Hybridizing direct and indirect optimal control approaches for aircraft conflict avoidance

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***Abstract*—Aircraft conflict avoidance is a crucial issue arising in air traffic management. The problem is to keep a given separation distance for aircraft along their trajectories. We focus on an optimal control model based on speed regulation to achieve aircraft separation. We propose a solution strategy based on the decomposition of the problem and on the hybridization of a direct and an indirect method applied on the obtained subproblems. Numerical results show that the proposed approach is promising in terms of reduction of computing time for conflict avoidance.**

***Keywords*-air traffic management; conflict avoidance; speed regulation; optimal control; Pontryagin's maximum principle.**

I. INTRODUCTION

In the context of Air Traffic Control (ATC), motivated by safety and efficiency reasons, tools for decision support are requested. To avoid the risk of collision, distances of separation must be respected. It is said that two aircraft are *in conflict* if the distances between them are less than 5 NM horizontally and 1000 ft vertically (1 NM (nautical mile) = 1852 m and 1 ft (feet) = 0.3048 m).

Various methods of conflict detection and resolution have been proposed (see *e.g.*, $[8]$). They are based on different strategies that can be exploited to achieve aircraft separation, such as trajectory (heading), flight level or velocity changes.

Evolutionary computation based algorithms are widely studied in this context $[5]$. They are, in general, low time-consuming, but the global optimal solution and even a feasible solution (without conflicts) is not garanteed to be achieved in a given time. Several models of *optimal control* also appeared in this domain $[1, 10, 11]$. They are mainly based on changes of aircraft trajectories, putting the trajectory as a command on the system.

Recently, the European project ERASMUS (En-Route Air traffic Soft Management Ultimate System) $[2]$ considered speed regulation and suggested a small velocity change range to enable a *subliminal control*, that is a speed control which is even not perceived and is performed without informing air traffic controllers. Velocity change approaches have also been recently studied in the context of mixed integer linear and nonlinear programming $[4, 9, 11]$.

This work focuses on aircraft conflict avoidance problems solved through speed regulation. We propose an optimal control approach keeping the flight trajectories and focusing on velocity variations.

The paper is organized as follows. First, in Section II, we present an optimal control model for the addressed air traffic control problem which should be solved by small speed changes. In Section III, we propose stategies to deal with computational complexity. Particularly, we propose a *decomposition* of the problem by considering different time periods such that the separation constraints have to be imposed only in the time periods when the aircraft conflicts potentially occurr. In Section IV, we present an analytical resolution based on the *Pontryagin's maximum principle* (PMP) in the other time periods, *i.e.*, the ones where conflict have already been solved. In Section V, we discuss numerical results issued to the methods. In Section VI, conclusions are drawn.

II. OPTIMAL CONTROL MODEL THROUGH VELOCITY REGULATION

We present an optimal control model to achieve separation based on speed changes only, keeping the aircraft trajectories unchanged. The *acceleration*, u_i respective to each aircraft i, is then the command on the system.

In model $(\mathcal{P})$, x_i, v_i and u_i are respectively the position, the velocity and the acceleration (control) of aircraft i, with $I = \{1, ..., n\}$ and n the number of aircraft involved; aircraft are expected to be at the same altitude (planar configuration, same flight level). For each aircraft i, velocity v_i and acceleration u_i are bounded (*i.e.*, belonging to $[\underline{v_i}, \overline{v_i}]$ and $[\underline{u_i}, \overline{u_i}]$ respectively).

We note by t, t_0 and t_f the time, the initial time and final time respectively. Moreover, D is the minimum required horizontal separation distance between two aircraft and d_i is the direction (heading) of the i^{th} aircraft. The final time t_f of maneuvers is fixed and identical for all aircraft. The mathematical model is the following:

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$$
(\mathcal{P}) \begin{cases} \min_{u} \quad \sum_{i=1}^{n} \int_{t_0}^{t_f} u_i^2(t)dt \\ \dot{v}_i(t) = u_i(t) & \forall t \in [t_0, t_f], \forall i \in I \\ \dot{x}_i(t) = v_i(t)d_i & \forall t \in [t_0, t_f], \forall i \in I \\ \underline{u_i} \leq u_i(t) \leq \overline{u_i} & \forall t \in [t_0, t_f], \forall i \in I \\ \underline{v_i} \leq v_i(t) \leq \overline{v_i} & \forall t \in [t_0, t_f], \forall i \in I \\ x_i(t_0) = x_i^0 \qquad v_i(t_0) = v_i^0 & \forall i \in I \\ x_i(t_f) = x_i^f \qquad v_i(t_f) = v_i^f & \forall i \in I \\ D^2 - \parallel x_i(t) - x_j(t) \parallel^2 \leq 0 & \forall t \in [t_0, t_f], \forall i < j \end{cases}
$$

We choose to minimize a quadratic energy-dependent cost function depending on speed variations. This criterion takes into account the contribution of each aircraft and also limits the penalization inequality between the aircraft.

Note that one of the main difficulties on this optimal control model is given by the constraints on the state variables v and x. In the next section, we present resolution approaches tailored on the problem to achieve its efficient solution.

III. Solution approach by decomposition of the problem

A typical solution approach for an optimal control problem like $(\mathcal{P})$ is based on the application of a *direct* method. It is based on a time discretization and leads to the solution of a nonlinear (continuous) optimization problem (NLP), which can be solved by standard NLP local solvers. For $(\mathcal{P})$, the corresponding NLP problem can be difficult to solve for large-scale problems, mainly due to the large number of variables and constraints. The *complexity* of the NLP corresponding to the *direct* method is $O(np)$ for the number of variables and $O(n^2p+p^2)$ for the number of constraints, where n and p are the number of aircraft and the number of time subdivisions respectively. For example, even on a simple conflict problem, with only 2 aircraft and a time window equals to $30'$ (with time subdivision equals to $15''$) the corresponding nonlinear problem has more than 240 variables and 9000 constraints.

We first recall that, in order to perform aircraft separation, a *detection* of potential conflict regions and a *resolution* step have to be carried out. The two steps can be performed at the same time by applying a direct method.

We propose to distinguish two discretization steps. The first one, for the detection, has to be tight enough to check if all constraints are respected. The second one, for the resolution, is used to decide the time frequency at which values the controls are computed and it can be larger than the previous one. For example, we used $15''$ for the detection and $1'$ or $5'$ for the resolution. As discussed in Section V, this strategy allows to reduce the number of variables and constraints of the nonlinear optimization problem to be solved.

Another possibility is to perform a pre-processing step to detect potential conficts. Given aircraft predicted trajectories, one can check intersections of the trajectories and identify spatial regions where the separation constraints must be checked [7]. Once the different regions have been localized, one can exploit this information to devise a specific strategy of resolution aimed at reducing the computational complexity of the problem at hand. The main contribution of this paper is a strategy based on problem decomposition and related hybridization of optimal control solution methods.

Let *zone* be the region where for an aircraft pair separation constraints have to be verified and *postzone* be the following region where all the conflicts have been solved and when the aircraft are already separated.

For each aircraft i, let x_i^{enter} be the *first* (by chronological order) $3D$ trajectory point for which there exists an aircraft j ($j \neq i$) such that the Euclidean distance between x_i^{enter} and the straight line corresponding to the j^{th} aircraft predicted $3D$ trajectory is equal to the separation standard D. For each aircraft i, let t_i^1 be the time to reach x_i^{enter} using the highest speed $\overline{v_i}$. Dually, for each aircraft i, let x_i^{exit} be the *last* $3D$ trajectory point for which there exists an aircraft j such that the Euclidean distance between x_i^{exit} and the straight line corresponding to the j^{th} aircraft predicted $3D$ trajectory is equal to the separation standard D. For each aircraft i, let t_i^2 the time to reach x_i^{exit} using the lower speed $\underline{v_i}$.

For n aircraft, setting the entry zone time equals to $t_1 := \min_{i\in\{1,\ldots,n\}} t_i^1$ and the exit zone time $t_2 := \max_{i\in\{1,\ldots,n\}} t_i^2$, we define conflict time phases for the whole problem. The *zone* and *postzone* correspond respectively to the time periods $[t_1, t_2]$ and $[t_2, t_f]$,

The *postzone* being characterized by the absence of separation contraints, it represents a subproblem easier to solve than the initial problem defined on the whole time horizon. We can apply the PMP [3] as discussed in the next section on the *postzone*. On the remaining time window, the direct method is applied. Numerical integrators of Euler-type are used to approximate the ordinary differential equations describing the system dynamic and different time discretization steps mentioned above are exploited.

IV. Application of the Pontryagin's maximum principle

Without the separation constraint (difficult state constraints), we can easily apply the PMP, which gives us an analytical solution. In the *postzone* (time window $[t_2, t_f]$), as the aircraft conflicts have been solved, the necessity to check separation constraint does not exist anymore. The velocity and acceleration constraints are checked *a posteriori*. Hence, for each aircraft i the following optimal control sub-problem $(\mathcal{P}_i)$ can be solved independently. We recall the assumption that aircraft are expected to be at the same altitude (planar configuration, same flight level) so that two-components vectors appear in the formulation. The distinction between the two components of the direction (heading) vector $d_i = (d_i^X, d_i^Y)^T$ and the distinction between the position components $x_i = (x_i^X, x_i^Y)^T$ have been done to make easier the formalism.

$$(\mathcal{P}_i)\begin{cases} \min\limits_{u_i} \quad \displaystyle\int_{t_2}^{t_f} u_i^2(t)dt \\ \dot{v}_i(t) = u_i(t) \qquad \forall t \in [t_2, t_f] \\ \dot{x}_i^X(t) = v_i(t)d_i^X \qquad \forall t \in [t_2, t_f] \\ \dot{x}_i^Y(t) = v_i(t)d_i^Y \qquad \forall t \in [t_2, t_f] \\ x_i^X(t_2) = x_i^{X_{t_2}} \quad x_i^Y(t_2) = x_i^{Y_{t_2}} \quad v_i(t_2) = v_i^{t_2} \\ x_i^X(t_f) \text{ free} \quad x_i^Y(t_f) \text{ free} \quad v_i(t_f) = v_i^{t_f} \end{cases}$$

We apply on $(\mathcal{P}_i)$ the *indirect* method. We introduce the co-state variables $z_0^i, z_1^i, z_2^i, z_3^i$, where z_1^i, z_2^i, z_3^i are associated to x_i^X, x_i^Y and respectively v_i. Writing the Hamiltonian

$$H_i = z_0^i u_i^2 + z_1^i v_i d_i^X + z_2^i v_i d_i^Y + z_3^i u_i \ ,$$

the co-state equations are:

$$\dot{z}_1^i = -\frac{\partial H_i}{\partial x_i^X} = 0 \ , \ \dot{z}_2^i = -\frac{\partial H_i}{\partial x_i^Y} = 0 \ ,$$
$$\dot{z}_3^i = -\frac{\partial H_i}{\partial v_i} = -(z_1^i d_1^X + z_2^i d_1^Y) \ .$$

By fixing $z_0^i = -1$, by using the PMP [3], we obtain:

$$u_i^* = \operatorname*{argmin}_{u_i} H_i = \frac{z_3^i}{2}.$$

Solving the differential system composed by state and co-state equations and introducing six real constants A_i, B_i, C_i, D_i, E_i and F_i, we obtain:

$$(\mathcal{S}_i)\begin{cases} z_1^i(t) = A_i \quad \text{and} \quad z_2^i(t) = B_i \ , \\ z_3^i(t) = -(A_i d_i^X + B_i d_i^Y)t + C_i \ , \\ u_i(t) = -\dfrac{A_i d_i^X + B_i d_i^Y}{2} t + \dfrac{C_i}{2} \ , \\ v_i(t) = -\dfrac{A_i d_i^X + B_i d_i^Y}{4} t^2 + \dfrac{C_i}{2} t + D_i \ , \\ x_i^X(t) = -\dfrac{A_i (d_i^X)^2 + B_i d_i^X d_i^Y}{12} t^3 \\ \qquad + \dfrac{C_i}{4} d_i^X t^2 + D_i d_i^X t + E_i \ , \\ x_i^Y(t) = -\dfrac{A_i d_i^X d_i^Y + B_i (d_i^Y)^2}{12} t^3 \\ \qquad + \dfrac{C_i}{4} d_i^Y t^2 + D_i d_i^Y t + F_i \ . \end{cases}$$

From the terminal (position) conditions, $x_i^X(t_f)$ is free and $x_i^Y(t_f)$ is free, the PMP implies (*transversality conditions*: $z_1^i(t_f) = 0, z_2^i(t_f) = 0$, see [3]) that the real constants A_i and B_i are both equal to zero. This reveals that the optimal control corresponds to a constant acceleration.

This optimal acceleration depends only on the initial and final velocities ($v_i^{t_2}$ and $v_i^{t_f}$) and on the time window extremities (t_2 and t_f). More precisely, we obtain the following solution system for each instant t belonging to $[t_2, t_f]$:

$$\begin{cases} u_i(t) = \dfrac{v_i^{t_f} - v_i^{t_2}}{t_f - t_2} \ , \\ v_i(t) = \dfrac{v_i^{t_f} - v_i^{t_2}}{t_f - t_2}(t - t_f) + v_i^{t_f} \ , \\ x_i^X(t) = \dfrac{v_i^{t_f} - v_i^{t_2}}{t_f - t_2} d_i^X \dfrac{t^2}{2} + (v_i^{t_f} - \dfrac{v_i^{t_f} - v_i^{t_2}}{t_f - t_2} t_f) d_i^X t \\ \qquad - (\dfrac{v_i^{t_f} - v_i^{t_2}}{t_f - t_2}(t_2 - t_f) + v_i^{t_f}) d_i^X t_2 + x_i^{X_{t_2}} \ , \\ x_i^Y(t) = \dfrac{v_i^{t_f} - v_i^{t_2}}{t_f - t_2} d_i^Y \dfrac{t^2}{2} + (v_i^{t_f} - \dfrac{v_i^{t_f} - v_i^{t_2}}{t_f - t_2} t_f) d_i^Y t \\ \qquad - (\dfrac{v_i^{t_f} - v_i^{t_2}}{t_f - t_2}(t_2 - t_f) + v_i^{t_f}) d_i^Y t_2 + x_i^{Y_{t_2}} \ . \end{cases}$$

Hence, starting from t_2, the problem can be analytically solved. Thus, just a discretization of the time window $[t_0, t_2]$ is needed.

V. Numerical Results

In this section, we discuss numerical results obtained by applying the proposed strategies to solve the conflict avoidance problem. A computer 2.53 GHz / 4 Go RAM and the *MatLab v.* 7 environment are used. Data problems were randomly generated with the following characteristics. The trajectory paths are straight. The horizontally separation norm is 5 NM. Most of the aircraft have a small *operating time* (*i.e.*, time before the first potential conflict), which is less than $15'$. Velocities are bounded, based on the ERASMUS project, by a small speed range, namelly: $[v_i^{t_0} - 6\% v_i^{t_0},\ v_i^{t_0} + 3\% v_i^{t_0}]$ (where $v_i^{t_0}$ is the initial velocity of aircraft i). Acceleration are bounded, based on Eurocontrol's base of aircraft data [6], namelly $\overline{u_i} = -\underline{u_i} = 4000$ NM / h^2. Terminal conditions are returning to the initial velocities ($v_i^{t_0}$) at final time ($t_f = 30'$). The number of aircraft, the *collision proximity* (*i.e.*, the minimal distance between aircraft which could occur if no maneuvers are done), and the initial aircraft velocities are reported in Table I.

In Table II, we compare the results obtained by applying a direct method, with detection step and resolution step equal to $15''$ and $1'$ respectively, on the whole time window (without considering the *postzone*) and the results obtained by decomposing the problem and applying the direct and the indirect methods as described in the previous sections.

Table I
TEST PROBLEMS CHARACTERISTICS: NUMBER OF AIRCRAFT, COLLISION PROXIMITY, AND INITIAL VELOCITY FOR PROBLEMS WITH 4 AND 6 AIRCRAFT.

instances	number of aircraft	collision proximity	initial velocity
pb_n4a	4	0 NM	400 NM / h
pb_n4b	4	2 NM	400 NM / h
pb_n6b	6	2 NM	400 NM / h
pb_n6c	6	3 NM	400 NM / h

Table II
COMPARISON OF NUMERICAL RESULTS OBTAINED WITHOUT AND WITH APPLICATION OF THE PMP ON THE POSTZONE : VALUE OF OBJECTIVE FUNCTION, NUMBER OF ITERATIONS, CPU TIME, FOR 4 AIRCRAFT PROBLEMS.

	Application of the PMP on the *POSTZONE*					
	without			**with**		
instances	objective	it.	time	objective	it.	time
pb_n4a	1.8×10^4	258	$22'$	1.9×10^4	148	$\mathbf{1'54''}$
pb_n4b	8.0×10^3	228	$16'30''$	1.0×10^4	188	$\mathbf{3'20''}$

From Table II, we can see that with the application of the PMP on the *postzone*, the CPU times are significantly reduced with respect to the classical resolution based on the direct method applied on the whole time window, up to 90% (see pb_n4a). The application of the PMP on the *postzone* allows us to tackle larger aircraft conflict avoidance problems because it reduces the time window where the direct method is applied, hence it reduces the number of variables and constraints of the NLP. We can then solve 6 aircraft conflict avoidance problems, as show in Table III, we compare results obtained by using different *resolution* time discretization steps. Like in Table II, the *detection* step is $15''$ and we applied the PMP on the *postzone*.

Table III
COMPARISON OF NUMERICAL RESULTS WITH DIFFERENT CONTROL TIME DISCRETIZATION STEPS: VALUE OF OBJECTIVE FUNCTION, NUMBER OF ITERATIONS, CPU TIME, FOR 6 AIRCRAFT PROBLEMS.

	Control time DISCRETIZATION step					
	$1'$			$\mathbf{5'}$		
instances	objective	it.	time	objective	it.	time
pb_n6b	1.6×10^4	342	$21'$	1.7×10^4	**43**	$\mathbf{0'35''}$
pb_n6c	1.0×10^4	317	$20'$	1.0×10^4	**51**	$\mathbf{0'44''}$

From Table III, we emphasize the importance of the *resolution* time discretization step. We can see that with the *resolution* step equals to $5'$, the CPU times are significantly reduced with respect to the configuration with the *resolution* step equals to $1'$, up to 97% (see pb_n6b). The two above comparisons (Tables II and III) show, on the one hand, the advantage of the application of the PMP on the *postzone*, and on the other hand, the benefit to hybridize the two methods to solve larger aircraft conflict avoidance problems.

VI. CONCLUSION

We considered an optimal control model for aircraft conflict resolution based on speed changes. We proposed a strategy based on hybridization of the direct method applied to the conflict *zone* and the indirect method applied to *postzone* where conflicts have been solved. First numerical results validate our approach. They show that the proposed decomposition strategy is beneficial in the context of the considered control problem, significantly reducing the computational time for solving the aircraft conflict avoidance problem.

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