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A fuzzy approach of decision making for an airline

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Abstract

In the Air Transport sector where the uncertainty is very frequent due to the high sensitivity of the activity to many factors that influence the behavior of the consumers. Fuzzy logic can be used to produce estimates which take into account the vagueness of the operating environment. In this sector, many important decisions about the supply of service are based on these estimates.

In this paper an analysis of the optimal supply conditions under fuzzy demand information is considered for a market where only one carrier is operating. The carrier's supply decision making process is investigated, first when the demand function is obtained through classical methods then when it is given by a fuzzy function. The main advantages of the new approach are then discussed.

Keywords: decision making, fuzzy logic, fuzzy optimization.

1. Introduction

Air transport markets are characterized by very tight margins of profit. Besides that, the demand for these markets is very sensitive to many variables (wars, economic crisis, cews strikes,...). So an airline needs an efficient decision making system in order to make positive profits. The decision making and the choice of the strategies for an operator such as the maximization of his profit need estimation of his own demand and of the demand of the possible competitors. This estimation of the demand is in general and in the classical approach, obtained through econometric regressions based on historical data and through statistical methods, leading to the definition of a crisp demand function. Some fuzzy modeling techniques ([11] and [4]) have recently been developed to take into account the uncertainty related with these predictions, leading to the definition of fuzzy demand

functions. This last approach allows in particular introducing more uncertainty in the estimation of demand of the competitor as it happens in general due to the lack of direct information.

Through the decision process, the fuzziness of demand propagates to the profit estimation. A fuzzy optimization process is then necessary to generate sets of possible solutions.

Here an air transport market is considered where it is supposed that there is only one carrier operating so that we focus on the fuzzy nature of the estimation of the demand. The decision here consists to choose the fare and the frequency of flights to put on the market for a given period of time.

Starting from the analysis of the decision making process in the deterministic case, a new decision making process is designed to cope with a fuzzy estimation of the demand function and then an example is used to illustrate the proposed method.

2. Deterministic Demand Function model

Let F denote the set of possible frequency levels (F is a finite set of positive integers). The demand function is supposed to be increasing with respect to frequency and decreasing with respect to fare. A possible model is given by:

for $p_{\min} \leq p \leq p_{\max}$ and $f \in F$,

$$D(p, f) = D_0 - \lambda(p - \varphi(f)) \quad (1)$$

where D_0 and λ are strictly positive parameters, D_0 is the potential gross demand, while $\lambda * (p - \varphi(f))$ (supposed to be a positive entity) is the degradation of the demand level resulting from the offer of the airline (generalized cost composed by the price and cost of the time of travel expressed as a function of the frequency). φ is an increasing and strictly concave function, for example $\varphi(f) = \frac{-\tau}{f}$ with $\tau > 0$ as in ref [1].

$[p_{\min}, p_{\max}]$ is the interval of possible fare values.

The operational cost on this market is supposed to be linear with respect to the frequency and to the level of the satisfied demand. A classical model [10] which is used in this study is given by:

$$C(p, f) = c_0 + c_f \cdot f + c_d \min\{D(p, f), K \cdot f\} \quad (2)$$

Here c_0 is the fixed operations cost for the considered market,

c_f traduces the increase of cost resulting from the establishment of a new frequency,

c_d is the marginal cost related to an additional transported passenger and

K is the capacity of the fleet of airplanes disserving the city pair O-D, this fleet is supposed to be homogenous.

The profit resulting from the operation on this single origin-destination market can then be written as:

$$\pi = (p - c_d) \min\{D(p, f), K \cdot f\} - c_0 - c_f \cdot f \quad (3)$$

The target of the airline is to choose a fare p from a possible set $[p_{\min}, p_{\max}] \subset \mathbf{R}$ and the frequency $f \in F$ that maximize this profit. Since it is easier to change fares than to change frequencies, a current practice is to solve this problem in two stages: first the frequency is taken as a parameter while the fare is chosen as a function of it and then the frequency is optimized as in [2]. For a given, level of f , the problem of the carrier becomes:

$$\max_p \pi = (p - c_d) \min\{D(p, f), K \cdot f\} - c_0 - c_f \cdot f \quad (4)$$

The search for a maximum profit leads at first sight to consider the cases where $D(p, f) < K \cdot f$ and where $D(p, f) \geq K \cdot f$. When $D(p, f) \geq K \cdot f$, the level of the satisfied demand is exactly equal to Kf , then the profit of the airline is given by the expression: $\pi = (p - c_d) \cdot K \cdot f - c_0 - c_f \cdot f$.

Since according to this expression the profit is increasing with respect to the fare p , the fare will be chosen as high as possible, this will be achieved when the fare is such that: ' $D(p, f) = K \cdot f$ '. So the two initial cases can be grouped under the unique condition: $D(p, f) \leq K \cdot f$. Then the expression (1) of the profit can be reduced to:

$$\pi = (p - c_d) \cdot D(p, f) - c_0 - c_f \cdot f \quad (5)$$

With this last expression, it is possible to use the classical optimality conditions to determine the

optimum solution; the first order condition of Lagrange is then given by:

$$\partial \pi / \partial p = D_0 + \lambda c_d - 2\lambda p + \lambda \varphi(f) \quad (6)$$

while the second order condition, in fact the concavity condition for π can be traduced by:

$$\partial^2 \pi / \partial p^2 < 0 \quad (7)$$

Observe that this last condition is always satisfied since here: $\partial^2 \pi / \partial p^2 = -2\lambda$.

So for every feasible $f \in F$, there is an optimal fare p_f^* that maximizes the profit π . The resulting optimized profit can be denoted as: $\pi_f^* = \pi(p_f^*, f)$ where p_f^* is given by:

$$p_f^* = c_d / 2 + D_0 / (2\lambda) + \varphi(f) / 2 \quad (8)$$

For this two step approach, the retained frequency f^* that will be such as:

$$f^* = \arg \max_{f \in F} \{\pi_f^*, f \in F\} \quad (9)$$

where

$$\pi_f^* = (D_0 + \lambda \varphi(f) + \lambda c_d)^2 / 4\lambda - c_f f - c_0 \quad (10)$$

Then the optimal price will be given by:

$$p^* = c_d / 2 + D_0 / (2\lambda) + \varphi(f^*) / 2 \quad (11)$$

Observe that the solution obtained through these two stages is a global solution for the problem. This is due to the fact that F is a finite set and all the cases are then explored.

In practical grounds it appears that this approach is difficult to be implemented since the adopted demand model provides estimates that are "around" the observed values, but with uncontrolled variations. This is mainly a consequence of the dependency of the demand levels to many qualitative factors and an intensification of statistical efforts may result no appreciable improvement of the accuracy of the estimates.

3. The case of a fuzzy demand function

In this subsection, it is then assumed that the available estimates of demand for the airline are represented by fuzzy sets allowing the inclusion of qualitative considerations in the building of these estimates: a fuzzy demand function is obtained. For every configuration (price, frequency), a fuzzy set is associated (and not only one crisp value as in the classical approach). The basis of the fuzzy set

represents the possible values of the demand relative to this configuration. They can be obtained from the subjective beliefs of the experts which are mainly based on past experience and data (see for example ref [4]). The membership function of this fuzzy set can take different shapes: triangular, trapezoidal. This fuzzy set has to cover the main possibilities which can result from different attitudes (optimistic, medium and pessimistic ones).

3.1. A fuzzy demand function model

For a given couple, $(p, f) \in [p_{\min}, p_{\max}] \times F$ the associated fuzzy estimate of demand $\tilde{D}(p, f)$ is here assumed to be represented by a trapezoidal fuzzy number and the fuzzy function \tilde{D} is defined by four level 'crisp' functions D^a , D^b , D^c and D^d .

Then, choosing a frequency f in F , a fuzzy function of price, parameterized by the frequency can be defined by: $\tilde{D}_f(p) = \tilde{D}(p, f)$ and is then composed of four "level functions" in the sense of Zimmermann [12]. Figure 2 sketches such a fuzzy function by showing for the interval $[p_{\min}, p_{\max}]$ "level functions" of $\tilde{D}_f(p)$.

There D_f^b and D_f^c are the functions associated to a degree of membership equal to 1. D_f^a and D_f^d are the functions associated to the limits of the base of the corresponding fuzzy numbers (the membership degree starts there from zero). Consistency implies that these four functions cannot intersect each other over the domain $[p_{\min}, p_{\max}]$.

For every price p in the allowed domain, the membership function $\mu_{D_f}^p$ of the demand is, according to the trapezoidal assumption, so that $\mu_{D_f}^p(d)$ is equal to:

$$\begin{cases} (d - D_f^a(p)) / (D_f^b(p) - D_f^a(p)) & \text{if } D_f^a(p) \leq d \leq D_f^b(p) \\ 1 & \text{if } D_f^b(p) \leq d \leq D_f^c(p) \\ (D_f^d(p) - d) / (D_f^d(p) - D_f^c(p)) & \text{if } D_f^c(p) \leq d \leq D_f^d(p) \\ 0 & \text{if } d \leq D_f^a(p) \text{ or } d \geq D_f^d(p) \end{cases} \quad (12)$$

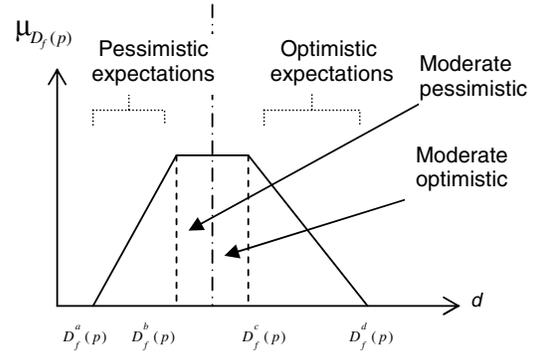


Figure 1. Trapezoidal membership function

Here the function D^b can be associated to pessimistic expectations and D^c to optimistic ones, it is much expected that the effective value of demand will be within their range, while D^a is associated to extremely pessimistic expectations and D^d to extremely optimistic ones.

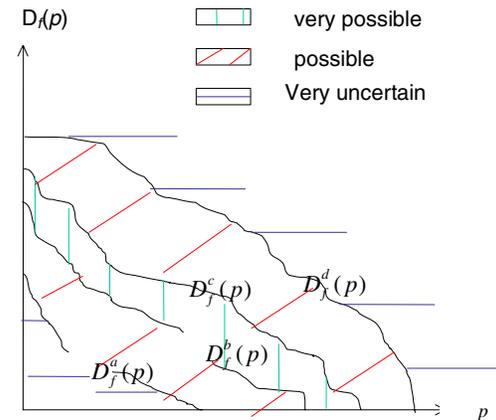


Figure 2. Fuzzy demand function

It is further assumed that the level functions adopts model (1) with different parameters values:

$$\text{For every } k \in \{a, b, c, d\}, D_f^k(p) = D_0^k - \lambda^k (p + \varphi(f)) \quad (13)$$

where the numerical sequences $(D_0^k + \lambda^k \varphi(f))_{k \in \{a, b, c, d\}}$ and $(D_0^k / \lambda^k)_{k \in \{a, b, c, d\}}$ increase with k varying along $\{a, b, c, d\}$.

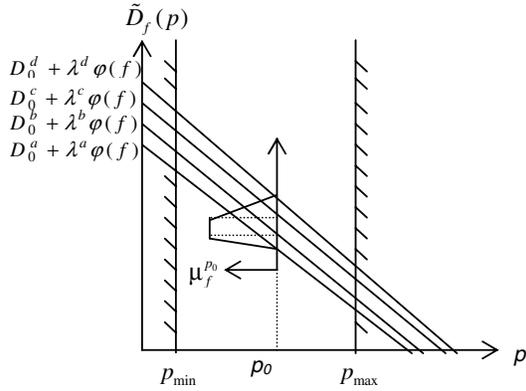


Figure 3. Fuzzy linear demand function for a given frequency f

3.2. Fuzzy expression of profit

For the operator, the fuzziness of demand propagates to the estimate of his profit resulting from a choice of the fare and the frequency. Let the fuzzy estimate of his own profit be written as $\tilde{\pi}$.

Let $\tilde{s}^{p,f} = \min\{\tilde{D}_f(p), Kf\}$ be the fuzzy estimation of the satisfied demand corresponding to the couple (p, f) ; the membership function of $\tilde{s}^{p,f}$ is denoted $\mu_s^{p,f}$, this membership function is deduced from the one of $\tilde{D}_f(p)$ by:

$$\mu_s^{p,f}(s) = \begin{cases} \mu_{D_f}^p(s) & \text{if } s < Kf, \\ \max_{d \geq Kf} \mu_{D_f}^p(d) & \text{if } s = Kf \\ 0 & \text{if } s > Kf \end{cases} \quad (14)$$

Then the fuzzy estimation of the profit of the carrier corresponding to the couple (p, f) is given by:

$$\tilde{\pi}_f(p) = p \cdot \tilde{s}^{p,f} - C(p, f) \quad (15)$$

And the membership function $\mu_{\pi}^{p,f}(\pi)$ of $\tilde{\pi}_f(p)$ is obtained from the one of \tilde{s} as follows:

$$\mu_{\pi}^{p,f}(\pi) = \mu_s^{p,f}[(\pi + C(p, f)) / p] \quad (16)$$

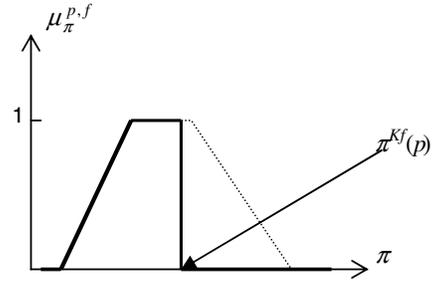


Figure 4. Membership function of the profit

For a given frequency level f , the program of the operator who tries to maximize his profit while considering a capacity constraint is here formulated such as:

$$\begin{cases} \text{Maximize } \tilde{\pi}_f(p) \\ \text{st } \tilde{D}_f(p) \lesssim Kf \end{cases} \quad (17)$$

where “Max” is a fuzzy maximization and “ \lesssim ” is a fuzzy operator associated to the above fuzzy inequality denoted \mathfrak{S} with a membership function given by:

$$\begin{cases} \text{if } Kf \leq D_f^a(p) \text{ then } \mu(\mathfrak{S})=0 \\ \text{if } D_f^a(p) < Kf < D_f^d(p) \text{ then } \\ \mu(\mathfrak{S}) = (D_f^d(p) - Kf) / (D_f^d(p) - D_f^a(p)) \\ \text{if } Kf \geq D_f^d(p) \text{ then } \mu(\mathfrak{S})=1 \end{cases} \quad (18)$$

Figure 5 provides a graphical representation of the fuzzy constraint of problem (17).

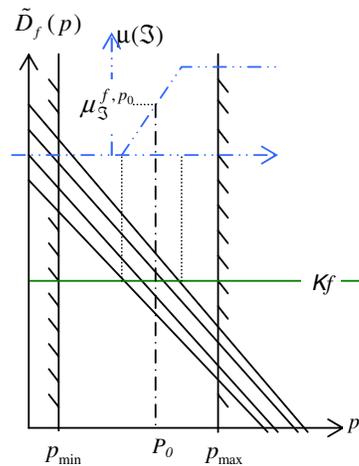


Figure 5. Fuzzy linear demand function for a given frequency f and the available capacity level

When the estimated demand is given by a fuzzy linear function, the problem (17) is a fuzzy linear programming. Several methods are given in the literature to resolve fuzzy linear programming problems (for example [7] and [8]) depending on the particularities of the problem (the most treated case is when the objective function is crisp and only the constraints bounds are fuzzy). In this paper, all the coefficients are fuzzy but the bounds of the constraints. That is why a method of resolution is given in this particular configuration, based on the techniques of ranking fuzzy numbers which will be treated in next subsection.

3.3. Fuzzy decision making

The final choice of the decision maker, even based on fuzzy representations, to be practical will be a crisp one. For that, he can perform an analysis of risk (spoilage and spill ones) using the above fuzzy representation of demand. For a given frequency level f and for each feasible fare p , a fuzzy set representative of the distribution of the estimate of the resulting profit is associated. To choose a price p_f^* , it should be necessary to compare the fuzzy numbers associated to the profit. Since demand is here considered to be represented by convex fuzzy sets, it will be also the case for the profit. Different techniques are available to rank convex fuzzy sets for example [6]. The main adopted approaches perform a defuzzification of the problem through a relationship that associates to each fuzzy number a crisp value. Here the defuzzification is making use of the barycenter of the fuzzy profit $\tilde{\pi}_f(p)$:

if $\mu(\mathfrak{S}) \neq 0$ then

$$\pi_f^e(p) = \frac{\int_{\pi_f^l(p)}^{\min(\pi^{Kf}, \pi_f^d(p))} \pi \cdot \mu_{\pi}^{p,f}(\pi) d\pi}{\int_{\pi_f^l(p)}^{\min(\pi^{Kf}, \pi_f^d(p))} \mu_{\pi}^{p,f}(\pi) d\pi} \quad (19)$$

else $\pi_f^e = 0$

The problem is transformed into the maximization of the above non linear (and non concave) expression with respect to p . An optimal $(\pi_f^e)^* = \max_p \pi_f^e$ can be obtained by inspection over a discretization of the feasible set of p . Let p_f^* be the corresponding fare.

The final decision step is to choose the frequency level denoted f^* . It is given by the expression:

$$f^* = \arg \max_{f \in F} \{(\pi_f^e)^*, f \in F\} \quad (20)$$

to which is associated the selected fare $p^* = p_{f^*}^*$. The expected profit is then represented by the fuzzy set: $\tilde{\pi}_{f^*}(p^*)$.

3.4. Risk analysis

The main risks to which the decider is confronted are:

- A risk of spoilage: when the offered capacity exceeds the effective demand level d_{eff} , then aircraft will fly with empty seats (see figure 6).
- A risk of spill: when the effective demand d_{eff} is larger than the offered capacity, so a large number of potential passengers will be refused at booking (see figure 7).

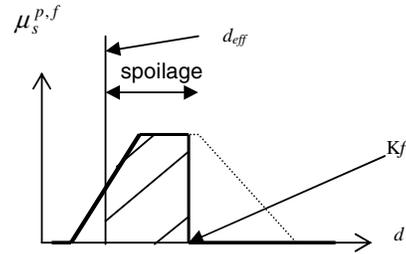


Figure 6. Representation of risk example : spoilage

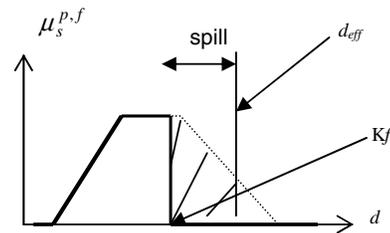


Figure 7. Representation of risk example: Spill

The decision maker could have a preference with respect to these risks. For that, he can introduce in the defuzzification method given by (19) a weight expressing his preferences as below:

if $\mu(\mathfrak{S}) \neq 0$ then

$$\pi_f^e(p) = \mu(\mathfrak{S})^q \frac{\int_{\pi_f^l(p)}^{\min(\pi^{Kf}, \pi_f^d(p))} \pi \cdot \mu_{\pi}^{p,f}(\pi) d\pi}{\int_{\pi_f^l(p)}^{\min(\pi^{Kf}, \pi_f^d(p))} \mu_{\pi}^{p,f}(\pi) d\pi} \quad (21)$$

Where q is a real number that can be chosen by the decider as follows:

- ✓ $q > 0$ if the goal is to avoid spoilage and empty seats.
- ✓ $q < 0$ if the priority is to avoid refusing demand and to satisfy consumers requests.
- ✓ $q = 0$ corresponds to (19) and represents an equal treatment for two risks.

Both studied approaches provide the decider with a couple of crisp values for action. The first value represents the fare of travel and the second represents the frequency of flights to be delivered to the market in order to get a somehow maximized profit. In the case of a deterministic approach, the expected profit is a crisp value while in the second approach, although the final decision is crisp, the expected profit is given by a fuzzy set. This fuzzy set displays to the decision maker a view of uncertainty in which lies the solution. Applying the proposed fuzzy approach, the experts can intervene at different stages and in several ways in the decision process. On one hand and depending on their opinion about the trend of the activity, they can decide about the level functions representing the fuzzy demand. On the other hand, they can be risk averse and choose the criteria concerning the minimization of risk in the profit maximization stage. In the next section, it is focused on the possibility of putting on execution the proposed method.

4. Case study

No data are available that is why we generate them through a Cobb Douglass model.

$$D(p, f) = \delta \cdot p^\varepsilon \cdot f^\varepsilon \quad (22)$$

$$\begin{aligned} p_{min} &= 400 \text{ u.m} \\ p_{max} &= 600 \text{ u.m} \\ f_{min} &= 20 \\ f_{max} &= 60 \end{aligned}$$

The elasticity values to respectively the fare and to the frequency are supposed to be given by $\varepsilon_p = -1,1$ and $\varepsilon_f = 0,8$ (see [3]).

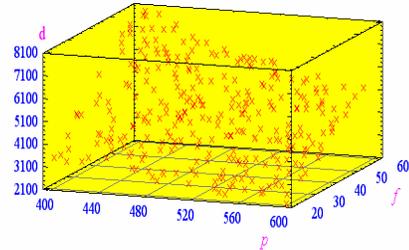


Figure 8. Generated data (demand in terms of fare and frequency)

Here we consider the model given in this paper where

$$\varphi(f) = \frac{-\tau}{f} \quad \text{where } \tau > 0, \text{ so the model (1) becomes:}$$

$$D(p, f) = D_0 - \lambda p - \theta / f \quad (23)$$

4.1. Estimation of the demand function

4.1.1. Econometric estimation

Using the generated data, Minimum Least Square regression gives the estimated coefficients of the model ($\hat{D}_0 = 13351$, $\hat{\lambda} = 10,9$, $\hat{\theta} = 117965$). The obtained function of demand with these estimated parameters is sketched by figure 9. So the function φ is given by

$$\varphi(f) = -10862,3/f \quad (= -(\hat{\theta} / \hat{\lambda}) / f)$$

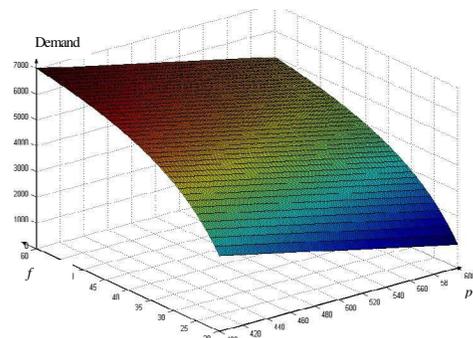


Figure 9. Estimated demand function through econometric regression

4.1.2. Fuzzy estimation

Here we make use of fuzzy regression analysis proposed by Charfeddine *et al.* [Charfeddine *et al.*, 2003] in order to build the four levels defining the fuzzy function based on the same data samples. Figures 10 and 11 give the estimated levels of the fuzzy demand function for respectively frequency values 25 and 40. ($D_0^a = 12828,8$, $\lambda^a = 10,8$, $D_0^d = 14077,4$, $\lambda^d = 10,4$, $D_0^b = \hat{D}_0 - \sigma$, $\lambda^b = \hat{\lambda}$, $D_0^c = \hat{D}_0 + \sigma$, $\lambda^c = \hat{\lambda}$).

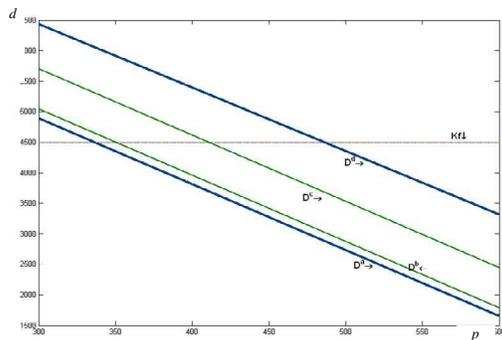


Figure 10. Estimation of levels of fuzzy demand function for $f=25$

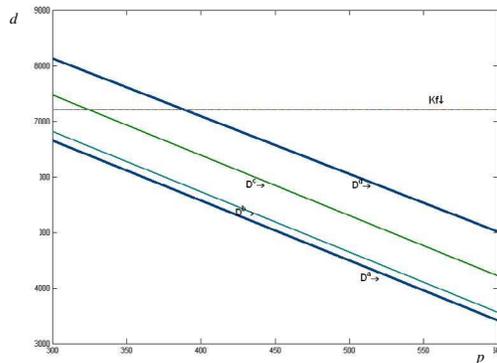


Figure 11. Estimation of levels of fuzzy demand function for $f=40$

4.2. Profit maximization

The cost function's parameters are supposed to have the following values: $c_d = 15$; $c_0 = 100000$; $c_f = 35000$;

4.2.1. Crisp profit

In the crisp case, the profit deduced from the estimation of the demand is sketched by figure 11. This function is concave so it has a maximum.

The solution for this crisp model, found by the optimization section of this paper is ($f^* = 40$ and $p^* = 486,4$).

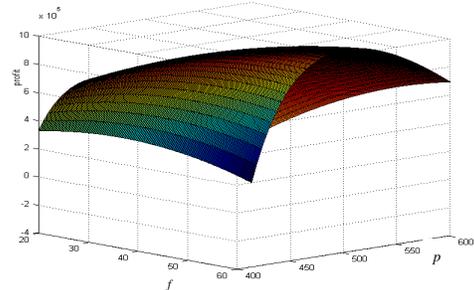


Figure 12. Profit found by econometrical method

4.2.2. Fuzzy estimated profit

Since the maximization of the profit in this case, is made through two stages, we fix first the frequency level, and then we vary the fare level.

Here to illustrate the obtained estimation for the profit, we consider only the case where f is equal to 25. Figure 13 gives in this case the level functions of the fuzzy demand function and it sketches the satisfied demand shapes for two values for the fare in order to have an idea about the different possibilities that can occur.

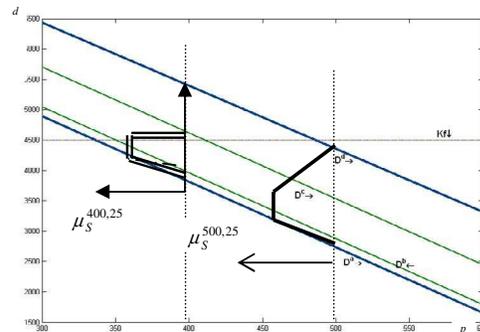


Figure 13. Levels of demand for $f=25$ and the satisfied demand for $p=400$ and $p=500$.

Depending on the shape of the demand, the one of the profit can be deduced (see figures 14 and 15).

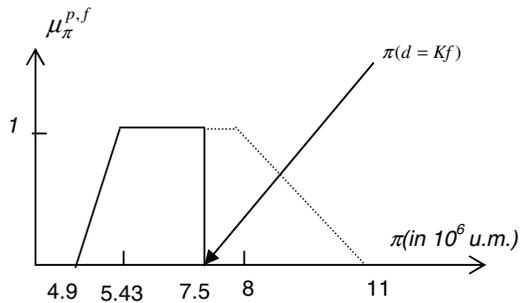


Figure 14. Fuzzy estimation of the profit for $f=25$ and $p=400$

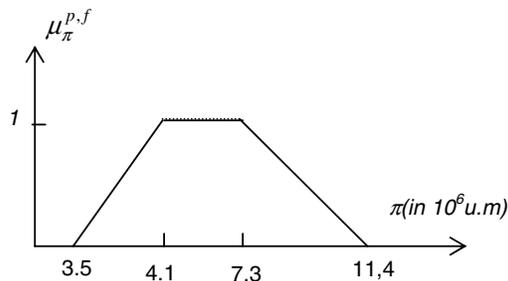


Figure 15. Fuzzy estimation of the profit for $p=500$ and $f=25$

Then the techniques of defuzzification and of ranking of the fuzzy numbers can be used. This subsection was not given to resolve the problem but to illustrate some steps of the proposed method.

5. Conclusion

This communication has considered the case in which demand functions for airlines are given in a fuzzy way. This representation of demand seems opportune since air transportation demand is very sensitive to many unpredictable factors and airlines operate in a very imperfect information environment. It has been shown in this communication that decision making with fuzzy information can be formalized, without too much intricacy in the monopoly situation. Future works should be turned towards a better evaluation of risk associated to better understanding of this last concept and when there are many operators on the same market.

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