MANAGEMENT OF CONTROL CHANNELS UNDER ACTUATOR FAILURE: AN OPTIMIZATION APPROACH

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Abstract

In this communication we consider a transportation airplane in the situation in which a main aerodynamic actuator failure occurs while he has either to maintain trim conditions or to perform guidance maneuvers. Depending of the extension and the importance of the failure and considering the redundant effects from other actuators, three cases can be considered: the case in which maneuvers can be performed perfectly, the case in which maneuvers can be performed approximately and the case in which maneuvers cannot be performed anymore. In this paper, the two first cases are tackled. Using dynamic inversion of flight dynamics, the necessary aerodynamic torques to perform a given basic maneuver, are computed. Then an on-line optimization problem whose solution provides the necessary deflection of the available actuators is introduced.

Introduction

In this communication we consider a transportation airplane in the situation in which a main aerodynamic actuator failure occurs while he has either to maintain trim conditions or to perform guidance maneuvers. Depending on the extension and the importance of the failure and considering the redundant effects from other actuators, three cases can be considered: the maneuver can be performed perfectly, the maneuver can be performed approximately and the maneuver cannot be performed anymore. In this paper, the two first cases are tackled: using dynamic inversion of flight dynamics, the necessary moments to perform a given guidance maneuver are computed, then an optimization problem is considered to generate on-line reference values for the remaining actuators. This optimization problem, to be solved on-line, considers explicitly guidance constraints as well as structural constraints from the aircraft and the different limitations of actuators. This represents the main difference with other previous approaches to actuator fault management [1-4]. In the cases considered, a linear quadratic programming formulation of the optimization problem can be adopted and different approaches to get on-line solutions, are discussed.

The case of cruise trim which is a very common situation during a flight and the case of a turn maneuver which may be quite demanding for the aircraft wings, are more particularly considered. The wings actuators considered are the ailerons, the spoilers and the flaps, but the other aerodynamic actuators, elevators and rudders, must be considered to take into account the existing coupling effects along the three body axis. The limitations of these actuators, such as position, speed and response time, are considered explicitly in this study as well as structural limitations expressed by maximum wing bending and flexion torques.

The proposed approach should be generalized to tackle in a systematic way the diversity of possible actuator failures while considering the criticality of this issue with respect to flight safety.

Aircraft Flight Dynamics

Here we consider the flight dynamics of a transportation aircraft with some failed aerodynamic actuators (they are either stuck to a fixed position, or free to rotate creating then no significant torque). The case of a twin engine aircraft is particularly considered but the analysis can be extended to other multi-engine aircraft. The transportation aircraft is assumed to be a rigid body flying in no wind standard atmosphere over a flat Earth. The motion of the center of gravity of the aircraft is then given in body axis by the following force equations [5] :

$$m(\dot{u} + qw - rv) = -mg\sin\theta - 1/2\,\rho V^2 S\, C_x + (P_L + P_R)\cos\sigma \quad (1.1)$$

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$$m(\dot{v} + ur - pw) =$$

$$mg \cos\theta \ \sin\phi + 1/2\ \rho V^2 S\ C_y \quad (1.2)$$

$$m(\dot{w} + pv - qu) =$$

$$mg \cos\theta \cos\phi + 1/2\ \rho V^2 S\ C_z + (P_L + P_R) \sin\sigma \quad (1.3)$$

The rotational motion is the result of the different torques applied to the whole aircraft. It is given by the equations:

$$A\dot{p} - E\dot{r} + (C - B)rq - Epq =$$

$$1/2\ \rho V^2 S\, l\, C_l - a(P_L - P_R)s \quad (2.1)$$

$$B\dot{q} + (A - C)rp - E(p^2 - r^2) =$$

$$1/2\ \rho V^2 S\, l\, C_m + b\,(P_L + P) \quad (2.2)$$

$$C\dot{r} - E\dot{p} + (B - A)qp =$$

$$1/2\ \rho V^2 S\, l\, C_n + a\,(P_L - P_R) \cos\sigma \quad (2.3)$$

where *A*, *B*, *C* and *E* are the inertia moments which compose the aircraft inertia matrix :

$$\begin{bmatrix} A & 0 & -E \\ 0 & B & 0 \\ -E & 0 & C \end{bmatrix} \quad (3)$$

where $\underline{V} = (u, v, w)'$ and $\underline{\Omega} = (p, q, r)'$ are respectively the instant translation speed vector and the rotation speed vector expressed both in the body frame. We will assume in the following that σ, the angle between the thrust direction and the aircraft longitudinal axis, is equal to zero. Here C_x, C_y, C_z, C_l, C_m and C_n are the aerodynamic coefficients, *m* is the mass of the aircraft. Here θ and ϕ are respectively the pitch angle and the bank angle while ψ is the heading angle.

The Euler equations provide the relations between the derivatives with respect to time of the attitude and the heading angles (ϕ, θ and ψ) and the body components (p,q,r) of the rotational speed of the aircraft:

$$\dot{\theta} = -r \sin\emptyset + q \cos\emptyset \quad (4.1)$$

$$\dot{\psi} = \frac{q \sin\phi + r \cos\phi}{\cos\theta} \quad (4.2)$$

$$\dot{\phi} = p + tg\theta(q \sin\emptyset + r \cos\phi) \quad (4.3)$$

Here V is the modulus of the aircraft airspeed, α is the angle of attack and β is the sideslip angle, where:

$$u = V \cos\beta \cos\alpha\,, \quad v = V \sin\beta\,, \quad w = V \cos\beta \sin\alpha \quad (5)$$

Observe that, since α should remain small and β should remain very small, from this last relation, α can be approximated by:

$$\alpha \approx w/V \quad (6)$$

Modeling the Effectiveness of Actuators

The effectiveness of the control surfaces is made apparent by their contribution to the dimensionless coefficients appearing in the expressions of the aerodynamic forces and torques. Then the angular deflections of these control surfaces produce a collective effect over the aircraft which should satisfy structural constraints.

Aerodynamic Coefficients

The global dimensionless coefficients used to express aerodynamic forces can be given by:

$$C_x = C_{x0l} + k\, C_z^2 \quad (7.1)$$

$$C_y = C_{y\beta}.\beta \ ++ C_{yp}\,.p.l_A/V_0 + C_{yr}\,.r.l_A/V_0$$

$$+ \underline{C}_{y\delta p}{}' .\underline{\delta}_p + \underline{C}_{y\delta r}{}' .\underline{\delta}_r \quad (7.2)$$

$$C_z = C_{z0} + C_{z\alpha}.\alpha + C_{zphr}\,.\delta_{ths} + \underline{C}_{zq}{}' .\underline{\delta}_q \quad (7.3)$$

where the different coefficient C_{ij} are also dimensionless.

Here standard scalar notation for main aerodynamic actuators is replaced by a vector one where each elementary aerodynamic surface is distinguished and assigned to its main effect (roll, pitch or yaw effect). The non dimensional coefficients of the different aerodynamic torques can in general be expressed such as:

$$C_m = C_{m\alpha}.\alpha + C_{mq} q.l_A/V_0$$

$$+ C_{m\delta phr}\,.\delta_{ths} + \underline{C}_{m\delta m}{}'\underline{\delta}_q \quad (8.1)$$

$$C_l = C_{l\beta}\,\beta + C_{lp}\, p.l_A/V_0 + C_{lr}\, rl_A/V_0$$

$$+ \underline{C}_{l\delta p}{}'\underline{\delta}_p + \underline{C}_{l\delta r}{}'\underline{\delta}_r \quad (8.2)$$

$$C_n = C_{n\beta}\,\beta + C_{np}\, pl_A/V_0 + C_{nr}\, r\, l_A/V_0$$

$$+ \underline{C}_{n\delta p}{}'\underline{\delta}_p + C_{n\delta r}\delta_r \quad (8.3)$$

Aerodynamic Torques

The expression of the different aerodynamic torques generated by the control surfaces can be approximated by an affine form with respect to the corresponding deflections of the different aerodynamic actuators (see Figure 1), so that we get expressions such as:

$$M_{ik} = M_{ik}^0 + \mu_{ik}\, \delta_k \qquad (9)$$

where M_{ik} is the i^{th} considered moment (roll, pitch, yaw, bending, flexion), δ_k is the deflection of the k^{th} aerodynamic actuator ($k \in K$={aileron, flap, right spoilers, left spoilers, elevator, rudder }) and μ_{ik} is the current effectiveness of actuator k to produce moment i. The current values $M_{ik}^0(t)$ and $\mu_{ik}(t)$ depend of the airspeed V of the aircraft, its flight level z and of the values of the main motion variables α, β, p, q and r.

Global aerodynamic torques generated by aircraft aerodynamic actuators can be rewritten in an affine form as:

$$L(t) = L^0(t) + \sum_{i \in I^L} X_i^L(t)\ \delta_i(t) \qquad (10.1)$$

$$M(t) = M^0(t) + \sum_{i \in I^M} X_i^M(t)\ \delta_i(t) \qquad (10.2)$$

$$N(t) = N^0(t) + \sum_{i \in I^N} X_i^N(t)\ \delta_i(t) \qquad (10.3)$$

with $I = I^L \cup I^M \cup I^N$, where I^L is the set of actuators able to generate roll moments, I^N is the set of actuators able to generate yaw torques, while I^M is the set of actuators generating pitch moments.The current values of $L^0(t)$, $X_i^L(t)$, $M^0(t)$, $X_i^M(t)$, $N^0(t)$ and $X_i^N(t)$ depend of the airspeed V of the aircraft, its flight level z and of the values of motion variables α, β, p, q and r.

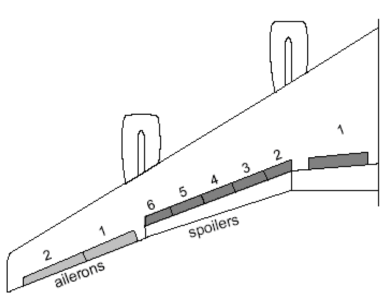

Figure 1. Example of Wing Actuators (A340)

Actuators Constraints and Limitations

The operation of the different actuators must satisfy to global and local physical constraints which must be taken into account by any flight control system.

Position and Speed Actuator Limitations

With respect to control surfaces, the following typical constraints should be met:

$$\delta_i^{\min} \le \delta_i \le \delta_i^{\max} \quad i \in I \qquad (11.1)$$

$$\dot{\delta}_i^{\min} \le \dot{\delta}_i \le \dot{\delta}_i^{\max} \quad i \in I \qquad (11.2)$$

where $\delta_i^{\min}$, $\delta_i^{\max}$, $\dot{\delta}_i^{\min}$ and $\dot{\delta}_i^{\max}$ are extreme position and speed values.

These last conditions can be considered at discrete instants, then they become:

$$\max\left\{\delta_i^{\min}, \delta_i(t-\Delta t) + \dot{\delta}_i^{\min}\Delta t\right\} \le \delta_i(t) \qquad (12.1)$$

$$\delta_i(t) \le \min\left\{\delta_i^{\max}, \delta_i(t-\Delta t) + \dot{\delta}_i^{\max}\Delta t\right\} \qquad (12.2)$$

With respect to turbo fan engines, the thrust of the left and right engines P_L and P_R can be considered to follow first order linear dynamics such as :

$$\tau_M \dot{P}_L + P_L = P_{LC} \quad \text{and} \quad \tau_M \dot{P}_R + P_R = P_{RC} \qquad (13)$$

where τ_M is a time constant and P_{LC} and P_{RC} are reference values for the thrust of the left and right engines. However, for simplicity, in the subsequent study these dynamics will be considered to be fast with respect to the aircraft flight dynamics.

After occurrence of a major aerodynamic actuator failure, the turbofan engines could contribute to flight control and guidance, even if these actuators present important limitations:

- a limited in flight range of values :

$$P_{\min}(z,V) \le P_L, P_R \le P_{\max}(z,V) \qquad (14)$$

- a varying response time (with flight level and airspeed) to throttle lever settings.
- limited effect on pitch torque $b(P_L+P_R)$ with a rather small lever length b .

However the yaw torque $a\ (P_L\text{-}P_R)$ generated by a differential thrust can be of interest to generate an additional yaw moment.

Global Constraints

Global constraints are in general related with structural considerations. It can be shown [6] that total wing bending and flexion torques during maneuver can be written in an affine form as:

$$M_b(t) = A_b(t) + \sum_{i \in I^{wing}} Y_{bi}(t)\ \delta_i(t) \qquad (15.1)$$

and

$$M_f(t) = A_f(t) + \sum_{i \in I^{wing}} Y_{fi}(t)\ \delta_i(t) \qquad (15.2)$$

with $I^{wing} \subset I$ is the set of wing actuators contributing to the bending and flexion torques, where M_b, Y_{bi}, A_f and Y_{fi} depend also of the airspeed V of the aircraft, its flight level z and of the values of motion variables α, β, p, q and r.

Then the global wing bending and flexion constraints can be written as:

$$A_b(t) + \sum_{i \in I^{Ail}} Y_{bi}(t)\ \delta_i(t) \le M_{bend}^{\max} \qquad (16.1)$$

$$A_f(t) + \sum_{i \in I^{wing}} Y_{fi}(t)\ \delta_i(t) \le M_{flex}^{\max} \qquad (16.2)$$

where $M_{emp}^{\max}$ and $M_{flex}^{\max}$ are maximum acceptable bending and flexion torques at the wing basis. Here it is supposed that the satisfaction of these global constraints implies the satisfaction of local bending and flexion torque constraints.

Non Linear Inverse Control Theory

While twenty years ago only linear control law design techniques where available, today different non linear control law design techniques are available to master aircraft dynamics and perform safe and accurate flights: sliding mode and robust control , nonlinear inverse control [7], backstepping control, differentially flat control and neural [8] as well as combinations of these techniques. One of these techniques, non linear inverse control, formalized by the theoretical work of *Isidori*, has been of particular interest in the field of flight control]. In this study we adopt this technique to develop a solution path to the considered fault situations.

Non linear inverse control basics

Consider now a non-linear dynamic system given by:

$$\dot{\underline{X}} = f(\underline{X}) + g(\underline{X}).\underline{U} \qquad (17\text{-}1)$$

$$\underline{Y} = h(\underline{X}) \qquad (17\text{-}2)$$

where $\underline{X} \in \mathbb{R}^n$, $\underline{U} \in \mathbb{R}^m$, $\underline{Y} \in \mathbb{R}^m$, f and g are smooth vector fields of $\underline{X}$ and h is a smooth vector field of $\underline{X}$. The system has, with respect to each independent output Y_i, a relative degree r_i ($\sum_{i=1}^{m}(r_i+1) \le n$, $i = 1, \dots, m$) around the state $\underline{X_0}$ if the output dynamics can be written as:

$$\begin{pmatrix} Y_1^{(r_1+1)} \\ \vdots \\ Y_m^{(r_m+1)} \end{pmatrix} = \underline{A}(\underline{X}) + B(\underline{X}).\underline{U} \qquad (18)$$

If $B(\underline{X})$ is invertible, a feedback control law such as:

$$\underline{U} = B^{-1}(\underline{X})\left(\underline{v} - \underline{A}(\underline{X})\right) \qquad (19)$$

can be obtained. Here the new control input $\underline{v} = [v_1, \dots, v_m]'$ is chosen such as:

$$v_i = Y_{di}^{(r_i)} - \sum_{k=0}^{r_i-1} c_{ik}\left(Y_i^{(k)} - Y_{di}^{(k)}\right) \quad i=1 \text{ to } m \qquad (20)$$

where Y_{di} is the reference control input for the output dynamics. Then the dynamics of the tracking error given by $e_i = Y_i - Y_{di}$ $i=1$ to m , are such as:

$$e_i^{(r_i)} + c_{ir_i-1} e_i^{(r_i-1)} + \dots + c_{i1} e_i^{(1)} + c_{i0} e_i = 0 \qquad (21)$$

where the coefficients c_{ik} can be chosen to make the output dynamics asymptotically stable and ensure the tracking of output y_i towards the reference output y_{di}. However the derived feedback control law works only if either no internal dynamics ($\sum_{i=1}^{m}(r_i+1) = n$) are present or if the internal dynamics ($\sum_{i=1}^{m}(r_i+1) < n$) are stable. To cope with the saturation of the actuators, the choice of the coefficients c_{ik} should be the result of a trade-off between the characteristics of the transient dynamics of the different outputs and the solicitations of the inputs.

Considering the flight equations displayed above, it appears that adopting as state vector :

$$X = (u, v, w, p, q, r, \phi, \theta, \psi)' \qquad (22)$$

we get an input affine state representation. Considering ϕ and θ as output variables for the basic flight control loop, their relative degrees being equal to 1. Then this control technique will help to build efficient second order dynamics for the controlled aircraft attitude.

Trim Management

The trim conditions considered here correspond to situations in which the rotation speed is null while the aircraft is straight level flight at altitude z_0 with a constant airspeed speed V_0. This situation can be encountered in different phases of the flight but mainly in cruise. Regulations with respect to transport aircraft flight qualities require that such equilibrium can be reached for adequate setting of thrust while aerodynamic actuators deflection remains at a given position. Furthermore this equilibrium state should be stable. In general the stability region around the chosen trim condition is large enough so that starting from slightly different initial flight conditions, under natural damped oscillations (short period oscillation, Dutch roll and phugoïd) , the aircraft returns asymptotically towards such an equilibrium state. However in the case of turbulent atmosphere only will remain the natural tendency to reach such a state.

Trim without Actuator Failures

In this case, the trim equations can then be written such as:

Force equations:

$$1/2\rho(z_0)V_0^2 S\, C_x(\alpha,\beta,p,q,r,\delta_{ths},\underline{\delta}_p,\underline{\delta}_q,\underline{\delta}_r) = mg\sin\theta - (P_L + P_R) \qquad (23.1)$$

$$1/2\ \rho(z_0)V_0^2 S\, C_y(\alpha,\beta,p,q,r,\delta_{ths},\underline{\delta}_p,\underline{\delta}_q,\underline{\delta}_r) = -mg\ \cos\theta\ \sin\phi \qquad (23.2)$$

$$1/2\ \rho(z_0)V_0^2 S\, C_z(\alpha,\beta,p,q,r,\delta_{ths},\underline{\delta}_p,\underline{\delta}_q,\underline{\delta}_r) = -mg\cos\theta\cos\phi \qquad (23.3)$$

and torque equations:

$$1/2\ \rho(z_0)V_0^2 S\, l\, C_l(\beta,p,r,\underline{\delta}_p,\underline{\delta}_r) = 0 \qquad (24.1)$$

$$1/2\ \rho(z_0)V_0^2 S\, l\, C_m(\alpha,q,\delta_{ths},\underline{\delta}_q) = -b\,(P_L + P_R) \qquad (24.2)$$

$$1/2\ \rho(z_0)V_0^2 S\, l\, C_n(\beta,p,r,\underline{\delta}_p,\underline{\delta}_r) = -a\,(P_L - P_R) \qquad (24.3)$$

with

$$\gamma = \theta - \alpha = 0 \qquad (25)$$

and

$$\dot{\theta} = q = 0, \dot{\psi} = r = 0, \dot{\phi} = p = 0 \qquad (26)$$

In the no-failure actuator case with no lateral wind, the trim values of the main actuators will be such as:

$$\underline{\delta}_p = \underline{0}_p, \underline{\delta}_q = \underline{0}_q, \underline{\delta}_r = \underline{0}_r \text{ and } \delta_{ths} = \delta_{ths0} \qquad (27)$$

with $\alpha_0 = \theta_0 \quad \beta_0 = 0$ and $\phi_0 = 0 \qquad (28)$

while the following conditions must be met :

$$1/2\ \rho(z_0)V_0^2 S\, C_x(\theta_0,\delta_{ths0}) = -mg\sin\theta_0 + (P_L + P_R) \qquad (29.1)$$

$$1/2\ \rho(z_0)V_0^2 S\, C_z(\theta_0,\delta_{ths0}) = -mg\cos\theta_0 \qquad (29.2)$$

$$1/2\ \rho(z_0)V_0^2 S\, l\, C_m(\theta_0,\delta_{ths0}) = -b\,(P_L + P_R) = 0 \qquad (29.3)$$

with $P_L = P_R$

Eliminating total thrust from 1 and 3, the equilibrium pitch angle θ_0 and the trim position of the THS are solution of the set of non linear equations:

$$mg\cos\theta_0 + 1/2\ \rho(z_0)V_0^2 S\, C_z(\theta_0,\delta_{ths0}) = 0 \qquad (30.1)$$

$$mg\sin\theta_0 + 1/2\ \rho(z_0)V_0^2 S(C_x(\theta_0,\delta_{phr0}) + (1/2)\rho(z_0)V_0^2(l/b)C_m(\theta_0,\delta_{phr0})) = 0 \qquad (30.2)$$

while the equilibrium thrust is such that :

$$P_{L0} = P_{R0} = (1/2)(mg\sin\theta_0 + (1/2)\ \rho(z_0)V_0^2 S\, C_x(\theta_0,\delta_{ths0})) \qquad (30.3)$$

In general, the trim conditions chosen either by the pilot or the Flight Management system correspond to a point in the normal flight envelop of the aircraft and constraints such as (11), (12) and (16) are naturally satisfied by the solution of the above equations.

Trim in the Presence of Faulty Actuators

In the case of actuators failures, the solution of the trim conditions must be revisited since some actuator may present abnormal behaviours. Here we consider three cases:

- The actuator is stuck at a known angular deflection which can be either null or different from zero:

$$\delta_i = \bar{\delta}_i \qquad i \in I_{FP} \tag{31}$$

where $\bar{\delta}_i$ is a known value.

- The actuator is not subject to a torque from its servo-control. This case is here equivalent to the previous case with a zero deflection for the considered actuator:

$$\delta_i = 0 \qquad i \in I_{FF} \tag{32}$$

- The angular position and speed are subject to additional limitations:

$$\delta_i^{\min} < \tilde{\delta}_i^{\min} \le \delta_i \le \tilde{\delta}_i^{\max} < \delta_i^{\max} \quad i \in I_{FL} \tag{33.1}$$

$$\dot{\delta}_i^{\min} < \tilde{\dot{\delta}}_i^{\min} \le \dot{\delta}_i \le \tilde{\dot{\delta}}_i^{\max} < \dot{\delta}_i^{\max} \quad i \in I_{FS} \tag{33.2}$$

where $\tilde{\delta}_i^{\min}$, $\tilde{\delta}_i^{\max}$, $\tilde{\dot{\delta}}_i^{\min}$ and $\tilde{\dot{\delta}}_i^{\max}$ are new extreme position and speed values.

Then:

$$I = I_F \cup I_{\bar{F}} \qquad \text{with} \qquad I_F = I_{FP} \cup I_{FS} \cup I_{FF} \cup I_{FL} \tag{34}$$

where $I_{\bar{F}}$ is the set of fully operational actuators. Let us write:

$$\underline{\tilde{\delta}}_j = \underline{\delta}_j \qquad j \in \{p, q, r, ths\} \tag{35}$$

with

$$\tilde{\delta}_{i_j} = 0 \quad \text{if} \quad i_j \in I_{FF},\ j \in \{p, q, r, ths\} \tag{36.1}$$

$$\tilde{\delta}_{i_j} = \bar{\delta}_{i_j} \quad \text{if} \quad i_j \in I_{FP},\ j \in \{p, q, r, ths\} \tag{36.2}$$

$$\tilde{\delta}_{i_j}^{\min} \le \delta_{i_j} \le \tilde{\delta}_{i_j}^{\max} \qquad i_j \in I_{FL},\ j \in \{p, q, r, ths\} \tag{36.3}$$

$$\tilde{\dot{\delta}}_{i_j}^{\min} \le \dot{\delta}_{i_j} \le \tilde{\dot{\delta}}_{i_j}^{\max} \qquad i_j \in I_{FS},\ j \in \{p, q, r, ths\} \tag{36.4}$$

Now, if any solution exists for a given trim condition relative to flight level, airspeed and heading hold with $p = q = r = 0$, it will satisfy the following conditions:

$$\begin{aligned} &1/2\rho(z_0)V_0^2 S\, C_x(\alpha_0, \beta_0, \tilde{\delta}_{ths}, \underline{\tilde{\delta}}_p, \underline{\delta}_q, \underline{\tilde{\delta}}_r) \\ &= mg\sin\theta_0 - (P_{L0} + P_{R0}) \end{aligned} \tag{37.1}$$

$$\begin{aligned} &1/2\,\rho(z_0)V_0^2 S\, C_y(\alpha_0, \beta_0, \tilde{\delta}_{ths}, \underline{\tilde{\delta}}_p, \underline{\tilde{\delta}}_q, \underline{\tilde{\delta}}_r) \\ &= -mg\,\cos\theta_0\ \sin\phi_0 \end{aligned} \tag{37.2}$$

$$\begin{aligned} &1/2\,\rho(z_0)V_0^2 S\, C_z(\alpha_0, \beta_0, \tilde{\delta}_{ths}, \underline{\tilde{\delta}}_p, \underline{\tilde{\delta}}_q, \underline{\tilde{\delta}}_r) \\ &= -mg\cos\theta_0\cos\phi_0 \end{aligned} \tag{37.3}$$

$$1/2\,\rho(z_0)V_0^2 S\,l\,C_l(\beta_0, \underline{\tilde{\delta}}_p, \underline{\tilde{\delta}}_r) = 0 \tag{38.1}$$

$$1/2\,\rho(z_0)V_0^2 S\,l\,C_m(\alpha, q, \tilde{\delta}_{ths}, \underline{\tilde{\delta}}_q) = -b\,(P_{L0} + P_{R0}) \tag{38.2}$$

$$1/2\,\rho(z_0)V_0^2 S\,l\,C_n(\beta_0, \underline{\tilde{\delta}}_p, \underline{\tilde{\delta}}_r) = -a\,(P_{L0} - P_{R0}) \tag{38.3}$$

with
$$\sin\theta_0 - \sin\alpha_0\ \cos\phi_0 = 0 \tag{39}$$

and additional constraints such as :

$$\alpha_{\min}^{trim} \le \alpha_0 \le \alpha_{\max}^{trim} \tag{40.1}$$

$$|\phi_0| \le \phi_{\max}^{trim} \ \text{ and } \ |\beta_0| \le \beta_{\max}^{trim} \tag{40.2}$$

with the actuator constraints (11), (12) and (16).

Depending of the extent and seriousness of actuator failures, the above set of nonlinear equations may have no solution, a unique solution or several solutions. In the case in which several solutions are available, different criteria can be adopted to retain one of them. For instance, solutions minimizing the total thrust $P_{L0} + P_{R0}$ or the total drag given by:

$$\begin{aligned} &1/2\rho(z_0)V_0^2 S\,(\cos\alpha_0\ C_x(\alpha_0, \beta_0, \tilde{\delta}_{ths}, \underline{\tilde{\delta}}_p, \underline{\delta}_q, \underline{\tilde{\delta}}_r) \\ &+ \sin\alpha_0\ C_z(\alpha_0, \beta_0, \tilde{\delta}_{ths}, \underline{\tilde{\delta}}_p, \underline{\tilde{\delta}}_q, \underline{\tilde{\delta}}_r)) \end{aligned} \tag{41}$$

The resulting optimization problem should be difficult to be solved considering its non convexity and the presence of nonlinear equality constraints. However, approximating the sine of small angles (α, β, θ and ϕ) by its value in radian and they cosine by 1, linearity and convexity are restored for the feasible set of solutions and linear programming (total thrust minimization) or linear quadratic programming (total drag minimization) can be used to fully establish trim conditions. Then standard optimization techniques can be used to optimize the reallocation and settings of remaining actuators.

When acceptable trim conditions cannot be reached and held, feasible trajectories mastering at least speed, incidence and height and keeping them at safe values should be performed.

Actuator Allocation for Manoeuvring Aircraft

Now we consider the case of an aircraft which has to perform a standard maneuver along one axis. If a reasonable degree of controllability along each axis is maintained despite the presence of actuator failures, basic guidance functions will remain available. The problem is quite similar to the previous one, except that the time dimension is introduced and dynamic inversion becomes necessary when following the nonlinear inverse control approach.

Stabilized Roll Maneuver

Here we study the case of a pure stabilized roll maneuver where the following conditions should be met by the body angular rates of the aircraft:

$$\tau_p \dot{p} + p = p_c \qquad (42.1)$$

$$q = 0 \qquad (42.2)$$

$$\tau_r \dot{r} + r = (g/V)\sin\phi \qquad (42.3)$$

where roll and yaw motions follow first order dynamics while pitch dynamics remains frozen. Here p_c is the desired roll rate and τ_p and τ_r are time constants. The dynamic constraint relative to the yaw rate is characteristic of an equilibrated turn, its completion should allow to avoid noticeable lateral load factors during this roll maneuver.

Applying the non linear inverse control approach displayed in a previous paragraph, we get the necessary on line values for each aerodynamic torque:

$$\widetilde{M}(t) = -(I_{xx} - I_{zz})\, r(t)\, p(t) + I_{xz}\,(p(t)^2 - r(t)^2) \qquad (43.1)$$

$$\begin{bmatrix} \widetilde{L}(t) \\ \widetilde{N}(t) \end{bmatrix} = \begin{bmatrix} I_{xx} & -I_{xz} \\ -I_{xz} & I_{zz} \end{bmatrix} \begin{bmatrix} \dfrac{1}{\tau_p}(p_c - p(t)) \\ \dfrac{1}{\tau_r}((g/V(t))\sin\phi(t) - r(t)) \end{bmatrix} \qquad (43.2)$$

Different situations may arise:

- The failure does not affect the control channels and actuators commonly used to perform the maneuver which continue to be performed in a standard way.
- The failure affects some of the commonly used actuators but some actuator redundancy remains which may imply an unusual activation of aerodynamic surfaces.

In the first case, an on-line solution to the following set of instant equality constraints involving only deflections of aileron, elevator and rudder surfaces is adopted:

$$\sum_{i \in I^L} X_i^L(t)\, \widetilde{\delta}_i(t) = \widetilde{L}(t) - L^0(t) \qquad (44.1)$$

$$\sum_{i \in I^M} X_i^M(t)\, \widetilde{\delta}_i(t) = \widetilde{M}(t) - M^0(t) \qquad (44.2)$$

$$\sum_{i \in I^N} X_i^N(t)\, \widetilde{\delta}_i(t) = \widetilde{N}(t) - N^0(t) \qquad (44.3)$$

In general the design of these aerodynamic surfaces is such that for standard roll rate demands, the solution presents no saturation and satisfies actuators position and speed constraints (11.1), (12.1) and (12.2) as well as structural constraints such as (16.1) and (16.2).

In the second case, when some aileron, elevator or rudder surfaces are no more fully available, other aerodynamic surfaces must be involved in the maneuver . Depending of the remaining degree of redundancy between elementary actuators, a solution matching exactly constraints (44.1), (44.2) and (44.3) can be constructed, in that case the maneuver will be performed still in a standard way. Otherwise, an approximate maneuver should be defined. Here we propose to solve on-line the following linear quadratic problem given by:

$$\begin{aligned} \min_{\underline{\delta}(t)} \quad & \omega_L \Big(\sum_{i \in I^L} X_i^L(t)\, \widetilde{\delta}_i(t) - \widetilde{L}(t) + L^0(t)\Big)^2 \\ & + \\ & \omega_M \Big(\sum_{i \in I^M} X_i^M(t)\, \widetilde{\delta}_i(t) - \widetilde{M}(t) + M^0(t)\Big)^2 \\ & + \\ & \omega_N \Big(\sum_{i \in I^N} X_i^N(t)\, \widetilde{\delta}_i(t) - \widetilde{N}(t) + N^0(t)\Big)^2 \end{aligned} \qquad (45)$$

with the following constraints:

$$A_b(t) + \sum_{i \in I^{Ail}} Y_{b_i}(t)\, \widetilde{\delta}_i(t) \le M_{bend}^{\max} \qquad (46.1)$$

$$A_f(t) + \sum_{i \in I^{wing}} Y_{fi}(t)\ \tilde{\delta}_i(t) \le M_{flex}^{\max} \qquad (46.2)$$

$$\delta_i^{\min} \le \tilde{\delta}_i \le \delta_i^{\max} \qquad i \in I_{\overline{F}} \qquad (47.1)$$

$$\tilde{\delta}_i^{\min} \le \tilde{\delta}_i \le \tilde{\delta}_i^{\max} \qquad i \in I_{FL} \qquad (47.2)$$

$$\max\left\{\delta_i^{\min}, \tilde{\delta}_i(t-\Delta t) + \dot{\delta}_i^{\min}\Delta t\right\} \le \tilde{\delta}_i(t) \quad i \in I_{\overline{F}} \qquad (47.3)$$

$$\tilde{\delta}_i(t) \le \min\left\{\delta_i^{\max}, \tilde{\delta}_i(t-\Delta t) + \dot{\delta}_i^{\max}\Delta t\right\} \quad i \in I_{\overline{F}} \qquad (47.4)$$

$$\max\left\{\tilde{\delta}_i^{\min}, \tilde{\delta}_i(t-\Delta t) + \dot{\tilde{\delta}}_i^{\min}\Delta t\right\} \le \tilde{\delta}_i(t) \quad i \in I_{FS} \qquad (47.5)$$

$$\tilde{\delta}_i(t) \le \min\left\{\tilde{\delta}_i^{\max}, \tilde{\delta}_i(t-\Delta t) + \dot{\tilde{\delta}}_i^{\max}\Delta t\right\} \quad i \in I_{FS} \qquad (47.6)$$

with

$$\tilde{\delta}_{i_j} = 0 \quad \text{if} \quad i_j \in I_{FF},\ j \in \{p, q, r, ths\} \qquad (48.1)$$

$$\tilde{\delta}_{i_j} = \overline{\delta}_{i_j} \quad \text{if} \quad i_j \in I_{FP},\ j \in \{p, q, r, ths\} \qquad (48.2)$$

Here ω_L, ω_M and ω_N are positive weightings which, in the case of a roll maneuver, can be such as:

$$\omega_L >> \omega_M \quad \text{and} \quad \omega_L >> \omega_N \qquad (49)$$

The above mathematical programming problem can be solved using standard programming techniques [9] and making use as a start of the previous value of the deflections of the actuators. Then after a reduced number of iterations, the solution of this small size linear quadratic problem is obtained for each successive elementary time periods.

Application

To illustrate the proposed approach, we have considered the case of an aileron failure (the aileron remains stuck at neutral position) for an aircraft [10] with the actuators structure displayed in Figure 2. The objective here is to perform the complex roll maneuver displayed in Figure 3 (continuous line). The step adopted for time discretization is ΔT= 0.05s, when the response time of the different actuators is taken equal to 1/30 s. The solution of each instance of the linear quadratic problem takes about 0.005 s. Figures 4, 5 and 6 display the time evolution of the deflection for the different actuators during the roll maneuver. Figure 3 displays also the effective roll time history (dashed line) where it appears that, despite a small delay, the result is quite satisfactory.

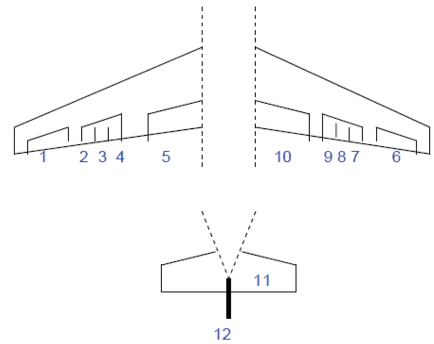

Figure 2. Aircraft Elementary Actuators

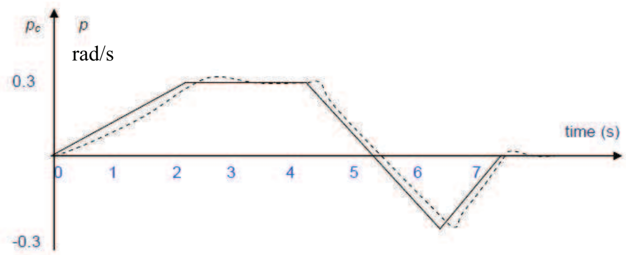

Figure 3. Roll Rate Maneuver

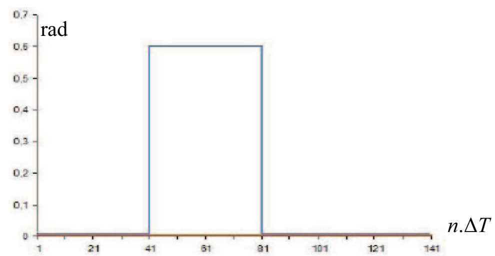

Figure 4. Deflection of Ailerons

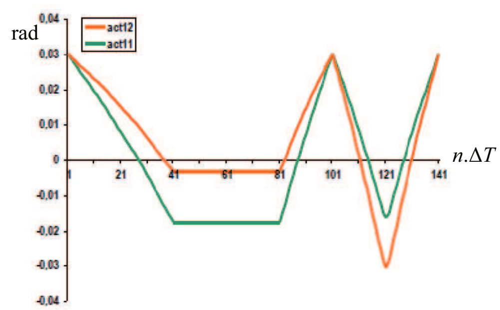

Figure 5. Deflections of Rudder and Elevator

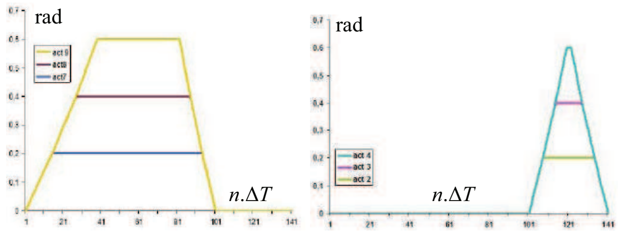

Figure 6. Deflections of Spoilers

Conclusion

In this communication an approach to manage the control surfaces of an aircraft under different actuator failure scenarios has been developed. The main objective has been to maintain the possibility to perform with the remaining fault free actuators standard maneuvers while limiting the structural strain (maximum wing bending and flexion torques) of the aircraft. Flight situations such as trimmed flight and basic maneuvers have been of interest.

Here, once the necessary aerodynamic torques have been computed by inversion of the flight dynamics, the contributions of each actuator to the aerodynamic forces and torques can be determined on-line by solving a linear quadratic optimization problem. An example of application considering an aileron failure while performing a pure roll maneuver has been displayed.

This study points out the necessity for the development of new global fault management systems for the control channels of a manoeuvring aircraft.

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