(\delta_{ail})$  caused by the deflexion of the ailerons is given by:

$$M_r(\delta_{ail}) = 2 \times L_{\delta_{ail}} \cdot \frac{1}{2} \rho S_{ail} V_{ail}^2 C_{Z_{ail}} \delta_{ail} \quad (10)$$

$$\text{with the aileron force} \quad F_{\delta_{ail}}(\delta_{ail}) = \frac{1}{2} \rho S_{ail} V_{ail}^2 C_{Z_{ail}} \delta_{ail} \quad (11)$$

where:  $C_{Z_{ail}}$  is the lift coefficient of the aileron,  $S_{ail}$  is the aileron surface and  $L_{ail}$  is the distance between the centre of the fuselage and the point of application of the lifting power due to the deflexion of the control surface.

Flexible roll moment: The wings are flexible and become deformed at the same time in torsion and in bending under the application of the  $F_{\delta_{ail}}$  forces. Le roll moment created by this strain is known as flexible moment of roll  $M_s$ :

$$M_s = 2 \times \int_{R_{fus}}^{b/2} y \cdot \frac{1}{2} \rho \cdot V^2 C_z \cdot \Delta\alpha(y) \cdot C(y) \cdot dy \quad (12)$$

Considering again a linear variation of the aerodynamic wing cord from its base, we can write the flexible moment of roll according to the deflection of the control surface as:

$$M_s = \rho V^2 C_z \cdot \int_{R_{fus}}^{b/2} y \cdot \frac{d_{ail} \cdot F_{\delta_{ail}}(\delta_{ail})}{G \cdot J} \cdot \left( \frac{y - R_{fus}}{\cos \varphi} \right) \cdot C_{emp} \left( 1 + (\varepsilon - 1) \frac{y - R_{fus}}{b/2 - R_{fus}} \right) \cdot dy \quad (13)$$

The total moment of roll create by a control surface deflexion is:

$$M_{Tail} = M_r + M_s \quad (14)$$

$$\text{or} \quad M_{Tail} = K_{ail} \cdot \delta_{ail} \quad \text{where} \quad C_{l\delta_{ail}} = \frac{K_{ail}}{1/2 \rho S V^2 c_{am}} \quad (15)$$

- Roll moment due of spoilers deflexions ( $\delta_{spl}$ ):

Let us suppose that the pressure is the same one in any point of the plate,  $\bar{F}_{\text{elementary}} = P_2 \cdot d\bar{S} = \frac{1}{2} \rho V_1^2 \cdot d\bar{S}$ , what gives for all the surface plate:

$$F_p(\delta_{spl}) = \frac{1}{2} \rho V_1^2 \cdot S_{spl} \sin(\delta_{spl}) \quad (16)$$

where  $F_p$  is the pressure force,  $S_{spl}$  is the spoiler surface and  $\rho$  is the volume mass of the air. By projecting the power created at the spoiler centre on the X and Z axis we get respectively for the lift and the drag:

$$F_{pZ}(\delta_{spl}) = \frac{1}{2} \rho V_1^2 \cdot S_{spl} \sin(\delta_{spl}) \cos(\delta_{spl}) \quad (17)$$

$$F_{pX}(\delta_{spl}) = \frac{1}{2} \rho V_1^2 \cdot S_{spl} \sin^2(\delta_{spl}) \quad (18)$$

The total roll moment created by the spoilers deflection is then given by:

$$M_{Tspl} = \sum_1^6 L_{spl-i} \times (F_{spl-i}(\delta_{spl-i}) + F_{pZ-i}(\delta_{spl-i})) \quad (19)$$

$$\text{with } C_{l\delta_{spl-i}} = \frac{K_{spl-i}}{1/2\rho SV^2 c_{am}} \quad (20)$$

where  $L_{spl}$  is the distance between the fuselage and the centre of the considered spoiler,  $M_{Tspl}$  is the total roll moment created by the spoilers deflection,  $C_{l\delta_{spl-i}}$  is the roll moment coefficient due of spoilers deflection.

#### 4. The Wing Bending Moment

To compute the bending aerodynamic moment of the wing, we consider all the forces created by all the portion of the wing located at the right of this point. The expression of this bending moment is the integral of  $dP$  multiplied by the distance to the fuselage:

$$M_f(y) = \int_y^{b/2} \left( \frac{\tau}{\cos\varphi} - \frac{y}{\cos\varphi} \right) dP(\tau) \quad (21)$$

$$M_f(y) = \frac{2}{3} \cdot \frac{b/2}{\pi \cos\varphi} \cdot \frac{1}{2} \rho SV^2 C_z \left( 1 - \left( \frac{y}{b/2} \right)^2 \right)^{\frac{3}{2}} - \frac{y}{\cos\varphi} \cdot \frac{1}{2} \rho SV_a^2 C_z \cdot \frac{1}{2} \left( 1 - \frac{2y}{\pi b/2} \sqrt{1 - \left( \frac{y}{b/2} \right)^2} - \frac{2}{\pi} \arcsin\left( \frac{y}{b/2} \right) \right) \quad (22)$$

The bending moment supported by the aircraft fuselage is:

$$M_f(y=0) = \frac{2}{3\pi} \cdot \frac{b/2}{\cos\varphi} \cdot \frac{1}{2} \rho SV^2 C_z \quad (23)$$

The wing root, which transmits the forces to the fuselage, supports the bending moment and he is written of such way:

$$M_f(y = R_{fus}) = M_f(y = 0) \quad (24)$$

- The bending moment of the wing at its base is the sum of the moment due to the weight of the wing, the moment due to the presence of the engine and the moment due to the weight of the fuel in the wing:

$$M_{emp} = \frac{2}{3\pi} \cdot \frac{b/2}{\cos\varphi} \cdot \frac{1}{2} \rho SV^2 C_z + \sum_1^2 F_{\delta_{ail-i}} \times \left( \frac{L_{\delta_{ail-i}} - R_{fus}}{\cos\varphi} \right) - \sum_1^6 \left( \frac{L_{spl-i} - R_{fus}}{\cos\varphi} \right) \times (F_{spl-i}) - M_{masses} \quad (25)$$

where  $F_{\delta_{ail}} \times \left( \frac{L_{\delta_{ail}} - R_{fus}}{\cos\varphi} \right)$  is the contribution of an aileron,  $(F_{spl-i}) \times \left( \frac{L_{spl-i} - R_{fus}}{\cos\varphi} \right)$  is the contribution of a spoiler.

The value of the bending moment at the wing base depends on the deflexion of the ailerons and the spoilers. We can then write the resulting moment expression as:

$$M_{emp} = A + \sum_{i \in I^L} \delta_i Y_i \quad (26)$$

#### 5. Formulation for the Actuator Optimal Allocation Problem

As it has been shown above, the expression of the different moments can be written in an affine form with respect to the corresponding deflection, so that we get expressions such as:  $M_{ik} = M_{IK}^0 + \mu_{ik} \delta_k$  where  $M_{ik}$  is the  $i$  considered moment (roll, pitch, yaw, bending, flexion),  $\delta_k$  is the deflection of the  $k^{th}$  actuator ( $k \in K = \{\text{aileron, flap, right spoilers 1 to } ns, \text{ left spoilers 1 to } ns, \text{ elevator, rudder}\}$ ) and  $\mu_{ik}$  is the current effectiveness of actuator  $k$  to produce moment  $i$ .

Here different objectives can be adopted:

- To use spoilers as less as possible. In general, aileron deflection is controlled by three servo actuators and present a smaller response time than spoilers which are in general controlled by a single servo actuator. In that case the problem to be solved is to check that using only ailerons, elevator and rudder, all the objectives can be reached and all constraints are satisfied. If this is not possible this approach must be left aside to consider the use of the additional actuators.

- To minimize the mean deflection of the actuators given by the index:  $\sqrt{\frac{\sum_{i \in I^{Ail}} S_i^2 \delta_i^2}{\sum_{i \in I^{Ail}} S_i^2}}$

Then the following optimization problem must be solved on line:

$$\min_{\delta_i} \sqrt{\frac{\sum_{i \in I^{Ail}} S_i^2 \delta_i^2}{\sum_{i \in I^{Ail}} S_i^2}} \quad (27)$$

under the constraints:

$$\text{Roll manoeuvre constraint: } \sum_{i \in I^{Ail}} X_i^L = L(t) - L_0(\beta, p, r, \phi) \quad (28)$$

$$\text{Bending moment constraint: } A + \sum_{i \in I^{Ail}} Y_i \delta_i \leq M_{emp}^{\max} \quad (29)$$

$$\text{Actuators constraints:} \quad \delta_i^{\min} \leq \delta_i \leq \delta_i^{\max} \quad i \in I^{Ail} \quad (30)$$

Here  $I^{Ail}$  is the set of currently available lateral actuators. In this case the decision problem turns out to be a classical small scale linear-quadratic optimization problem for which many fast solution methods are available. When no feasible solution is available, the nominal manoeuvre must be abandoned and a modified manoeuvre should be adopted.

## 6. Conclusions

In this communication an approach to manage the control surfaces of an aircraft, with or without actuator failures, during manoeuvres has been proposed. The main objective is to perform the planned manoeuvre while limiting the structural strain. The case of a pure roll manoeuvre has been considered. The contributions of each actuator, spoilers and ailerons, to the aerodynamic forces and moments and to the bending and the flexion moments  $M_f$  and  $M_s$  are considered within an initial additive framework. The proposed approach leads to the formulation of a linear quadratic problem which should be solved on-line. This approach leads to the development of a new supervision function for the control channels of a manoeuvring aircraft.

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