Princing ATC/ATM services: Part I: the case of a public provider

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PRICING OF ATC/ATM SERVICES

Part I: THE CASE OF A PUBLIC PROVIDER

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RESUMO
Neste estudo uma abordagem multi niveis é proposta para estudar o problema da definição das taxas a serem aplicadas por un provedor publico de serviços de controle e gerenciamento do trafego aéreo. O provedor de serviço procura, dentro de toda segurança, fomentar a demanda final dos usuários do sistema de transporte aéreo. Esta analise lea a considerar a resolução de um problema de programação matemática de tipo bi nivel onde o provedor do serviço é o "líder" e o setor das empresas aéreas é o "seguidor". No caso mono dimensional, uma solução analítica é estabelecida e analizada para este problema. Um esquema natural de negociação entre o provedor de serviço e as empreasas aéreas é então considerado e condições suficientes de convergência do processo para a solução ôtima são estabelecidas.

ABSTRACT
In this study a multilevel framework is proposed to analyze the charges definition problem for a public ATC/ATM service provider whose final objective is, while guarranteeing safety, to promote the level of end users demand. This leads to the formulation of a bi-level optimization program involving ATC/ATM service provider as the leader and the whole airline sector as the follower. A direct optimal solution has been obtained in the simplistic considered one dimensional case. Then a natural negotiation process between the ATC/ATM service provider and the airline sector is introduced. Sufficient conditions are established so that the proposed negotiation process converges towards the optimal solution.

Keywords: pricing, bilevel programming, ATC/ATM, air transportation

1. INTRODUCTION

Along the last decades, many studies in the fields of Operations Research, Systems Management and Applied Economics have been devoted to air transportation planning , tariffs and operations related issues. In general the analysis which have been performed are limited to direct effects so that the scope of the adopted models are in general too limited. This implies that feedback phenomena between the different actors and involved air transportation activities cannot be fully taken into account to perform a comprehensive analysis and to design efficient plans and policies . In this study a multilevel approach is developped.

The objective assumed for airlines in this study is of a pure economic nature: profit maximization. The main concern of this study being with the definition of efficient ATC/ATM charges , the whole airline sector is taken as a whole, so that market competition between airlines is not contemplated. This is a limitation of the study, which is done in sake of limited complexity, since in fact, ATC/ATM charges may have some influence on the equilibrium state of different air transportation markets. However, it is also worth to observe that these airlines are in general represented by a unique entity during negotiations with ATC/ATM authorities.

In the case of a public ATC/ATM service provider, it is considered that the main objective is to promote air transportation for end users, i.e. the passengers (freight is not considered explicitly in this study), through a safe and efficient transportation supply by airlines and ATM authorities. Hence a passengers demand model, reactive to airlines tariffs is introduced to take into account indirect influence of ATC/ATM charges on passengers demand levels over different air transportation markets. Another objective, which is taken into account through inequality constraints, is that the economic performance of the airline sector is not impaired by the retained levels of ATC/ATM charges. Also, ATC/ATM costs related directly with the current traffic situation (investments costs leading to enlarged ATC/ATM capacity to face future traffic situations are not considered) should be adequately covered. This results in a bilevel optimization problem which is of the upper linear-lower linear class.

To illustrate the approach a one dimensionnal system is considered. In that case it is possible to establish directly its optimal solution while sufficient conditions are established so that a negotiation process between ATM authorities and the airline sector leads to this optimal solution. There, the problem is split in two dependent problems: one where ATM authorities determine ATC/ATM charges for given airlines tariffs and one where the airline sector determines the tariffs and seat capacity supplies over the different markets for given ATC/ATM charge levels.

2. BASIC ASSUMPTIONS

Here is considered the case of an elementary air transportation system composed of a single pair of airports linked by a single air route. There is a unique ATC/ATM operator and a unique airline operating between these two airports.

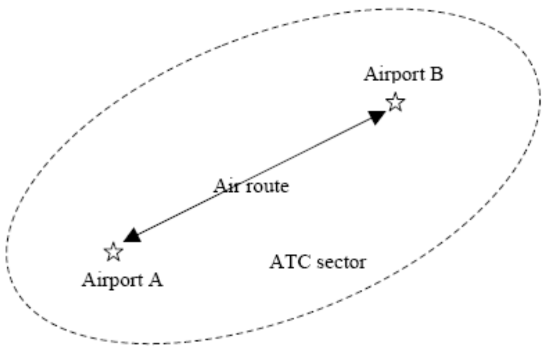

Figure 1: The elementary ATC/ATM case

The potential demand is supposed to be composed of round trips and to obey, for the sake of simplicity and the ability to develop clear analytical and graphical results, to the following affine demand function:

$$\phi = D_0 - \rho\, \pi(1+\lambda) \tag{1}$$

where ϕ is the effective level of potential demand. Here it is supposed that there is a unique class of travelers and a unique apparent price $\tilde{\pi} = \pi(1+\lambda)$ is adopted for round trips. D_0 is an absolute potential demand and ρ is a constant positive parameter characteristic of the response of the market to price changes. The parameter λ represents a tax index applied to each trip ticket.

Other demand models such as the exponential one with constant price elasticity :

$$\phi = D_0 \, e^{-\rho \, \tilde{\pi}} \tag{2}$$

or either unspecified models such as:

$$\phi = D(\tilde{\pi}) \tag{3}$$

with adequate assumptions such as:

$$\partial D / \partial \tilde{\pi} \leq 0 \quad \text{and} \quad \partial^2 D / \partial \tilde{\pi}^2 \geq 0 \tag{4}$$

could have been adopted.

Beyond the ATC charges, an additional way to fund the ATC services which appears natural, is to assign a proportion α ($\alpha \in [0,1]$) of the above tax, which is paid by the final users, to the ATC service provider.

The transport capacity of the airline is given by the maximum affordable frequency of service $f_{\max}$ which is related with the size of the fleet of the airline. Here for simplicity and considering that for the given time period f can be a high number, f will be taken as real. When a frequency of service f is adopted, the operations costs are supposed to be given by:

$$(c+v)\, f + C_{ALN}^{F} \tag{5}$$

here:

- c is a positive parameter (a mean variable cost with respect to frequency). It is related with the price of fuel, the cost of the crew and the length of the flights.
- C_{ALN}^{F} is a fixed cost related with the sizes of the fleet and the crews of the airline as well as with the characteristics of the operated network.
- v is the ATC tariff applied to a round flight, including en route control and airport control. Here no distinction will be made between airport taxes and approach and en route charges.

The available seat capacity is given by:

$$q\, f \tag{6}$$

where q is the mean seat capacity of an aircraft of the fleet of the airline.

The operating costs of the ATC supplier are given by:

$$\sigma\, f + C_{ATC}^{F} \tag{7}$$

here:

- σ is a positive parameter (a mean variable cost with respect to frequency). It is related mainly with the length of the flights.
- C_{ATC}^{F} is a fixed cost related with the characteristics of the controlled airspace and with the size of the ATC staff.

No saturation effects with consequences over the cost functions of the ATC service provider and the airline are considered in this study.

3. OPTIMIZING CHARGES FOR A PUBLIC ATC/ATM SERVICE SUPPLIER

Here it is supposed that the final objective of the public ATC/ATM service supplier is to maximize the satisfied demand while guarranteeing a minimum economic return for the ATC/ATM services, R_{ATC}, and a minimum economic return R_{ALN} for the airline. It is supposed also that the airline tries to maximize her benefit taking into account her cost function and the ATC/ATM tariff.

3.1 A bilivel program for public ATC/ATM pricing

According to the above assumptions, a bilivel program can be established:

$$\max_{v \geq 0} \quad \phi \tag{8}$$

with

$$vf + \alpha\lambda\pi\phi - (\sigma f + C_{ATC}^{F}) \geq R_{ATC} \tag{9}$$

where λ is the tax rate applied to air travellers, and the airline's profit constraint:

$$\pi\,\phi - ((c+v)f + C_{ALN}^{F}) \geq R_{ALN} \tag{10}$$

where π and f are given by the solution of:

$$\max_{\pi,\, f} \quad \pi\,\phi - ((c+v)f + C_{ALN}^{F}) \tag{11}$$

with

$$\phi = \max\{0, \min\{q\ f, D_0 - \rho\ \pi(1+\lambda)\}\ \} \tag{12}$$

and

$$0 \leq f \leq f_{\max} \tag{13}$$

3.2 Solving the airlines profit maximization problem

To solve the airline's profit maximization, two cases must be considered with respect to the effective level of passengers demand.

Either :

$$\phi = q\ f \tag{14}$$

or:

$$\phi = D_0 - \rho\ \pi(1+\lambda) \tag{15}$$

3.2.a Case in which effective demand is determined by the seat capacity

In this first case, we have:

$$q\ f \leq D_0 - \rho\ \pi(1+\lambda) = D_0 - \tilde{\rho}\,\pi \tag{16}$$

where

$$\tilde{\rho} = (1+\lambda)\,\rho \tag{17}$$

Now, considering airline's profit level curves p_{ALN}, we get:

$$f = \frac{p_{ALN} + C_{ALN}^{F}}{q\pi - (c+v)} \tag{18}$$

where p_{ALN} is a chosen level of profit for the airline. Changing the value of p_{ALN} and considering constraint (13) in the (π, f) plane , we get hyperbola arcs for the profit level curves.

It appears clearly in figure 2 that the maximum profit is obtained when constraint (13) reduces to equality. This result is also valid (see figure 3) when the tangency of a profit level curve and the demand ligne provides a frequency above f_{max}.
In the case of no active fleet constraints, we get a double solution for the following equation:

$$\frac{p_{ALN} + C^F_{ALN}}{q\pi - (c+v)} = \frac{1}{q}(D_0 - \tilde{\rho}\,\pi) \tag{19}$$

when p_{ALN} reaches the value:

$$p^{\max}_{ALN} = \frac{1}{\tilde{\rho}}\left(\frac{(c+v)\tilde{\rho} + qD_0}{2q}\right)^2 - (c+v)D_0 / q - C^F_{ALN} \tag{20}$$

Then:

$$f^* = \max\left\{0, \min\left\{ f_{\max}, \frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c+v)\right\}\right\} \tag{21}$$

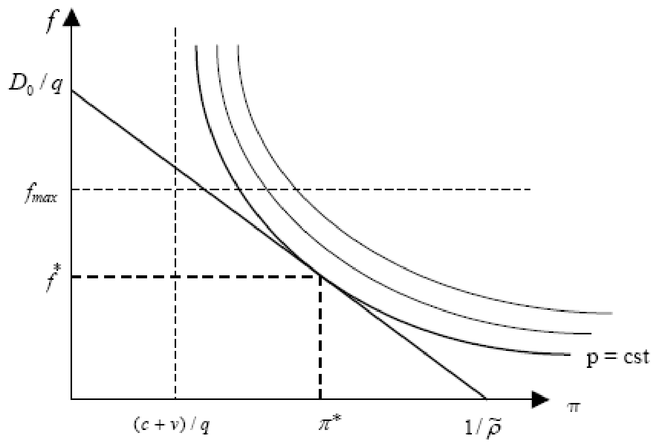

Figure 2: Optimization of airline profit, no active fleet constraint, full capacity

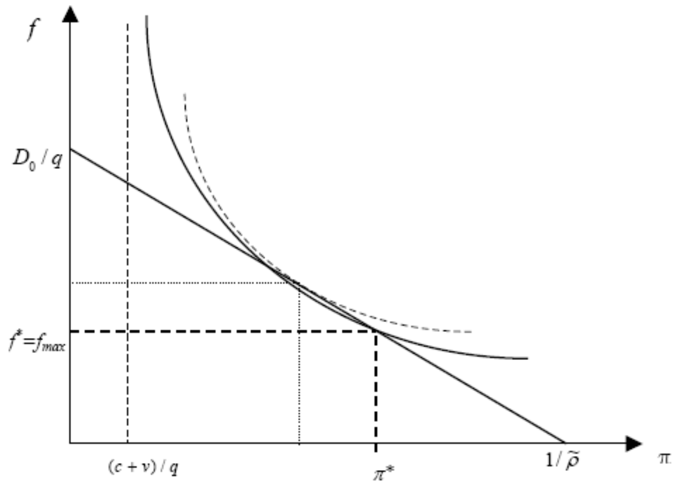

Figure 3: Optimization of airline profit with an active fleet constraint at full capacity

and either (no activation of the fleet constraint):

$$\pi^* = \frac{c+v}{2q} + \frac{D_0}{2\tilde{\rho}} \tag{22}$$

or (activation of the fleet constraint):

$$\pi^* = \frac{D_0}{\rho} - \frac{q}{\tilde{\rho}} f_{\max} \tag{23}$$

while the optimal airline profit is given either by $p^*_{ALN} = p^{\max}_{ALN}$ (relation (16)) or by:

$$p^*_{ALN} = (\frac{q}{\tilde{\rho}} D_0 - c) f_{\max} - \frac{q^2}{\tilde{\rho}} f_{\max}{}^2 - C^F_{ALN} \tag{24}$$

3.2.b Case in which effective demand is determined by the price level

In this second case, we have:

$$q\ f \geq D_0 - \tilde{\rho}\,\pi \tag{25}$$

Now, considering airline's profit level curves p_{ALN}, we get:

$$f = \frac{D_0}{c+v}(1 - \tilde{\rho}\,\pi)\pi - \frac{p_{ALN} + C^F_{ALN}}{c+v} \tag{26}$$

where p_{ALN} is a chosen level of profit for the airline. Changing the value of p_{ALN} and considering constraint (25) in the (π, f) plane , we get parabola arcs for the profit level curves.

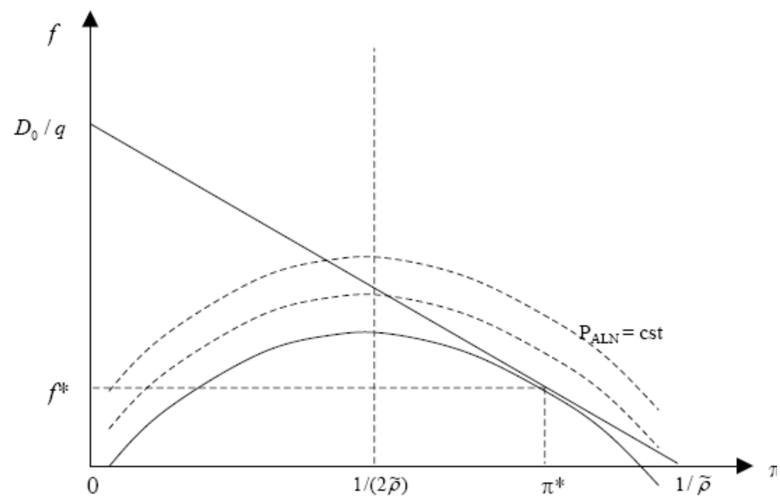

Figure 4: Optimization of airline profit with priced constrainted demand

It appears that here again the optimal solution is given by relations:

$$f^* = \frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c+v) \tag{27}$$

with again (20), (22) and :

$$\phi = \frac{D_0}{2} - \frac{\tilde{\rho}}{2q}(c+v) \tag{28}$$

Relation (22) reveals the influence of ATC/ATM tolls over air transportation tariff and consequently over satisfied demand (relation (28)). In the following, it will be assumed that the fleet constraint remains inactive.

3.3 Solving the public ATC/ATM service provider problem

Considering the solution of the airline profit maximization problem, problem (8), (9), (10) with (11), (12) and (13) becomes:

$$\max_{v\geq 0} \quad \frac{D_0}{2} - \frac{\tilde{\rho}}{2q}(c+v) \tag{29}$$

under the constraints:

$$\frac{D_0 v}{2q} - \frac{\tilde{\rho} v}{2q^2}(c+v) + \alpha\lambda\left(\frac{c+v}{2q} + \frac{D_0}{2\tilde{\rho}}\right)\left(\frac{D_0}{2} - \frac{\tilde{\rho}}{2q}(c+v)\right) + \sigma\left(\frac{\tilde{\rho}}{2q^2}(c+v) - \frac{D_0}{2q}\right) - C_{ATC}^F \geq R_{ATC} \tag{30}$$

$$\left(\frac{c+v}{2q} + \frac{D_0}{2\tilde{\rho}}\right)\left(\frac{D_0}{2} - \frac{\tilde{\rho}}{2q}(c+v)\right) - (c+v)\left(\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c+v)\right) - C_{ALN}^F \geq R_{ALN} \tag{31}$$

and

$$0 \leq v \leq (q/\tilde{\rho})D_0 - c \tag{32}$$

This last condition insures that there is some satisfied demand.
This problem can be rewritten as:

$$\min_{x} \quad x \tag{33}$$

under the constraints:

$$(x-c)\left(\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}x\right) + \alpha\lambda\left(\frac{x}{2q} + \frac{D_0}{2\tilde{\rho}}\right)\left(\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q}x\right) + \sigma\left(\frac{\tilde{\rho}}{2q^2}x - \frac{D_0}{2q}\right) - C_{ATC}^F \geq R_{ATC} \tag{34}$$

$$\left(\frac{x}{2q} + \frac{D_0}{2\tilde{\rho}}\right)\left(\frac{D_0}{2} - \frac{\tilde{\rho}}{2q}x\right) - x\left(\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}x\right) - C_{ALN}^F \geq R_{ALN} \tag{35}$$

and

$$x_{\min} = c \leq x \leq (q/\tilde{\rho})D_0 = x_{\max} \tag{36}$$

Constraints (34) and (35) can be rewritten as:

$$-ax^2 + bx - c \geq 0 \tag{37}$$

with
$$\begin{cases} a = \dfrac{\tilde{\rho}}{2q^2}(1+\alpha\,\lambda/2) \\ b = \dfrac{\tilde{\rho}}{2q^2}(\sigma + c + \dfrac{q}{\tilde{\rho}}D_0) \\ c = \dfrac{D_0}{2q}(c+\sigma+\dfrac{\alpha\,\lambda}{2\tilde{\rho}}D_0) + C_{ATC}^F + R_{ATC} \end{cases} \tag{38}$$

and

$$\frac{\tilde{\rho}}{4q^2}x^2 - \frac{D_0}{2q}x + \frac{{D_0}^2}{4\tilde{\rho}} - C_{ALN}^F - R_{ALN} \geq 0 \tag{39}$$

The equation:
$$-ax^2 + bx - c = 0 \tag{40}$$

presents real roots which are then positive, when:

$$b \geq 2\sqrt{a\ c} \tag{41}$$

or

$$C_{ATC}^F + R_{ATC} \leq \frac{\rho(1+\lambda)}{8q^2}\frac{(c+\sigma+(q/\rho(1+\lambda))D_0)^2}{1+\alpha\,\lambda/2} - \frac{D_0}{2q}(c+\sigma+\frac{\alpha\lambda}{2\rho(1+\lambda)}D_0) \tag{42}$$

When $\alpha = 0$, this condition reduces to:

$$C_{ATC}^F + R_{ATC} \leq \frac{\rho(1+\lambda)}{8q}(\frac{D_0}{\rho(1+\lambda)} - \frac{c+\sigma}{q})^2 \tag{43}$$

Let x_1 be the minimum real root of (40) when it exists:

$$x_1 = \frac{(\sigma + c + (q/\tilde{\rho})D_0) - \sqrt{\Delta(\alpha)}}{2(1+\alpha\lambda/2)}$$

with
$$\begin{gathered} \Delta(\alpha) = (c+\sigma+(q/\tilde{\rho})D_0)^2 \\ -4(1+\alpha\lambda/2)((q/\tilde{\rho})D_0(c+\sigma+(\alpha\lambda/(2\tilde{\rho}))D_0) + (2q^2/\tilde{\rho})(C_{ATC}^F + R_{ATC})) \end{gathered} \tag{44}$$

When α =0, we get:

$$x_1 = \frac{(\sigma + c + (q/\tilde{\rho})D_0) - \sqrt{\Delta(0)}}{2} \tag{45}$$

with

$$\Delta(0) = (c+\sigma+(q/\tilde{\rho})D_0)^2 - 4((q/\tilde{\rho})D_0(c+\sigma) + (2q^2/\tilde{\rho})(C_{ATC}^F + R_{ATC})) \tag{46}$$

and x_1 is then less than x_{max} if:

$$(q/\tilde{\rho})D_0 + \sqrt{\Delta(0)} - (c+\sigma) \geq 0 \tag{47}$$

It will be supposed in the following that α, λ and R_{ATC} are chosen so that x_1 is less than x_{max} .

The roots of equation:

$$\frac{\tilde{\rho}}{4q^2}x^2 - \frac{D_0}{2q}x + \frac{D_0^{\ 2}}{4\tilde{\rho}} - C_{ALN}^F - R_{ALN} = 0 \tag{48}$$

are always real, one of them being always positive:

$$x_2^+ = \frac{q}{\tilde{\rho}}(D_0 + \sqrt{\tilde{\rho}(C_{ALN}^F + R_{ALN})}) \tag{49}$$

The smallest of these roots, x_2^-, is such as:

$$x_2^- = \frac{q}{\tilde{\rho}}(D_0 - \sqrt{\tilde{\rho}(C_{ALN}^F + R_{ALN})}) \tag{50}$$

Considering (36), x_2^- is such as:

$$x_2^- < x_{\max} \tag{51}$$

Here also,it will be supposed in the following that λ and R_{ALN} are chosen so that x_2 is less than x_{max} . By inspection of all the possiblities, only two situations lead to a solution for problem (33), (34), (35) and (36).

They are represented graphically bellow:

- In the case in which x_2 is positive superior to c, to have a solution , x_1 must be inferior or equal to x_2 and superior to c, then the solution x^* is equal to x_1.

- In the case in which x_2^- is less than c, there is a solution given by $x^* = \max\{x_1, x_2^+\}$ when:

$$c \le \max\{x_1, x_2^+\} \le x_{\max} \tag{52}$$

The ordinates in figure 5 and figure 6 bellow represent the profits above the guarranteed values R_{ATC} and R_{ANL}.

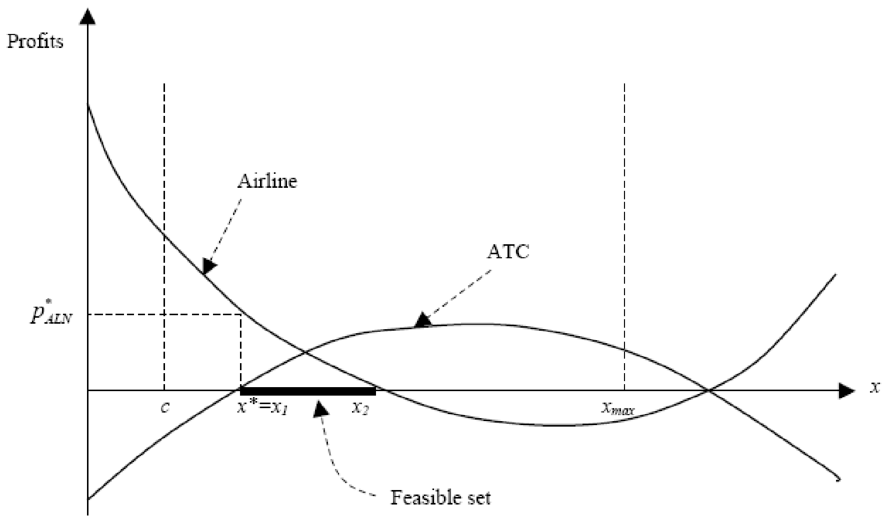

Figure 5: Solution of the public ATC/ATM supplier problem (case in which x_2^- is superior to *c*)

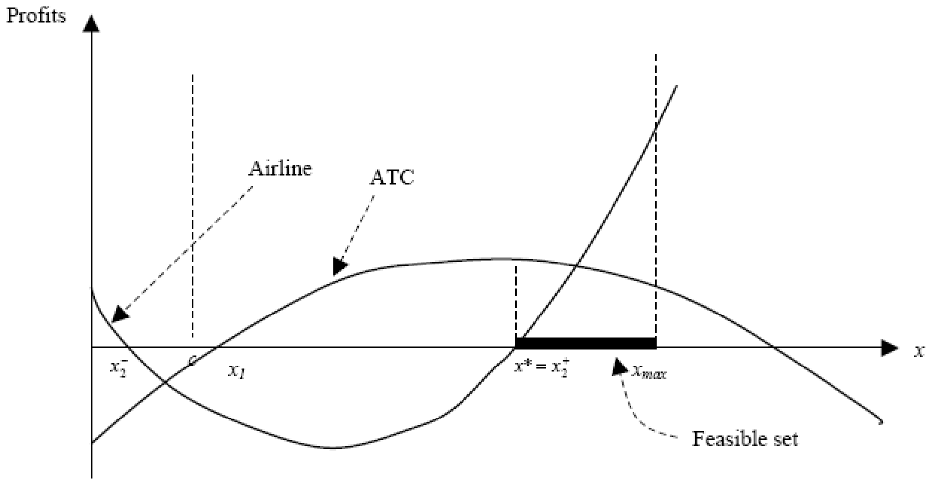

Figure 6: Solution of the public ATC/ATM supplier problem (case in which x_2^- is inferior to *c*)

4. GLOBAL SOLUTION FOR PUBLIC ATC/ATM SERVICE PROVIDER

It will be supposed for the following that λ and R_{ALN} are chosen so that $x_2^- \geq c$ (figure 5):

$$0 \leq \lambda \leq (\frac{q}{c\rho} D_0 + \frac{q^2}{2\rho c^2}(C_{ALN}^F + R_{ALN} - \sqrt{\frac{4c}{q} D_0(C_{ALN}^F + R_{ALN}) + (C_{ALN}^F + R_{ALN})^2}\,) - 1 \qquad (53)$$

then, only the first case is to be considered and the solution of the whole problem (8) to (13) is given by:

$$v^* = \frac{(\sigma + c + (q/\tilde{\rho})D_0) - \sqrt{\Delta(\alpha)}}{2(1 + \alpha\lambda/2)} - c \qquad (54)$$

The level of the satisfied demand is given by:

$$\phi^* = \frac{D_0}{2} - \frac{\rho(1+\lambda)}{2q}(c + v^*) \qquad (55)$$

The profit of the public ATC supplier is equal (see figure 5) to the minimum guarranteed level R_{ATC} while the profit of the airline is given by:

$$p^* = \frac{1}{\rho(1+\lambda)}(\frac{(c+v^*)\rho(1+\lambda) + qD_0}{2q})^2 - (c+v^*)D_0/q - C_{ALN}^F \qquad (56)$$

which is superior or equal (see figure 4) to the minimum guarranted level R_{ALN}.

From relations (54) and (55) , it is easy to show ($\partial v^* / \partial \alpha < 0$) that for a given value of λ, the value of α which maximizes ϕ is $\alpha^* = 1$.
Then, a further step towards the optimization of the sector would be to choose efficiently the rate of the tax applied to the trips. In that case, considering (54) with α =1 , we should define $v^*(\lambda)$ by :

$$v^*(\lambda) = \frac{1}{2}((\sigma - c + (q / \tilde{\rho}(\lambda))D_0) - \sqrt{(c + \sigma - qD_0 / \tilde{\rho}(\lambda)) - 8(C_{ATC}^F + R_{ATC}))q^2 / \tilde{\rho}(\lambda))} \tag{57}$$

and the best value of λ would be solution of:

$$\max_{\lambda} \frac{D_0}{2} - \frac{\rho(1+\lambda)}{2q}(c + v^*(\lambda)) \tag{58}$$

with

$$\frac{1}{\rho(1+\lambda)}(\frac{(c + v^*(\lambda))\rho(1+\lambda) + qD_0}{2q})^2 - (c + v^*(\lambda))D_0 / q - C_{ALN}^F \geq R_{ALN} \tag{59}$$

and

$$0 \leq \lambda \leq (D_0 - (2qc + \sqrt{4\rho(C_{ALN}^F + R_{ALN})}) \tag{60}$$

However, the value of parameter λ is the result of an exogenous choice process where overall economic as well as political considerations are taken into account.

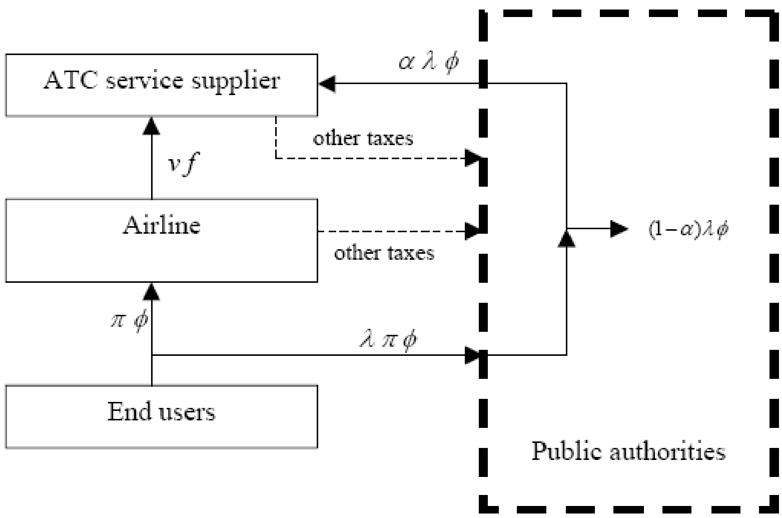

Figure 7: Financial flows and activity levels with public ATC/ATM service supplier

5. OPTIMALITY OF A PRICING NEGOTIATION PROCESS

Since the considered pricing problem involves two major economic agents, the ATC/ATM service provider and the airline sector, a solution of the considered bilevel problem resulting from the hypothesis with respect to the nature, public or private, and the goals of the ATC/ATM service provider, through a negociation process can be of interest. Following the general formulation (5) to (10) , a natural negociation process based on the objectives of the involved economic agents could be the following:

The public ATC/ATM service supplier solves at iteration n+1 the following problem with respect to v , given π^n and f^n:

$$v^{n+1} = \arg\left\{ \max_{v \geq 0} \quad \phi(v) \right\} \tag{61}$$

with

$$vf^n + \alpha\lambda\,\pi^n\,\phi^n - (\sigma\,f^n + C^F_{ATC}) \geq R_{ATC} \tag{62}$$

and the airline's profit constraint:

$$\pi^n\;\phi(v) - ((c+v)f^n + C^F_{ALN}) \geq R_{ALN} \tag{63}$$

where π^n and f^n are provided by the airline sector which solves the following problem given v^n:

$$\max_{\pi,\,f} \quad \pi\,\min\{q\,f, D_0 - \widetilde{\rho}\,\pi\} - ((c+v^n)f + C^F_{ALN}) \tag{64}$$

with

$$0 \leq f \leq f_{\max} \tag{65}$$

This process is represented in figure 8:

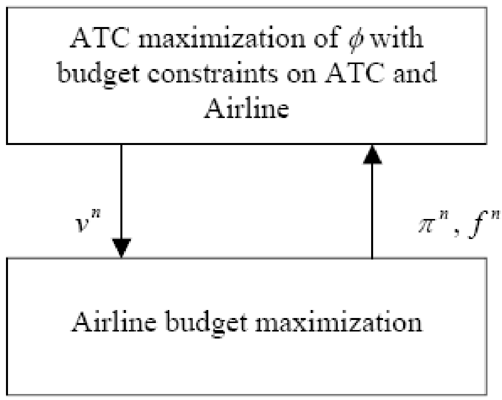

Figure 8: Negotiacion process between public ATC/ATM and airlines

Since $\phi(v) = \dfrac{D_0}{2} - \dfrac{\widetilde{\rho}}{2q}(c+v)$, the ATC/ATM service supplier has to satisfy the two budget constraints with the minimum value for v, since the profit of the airline is a decreasing function of v, the solution of the problem corresponds to the saturation of his own budget constraint, so that:

$$v^{n+1} = \frac{R_{ATC} + C_{ATC}^{F} + \sigma f^{n} + \alpha \lambda \pi^{n}((\tilde{\rho} c / q) - D_{0})/2}{f^{n} - (\tilde{\rho}/2q)\alpha \lambda \pi^{n}} \tag{66}$$

Here we consider the case in which the solution of the airline problem is given by:

$$f^{n} = \frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c + v^{n}) \tag{67}$$

with

$$\pi^{n} = \frac{c + v^{n}}{2q} + \frac{D_0}{2\tilde{\rho}} \tag{68}$$

then we get a recurent formula for v^n:

$$v^{n+1} = \frac{R_{ATC} + C_{ATC}^{F} + \sigma(\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c + v^{n})) + \alpha \lambda (\frac{c + v^{n}}{2q} + \frac{D_0}{2\tilde{\rho}})((\tilde{\rho} c / q) - D_0)/2}{\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c + v^{n}) - (\tilde{\rho}/2q)\alpha \lambda (\frac{c + v^{n}}{2q} + \frac{D_0}{2\tilde{\rho}})} \tag{69}$$

For simplicity we consider here only the case in which $\alpha = 0$, then:

$$v^{n+1} = \frac{R_{ATC} + C_{ATC}^{F} + \sigma(\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c + v^{n}))}{\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c + v^{n})} = \sigma + \frac{R_{ATC} + C_{ATC}^{F}}{\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}(c + v^{n})} \tag{70}$$

or

$$v^{n+1} = \sigma + \frac{R_{ATC} + C_{ATC}^{F}}{(\frac{D_0}{2q} - \frac{\tilde{\rho}}{2q^2}c) - \frac{\tilde{\rho}}{2q^2}v^{n}} \tag{71}$$

Here we consider that the following condition is satisfied:

$$D_0 > \tilde{\rho} c / q \tag{72}$$

Then in figure 9, the convergence of the negociation process is analyzed graphically.

It appears that if:

$$(c+\sigma+(q/\tilde{\rho})D_0)^2-4((q/\tilde{\rho})D_0(c+\sigma)+(2q^2/\tilde{\rho})(C_{ATC}^F+R_{ATC}))>0 \qquad (73)$$

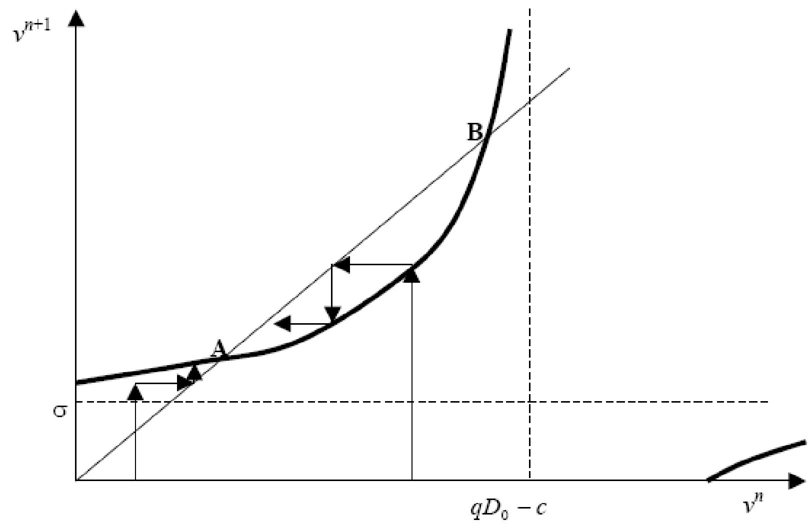

Figure 9: Convergence of the public ATC/ATM- Airline negociation process

or

$$R_{ATC}<\frac{(c+\sigma+(q/\tilde{\rho})D_0)^2}{8q^2/\tilde{\rho}}-C_{ATC}^F-\frac{D_0(c+\sigma)}{2q} \qquad (74)$$

then there are two equilibrium points: point A which is a stable equilibrium solution and point B which is an unstable equilibrium solution. It appears that point A corresponds to the optimal solution of the bilevel problem treated in section III.3. In that case the proposed negociation schemes leads to the optimal solution.

6. CONCLUSION

In this study a multilevel framework has been proposed to analyze the ATC/ATM charges definition problem in the case of a public ATC/ATM service provider in a deregulated market. Here the one dimensional case has been considered. The objective which has been assumed for the ATC/ATM public service provider is to promote air transportation for end users while the objective of the airlines sector is profit maximization. This has led to the formulation of a bi-level optimization program involving ATC/ATM service provider as the leader and the whole airline sector as the follower. A direct optimal solution has been obtained in the simplistic considered one dimensional case. Then a natural negotiation process between the ATC/ATM service provider and the airline sector has been introduced, splitting this problem in two dependent problems: one where ATM authorities determine ATC/ATM charges for given airlines tariffs and one where the airline sector determines the tariffs and seat capacity supplies over the different markets for a given ATC/ATM charge level. Sufficient conditions have been established so that the proposed negotiation process converges towards the optimal solution. Then it appears of interest to the service provider to adopt a similar negotiation process when dealing with the full scale networked pricing problem.

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