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Automatic conflict solving using biharmonic navigation functions

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Abstract

Automatic conflict solving is an old standing problem within the field of ATM. Proposed algorithms fall into two categories :

- Deterministic ones that have a provable property of collision avoidance. For all known algorithms, trajectories produced are generally not flyable because no bounds on speed and curvature can be imposed.
- Stochastic methods that select an optimal sequence of manoeuvres. By design, trajectories are flyable, but no guarantee can be given on the fact that a collision-free planning can be found in finite time.

It is highly desirable for a wide social acceptance of automated trajectory planning, even at a strategical level, that the algorithms in use have by-design the collision avoidance property and, at the same time, a mean of keeping the speed within a given interval. Navigation functions are common in the field of robotics but do not have the last property. We present in this paper a new approach based on biharmonic functions yielding a navigation field with constant speed. Such functions have been considered previously, but proof of collision avoidance is lacking : we address this problem in this work as summarized below. Navigation functions produce a speed field by taking the gradient of a potential function : if the obstacles to be avoided are at a higher potential than inner points of the domain (including destination), collision avoidance is guaranteed. If the potential has the Morse property (no critical point is degenerated) then there exists a descent direction at every point of the admissible domain, making the destination reachable. In the framework of biharmonic functions, a tensor field is produced instead of a vector one ; the Morse property is no longer relevant. We show here that all benefits of navigation functions can be recovered through the use of the bienergy density τ^2, with the ability to get constant speed fields.

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Keywords: Biharmonic function, Navigation function, Plane trajectories, Conflict avoidance

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1. Introduction

Air Traffic Management (ATM) is currently managed at a national level through the use of sectors and air routes. Each sector is supervised by a team of controllers. Their work can be divided into three tasks. The first one consists in detecting and solving conflicts by deviating some aircraft from their planed route. The second task consists in communication with neighbouring sectors to transfer the responsibility of an aircraft when it changes sector. The third one is a monitoring task, to insure that aircraft do not deviate from their planed route. To manage these three tasks safely, controllers can only supervised a limited number of aircraft at the same time. In order to simplify controllers work, air traffic is organized through the use of air routes consisting in a list of beacons, where aircraft are required to be at a given time. This structure makes the monitoring and deconfliction tasks easier for the controllers. As the traffic is growing, the workload incurring in a sector may exceed controllers capabilities. To deal with this issue, the number of sectors has been increased and their size reduced thus lowering the number of aircraft simultaneously under the responsibility of a single controller. This procedure can not be pushed beyond a given threshold where the transfer task takes priority over the other two or where controllers lack time to solve conflicts and provide the next sector with an already critical situation.

Since a two to three times increase in the number of flights is expected by years 2020-2030, a paradigm shift has to be made in the ATM system. The SESAR[1] program in Europe and the NextGen program in USA have been initiated to address this problem and will design new rules and tools for ATM. One of the main challenges of SESAR and NextGen is to cope with the increase in the number of flights while maintaining a safety level at least equal to the present one (an improvement is in fact a target for the two projects). To reach this goal, most of the separation task will be delegated to the aircraft themselves and it will, in turn, introduce a higher level of automation in the system. In this context, fixed air routes are no longer imposed and are replaced by Reference Business Trajectories (RBT) that are negotiated among the stakeholders of the system using a Collaborative Decision Making process (CDM). Trajectories are planned in 4D, meaning that at each time during the flight, the aircraft has to be at a determined point. A lot of researches were conducted to create such trajectories several days ahead the departure date so as to avoid conflicts by design. Unfortunately, no existing algorithm is known to produce planning such that trajectories are flyable and have at the same time a theoretical proof of conflict avoidance. Navigation functions, introduced originally for robots trajectory planning, were extended to generate aircraft trajectories. A proof of conflict avoidance can be given for navigation functions, but the trajectories obtained do not meet ATM requirements on both speed and curvature. We present in this paper a new kind of navigation function based on biharmonic functions that maintains the collision avoidance property under bounded speed assumption.

After a state of the art on the navigation function in part 2, we present the biharmonic approach in part 3. We then show in part 4 how biharmonic functions can be used as navigation functions and used to obtained smooth trajectories with a bounded speed.

2. State of the art

2.1. Navigation functions for robots

Navigation functions were introduced by Rimon and Koditschek [1] to plan robot navigation. Their goal was to generate trajectories with a guarantee of obstacle avoidance. Navigation functions are special instances of potential functions, with their maximum on the boundary of the obstacles, their minimum at the destination and no local minimum or maximum in the free space (critical points with vanishing gradient of the potential can only be saddle points). The standard assumption for a navigation function is that it has to be Morse : such a function is a smooth mapping defined on a domain with boundary such that all its interior critical points have a non-degenerate Hessian. It is known that on a compact smooth manifold with boundary

[1] Single European Sky ATM Research

any smooth real-valued function is generically Morse [2]. The quasi-gradient of a navigation function exists if it is Morse. By following the quasi-gradient field one arrives at the destination without entering any obstacle. It can be proven that there always exists a navigation function for any smooth connected and compact manifold [3]. It implies that if the destination is reachable from the starting point, it exists a path along the quasi-gradient connecting those two points. Furthermore, it has been proven that this trajectory avoids obstacles and reaches the destination in a finite time. The definition of the navigation function is given the following way [3] :

Definition 1. *A function $\phi : E^n \to [0,1]$ is a navigation function on a smooth, connected and compact manifold with boundary $\mathcal{F} \subset E^n$ if ϕ is:*

- *Analytic in the interior of $\mathcal{F}$*
- *Polar at q_d in $\mathcal{F}$, meaning that ϕ admits a unique minimum at q_d.*
- *Uniformly maximal on the boundary of $\mathcal{F}$. ϕ is said to be admissible on $\mathcal{F}$ in such a case.*
- *Morse on $\mathcal{F}$*

In practice, a navigation function does not need to be analytic, belonging to C^2 class is enough. Once the navigation function ϕ is computed, one just has to follow $-\nabla\phi$ at non critical points or the steepest descent given by the Hessian at critical points to reach the destination while avoiding obstacles.

2.2. Navigation functions for aircraft trajectories planning

Kyriakopoulos *et al.* extend Rimon and Koditschek's navigation functions for aircraft trajectory planning [4]. But some ATM constraints, as bounded speed and curvature are not relevant for robots navigation and were not addressed in the original work of Rimon and Koditschek. One can ask a robot to stop to let an other robot pass, but a aircraft can neither stop, nor even fly under a given speed in order to let an other aircraft pass. Kyriakopoulos *et al.* tried first to use navigation functions to compute trajectories and then to reach a bounded speed for the aircraft through specially tailored command laws. However, the velocity obtained that way has many heading changes, which is not realistic for aircraft navigation. Bounding the overall curvature along the trajectory is still an open issue in the work of Kyriakopoulos.

2.3. Harmonic functions as navigation functions

Definition 2. *Harmonic functions V are functions satisfying : $\Delta V = 0$ in a domain $\Omega \subset \mathbb{R}^k$.*

Property 1. *Let $\mathcal{D}$ be a closed disk and let $\mathcal{F}$ the manifold with boundary obtained from $\mathcal{D}$ by removing a finite number of non-overlapping open disks (these disks represent obstacles or forbidden areas). It exists an harmonic navigation function ϕ on $\mathcal{F}$ with destination q_d in the interior of $\mathcal{F}$.*

Proof. The Dirichlet problem:

$$\begin{cases} \Delta\phi = 0 \text{ in the interior of } \mathcal{F} \\ \phi(x) = c > 0, \quad x \in \partial\mathcal{F} \\ \phi(q_d) = 0 \end{cases}$$

has an analytic solution by the Weyl lemma. Since ϕ is harmonic, it satisfies the maximum principle so that no interior point can be a local minimum or maximum, which implies that ϕ is polar. It is admissible by construction. Furthermore, it can be proven to be Morse with the previous boundary conditions. □

Harmonic functions can be used as navigation functions, but they have a major drawback : the navigation field they give is evanescent. The obstacle avoidance property is kept, but the aircraft will exponentially slow down as it gets closer to the destination. An easy way to avoid the evanescence of the field is to norm it. The trajectories created with the normed field meets ATM requirements on the bounded speed as the speed obtained is constant. However, the curvature requirement may not be respected. Some trajectories indeed

present sharp turns, making them unflyable, as can be seen Fig.2.

An other drawback of harmonic functions is their tendency to compute trajectories going very close to the obstacles, as can be seen Fig.3. This kind of trajectory is not robust against uncertainties which renders such an algorithm prone to failures in an operational context. If the position of an obstacle is not evaluated properly as a consequence of stochastic phenomenons (wind for example) or the forecast of aircraft position, then the separation norms may be violated and a conflict will occur. Furthermore, it is highly advisable for safety to avoid trajectories entering areas of high aircraft density if possible, as it increases greatly the monitoring needed and is not resilient in case of failure of the surveillance system. Unfortunately, many trajectories obtained with harmonic navigation functions have a tendency to slip through obstacles, which is not compatible with this requirement.

3. The biharmonic function : a mechanical approach

After studying harmonic functions for robot navigation [5], Masoud and Masoud had the idea of using a biharmonic function F satisfying $\Delta^2 F = 0$ in $\Omega \subset \mathbb{R}^k$ to create trajectories [6]. The concept is borrowed to the elasticity theory. Consider a plate of an elastic material with holes representing the obstacles and the destination. In the destination hole, put normal stresses to compress the material, as if there was a ball swelling in it. The stress field spreads in all plate in a continuous way. From any point in the plate, one just has to track this field backwards in order to reach the destination. This phenomenon is governed by the equations of elasticity theory [7] : the Hook's laws linking the strains to the stresses, the compatibility equation and the equation of equilibrium of moments detailed in 4.1. With these equations and some boundary conditions on the displacements and the stresses (see [6]), the solution of the biharmonic equation $\Delta^2 F = 0$ can be computed. In this method, the navigation field is not given by the gradient of the solution, but by the direction associated with the minimal eigenvalue of the Hessian of F. Masoud and Masoud used an finite element simulation software to solve their system.

4. The biharmonic function as a navigation function

The original approach of Masoud and Masoud is not well suited to have a proof of the collision avoidance property. We will present now a new formulation of the same underlying biharmonic system but with different boundary conditions and expressed using only stresses.

4.1. Construction of the system to solve

For the sake of simplicity, we work for now in two dimensions. This is also the generic setting for automating the conflict solving process since all manœuvres an aircraft can perform without an adverse impact on efficiency are those keeping it at constant altitude. Changing flight levels implies a change in engine speed and greatly increases the fuel flow.

In usual mechanics equations, there are two sets of unknowns : strains and stresses. Our goal was to construct a system with only one set of unknowns : the stresses. The compatibility equation and the equations of equilibrium of moments can be written with stresses only. The Hook's law is used to link strains and stresses, so we do not need to use it. The stresses in a material can be divided in two categories : the normal stresses, denoted σ_{xx}, σ_{yy} and the shear stresses, denoted σ_{xy}, σ_{yx}. It is well known that $\sigma_{xy} = \sigma_{yx}$ and so we replace σ_{yx} by σ_{xy} into the next equations.

Equations of equilibrium of moment. We consider there is no volumic force acting on the plate, which gives the following equations :

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0 \qquad \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0 \tag{1}$$

Equation of compatibility. This equation enforces the continuity of the stress field in the plate.

$$\Delta(\sigma_{xx} + \sigma_{yy}) = 0 \tag{2}$$

Airy's function. To make the system easier to solve, the Airy's function F is introduced :

$$\sigma_{xx}(x, y) = \frac{\partial^2 F(x, y)}{\partial y^2} \qquad \sigma_{yy} = \frac{\partial^2 F(x, y)}{\partial x^2} \qquad \sigma_{xy} = -\frac{\partial^2 F(x, y)}{\partial x \partial y} \tag{3}$$

The Airy's function satisfies the equations (1). By injecting equation (3) in equation (2), we obtained the biharmonic equation :

$$\Delta^2 F(x, y) = 0 \tag{4}$$

Boundary conditions. Now that we have the equation to solve on the domain, we have to rewrite the boundary conditions with the Airy's function. At the destination point, there is no shear stress and a normal stress $P > 0$ is imposed. With n_x and n_y the x and y components of the unity vector normal to the boundary of the destination, the boundary conditions can be written as :

$$\sigma_{xx} = \frac{\partial^2 F(x, y)}{\partial y^2} = P.n_x \qquad \sigma_{yy} = \frac{\partial^2 F(x, y)}{\partial x^2} = P.n_y \qquad \sigma_{xy} = -\frac{\partial^2 F(x, y)}{\partial x \partial y} = 0$$

In order to obtain only one boundary condition, we rewrite it as :

$$\Delta F(x, y) = P \tag{5}$$

On all other boundaries, there is no stresses :

$$\sigma_{xx} = \frac{\partial^2 F(x, y)}{\partial y^2} = 0 \qquad \sigma_{yy} = \frac{\partial^2 F(x, y)}{\partial x^2} = 0 \qquad \sigma_{xy} = -\frac{\partial^2 F(x, y)}{\partial x \partial y} = 0$$

Which can be rewritten as :

$$\Delta F(x, y) = 0 \tag{6}$$

4.2. *Solving of the system*

Both boundary conditions where written in terms of laplacian to make the system easier to solve. The biharmonic system :

$$\begin{cases} \Delta^2 F(x, y) = 0 \\ \Delta F(x, y) = P \text{ at the destination} \\ \Delta F(x, y) = 0 \text{ on the boundaries} \end{cases} \tag{7}$$

can indeed be solved as two coupled harmonic equations :

$$\begin{cases} \Delta L_F(x, y) = 0 \\ L_F(x, y) = P \text{ at the destination} \\ L_F(x, y) = 0 \text{ on the boundaries} \end{cases} \quad \text{and} \quad \begin{cases} \Delta F(x, y) = L_F \\ F(x, y) = P(x^2 + y^2) \text{ at the destination} \\ F(x, y) = 0 \text{ on the boundaries} \end{cases} \tag{8}$$

The boundary conditions for the second harmonic system are obtained by integrating the boundary conditions of the first one. The integration of the boundary condition at the destination is performed so as to satisfy the condition $\sigma_{xy} = 0$ which could not be included directly in the biharmonic system.

4.3. Computation of the navigation field

Once the system is solved through the use of a finite element method, we compute the stresses matrix Σ with finite differences :

$$\Sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 F(x,y)}{\partial y^2} & -\frac{\partial^2 F(x,y)}{\partial x \partial y} \\ -\frac{\partial^2 F(x,y)}{\partial x \partial y} & \frac{\partial^2 F(x,y)}{\partial x^2} \end{pmatrix}$$

At each point of the space, we have a two-by-two matrix. In other terms we have a tensor field unusable as it. To derive a vector field from this tensor field, the eigenvalues of the stresses matrix are computed pointwise by :

$$\begin{aligned} \sigma_{max} &= \frac{\sigma_{xx} + \sigma_{yy}}{2} + \sqrt{\frac{(\sigma_{xx} - \sigma_{yy})^2}{4} + \sigma_{xy}^2} \\ \sigma_{min} &= \frac{\sigma_{xx} + \sigma_{yy}}{2} - \sqrt{\frac{(\sigma_{xx} - \sigma_{yy})^2}{4} + \sigma_{xy}^2} \end{aligned} \tag{9}$$

Of these two eigenvalues, the minimal one is the only usable to compute our navigation field. The reason of the choice of σ_{min} is explained in details in 4.6. By computing, at each point, the unit eigenvector associated with the minimal eigenvalue, we recover a vector field. Unfortunately this vector field can not be used directly for navigation. An eigenvector being defined up to a constant, the eigenvector can indeed point in the direction of the destination or in the opposite way. The vector field has to be rectified before it can be used to generate trajectories.

4.4. Rectification of the field

Creating a good vector field from a tensor one is a recurrent problem in medical imaging and diffusion MRI. There is no easy way to rectify the vector field obtained with the eigenvector except by propagating orientations by continuity. For our problem, the vector field we seek after has some distinguished properties that allow for a more systematic algorithm. Below are listed four different ideas, with their own advantages and their own drawbacks. For now, a combination of at least two of these ideas has to be used in order to rectify all the field, mainly because the last one that is indeed the one yielding the collision avoidance property degenerates in the vicinity of boundaries.

- **By continuity from the destination :** Thanks to the compatibility equation, we know that the stress field has to be continuous, which means that our vector field has to be continuous too. The method begins with the rectification of the field at the points in a neighbourhood of the destination. Those points can then be used to rectify the points in their close neighbours and so on. This operation, easy when there is no obstacles, becomes very complex with obstacles and complex geometries.

- **By continuity from the obstacles :** This method uses the same idea then the previous one. But instead of rectifying the field at the destination in the first step, the field is rectified on all the boundaries. The field is then rectified from the boundaries to the destination by continuity. It is even more difficult to implement than the method above.

- **By keeping the direction minimizing the distance to the destination :** We know that the stress field minimizes the energy necessary to reach the destination. At each point, the two possible direction are tested and the one minimizing the distance to the destination is kept. Unfortunately, in order to avoid obstacles, the trajectory can steer away from the destination before reaching it. In this case, this method leads the field to point to the obstacles instead of avoiding them.

- **By using the vector field $-\nabla Tr(\Sigma)$:** This method is the most promising one. By construction $\Delta F = Tr(\Sigma)$ is harmonic. It means that it satisfies the maximum principle and that its gradient is a navigation field. This navigation field can be used to rectify the vector field obtained with the eigenvectors. The only drawback is that near the obstacle, these two vectors are nearly perpendicular. Which means that at those points, the navigation field can not help to rectify the vector field.

The main solution seems to be to use the vector field $-\nabla Tr(\Sigma)$ at all points where it is useful and then rectify the points left by continuity.

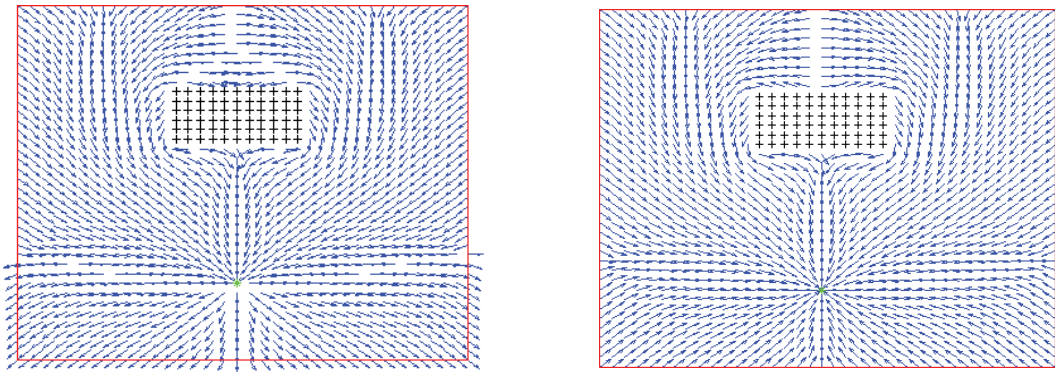

Fig. 1. Biharmonic navigation field, before and after rectification

4.5. Usefulness of biharmonic fields against harmonic ones

We compare here biharmonic navigation fields with two kinds of harmonic navigation fields : one obtained with a Dirichlet boundary condition and one obtained with a Neumann boundary condition. The behaviour of the biharmonic field is closer to the behaviour of a harmonic field with a Dirichlet boundary condition then a field with a Neumann one.

A biharmonic field tends to give trajectories smoother than a Dirichlet harmonic field. Trajectories are also shorter with a biharmonic field than with a Dirichlet harmonic. Fig.2 presents one of the many cases where the biharmonic field gives better trajectories than a Dirichlet harmonic field.

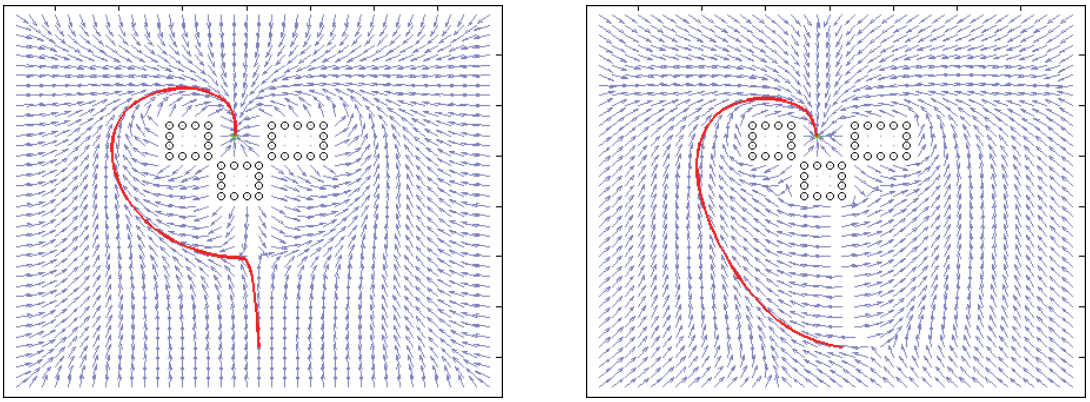

Fig. 2. Trajectories computed with a Dirichlet harmonic navigation field and a biharmonic navigation field. Starting point at the bottom and destination point between the obstacles.

Fig.3 shows why the biharmonic field is better for ATM than the harmonic field with a Neumann boundary condition. In this case there are four obstacles close to each others. The Neumann harmonic field goes through the pack of obstacles and navigates very close to one obstacle. The biharmonic field avoid the whole pack of obstacles. This behaviour tends to lengthen the trajectory, but it also makes it more robust. As the trajectory stays at a good distance from the obstacles, if the position of these obstacles is subject to uncertainties, the biharmonic trajectory will still be usable while the Neumann harmonic trajectory will

have to be computed again.

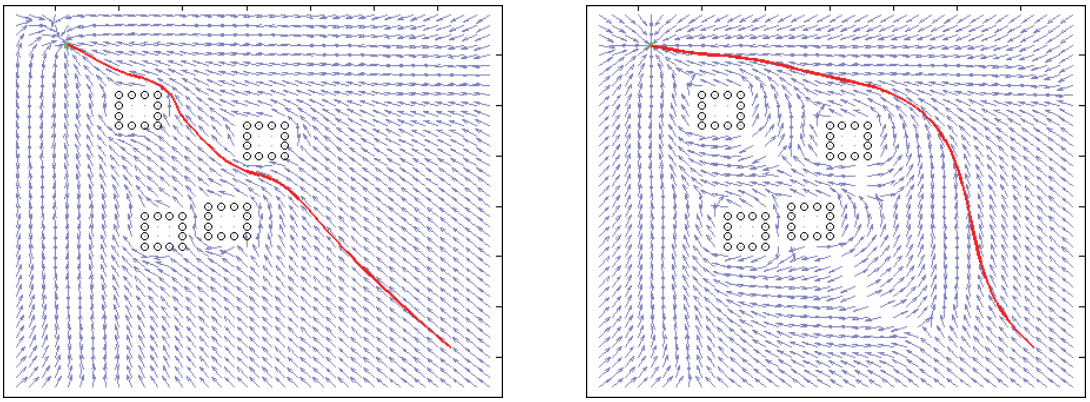

Fig. 3. Trajectories computed with a Neumann harmonic navigation field and a biharmonic navigation field. Starting point : bottom right, destination point : top left.

4.6. Properties of the biharmonic navigation field

Property 2. *For any connex free space, there always exists a biharmonic navigation field.*

Proof. As the navigation field is created using a eigenvector, we just have to prove that this eigenvector always exists. The only case in which the eigenvector does not exist is when the associated eigenvalue vanishes. By proving that the minimal eigenvalue is always non-zero, we prove that there will always be a navigation field. By construction $Tr(\Sigma) = \sigma_{xx} + \sigma_{yy}$ is harmonic. Thanks to the maximum principle and the boundary conditions imposed :

$$\begin{aligned} & -P < Tr(\Sigma) < 0 \text{ at any interior point} \\ & \Longrightarrow \sigma_{min} + \sigma_{max} < 0 \\ & \Longrightarrow \sigma_{min} < -\sigma_{max} \end{aligned} \tag{10}$$

By construction :

$$\sigma_{min} \leqslant \sigma_{max} \Longleftrightarrow \begin{cases} \sigma_{min} = \sigma_{max} \\ \text{ or} \\ \sigma_{min} < \sigma_{max} \end{cases} \Longrightarrow \begin{cases} \sigma_{min} = Tr(\Sigma)/2 < 0 \\ \text{ or} \\ \sigma_{min} < \sigma_{max} \end{cases} \tag{11}$$

Using (10) and (11), we obtain $\sigma_{min} < -|\sigma_{max}| \leqslant 0$ at any interior point. □

Property 3. *Let $\mathcal{F}$ be a manifold with boundary obtained like in property 1. For a smooth read-valued function ϕ defined on $\mathcal{F}$ the bienergy density is defined at any interior point x by $\tau^2(\phi)(x) = (\Delta\phi(x))^2$. Then the bienergy is a Lyapunov function for the flow associated with the biharmonic navigation field. Furthermore, the destination is an asymptotically locally stable point.*

This proposition shows that the biharmonic navigation field enjoys the same destination reachability property as standard navigation functions. Collision avoidance is in fact a direct consequence of $\Delta\phi$ being harmonic : since this function is uniformly maximal on the boundary, it can not reach this value in the interior of the domain. As the direction of the biharmonic navigation field is oriented along $-\nabla\Delta\phi$, the navigation path has to be minimizing for $\Delta\phi$ and thus cannot reach the boundary.

Proof. We give only the sketch of the proof here. The only thing that has to be verified is that the navigation field X is not orthogonal to $-\nabla Tr(\Sigma)$. Integration of the inner product of the two vectors on a domain Ω

sufficiently small to be inside the domain gives :

$$\int_{\Omega} \langle \nabla \Delta \phi, X \rangle = \int_{\Omega} \langle \nabla \Delta \phi, \nabla \nabla^T \sigma X \rangle$$

where $\sigma(x)$ is the minimal eigenvalue of the Hessian matrix of ϕ at x. The equality is obtained by using the fact that $X(x)$ is an eigenvector of the inverse of the Hessian at point x. Stokes formula and use of the biharmonicity of ϕ can be applied to show that this integral is equal to:

$$\int_{\partial\Omega} \left(\frac{\partial}{\partial x} \Delta \phi + \frac{\partial}{\partial y} \Delta \phi \right) \sigma div X$$

A transport relation along the flow X is recognized and by letting the domain of integration go to 0 a non zero value is obtained. □

5. Conclusion and prospects

We presented a new kind of navigation function : the biharmonic navigation function. It computes trajectories with constant speed and keeps the good properties of navigation functions : existence of a navigation field, obstacles avoidance, arrival at the destination. The trajectories obtained with biharmonic navigation functions are also smoother than trajectories obtained with harmonic ones. The method to construct trajectories with a biharmonic navigation function can be summarized the following way :

∘ Choice of the destination	
∘ Construction of the system	
∘ Solving of the system with FEM	} ⟶ Map of values
∘ Computation of the Hessian of the solution	} ⟶ Tensor field
∘ Computation of the eigenvector corresponding to the minimal eigenvalue at each space point	} ⟶ Vector field
∘ Recovery of the direction to follow	} ⟶ Navigation field
∘ Computation of the trajectory from any point in the free space	} ⟶ Trajectory with obstacle avoidance, proof of arrival and constant speed

Table 1. Trajectories generation algorithm

Future work : Now that we have a method with good properties to solve static problems, some issues have to be tackle in order to solve dynamic problems. In dynamic cases, the biharmonic equation has to be solved at each time step, to take into account the displacement of the obstacles. In this case, the finite element method may not be the best one, as the space will have to be remeshed for each time step, which is a time consuming operation. To avoid this remeshing, we are trying to solve our biharmonic system with a meshless method, as the radial basis functions network method.

In the case of trajectory planning for two or more aircraft, there is two different ways to create the trajectories of the aircraft. The first one is sequencement. One aircraft is selected to follow his trajectory without taking the other aircraft into account. The trajectory of the second aircraft sees the first aircraft as an obstacle and avoids it. The third aircraft avoids the first and second ones and so on. Sequencement is an unfair way of computing trajectories but it is the current way controllers are processing. An other solution is to find a method to plan a coordinated movements of the aircraft. This is possible through the use of the so-called configuration spaces. Briefly speaking, the configuration space associated with N aircraft is the

subset of $\mathbb{R}^{2N}$ obtained by removing the diagonal Δ:

$$\Delta = \{(x_1, y_1, \ldots x_N, y_N)\,,\ \exists i \neq j\, x_i = x_j, y_i = y_j\}$$

Any continuous path within the configuration space represents a feasible coordinated planning. The extension of our method to this setting will be investigated by using tensor products of elementary solutions of the biharmonic equation.

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