Maximizing the number of solved aircraft conflicts through velocity regulation

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Abstract We propose a model for the maximization of the number of aircraft conflicts that can be solved by performing velocity regulation. The model is mixed-integer as binary variables are used to count solved conflicts and to model alternative choices, while nonlinearities appear in the aircraft separation conditions. The main nonlinearities can however be relaxed by standard reformulations. Numerical results show that the model can be satisfactorly applied at least as a preprocessing step in a conflict avoidance procedure in a given airspace.

Keywords: Aircraft conflict avoidance, mixed-integer nonlinear optimization (MINLO), mathematical modelling, Air Traffic Management applications

1. Introduction

The growth of air traffic on the world scale leads to an increasing need for automatic decision-support tools able to integrate the work of air traffic controllers to guarantee flight safety. In this context, we focus on a crucial problem arising in Air Traffic Management, that of aircraft conflict detection and resolution.

Aircraft are said to be potentially *in conflict* when their horizontal or altitude distances are less than given standard separation distances (5NM, where 1 NM (nautical mile)= 1852 m, and 1000 ft). Thus, when a loss of separation occurs, aircraft have to be separated. Aircraft conflict avoidance can be performed by different strategies aimed to separate aircraft, including aircraft trajectory (heading angle) changes, flight level changes or aircraft velocity regulation. Corresponding mathematical models can be developed, leading to optimization or optimal control problems. A review is provided in [5]. In recent years, mixed-integer linear and nonlinear optimization (MILO, MINLO) have been proposed for aircraft conflict detection and resolution, with interesting results. See for example [1], [4], [6],[7]. In previous work [4], we proposed MINLO models for conflict avoidance based on velocity regulation, aiming at solving all the conflicts occurring in a given air sector observed during a time horizon (in a tactical flight phase). These models are quite complex and computationally challenging.

In this paper, we propose a mixed-integer nonlinear programming problem for maximizing the number of conflicts that are solved, in a time horizon, when only a velocity regulation is performed. The model can be easily relaxed using standard reformulations. The interest of the proposed model is twofold. First, it allows to easily discriminate between conflicts that can be solved by velocity changes and those that require the application of another separation strategy. Second, it can provide a starting point, and a corresponding feasible solution, for more

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complex models like those in [4], eventually simplifying the branch-and-bound procedure for their solution.

The paper is organized as follows. In Sect. 2 we present the proposed mixed-integer optimization model. In Sect. 3 we discuss the results of some numerical experiments. Sect. 4 concludes the paper.

2. Model: maximizing the number of solved conflicts

We model conflict avoidance in such a way to achieve aircraft separation by performing a speed change maneuver. This means that a conflict involving a pair of aircraft is solved by aircraft acceleration or deceleration, so that aircraft pass through the points of potential conflict at a different time with respect to what would occur if no maneuvers were performed. There are however a few situations where velocity regulation cannot solve all conflicts of a given aircraft configuration, that corresponds to infeasible optimization problems. In such a case, speed change maneuvers can be performed, leaving potentially some conflicts unsolved and needing the application of another separation maneuver, like heading angle changes.

In the present work we propose an optimization problem where the number of aircraft conflicts that can be solved by speed changes is maximized. The proposed model can then be used as a preprocessing step in a conflict resolution procedure in a target airspace.

Let A be the set of n aircraft. For all $i, j \in A$, let z_{ij} be binary decision variables defined as

$$z_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are separated (no conflict)} \\ 0 & \text{otherwise} \end{cases}$$

The other decision (continuous) variables of the problem are represented by the aircraft velocities, which are eventually modified with respect to the original ones (that are data of the problem) to solve conflicts:

$$v_{min} \leq \bar{v}_i \leq v_{max} \quad \forall i \in A,$$

where the bounds v_{min} and v_{max} are imposed to allow aircraft only small speed changes, following the idea of *subliminal control* of velocities suggested in the context of the aeronautic project ERASMUS [3], such that speeds can vary between -6% and +3% of the original speed. We obtain a mixed-integer model because of the presence of binary as well as continuous variables.

The objective function, to be maximized, is the sum of solved conflicts:

$$\sum_{i,j \in A,\ i<j} z_{ij}.$$

The constraints are given by the integrality constraints on z variables, the bounds on $\bar{v}$ variables, and the separation constraint on pairs of aircraft.

Let us assume that aircraft fly at the same flight level and are identified by 2-dimensional points on a plane. We know their initial position, their trajectory (heading) and their velocity. The aircraft separation between two aircraft i and j at time t is expressed by the condition

$$||x^r_{ij}(t)|| \geq d, \tag{1}$$

where d is the minimum required separation distance (usually, 5 NM) and $x^r_{ij}(t) = x_i(t) - x_j(t)$ is a vector representing the relative distance between aircraft i and j.

We assume that uniform motion laws apply, so the relative distance of aircraft i and j is expressed as the sum of their relative initial position and the product of their relative speed $\bar{v}^r_{ij}$ by the time:

$$x^r(t) = x^{r0}_{ij} + \bar{v}^r_{ij} t \qquad \forall t,$$

that, substituting into (1) and squaring, gives

$$(v^r_{ij})^2\ t^2 + 2(x^{r0}_{ij}\bar{v}^r_{ij})\ t + ((x^{r0}_{ij})^2 - d^2) \geq 0. \tag{2}$$

Notice that the associated equation is an equation of second degree in one unknown t (its graph is a parabola that, as $(\bar{v}^r_{ij})^2 > 0$, has a minimum point and opens upward), that has no solutions if the discriminant $\Delta = (x^{r0}_{ij}\bar{v}^r_{ij})^2 - (\bar{v}^r_{ij})^2((x^{r0}_{ij})^2 - d^2)$ is negative. The solutions of this equation, if they exist, are the times at which the aircraft are not separated. So, we consider $\Delta < 0$ as a first condition of separation of aircraft i and j. In the case when this condition is not satisfied, and so aircraft i and j can potentially be in conflict, we look at the form of trajectories. In this work, we assume that trajectories are straight lines intersecting in one point. As per the geometric interpretation of the scalar product, we can look at the sign of the scalar product $x^{r0}_{ij}\bar{v}^r_{ij}$ to infer if the vectors form an acute or an obtuse angle. In particular, when the scalar product $x^{r0}_{ij}\bar{v}^r_{ij}$ is negative, then the aircraft are converging, potentially generating a conflict, while they are separated when the product is positive.
Finally, we impose aircraft separation imposing that $\Delta < 0$ or $x^{r0}_{ij}\bar{v}^r_{ij} > 0$ for all $i, j,\ i < j$. Using again the z binary variables, the two constraints are written as

$$\left((x^{r0}_{ij}\bar{v}^r_{ij})^2 - (\bar{v}^r_{ij})^2((x^{r0}_{ij})^2 - d^2)\right)(2z_{ij} - 1) \leq 0$$

i.e.

$$(x^{r0}_{ij}\bar{v}^r_{ij})^2(2z_{ij} - 1) \leq (\bar{v}^r_{ij})^2((x^{r0}_{ij})^2 - d^2)(2z_{ij} - 1) \tag{3}$$

and, respectively,

$$(x^{r0}_{ij}\bar{v}^r_{ij})(2z_{ij} - 1) \geq 0. \tag{4}$$

Notice that the left hand sides of the two conditions differ only for a square. The same binary variable z_{ij} can be used to model the *or* condition:

$$(x^{r0}_{ij}\bar{v}^r_{ij})^2(2z_{ij} - 1) \leq (\bar{v}^r_{ij})^2((x^{r0}_{ij})^2 - d^2)(2z_{ij} - 1) \tag{5}$$

$$(x^{r0}_{ij}\bar{v}^r_{ij})(1 - z_{ij}) \geq 0 \tag{6}$$

then using an additional variable to account for a separated pair of aircraft when the second condition is satisfied.

The nonlinear terms appearing in the constraints come mainly from the products between continuous and binary variables, that can be easily relaxed using the Fortet linearization. This is commonly implemented in the most of the MINLO solvers.

3. Numerical experiments

We tested our model on instances built placing n aircraft on a circle of a given radius r, in 2-dimensional space, with speed v and a heading angle such that their trajectory is toward the center of the circle (or slightly deviated with respect to such direction). The zone of conflict is around the center of the circle where aircraft are placed, and each aircraft is in conflict with each other. We solve the problem using `COUENNE` [2], which implements a spatial Branch-and-Bound based on convex relaxations and provides the global optimal solution.

As an example of solution, let us consider an instance of the conflict avoidance problem with $n = 5$ aircraft having speed v = 400 NM/h (equal for all aircraft). There are 10 potential conflicts, that are all solved.
The ratio of the new speeds over the original ones for the 5 aircraft is shown in Table 1.
We see that 2 aircraft are accelerated and 3 of them are decelerated. The speed variation are in the small range [-6%, +3%] around the original velocity for a subliminal control as suggested by ERASMUS.
The global optimal solution is obtained in 0.16 seconds.

Table 1. Ratio of the aircraft velocities in the optimal solution over the original ones.

aircraft	v_{ratio}
1	1.00814
2	1.02809
3	0.941877
4	0.981939
5	0.962551

4. Summary

We proposed a mathematical model for the maximization of the number of aircraft conflicts that can be solved by velocity changes. The model gives a mixed-integer nonlinear optimization problem that can be efficiently solved by standard solvers for MINLO.

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