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A two-stage approach for managing actuators redundancy and its application to fault tolerant flight control

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Abstract In safety-critical systems such as transportation aircraft, redundancy of actuators is introduced to improve fault tolerance. How to make the best use of remaining actuators to allow the system to continue achieving a desired operation in the presence of some actuators failures is the main subject of this paper. Considering that many dynamical systems, including flight dynamics of a transportation aircraft, can be expressed as an input affine nonlinear system, a new state representation is adopted here where the output dynamics are related with virtual inputs associated with the intended operation. This representation, as well as the distribution matrix associated with the effectiveness of the remaining operational actuators, allows us to define different levels of fault tolerant governability with respect to actuators' failures. Then, a two-stage control approach is developed, leading first to the inversion of the output dynamics to get nominal values for the virtual inputs and then to the solution of a linear quadratic (LQ) problem to compute the solicitation of each operational actuator. The proposed approach is applied to the control of a transportation aircraft which performs a stabilized roll maneuver while a partial failure appears. Two fault scenarios are considered and the resulting performance of the proposed approach is displayed and discussed.

1. Introduction

To meet the stringent reliability and safety requirements for critical systems such as modern aircraft,[1] spacecraft,[2] hazardous material processing,[3] marine vehicles[4] and over-redundant actuator systems are often introduced. Many of these controlled systems are known to be complex nonlinear systems. Nowadays, direct nonlinear control law design techniques such as sliding mode control,[5] nonlinear inverse control,[6] backstepping control[7] as well as combinations of these techniques[8,9] are available

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to provide improved control performances. However, since these control design techniques do not consider explicitly actuators' failure scenarios, they should be completed to allow the management of these hazardous situations. Then, the control of these nonlinear systems while managing actuator redundancy when facing control channels' failures remains a challenge.

Two main active fault tolerant control strategies have been developed to manage these failure scenarios depending on the availability of a fault detection and identification (FDI) device[10]: adaptive control and two-stage approaches (control allocation). Adaptive control methods estimate online the effectiveness of actuators and redistribute effects accordingly, while control allocation techniques separate the control law synthesis task from the actuator distribution task which is dependent on the FDI diagnostic. It appears that the adaptive control strategy provides acceptable results only in the case of limited degradation of actuators' effectiveness[11] comparable with a parameter change scenario. When using a control allocation scheme, a large variety of failure cases can be handled successfully depending on the FDI performance and the controllability resulting from the remaining operational actuators.

Over the past two decades, different control allocation methods have been developed. The explicit ganging method[12] can be used when it is obvious as to how to combine redundant actuators. The direct allocation method[13,14] attempts to match the desired control efforts in both magnitude and direction. Daisy chaining[15] assumes a hierarchy of actuators and distributes the desired control efforts according to some priority. Besides these direct methods, optimization based control allocation methods making use of linear programming,[16] quadratic programming,[17] even nonlinear programming[18] have been proposed. The optimization based approaches try to use the healthy actuators to the most possible degree as well as guarantee the integrity of the system and performance when some actuator failure occurs and has been detected and identified successfully. Numerical approaches such as active set, interior point and neural networks can be used to effectively solve the resulting optimization problems[19–21] and implement them in an online control context. These approaches have been applied in reconfigurable control allocation such as flight control, control of marine vehicles and robots.[19–22]

This paper considers the case of a complex nonlinear system, provided with redundant actuators which are subject to some partial actuator failure while performing a standard maneuver. Here it is supposed that an FDI scheme produces a timely exact diagnostic for the different actuators and control channels. Then a general two-stage approach to deal with this situation is proposed. This approach makes use of an output based state representation which is associated with virtual inputs, which is output state representation with associated virtual inputs (OSVI), and the distribution of these virtual inputs among the operational actuators. This new representation allows us to introduce some new concepts with respect to fault tolerant governability and is in accordance with the adopted two-stage approach. At the first stage, according to the desired maneuver, virtual inputs are computed through the use of a nonlinear control technique, while at the second stage a control allocation problem, considering the remaining operational actuators, is solved.

2. Output state representation with associated virtual inputs

Many dynamical systems admit a state space representation termed input affine:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \sum_{j=1}^{m} g_j(\boldsymbol{x}) u_j \tag{1}$$

where $\boldsymbol{x} \in \mathbf{R}^n$ is the state vector representing the system dynamics, u_j $(j = 1, 2, \ldots, m)$ the control input, $\boldsymbol{f}(\boldsymbol{x})$ and $g_j(\boldsymbol{x})$ $(j = 1, 2, \ldots, m)$ the smooth vector fields of $\boldsymbol{x}$.

When considering an output trajectory tracking problem for this nonlinear system, a characteristic output vector must be chosen and the corresponding output based state representation can be adopted to compute the corresponding control signal according to a nonlinear control technique.[23] Let

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}), \quad \boldsymbol{y} \in \mathbf{R}^p \tag{2}$$

be the chosen independent outputs, where $\boldsymbol{h}(\boldsymbol{x})$ is a smooth vector field of $\boldsymbol{x}$. It is supposed that $p < n$ and $p < m$. The output based state vector is given by

$$\boldsymbol{X} = \left[y_1, \dot{y}_1, \ldots, y_1^{(r_1)}, \ldots, y_p, \dot{y}_p, \ldots, y_p^{(r_p)}, z_1, z_2, \ldots, z_q \right]^{\mathrm{T}} \tag{3}$$

$$\text{with } q + \sum_{j=1}^{p} (r_j + 1) = n \tag{4}$$

where r_j $(j = 1, 2, \ldots, p)$ is the relative degree of output y_j, and z_i $(i = 1, 2, \ldots, q)$ the inner dynamics. In the following text, it is supposed that the output variables are chosen so that there are no left internal dynamics and that the system is governable when adopting these outputs. Then the output based state representation can be written as

$$\dot{\boldsymbol{X}} = \boldsymbol{F}(\boldsymbol{X}) + \boldsymbol{G}(\boldsymbol{X})\boldsymbol{u} \tag{5}$$

where $\boldsymbol{u} \in \mathbf{R}^m$ is the control inputs' vector, $\boldsymbol{F}(\boldsymbol{X})$ a vector of dimension n, and $\boldsymbol{G}(\boldsymbol{X}) \in \mathbf{R}^{n \times m}$ a matrix with p non zero rows at positions $s_1, s_2, \ldots, s_p$ given by

$$\begin{cases} s_1 = \sum_{k=1}^{1} (r_k + 1) \\ s_2 = \sum_{k=1}^{2} (r_k + 1) \\ \vdots \\ s_p = \sum_{k=1}^{p} (r_k + 1) \end{cases} \tag{6}$$

and with zero rows for any other positions.

Then, the OSVI is written as

$$\dot{\boldsymbol{X}} = \boldsymbol{F}(\boldsymbol{X}) + \boldsymbol{H}\boldsymbol{v} \tag{7}$$

where $\boldsymbol{H} \in \mathbf{R}^{n \times p}$, whose (s_k, k)th elements are 1 and other elements are 0 which can be expressed as

$$\begin{cases} H_{s_k k} = 1 & (k = 1, 2, \ldots, p) \\ H_{ij} = 0 & \text{Otherwise} \end{cases}$$

$\boldsymbol{v}$ is a vector with element $v_i = \sum_{j=1}^{m} G_{s_i j}(\boldsymbol{X}) u_j$ $(i = 1, 2, \ldots, p)$, with $G_{s_i j}(\boldsymbol{X})$ the (s_i, j) th element of matrix $\boldsymbol{G}(\boldsymbol{X})$, or

$$\boldsymbol{v} = \boldsymbol{D}(\boldsymbol{X})\boldsymbol{u} \tag{8}$$

where $\boldsymbol{D}(\boldsymbol{X}) \in \mathbf{R}^{p\times m}$ is the distribution matrix.

Eq. (7) introduces the vector $\boldsymbol{v}$ of the p called virtual inputs. Here contrary to Ref.[24] the virtual inputs are defined according to the chosen p independent outputs. Then depending on the considered maneuver, they can be defined differently according to Eq. (3), (7) and (8).

From Eqs. (7) and (8), it appears that the governability of the chosen outputs as well as the degree of actuators redundancy may be treated in two steps. The system Eq. (5) will be called governable with respect to the virtual inputs defined by Eq. (8) since $\boldsymbol{H}$ is full rank. The virtual inputs will be called fully covered by the control inputs if $\boldsymbol{D}(\boldsymbol{X})$ is full rank. Then the target maneuver will be locally achievable and the system will be called locally governable. When failures appear, the column corresponding to the failed actuators should be removed from matrix $\boldsymbol{D}$. Let $\boldsymbol{D}_{ijk}$ be the resulting $\boldsymbol{D}(\boldsymbol{X})$ matrix without the columns corresponding to the failed actuators i, j and k . Then the system governability will be tolerant to faults (i, j, k) when matrix $\boldsymbol{D}_{ijk}$ $(\boldsymbol{X})$ is full rank. If this property is true with any k failed actuators, whatever these actuators, this fault tolerant governability property will be called kth order.

When an actuator failure case is identified, the set of control inputs $\boldsymbol{S}$ can be divided into the set of the operational inputs $\boldsymbol{S}_{\mathrm{o}}$ of dimension m_{o}, and the set of the failed inputs $\boldsymbol{S}_{\mathrm{f}}$ of dimension m_{f}. Then Eq. (8) is rewritten as

$$v_i = \sum_{j\in\boldsymbol{S}_{\mathrm{o}}} D_{ij}(\boldsymbol{X})u_j + \sum_{k\in\boldsymbol{S}_{\mathrm{f}}} D_{ik}(\boldsymbol{X})u_k \quad (i = 1, 2, \ldots, p)$$

or

$$\boldsymbol{v} = \boldsymbol{D}_{\mathrm{o}}(\boldsymbol{X})\boldsymbol{u}_{\mathrm{o}} + \boldsymbol{D}_{\mathrm{f}}(\boldsymbol{X})\boldsymbol{u}_{\mathrm{f}} \tag{9}$$

where D_{ij} and D_{ik} are respectively the contribution of the jth operational input and the contribution of the kth failed input to the ith virtual input, $\boldsymbol{D}_{\mathrm{o}} \in \mathbf{R}^{p\times m_{\mathrm{o}}}$ the distribution matrix associated to the identified failure case, $\boldsymbol{u}_{\mathrm{o}} \in \mathbf{R}^{m_{\mathrm{o}}}$ the vector of the remaining operational control inputs, $\boldsymbol{D}_{\mathrm{f}} \in \mathbf{R}^{p\times m_{\mathrm{f}}}$ the control effectiveness matrix related to the failed control inputs, and $\boldsymbol{u}_{\mathrm{f}} \in \mathbf{R}^{m_{\mathrm{f}}}$ the vector of the failed control inputs which are no more active and whose values are supposed known.

3. The proposed two-stage control approach

Now the control signal synthesis can be split into a virtual inputs' synthesis problem for system Eq. (7) and a control allocation problem to distribute the virtual inputs among the remaining operational inputs following Eq. (9). Some benefits of adopting such strategy can be found as following.

When solving the virtual inputs' control problem, it is not yet necessary to take into account the physical constraints attached to each operational actuator. This is fortunate since few control techniques are able to take explicitly into account input constraints. Then the actuator constraints as well as other operational limitations can be more easily taken into account when solving the control allocation problem. Moreover, additional constraints can be taken into account in the control allocation problem. Examples of possible additional constraints in flight control application are: maximum wing loading, maximum control surface deflection, maximum radar signature, maximum drag and minimum lift.

The computation of the necessary virtual inputs to achieve output trajectory tracking is illustrated here with the nonlinear inverse control technique.[6]

Then, with the adopted assumptions, the output dynamics Eq. (7) can be rewritten as

$$\begin{bmatrix} y_1^{(r_1+1)} \\ y_2^{(r_2+1)} \\ \vdots \\ y_p^{(r_p+1)} \end{bmatrix} = \begin{bmatrix} F_{s_1}(\boldsymbol{X}) \\ F_{s_2}(\boldsymbol{X}) \\ \vdots \\ F_{s_p}(\boldsymbol{X}) \end{bmatrix} + \boldsymbol{v} \tag{10}$$

where $F_{s_i}(\boldsymbol{X})$ denotes the s_ith $(i = 1, 2, \ldots, p)$ element of $\boldsymbol{F}(\boldsymbol{X})$.

For a trajectory tracking problem, according to the nonlinear inverse control technique, the virtual inputs can be chosen as

$$v_i^* = y_{\mathrm{d}i}^{(r_i+1)} - \sum_{k=0}^{r_i} c_{ik}\left(y_i^{(k)} - y_{\mathrm{d}i}^{(k)}\right) - F_{s_i}(\boldsymbol{X}) \quad (i = 1, 2, \ldots, p) \tag{11}$$

where v_i^* is the ith element of the chosen virtual inputs vector $\boldsymbol{v}^*$, $y_{\mathrm{d}i}$ the ith element of the desired output trajectory $\boldsymbol{y}_{\mathrm{d}}$, while the coefficients c_{ik} are chosen so that the dynamics of the tracking error defined by $e_i = y_i - y_{\mathrm{d}i}$ $(i = 1, 2, \ldots, p)$, is asymptotically stable and converges towards zero. The tracking errors dynamics are then given by

$$e_i^{(r_i+1)} + c_{ir_i}e_i^{(r_i)} + \cdots + c_{i1}e_i^{(1)} + c_{i0}e_i = 0 \quad (i = 1, 2, \ldots, p) \tag{12}$$

To limit the requested values of the virtual inputs, the choice of the coefficients c_{ik} may be the result of a trade-off between the characteristics of the transient dynamics of the different outputs and the corresponding solicitations of the remaining operational actuators.

A general scheme of the fault tolerant trajectory tracking system based on this two-stage control approach is represented in Fig. 1.

Once the virtual input values have been computed from Eq. (11), the control allocation problem will distribute the requested effects between the remaining operational actuators. This point is discussed in the next paragraph.

4. Control allocation

4.1. Problem statement

To be effective, a control allocation method must generate an online solution to Eq. (9). The virtual inputs' solution given by Eq. (11) provides the level constraints of the control allocation problem to be solved online:

$$\boldsymbol{D}_{\mathrm{o}}(\boldsymbol{X})\boldsymbol{u}_{\mathrm{o}} + \boldsymbol{D}_{\mathrm{f}}(\boldsymbol{X})\boldsymbol{u}_{\mathrm{f}} = \boldsymbol{v}^* \tag{13}$$

Meanwhile, the problem should take into account the remaining operational actuators constraints and limitations which in general can be written:

$$\begin{cases} \boldsymbol{u}_{\mathrm{o}}^{\min} \leqslant \boldsymbol{u}_{\mathrm{o}}(t) \leqslant \boldsymbol{u}_{\mathrm{o}}^{\max} \\ \dot{\boldsymbol{u}}_{\mathrm{o}}^{\min} \leqslant \dot{\boldsymbol{u}}_{\mathrm{o}}(t) \leqslant \dot{\boldsymbol{u}}_{\mathrm{o}}^{\max} \end{cases} \tag{14}$$

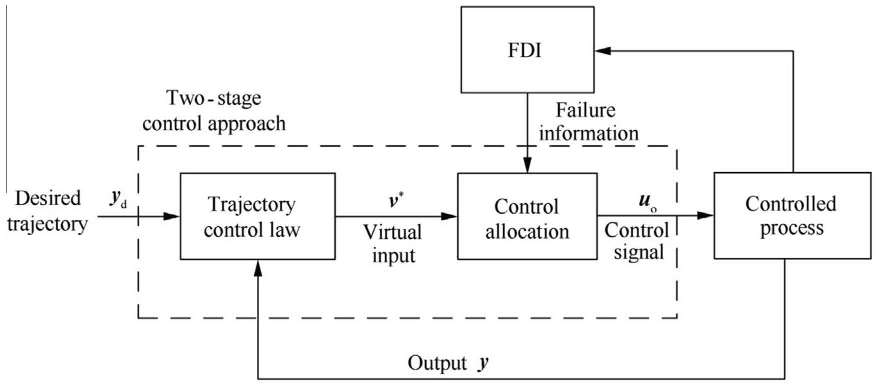

Fig. 1 The proposed fault tolerant trajectory tracking scheme.

where u_o^{min}, u_o^{max}, $\dot{u}_o^{min}$ and $\dot{u}_o^{max}$ are the limits of the minimum and maximum positions and speeds for the operational actuators, respectively.

When the controller operates with sampled signals at some very short period Δt, then the online control allocation problem consists in practice in finding a solution $u_{ot}^* \in \mathbf{R}^{m_o}$ to the constrained set of equations:

$$\tilde{v}_t = D_o(X_t)u_{ot} \text{ with } \tilde{v}_t = v_t^* - D_f(X_t)u_{ft} \tag{15}$$

with

$$\begin{cases} \max\left\{u_o^{min}(X_t), u_{ot-\Delta t} + \dot{u}_o^{min}(X_t)\Delta t\right\} \leqslant u_{ot} \\ u_{ot} \leqslant \min\left\{u_o^{max}(X_t), u_{ot-\Delta t} + \dot{u}_o^{max}(X_t)\Delta t\right\} \end{cases} \tag{16}$$

where v_t^* is the chosen virtual vector at time t, $\tilde{v}_t$ the contribution of operational inputs to v_t^*, X_t the output based state vector at time t, u_{ot} the current solution for operational actuators at time t, $u_{ot-\Delta t}$ the previous solution for operational actuators at time $t-\Delta t$, and u_{ft} the position of failed actuators at time t.

The above control allocation problem can be rewritten as

$$\text{Find} \quad u_{ot} \in \mathbf{R}^{m_o} : \tilde{v}_t = D_{ot}u_{ot} \tag{17}$$

with

$$L_t u_{ot} \leqslant l_t \tag{18}$$

where $D_{ot} \in \mathbf{R}^{p\times m_o}$ is the distribution matrix at the current time, $L_t \in \mathbf{R}^{2m_o\times m_o}$ the coefficient matrix, and $l_t \in \mathbf{R}^{2m_o}$ the restriction condition at the current time.

Since D_{ot} is supposed to be full rank with $p < m_o$, Eq. (17) has an infinite set of solutions Σ_t. Points in $\mathbf{R}^{m_o}$ satisfying constraints Eq. (18) constitute a feasible convex polyhedron Ω_t. According to the relationship between Σ_t and Ω_t, four cases (see Fig. 2) can be considered:

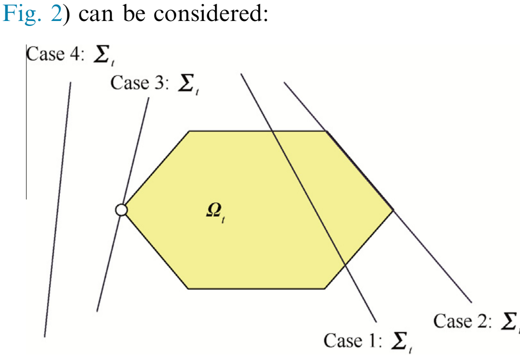

Fig. 2 Illustration of the four different cases.

Case 1: the interior of set $\Sigma_t \cap \Omega_t$ is not empty and there is a multiplicity of solutions inside it.
Case 2: the interior of set $\Sigma_t \cap \Omega_t$ is empty but there is a multiplicity of solutions at its border.
Case 3: the set $\Sigma_t \cap \Omega_t$ is reduced to a unique point and there is a unique solution.
Case 4: the set $\Sigma_t \cap \Omega_t$ is empty and there is not even a feasible solution.

4.2. Solution approaches

Rather simple methods such as explicit ganging,[12] direct allocation[13] and daisy chain[15] have been developed with the objective of solving the control allocation problems such as Eqs. (17) and (18). These methods which do not generate heavy computations are in general unable to tackle other objectives than input distribution (Eq. (17)) and have no capability to face additional inequality constraints which may be necessarily considered to guarantee the structural integrity of the system in a failure situation.[21]

In the first two cases considered above, where a solution must be selected among many feasible ones, an effective way is to choose the closest feasible solution to the previous solution $u_{ot-\Delta t}^*$ at time $t-\Delta t$ (see Fig. 3). Here, u_{ot}^* will be the solution of the linear quadratic (LQ) problem given by

$$u_{ot}^* = \arg\min\{\|u_{ot} - u_{ot-\Delta t}^*\|_{m_o}^2, u_{ot} \in \Sigma_t \cap \Omega_t\} \tag{19}$$

where $\|\cdot\|_{m_o}$ is an Euclidian distance over $\mathbf{R}^{m_o}$.

In the third case, the unique solution can be found either by solving directly Eqs. (17) and (18) or by searching a solution to problem Eq. (19) through a non-feasible search approach.

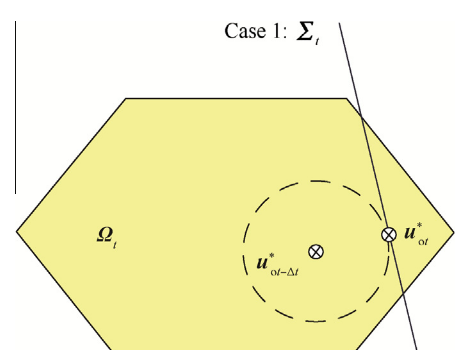

Fig. 3 Selection of the closest feasible solution.

In the fourth case, since $\boldsymbol{\Sigma}_t \cap \boldsymbol{\Omega}_t$ is empty, the control objective given by Eq. (15) is not reachable for time t. Two approaches can be considered:

(1) Choose the instant value of $\boldsymbol{u}^*_{ot}$ when the error in the instant control objective is minimized, this leads to solve the problem:

$$\boldsymbol{u}^*_{ot} = \arg\min\{\|\tilde{\boldsymbol{v}}_t - \boldsymbol{D}_{ot}\boldsymbol{u}_{ot}\|^2_{m_o},\ \boldsymbol{u}_{ot} \in \boldsymbol{\Omega}_t\} \tag{20}$$

which results in a small size LQ problem, in general easy to solve.

(2) Adopt an approach similar to one of the predictive controls;[25] choose $\boldsymbol{u}^*_{ot}$ as the first solution component of the following LQ problem over a time span $K\Delta t$:

$$\min_{\boldsymbol{u}_{ot+(k-1)\Delta t}} \sum_{k=1}^{K} \left\|\tilde{\boldsymbol{v}}_{t+(k-1)\Delta t} - \boldsymbol{D}_{ot+(k-1)\Delta t}\boldsymbol{u}_{ot+(k-1)\Delta t}\right\|^2_{m_o} \tag{21}$$

with

$$\boldsymbol{u}_{ot+(k-1)\Delta t} \in \boldsymbol{\Omega}_{t+(k-1)\Delta t} \quad (k = 1, 2, \ldots, K) \tag{22}$$

where $\tilde{\boldsymbol{v}}_{t+(k-1)\Delta t}$, $D_{ot+(k-1)\Delta t}$ and $\boldsymbol{\Omega}_{t+(k-1)\Delta t}$ $(k=1, 2, \ldots, K)$ can be computed or estimated in advance. In this case, the instant satisfaction of the control objective is shifted to an overall satisfaction over the period from t to $t+(K-1)\Delta t$ and the resulting LQ problem is no more so small.

4.3. Discussion

It appears that depending on the composition of the feasible set of the control allocation problem, different mathematical programming problems should be considered and solved online to provide an effective control allocation. This should be under the assumption that a device will be available to identify online the composition of the current feasible set. However, it should not be an easy task to be performed prior to solving the control allocation problem.

The proposed solution is to retain a global control allocation formulation, valid for all the feasible set cases considered above. This can be achieved for example by adopting an optimization problem formulation:

$$u^*_{ot} = \arg\min\left\{\gamma\|\tilde{v}_t - D_{ot}\ u_{ot}\|^2_{m_o} + \left\|u_{ot} - u^*_{ot-\Delta t}\right\|^2_{m_o}, u_{ot} \in \boldsymbol{\Omega}_t\right\} \tag{23}$$

which is again an LQ problem with a weighting factor γ. The value of γ should be chosen so that the original control objective is preserved.

In the discussion above, only physical constraints resulting from the limitations of the actuators are considered for the definition of the feasible set $\boldsymbol{\Omega}_t$. However, the feasible set $\boldsymbol{\Omega}_t$ may be completed by including additional constraints such as structural or operational constraints.

To get online numerical solutions of the resulting LQ problems, iterative methods such as active set,[20] interior point[17] or dedicated neural networks solvers[19,21] have been proposed recently.

5. Application to fault tolerant flight control

Considering the size of modern transportation aircraft as well as the safety issue, over-redundant control actuators which contribute to the roll, pitch and yaw moments around three main axes, are currently operated. Fig. 4 shows an A340's wing where six spoilers and two ailerons contribute mainly to the roll moment.

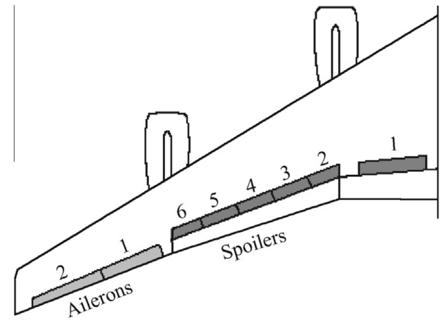

Fig. 4 Aerodynamic surface of A340 wing.

Then the proposed approach is illustrated with the case of a transportation aircraft supposed to perform a stabilized roll maneuver while subject to limited actuators' failures.

5.1. Formulation of actuator allocation problem

In the case of a stabilized turn the output variables are the angular rates p, q and r. Considering the equations of aircraft rotational dynamics displayed in Appendix A, Eq. (A1) can be rewritten as

$$\dot{\boldsymbol{\omega}} = -I_m^{-1}[\boldsymbol{\omega} \times (I_m\boldsymbol{\omega})] + I_m^{-1}\boldsymbol{\mu} \tag{24}$$

where $\boldsymbol{\omega} = [p, q, r]^{\mathrm{T}}$ is the inertial rotational velocity expressed in the body-fixed reference frame, with p, q, r the roll rate, the pitch rate and the yaw rate; $\boldsymbol{\mu} = [L, M, N]^{\mathrm{T}}$, with L, M, N the roll, pitch and yaw aerodynamic torques respectively; $\boldsymbol{I}_{\mathrm{m}}$ is the matrix of inertial moments; $\dot{\boldsymbol{\omega}}$ is the inertial rotational acceleration in the body-fixed reference frame; and $\times$ is the cross product operator.

This is an output based state representation with an input affine form where the three natural virtual inputs are given by

$$v = I_m^{-1}\boldsymbol{\mu} \tag{25}$$

which is a linear combination of the roll, pitch and yaw moments L, M and N. Here the output variables p, q and r have relative degrees equal to zero while there is no internal dynamics. The stabilized turn in a steady atmosphere is performed at a constant flight level z and at a constant air speed V which induce a pitch trim value θ_0. Assuming a no wind condition, then the stabilized turn is parameterized by the steady bank angle ϕ_0 which should be related with the target roll, pitch and yaw rates p_{c}, q_{c} and r_{c} by the relations (see Fig. 5, where g denotes the gravity acceleration, ψ is the heading angle and m is the total mass of the aircraft):

$$\begin{cases} p_{\mathrm{c}} = -(g/V)\tan\phi_0 \sin\theta_0 \\ q_{\mathrm{c}} = (g/V)\sin\phi_0 \tan\phi_0 \cos\theta_0 \\ r_{\mathrm{c}} = (g/V)\sin\phi_0 \cos\theta_0 \end{cases} \tag{26}$$

to avoid noticeable lateral load factors during this turn maneuver.

Since the pitch trim value θ_0 is very small in a steady turn, it will be considered that p_{c} and q_{c} remain equal to zero during it,

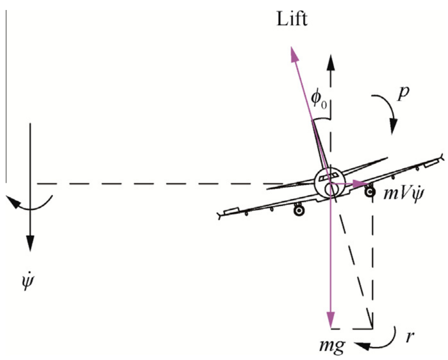

Fig. 5 Steady stabilized turn.

while p_c will be changed from zero when driving the aircraft into the entry and the exit of the stabilized turn. During the whole roll maneuver, the target yaw rate will be given by relation:

$$r_c = (g/V)\sin\phi \tag{27}$$

where ϕ is the current bank angle given by the Euler equations (Eq. (A2)). Then, according to the relative degrees of the chosen outputs, to perform the maneuver in this standard way, the following first order desired dynamics are adopted for the body angular rates of the aircraft:

$$\begin{cases} \dot{p}_d = (p_c - p_d)/\tau_p \\ \dot{q}_d = (q_c - q_d)/\tau_q \\ \dot{r}_d = (r_c - r_d)/\tau_r \end{cases} \tag{28}$$

where p_d, q_d and r_d denote the desired output trajectories; τ_p, τ_q and τ_r are time constants which are chosen to insure a short time response while limiting the corresponding efforts on the aircraft structure.

According to nonlinear inverse control and with Eqs. (24) and (25), the corresponding virtual inputs vector associated with the above defined stabilized roll maneuver is given by

$$v^* = \boldsymbol{\Lambda}^{-1}(\boldsymbol{\omega}_c - \boldsymbol{\omega}) + I_m^{-1}[\boldsymbol{\omega} \times (I_m\boldsymbol{\omega})] \tag{29}$$

where $\boldsymbol{\Lambda} = \mathrm{diag}(\tau_p, \tau_q, \tau_r)$; $\boldsymbol{\omega}_c = [p_c, q_c, r_c]^T$.

Here, the effectiveness of the different aerodynamic actuators to perform the considered roll maneuver appears through the contributions of their angular deflections to the roll, pitch and yaw torque, which according to Eqs. (A3) and (A4) can be written as

$$\begin{cases} L = L^0(\alpha, \beta, p, q, r, V, z) + \sum\limits_{i \in \boldsymbol{I}^L} C_i^L(V, z)\delta_i \\ M = M^0(\alpha, \beta, p, q, r, V, z) + \sum\limits_{i \in \boldsymbol{I}^M} C_i^M(V, z)\delta_i \\ N = N^0(\alpha, \beta, p, q, r, V, z) + \sum\limits_{i \in \boldsymbol{I}^N} C_i^N(V, z)\delta_i \end{cases} \tag{30}$$

where α is the angle of attack; β is the side slip angle; δ_i is the deflection of ith actuator; C_i^L, C_i^M and C_i^N denote the contribution of ith actuator to aerodynamic torques L, M and N, respectively; L^0, M^0 and N^0 denote the aerodynamic torques L, M and N except the contribution of actuators, respectively, and they can be expressed as the functions of states α, β, p, q, r, V and z; the set of all actuators $\boldsymbol{I}$ is

$$I = I^L \cup I^M \cup I^N \tag{31}$$

with I^L the set of actuators contributing to the roll moment, I^M the set of actuators contributing to the pitch moment , and I^N the set of actuators contributing to the yaw torque. Here it is supposed that each aerodynamic actuator is submitted to position and rate constraints given by

$$\begin{cases} \delta_i^{\min} \leqslant \delta_i \leqslant \delta_i^{\max} \\ \dot{\delta}_i^{\min} \leqslant \dot{\delta}_i(t) \leqslant \dot{\delta}_i^{\max} \end{cases} \quad (i \in \boldsymbol{I}) \tag{32}$$

where $\delta_i^{\min}$, $\delta_i^{\max}$, $\dot{\delta}_i^{\min}$ and $\dot{\delta}_i^{\max}$ are respectively the limits of minimum and maximum positions and speeds for the actuators.

Global aerodynamic torques Eq. (30) generated by aircraft aerodynamic actuators can be rewritten in a global affine form as

$$\boldsymbol{\mu} = \boldsymbol{\mu}^0 + C\boldsymbol{\delta} \tag{33}$$

where $\boldsymbol{\mu}^0 = [L^0, M^0, N^0]^T$; $C \in \mathbf{R}^{3\times|\boldsymbol{I}|}$ is the effectiveness matrix of actuators; $\boldsymbol{\delta} \in \mathbf{R}^{|\boldsymbol{I}|}$ is the vector of actuators' positions.

When some actuator failure case is identified, the actuators set $\boldsymbol{I}$ is divided into the set of operational actuators $\boldsymbol{I}_o$ and the set of failed actuators $\boldsymbol{I}_f$. According to Eqs. (25) and (33), the virtual inputs generated by aerodynamic actuators are given by

$$v = I_m^{-1}(\boldsymbol{\mu}^0 + C_f\boldsymbol{\delta}_f + C_o\boldsymbol{\delta}_o) \tag{34}$$

where $\boldsymbol{\delta}_f \in \mathbf{R}^{|I_f|}$ is the vector of the failed actuators which are no more active and whose values are supposed known, and $C_f \in \mathbf{R}^{3\times|I_f|}$ is their effectiveness matrix; $\boldsymbol{\delta}_o \in \mathbf{R}^{|I_o|}$ is the vector of the remaining operational actuators' inputs, and $C_o \in \mathbf{R}^{3\times|I_o|}$ is their effectiveness matrix.

When the controller operates in a short sampling period, the online control allocation problem is to find a solution $\boldsymbol{\delta}_{ot}^* \in \mathbf{R}^{|I_o|}$ to the constrained set of equations:

$$\tilde{v}_t = D_o(X_t)\boldsymbol{\delta}_{ot} \tag{35}$$

with

$$\begin{cases} \max\left\{\boldsymbol{\delta}_o^{\min}, \boldsymbol{\delta}_{ot-\Delta t} + \dot{\boldsymbol{\delta}}_o^{\min}\Delta t\right\} \leqslant \boldsymbol{\delta}_{ot} \\ \boldsymbol{\delta}_{ot} \leqslant \min\left\{\boldsymbol{\delta}_o^{\max}, \boldsymbol{\delta}_{ot-\Delta t} + \dot{\boldsymbol{\delta}}_o^{\max}\Delta t\right\} \end{cases} \tag{36}$$

where $\boldsymbol{\delta}_{ot} \in \mathbf{R}^{|I_o|}$ is the operational actuators' inputs vector at time t; $\boldsymbol{D}_o(\boldsymbol{X}_t) = I_m^{-1}\boldsymbol{C}_o$ and $\tilde{v}_t = \boldsymbol{\Lambda}^{-1}(\boldsymbol{\omega}_c - \boldsymbol{\omega}_t) + I_m^{-1}\cdot[\boldsymbol{\omega}_t\times(I_m\boldsymbol{\omega}_t)] - I_m^{-1}(\boldsymbol{\mu}^0 + C_f\boldsymbol{\delta}_{ft})$, with $\boldsymbol{\omega}_t$ a vector consists of p, q and r at time t and $\boldsymbol{\delta}_{ft} \in \mathbf{R}^{|I_f|}$ the failed actuators' inputs vector at time t.

Then, following the approach developed in Section 5.1, an LQ problem can be formulated as

$$\min_{\boldsymbol{\delta}_{ot}}\left\{\gamma\left\|\tilde{v}_t - I_m^{-1}C_o\boldsymbol{\delta}_{ot}\right\|^2 + \left\|\boldsymbol{\delta}_{ot} - \boldsymbol{\delta}_{ot-\Delta t}^*\right\|^2\right\} \tag{37}$$

with the constraints of Eq. (36). Here γ is a positive weight and $\boldsymbol{\delta}_{ot-\Delta t}^*$ is the solution at time $t - \Delta t$.

5.2. Solution of actuator allocation problem: numerical examples

The proposed control allocation approach is illustrated by adopting the flight mechanics of the research civil aircraft model (RCAM),[26] which has the characteristics of a wide body transportation aircraft with a constant mass of 120000 kg. Then it has been considered that this aircraft is equipped with two pairs of ailerons, two pairs of elevators and a pair of rudders. The considered control actuator architecture has been chosen to be first order with respect to fault tolerant governability. The position and rate limits of these actuators are given in Table 1.

The desired maneuver is a stabilized roll with target dynamics given by Eq. (28) with time constants τ_p, τ_q and τ_r equal to 1/3 s which is ten times larger than current time constants for these particular aircraft actuators. This maneuver is supposed to be performed in a no wind atmosphere at flight level 3000 m with a speed of 124 m/s.

To check the feasibility and performances of the proposed two-stage control approach for online flight fault tolerant control, two fault scenarios have been considered: a soft one where only a deflection rate is affected by a fault and a hard one where a main actuator remains stuck at a nonzero position.

In both numerical applications, the sampling time adopted by the digital control system of the different actuators is taken equal to 0.05 s. The retained values for the weights of the optimality criterion Eq. (37) have been: $\gamma = 10^6$ and 1 for all the weighting parameters. A dedicated neural networks solver[19] has been used online to find the solutions for the resulting successive LQ problem (Eqs. (36) and (37)). This has been possible considering the very short computation time (around 10^{-5} s) needed by this class of neural networks solver to converge to the solution of each actuator allocation problem instances expressed as LQ problems.

In the soft fault scenario, it is assumed that all actuators are operational except for the rate limits of the right outer aileron which changes to ± 5 (°)/s at $t = 1$ s. But since it is still able to operate positively, it is considered that it can still contribute to the solution of the actuator allocation problem. The input signals for ailerons are displayed in Fig. 6 which provide the main contribution to the roll maneuver, where C_{ai} $(i = 1, 2, 3, 4)$ represents the aileron commands. The resulting aircraft performance during the maneuver is displayed from Fig. 7, where p_{real} and p_{des} denote the real and desired roll angular rate, q_{real} and q_{des} the real and desired pitch angular rate and r_{real} and r_{des} the real and desired yaw angular rate. It appears that in spite of the rate degradation of an aileron, the aircraft is able to proceed in a standard way with the considered maneuver.

A more serious fault case (or hard fault scenario) has been considered where an actuator deactivates and cannot be used anymore in the actuator allocation process. Here it has been assumed that the right outer aileron remains stuck at its

Table 1 Parameters of actuators under nominal condition.

Actuator	Number of actuator	Position limit	Rate limit
Aileron	4	−25° to 25°	−25 °/s to 25 °/s
Elevator	4	−25° to 10°	−15 °/s to 15 °/s
Rudder	2	−30° to 30°	−25 °/s to 25 °/s

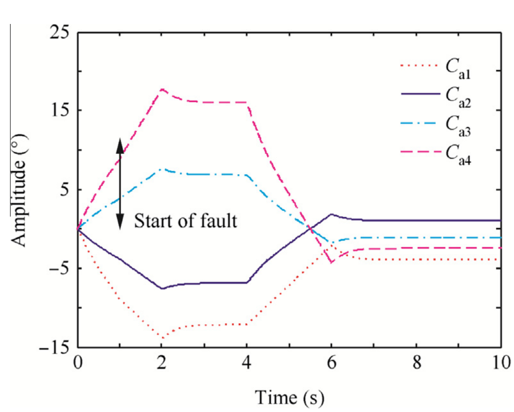

Fig. 6 Distribution of input signals for ailerons in soft fault scenario.

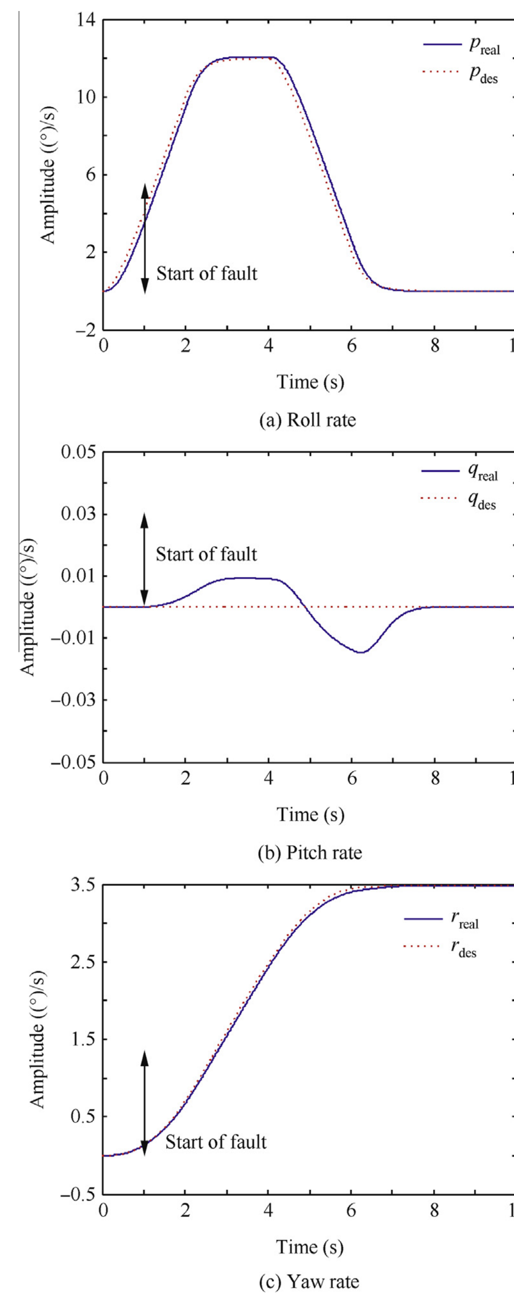

Fig. 7 Aircraft performance with actuator allocation in soft fault scenario.

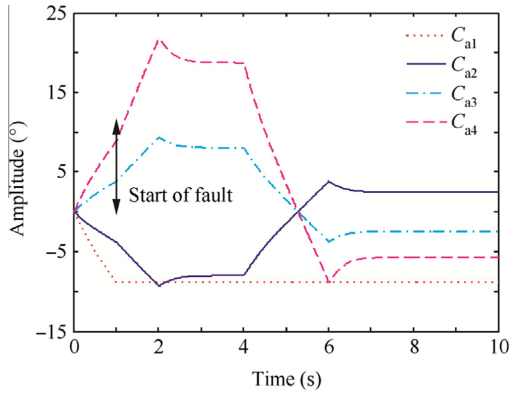

Fig. 8 Distribution of input signals for ailerons in hard fault scenario.

current position starting at $t = 1$ s. Fig. 8 displays the resulting time history for the different ailerons, including the failed one. Then from Fig. 9, the resulting aircraft performance is displayed showing also an acceptable performance during the desired maneuver.

It is clear that the satisfactory results obtained in these two failure scenarios are a direct consequence of the chosen control actuator architecture which is first order with respect to fault tolerant governability.

6. Conclusions

In this study, fault tolerant control has been considered in the case of output trajectory tracking for input affine nonlinear systems. A new state representation has been introduced to tackle this situation. One the one hand, this new state representation allows to introduce new concepts and conditions for fault tolerant governability and on the other hand it gives support to the proposed two-stage control strategy. Both stages have been analyzed and particular solution approaches (nonlinear inverse control and LQ programming) have been developed. Then it has been shown that the application of the proposed strategy to an aircraft performing a standard maneuver (a stabilized roll maneuver) is straightforward.

However, many questions remain to be solved to turn effective the proposed approach in the flight control domain. Some important ones are

– How to cope with the succession of standard maneuvers composed of a typical flight plan in the presence of actuator failures?
– How to introduce an aircraft reconfiguration scheme, based on the control of the secondary actuators, to improve the effectiveness of the remaining operational primary actuators to perform the current intended maneuver?

The answers to these questions should lead to the design of an effective actuator redundancy management (ARM) function.

Finally, in this study, FDI and ARM functions are considered to operate in sequence, while it appears of interest to study their integration, especially when FDI is also based on analytical redundancies extracted from the OSVI representation.

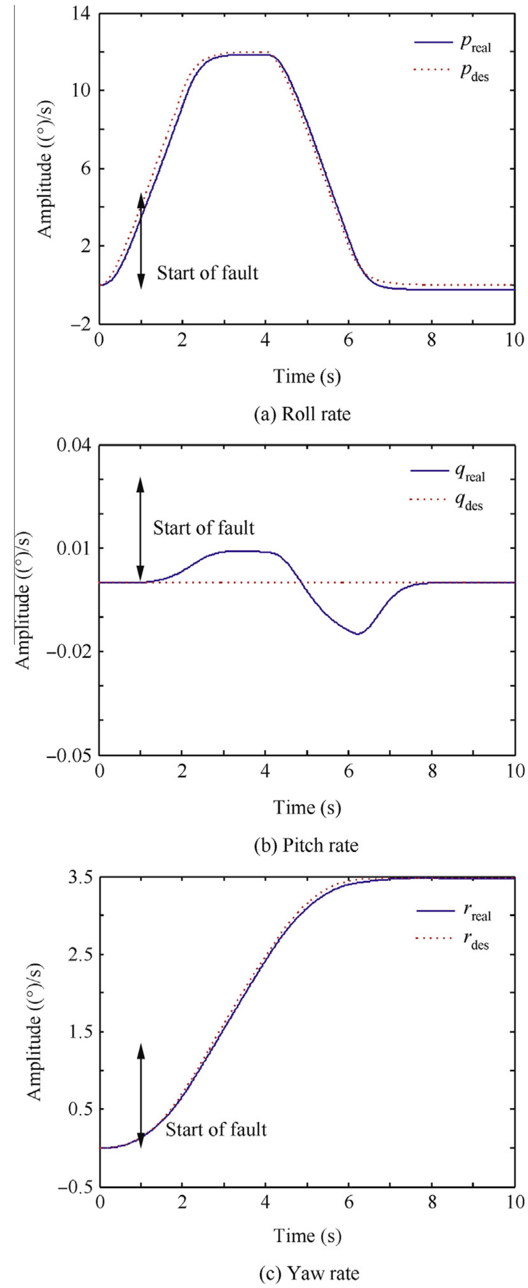

Fig. 9 Aircraft performance with actuator allocation in hard fault scenario.

Appendix A. Aircraft rotation dynamics

The equation for the rotational movement of a rigid aircraft in the body reference frame can be expressed as

$$I_m\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (I_m\boldsymbol{\omega}) = \boldsymbol{\mu} \tag{A1}$$

The angular rates $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$, of respectively the rates of the bank angle ϕ, the pitch angle θ and the heading angle ψ, are related with the p, q and r angular rates by the Euler equations:

$$\begin{cases} \dot{\phi} = p + \tan\theta(q\sin\phi + r\cos\phi) \\ \dot{\theta} = q\cos\phi - r\sin\phi \\ \dot{\psi} = (q\sin\phi + r\cos\phi)/\cos\theta \end{cases} \tag{A2}$$

The aerodynamic moments along each body axis are given by

$$\begin{cases} L = \rho(z)V^2SlC_l/2 \\ M = \rho(z)V^2SlC_m/2 \\ N = \rho(z)V^2SlC_n/2 \end{cases} \tag{A3}$$

where C_l, C_m and C_n are the roll, pitch and yaw dimensionless aerodynamic coefficients respectively; V is the airspeed; $\rho(z)$ is the density of air depending on the flight level z; S and l are the reference area and the length specific to a given aircraft, respectively.

The dimensionless coefficients of the main axis aerodynamic torques can in general be expressed as

$$\begin{cases} C_l = C_{l0} + C_{l\beta}\beta + C_{lp}pl/V + C_{lr}rl/V + C_{l\delta_p}\delta_p + C_{l\delta_r}\delta_r \\ C_m = C_{m0} + C_{m\alpha}\alpha + C_{mq}ql/V + C_{m\delta_{\text{ths}}}\delta_{\text{ths}} + C_{m\delta_q}\delta_q \\ C_n = C_{n0} + C_{n\beta}\beta + C_{np}pl/V + C_{nr}rl/V + C_{n\delta_p}\delta_p + C_{n\delta_r}\delta_r \end{cases} \tag{A4}$$

where C_{ij} $(i \in \{l, m, n\},\ j \in \{0, \alpha, \beta, p, q, r, \delta_p, \delta_q, \delta_r, \delta_{\text{ths}}\})$ are dimensionless aerodynamic derivatives; δ_p, δ_q, δ_r correspond to the aileron, elevator and rudder deflections, respectively; while δ_{ths} is the deflection of the trimmable horizontal stabilizer, if any.

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