A new concept for GPS phase ambiguity resolution on-the-fly : the maximum a posteriori ambiguity search (MAPAS) method

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A NEW CONCEPT FOR GPS PHASE AMBIGUITY RESOLUTION ON THE FLY : THE MAXIMUM A POSTERIORI AMBIGUITY SEARCH (MAPAS) METHOD

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Presented at ION GPS-95, September 13th 1995

BIOGRAPHY

Christophe MACABIAU was born in 1968 in Moissac, France. He graduated in 1992 as an electronics engineer at the Ecole Nationale de l'Aviation Civile (ENAC) in Toulouse, France. He is specialized in signal processing and in radionavigation electronics. After working in 1993 for the MLS Project Office in Ottawa, Canada, he became a Ph.D. candidate at the Laboratoire de Traitement du Signal et des Télécommunications of the ENAC in 1994. He is working on the application of precise GPS positioning techniques to aeronautics.

ABSTRACT

The use of the pseudorange information contained within the GPS carrier phase observables enables to achieve a high level of positioning accuracy, but requires the resolution of the intrinsic cycle ambiguities of the phase measurements. Several methods have been proposed that can solve the ambiguities without static initialization by performing a search of the most coherent values of the double difference ambiguities of four particular satellites. They belong to the class of the multiple hypotheses sequential tests, that check each envisaged hypothesis against a decision criterion. A new method of ambiguity resolution on-the-fly, designed to make an optimal use of all the available measurements, is proposed in this paper. The decision criterion used by this method is the a posteriori probability of each potential solution. The mathematical developments involved in the design of the method are exposed, and the first simulation results obtained are presented, showing the validity of the concept proposed in this paper.

1. INTRODUCTION

The GPS phase measurements delivered by a receiver are related to the geometrical distance between the transmitting satellite and the receiver. These measurements constitute a potential source for a very precise determination of position, as they can be achieved with an accuracy of the order of one centimeter. However, the full access to that accurate geometrical information requires the resolution of the intrinsic integer ambiguities of the measurements. Since fifteen years, several specialized techniques have been developed to achieve this resolution. Some of them assume the receiver is static during the determination, while the others can be carried out even when the receiver is moving. When the resolution is performed without static initialization, the resolution is said to be made *on-the-fly*. These positioning techniques are very useful tools for static applications, like static baselines surveying, and for mobile positioning in dynamic applications like satellite altimetry, aerial photogrammetry, airborne gravimetry or aircraft landing.

This paper describes a new method for ambiguity resolution on-the-fly of GPS phase measurements, called the *Maximum A Posteriori Ambiguity Search (MAPAS)* method. In section 2, a model of the GPS signal carrier phase measurements is presented and the problem of ambiguity resolution is introduced. Then, the Least Squares Ambiguity Searching Technique is described, followed by a discussion on the statistical aspect of the ambiguity searching techniques. In section 3, the principles of the MAPAS method are exposed, then a presentation of the input data used by the method is made, enabling us to derive the theoretical expressions of the decision criterion, and to build the algorithm. In section 4, the simulated data is described and the results obtained using the MAPAS method are presented and discussed.

2. AMBIGUITY RESOLUTION

The L_1 carrier phase measurements delivered by a suitably equipped civilian GPS receiver are the measurements of the phase of the low frequency signal generated by mixing the received L_1 satellite signal with the output of an oscillator tuned at the nominal carrier frequency. As described by Rocken (Rocken, 1988), a first order model of this beat phase measurement, for satellite i, at epoch k is :

$$\Phi_i(k) = f(\Delta t_R - \Delta t_{S_i}) - \frac{D_i(k)}{\lambda} - N_i + f\tau_{ion} - f\tau_{trop} + b_i(k) + \varepsilon_{mult}(k) \quad (1)$$

where

- f is the L_1 frequency and λ is the corresponding wavelength.
- Δt_R and Δt_{S_i} are respectively the receiver and satellite time equivalent phase offset with respect to GPS time.

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- $D_i(k)$ is the geometrical distance between the satellite i and the receiver.
- N_i is the initial ambiguity value of the measurement.
- τ_{ion} and τ_{trop} are the ionospheric and tropospheric propagation delays.
- $\varepsilon_{mult}(k)$ is the carrier phase multipath term.
- $b_i(k)$ is the phase measurement noise. In the following, we assume that $b_i(k)$ is a discrete white gaussian noise, having zero mean and variance σ^2.

As long as the lock on the signal is held, the phase measurement device can keep track of all the detected whole-cycle phase revolutions, and all the phase measurements delivered are biased by the same phase *ambiguity* N_i. When a *cycle slip* occurs, that is when a loss of lock on the signal is experienced, the phase measurements can not be performed any more. Once the signal is re-acquired, the integer number of whole-cycle revolutions has been lost, and the initial ambiguity has a different value.

The multipath propagation of the signal can be responsible for a loss of phase lock. It may also cause the ambiguities to be resolved to incorrect values. The effect of multipath on the performance of the method presented in this paper will not be investigated here. In consequence, the term $\varepsilon_{mult}(k)$ will be neglected.

Assume now that two receivers, denoted with the subscripts 1 and 2, make the carrier beat phase measurements of the signal transmitted by the satellite i at the same epochs. The receivers are supposed to be close to each other, so that the tropospheric and ionospheric propagation delays affecting their measurements can be considered as identical. This approximation will be valid as long as the distance between them is less than 20 km.

In order to eliminate the satellite clock offset, we can form the single differences of phase

$$\begin{aligned}\Delta\Phi_i(k) &= \Phi_{1_i}(k) - \Phi_{2_i}(k)\\ &= f\Delta t_{R_{12}} - \frac{\Delta D_i(k)}{\lambda} - \Delta N_i + \Delta b_i(k)\end{aligned}$$

where

- $\Delta t_{R_{12}} = \Delta t_{R_1} - \Delta t_{R_2}$
- $\Delta D_i(k) = D_{1_i}(k) - D_{2_i}(k)$
- $\Delta N_i = N_{1_i} - N_{2_i}$
- $\Delta b_i(k) = b_{1_i}(k) - b_{2_i}(k)$

Further on, to remove the receiver clock offset, we can form the double differences of phase, as shown in the following. This is achieved by choosing a reference satellite. For demonstration purposes, satellite 1 will be chosen as the reference satellite. If N satellites are being tracked at epoch k, we can form $N-1$ double differences with the $N-1$ remaining satellites:

$$\begin{aligned}\nabla\Delta\Phi_{1i}(k) &= \Delta\Phi_1(k) - \Delta\Phi_i(k)\\ &= -\frac{\nabla\Delta D_{1i}(k)}{\lambda} - \nabla\Delta N_{1i} + \nabla\Delta b_{1i}(k)\end{aligned} \quad (2)$$

where

- $\nabla\Delta D_{1i}(k) = \Delta D_1(k) - \Delta D_i(k)$
- $\nabla\Delta N_{1i} = \Delta N_1 - \Delta N_i$
- $\nabla\Delta b_{1i}(k) = \Delta b_1(k) - \Delta b_i(k)$

The model (2) for the double differences depends on the known coordinates of satellite 1 and satellite i, and on the coordinates of the two receivers. Thus, once the double difference ambiguities $\nabla\Delta N_{1i}$ are solved, we may be able to reach the desired positioning information contained within the double differences of phase.

Assume now that the coordinates of one receiver, say receiver 1, are well known. In the case where receiver 2 keeps a constant position during the ambiguity resolution procedure, then the number of unknowns in the system of the double differenced equations at any epoch is $3+N-1 = N+2$, while the number of observations is $N-1$. Then, the system can be solved using the observations gathered over two epochs if $2(N-1) \geq N+2$, that is if $N \geq 4$. The resolution yields an estimate of the double differences ambiguities as well as an estimate of the position of receiver 2.

The double difference model (2) is linearized and solved using an iterative procedure, as described by Remondi (Remondi, 1984), Rocken (Rocken, 1988), Blewitt (Blewitt, 1989), Leick (Leick, 1990) or Hofman et al. (Hofman et al., 1993) for example.

When the position of receiver 2 cannot be modelled as a constant, the system cannot be directly solved, as the number of unknowns increases over time. Performing an active search of the correct solution at each epoch is an adequate strategy for the resolution of the ambiguities. This search is carried out over a physical or a mathematical domain centered around an estimate of the solution. These methods require a great calculation power from the executing processor.

The Ambiguity Function Method, described by Remondi (Remondi, 1984) and by Mader (Mader, 1992) in the dynamic case, searches for the most coherent position in a physical volume, considering the phase measurements. The other searching techniques search for the most coherent combination of the double difference ambiguities in a mathematical set of probable discrete combinations. Numerous methods have been proposed so far. Among them, are the Least Squares Ambiguity Search Technique (LSAST) described by Hatch (Hatch, 1991) or Lachapelle et al. (Lachapelle et al., 1992), the Fast Ambiguity Resolution Approach (FARA) described by Frei and Beutler (Frei and Beutler, 1990), the Fast Ambiguity Search Filter (FASF) described by

Chen (Chen, 1993), the optimized Cholesky decomposition method described by Landau and Euler (Landau and Euler, 1992), or the integrated 'on-the-fly' technique described by H.Abiddin (Abidin, 1991) which achieves an integration of several techniques.

We will now have a closer look to the methods searching for the ambiguities related to only four particular satellites, like the method described by Hatch (Hatch, 1991) or Lachapelle et al. (Lachapelle et al., 1992). Let us denote N_k as the number of tracked satellites at epoch k. We can split the system of the $N_k - 1$ equations (2) in two parts :

- the system of the 3 double difference equations corresponding to four particular satellites. These satellites are called the *primary* satellites
- the system of the $N_k - 4$ remaining double difference equations corresponding to the other satellites. These satellites are in turn called the *secondary* satellites.

All the quantities related to the primary satellites (resp. the secondary satellites), like the observations, the ambiguities and the phase noise values will be qualified as primary (resp. secondary) quantities.

If the primary ambiguities are known, then the position of the moving receiver can be determined, and the secondary ambiguities can be known. Thus it is not necessary to search for the entire set of the unknown ambiguities, and some computation effort can be saved.

For example, the method described by Lachapelle et al. (Lachapelle et al., 1992) searches for the primary ambiguities that are associated, through the primary system of equations, with a physical position contained within a search cube built around a carrier-phase-smoothed code solution. At each measurement epoch k, for each potential three-integer combination, the difference between the actual secondary phase observations and some computed observations is formed. This secondary *prediction error*, called $z(k)$, together with the secondary observations covariance matrix $\Sigma_{SS}(k)$ can be used to form the *local variance factor*

$$\hat{\sigma}_0^2(k) = \frac{z(k)^\top \Sigma_{SS}(k)^{-1} z(k)}{N_k - 4}$$

A *global variance factor* can also be built using all of the local variance factors. A candidate yielding too high a local or global variance factor is rejected from the search set, and thus will not be tested for during the next search epoch. After several rejection epochs, the best combination can be isolated in the set.

Considering the statistical aspect of these methods, we see that the ambiguity searching procedures can be included in the large group of the *multiple hypotheses sequential tests*. The set of the potential three-integer vectors constitutes the set of the unknown parameters of the probability density function of the primary and secondary observations. Let us call this set $\mathcal{N}$. Thus, the procedure is built to decide between the hypotheses :

$$H_{abc} = \{[a\ b\ c] : [\nabla\Delta N_{12}\ \nabla\Delta N_{13}\ \nabla\Delta N_{14}] = [a\ b\ c]\}$$

for each three-integer vector $[a\ b\ c] \in \mathcal{N}$. In this discussion, satellites 1,2,3,4 are considered as being the 4 primary satellites.

The decision is taken using the raw data

$$\nabla\Delta\Phi_1^n = [\nabla\Delta\Phi_{11}(1) \ldots \nabla\Delta\Phi_{1N_1}(1) \ldots \nabla\Delta\Phi_{11}(n) \ldots \nabla\Delta\Phi_{1N_n}(n)]$$

The test is a mapping g that associates to the observation data $\nabla\Delta\Phi_1^n$ a particular hypothesis H_{abc} :

$$g\left(\nabla\Delta\Phi_1^n\right) = H_{abc}$$

The decision is taken at the epoch n when a preset decision condition is satisfied. Thus the size of the sample $\nabla\Delta\Phi_1^n$ is not known before the test is performed, and a compromise must be struck between the delay in making the decision and the accuracy of that decision by specifying the decision condition. This kind of test is called a *sequential test*. The important sets of parameters used to assess the quality of a sequential test are the set of the *error probabilities* and the set of the *Average Sample Numbers (ASNs)*.

The set of the error probabilities is the set of the conditional probabilities

$$\alpha_{abc} = P\left[g\left(\nabla\Delta\Phi_0^n\right) \neq H_{abc} \mid H_{abc}\ true\right]$$

We can build the weighted error probability as

$$\bar{\alpha}(g) = \sum_{abc \in \mathcal{N}} P\left[H_{abc}\ true\right] \alpha_{abc}$$

The set of the ASNs is the set of the conditional expectations :

$$ASN_{abc} = E\left[n \mid H_{abc}\ true\right]$$

A sequential test is usually built by specifying values for the error probabilities. These values are inserted into the theoretical expressions of the decision thresholds to design the test. The ASNs are also determined using their own theoretical expressions.

In the case of the ambiguity searching algorithms, like in the case of most multiple hypotheses sequential tests, these theoretical expressions are hard to derive, and the design is made with empirical threshold values. These values are set so that the measured error probabilities and ASNs are as low as required. The error probabilities and the ASNs thus become estimated criteria used to assess the performance of the test.

Several decision criteria can be chosen. The LSAST method uses the weighted least squares sum of the prediction errors for each hypothesis. The MAPAS method uses the a posteriori probability of each hypothesis knowing the prediction errors.

3. THE MAPAS METHOD

The Maximum A Posteriori Ambiguity Search (MAPAS) method is based on the same basic principles as the LSAST method described by Hatch (Hatch, 1991). The main difference resides in the decision criterion used by the test : the MAPAS method uses the *a posteriori probability* of each potential solution knowing all the past secondary prediction errors. Thus the criterion is a naturally global parameter, and all the measurements acquired up to the current epoch are considered when a decision is taken.

The principle of this method is inspired from the method described by Brown and Hwang (Brown and Hwang, 1983), where the ambiguities are estimated as unknown parameters of an observation model using a Kalman filter.

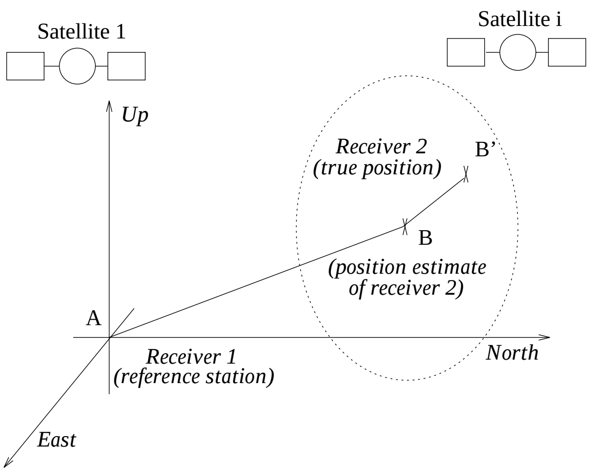

Figure 1: *Illustration of the situation of the receivers in a local coordinate system.*

The MAPAS method assumes that the measurements made by two receivers, located as described in figure 1, are available at each measurement epoch. Receiver 1 is called the reference station. Its position, denoted A, must be well known, and its code and phase measurements must be known by receiver 2. In the following discussions, we will consider that receiver 2 is located in B'. The ground distance between A and B' should be less than 20 km, and their altitude should not differ by more than 1.5 km. Thus the ionospheric and tropospheric propagation delays will be considered as identical for both receivers. At each measurement epoch, a good estimate of the position of receiver 2, called B, must be available. This estimation can be made using pseudo-range corrections delivered by the reference station, or using a carrier-phase-smoothed code technique. The standard deviation of the estimate should be less than 1m on either axis in order to restrict the number of ambiguities candidates.

The main steps of the MAPAS method are as follows. The procedure is initialized at epoch 0 by the selection of four particular satellites among the tracked satellites, and by the construction of the set of the candidate primary ambiguities. Then, at each measurement epoch, for each potential solution, a prediction of the secondary phase measurements is made, and subtracted from the actual secondary phase measurements. The a posteriori probability of each potential solution in the set is then computed from the *a priori probability* of each prediction error. That potential solution is rejected from the set if its a posteriori probability is lower than a preset acceptance threshold. If the largest a posteriori probability of the set is higher than a predefined threshold, then the combination related to that best value is elected as the correct solution.

The primary satellites are selected according to their PDOP factor, among the tracked satellites whose elevation is greater than 7.5 deg, in order to ensure their visibility during the whole resolution procedure.

The PDOP of the primary satellites is an important selection criterion, as it seriously affects the performance parameters of the test. If the primary satellites are the satellites meeting the elevation angle requirement with the lowest PDOP factor, then the number of initial potential solutions is at its maximum. Many observation epochs, as well as many computer operations are then required to isolate the correct ambiguities. A minimum PDOP of 5 is usually required. On the other hand, when the PDOP of the primary satellites is too high, the posterior probability of the true solution may be accidentally lowered because of inaccurate intermediate position estimates, as it will be pointed out further. In this case, a reasonable upper bound of 10 is set.

Thus the PDOP of the primary satellites has an opposite influence on the duration of the test and on its error probability, and an ideal mean value has to be selected. In the case of the MAPAS algorithm, the primary satellites are chosen as the satellites of elevation angle greater than 7.5 deg having the PDOP factor which is the closest to the arbitrary value of 7.5.

The double difference model (2) is linearized around the position estimate B. The phase data used by the procedure at each measurement epoch are the double differences of phase that can be formed using the actual phase measurements made by the moving receiver, and the computed phase measurements that could be made by receiver 1 if it was located in point B, corresponding to the position estimate. The computed measurements are denoted $\Phi_{3i}(k)$. Considering equation (1), we have:

Presented at ION GPS 95, Palm Springs, 1995

$\Phi_{1i}(k) - \Phi_{3i}(k) = -\frac{D_{1i}(k)-D_{3i}(k)}{\lambda}$

$$\Phi_{3i}(k) - \Phi_{2i}(k) = f\Delta t_{R_{12}} - \frac{D_{3i}(k)-D_{2i}(k)}{\lambda} - \Delta N_i + \Delta b_i(k) \quad (3)$$

Therefore, we can use the single differences of phase $\Delta\bar{\Phi}_i(k) = \Phi_{3i}(k) - \Phi_{2i}(k)$, where $\Phi_{3i}(k)$ is generated using ().

$\Delta\bar{\Phi}_i(k)$ has the same ambiguity and noise values as $\Delta\Phi_i(k)$.

As point B is very close to point B', we can linearize the model (3), assuming that the direction cosines $(C_x(k), C_y(k), C_z(k))$ of the satellites are identical for both receivers. This approximation is justified because the distance between B and B' is of the order of a few meters. We have

$$\begin{aligned} D_{3i}(k) - D_{2i}(k) = C_{xi}(k)\left(x_B(k) - x_{B'}(k)\right) \\ +C_{yi}(k)\left(y_B(k) - y_{B'}(k)\right) \\ +C_{zi}(k)\left(z_B(k) - z_{B'}(k)\right) \end{aligned}$$

and if we note

$$\Delta R(k) = [x_B(k) - x_{B'}(k) \quad y_B(k) - y_{B'}(k) \quad z_B(k) - z_{B'}(k)]^\top$$

and

$$C_i(k) = \frac{1}{\lambda}\left[C_{xi}(k) \quad C_{yi}(k) \quad C_{zi}(k)\right]$$

then we have

$$\Delta\bar{\Phi}_i(k) = f\Delta t_{R_{12}} - C_i(k)\Delta R(k) - \Delta N_i(k) + \Delta b_i(k)$$

For the N_k tracked satellites, we can form the following linearized models of the double differences of phase :

$$\nabla\Delta\bar{\Phi}_{1i}(k) = -C_{1i}(k)\Delta R(k) - \nabla\Delta N_{1i} + \nabla\Delta b_{1i}(k) \quad (4)$$

where $C_{1i}(k) = C_1(k) - C_i(k)$.

We can stack the $N_k - 1$ values $\nabla\Delta\bar{\Phi}_{1i}(k)$ obtained in (4) into two separate vectors $\Phi_P(k)$ and $\Phi_S(k)$. $\Phi_P(k)$ contains the 3 values related to the 4 primary satellites, and $\Phi_S(k)$ contains the $N_k - 4$ values related to the secondary satellites. Thus, we get the following primary and secondary systems of equations, using vectors and matrices denoted with the respective subscripts P and S :

$$\Phi_P(k) = -C_P(k)\Delta R(k) - N_P + b_P(k) \quad (5)$$

$$\Phi_S(k) = -C_S(k)\Delta R(k) - N_S + b_S(k) \quad (6)$$

The set of the acceptable three-integer solution vectors can be built using the primary system (5) at the initial epoch 0. The physical search volume can be chosen as an *ellipsoid* centered on the estimated position B. The semi-axes of the ellipsoid are set to three times the corresponding values of the standard deviations of the position estimate. The first step of the elaboration of the initial set $\mathcal{N}$ can be the calculation of the ambiguities corresponding to the eight corners of the cube englobing the ellipsoid using (5). Then we can build the set of ambiguities delimited by these eight values. Eventually, the set $\mathcal{N}$ is formed by rejecting those ambiguities associated with a position located outside the ellipsoid.

At each epoch k, the position associated with a particular combination $N_{P_{abc}} = [a\,b\,c]$ can be estimated using the primary phase observations. For example, using least squares estimation theory, we get :

$$\Delta\hat{R}_{abc}(k) = -S(k)\Phi_P(k) - S(k)N_{P_{abc}} \quad (7)$$

where $S(k)$ is

$$S(k) = \left[C_P(k)^\top\Sigma_{PP}^{-1}(k)C_P(k)\right]^{-1} C_P(k)^\top\Sigma_{PP}^{-1}(k)$$

and $\Sigma_{PP}(k)$ is the covariance matrix of the primary observations. Using (2) and the assumptions on the phase measurement noise process made in (1), the expression of this matrix can be shown to be

$$\Sigma_{PP}(k) = \sigma^2\begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix} \quad (8)$$

A prediction of the secondary phase measurements at the epoch k can be elaborated if we inspire from the linear measurement model (6). Assume a prediction of the secondary ambiguities $\hat{N}_{S_{abc}}(k)$ is available for each combination $[a\ b\ c]$ in $\mathcal{N}$.

We can then form the prediction

$$\hat{\Phi}_{S_{abc}}(k) = -C_S(k)\Delta\hat{R}_{abc}(k) - \hat{N}_{S_{abc}}(k) \quad (9)$$

The prediction $\hat{N}_{S_{abc}}(k)$ can be made using all the past secondary observations obtained from the satellites present in $\Phi_S(k)$.

For example, considering the fact that for the true hypothesis the prediction of the secondary ambiguities should be a vector of constant integers, we can choose to use the prediction

$$\hat{N}_{S_{abc}}(k) = [Round(\frac{1}{k-k_i}\sum_{j=k_i}^{k-1}[-C_{1i}(j)\Delta\hat{R}_{abc}(j) - \nabla\Delta\bar{\Phi}_{1i}(j)])]_{i\in\{5,..,N_k\}} \quad (10)$$

where k_i is the first epoch of lock on the signal transmitted by satellite i. A predicted value of secondary ambiguity concerning satellite i is then computed for the first time at epoch $k_i + 1$.

We need now to introduce the a posteriori probability of each potential value $[a\ b\ c]$, conditionally on the value of the prediction errors. That probability is the conditional probability for the true value of the primary

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ambiguities $[p\,q\,r]$ to be equal to $[a\,b\,c]$, given the values of the prediction errors obtained for that hypothesis $[a\,b\,c]$ up to the current epoch.

The actual secondary observations depend on the true primary ambiguities $N_{P_{pqr}} = [p\ q\ r]$. The true position can be expressed as :

$$\Delta R_{pqr}(k) = -S(k)\Phi_{P_{pqr}}(k) - S(k)N_{P_{pqr}} + S(k)b_P(k) \quad (11)$$

so that

$$\Phi_{S_{pqr}}(k) = -C_S(k)\Delta R_{pqr}(k) - N_{S_{pqr}} + b_S(k) \quad (12)$$

Thus, for each candidate $N_{P_{abc}}$, the prediction error can be written as :

$$z_{abc}(k) = \Phi_{S_{pqr}}(k) - \hat{\Phi}_{S_{abc}}(k) \quad (13)$$

Using Bayes' rule, we can write the a posteriori probability of a particular three-integer combination $[a\ b\ c]$ as :

$$P\left[N_P = [a\ b\ c] \mid {z_{abc}}_1^k\right] = \frac{f\left({z_{abc}}_1^k | N_P = [a\ b\ c]\right) P[N_P = [a\ b\ c]]}{\sum_{[a\ b\ c]\in\mathcal{N}} f\left({z_{abc}}_1^k | N_P = [a\ b\ c]\right) P[N_P = [a\ b\ c]]} \quad (14)$$

where ${z_{abc}}_1^k = \left[z_{abc}(1) \ldots z_{abc}(k)\right]$ and $\mathcal{N}$ is the set containing all the $[a\ b\ c]$ candidates.

$$f\left({z_{abc}}_1^k \mid N_P = [a\,b\,c]\right) \quad (15)$$

is the value of the a priori probability density function of the innovations at the point ${z_{abc}}_1^k$. We assume that each three-integer combination in $\mathcal{N}$ is equally probable, so that (14) can be reduced to

$$P\left[N_P = [a\ b\ c] \mid {z_{abc}}_1^k\right] = \frac{f\left({z_{abc}}_1^k | N_P = [a\ b\ c]\right)}{\sum_{[a\ b\ c]\in\mathcal{N}} f\left({z_{abc}}_1^k | N_P = [a\ b\ c]\right)} \quad (16)$$

Thus, at each epoch k, the a posteriori probability of each potential solution in the set can be computed once the current value of (15) is determined for every hypothesis.

The nature of the a priori law of probability of the recorded prediction errors ${z_{abc}}_1^k$, as well as the value of their conditional mean and covariance matrix has to be determined in order to perform the calculation of (15).

From (13), if we insert (7) into (9), and (11) into (12), we can derive the expression of the conditional prediction error for a candidate solution $[a\,b\,c]$:

$$z_{abc}(k) \mid N_P = [p\,q\,r] = [a\,b\,c] = \hat{N}_{S_{abc}}(k) - N_{S_{pqr}} - C_S(k)S(k)b_P(k) + b_S(k) \quad (17)$$

Moreover, assuming $N_P = [p\,q\,r] = [a\,b\,c]$, we have

$$\hat{N}_{S_{abc}}(k) = N_{S_{pqr}} \quad (18)$$

If no precaution is taken when using the predictions elaborated with (10), this assumption may not be verified for the true combination $[a\,b\,c]$, especially in the first stages of the procedure, when few measurements have been used to compute the average. A way to counter that is to allow the prediction several averaging epochs to get stabilized to the correct value before using it for statistical selection. However, it is very essential that the algorithm uses all the information available to feed its selection routines within the shortest delay, and the predictions elaborated should be used by the test as early as possible. Moreover, this prediction is not very noise sensitive, and it has proven to be highly accurate during all the trials performed.

Hence, the MAPAS algorithm releases the secondary ambiguity predictions concerning a given satellite only if there exists at least one $[a\,b\,c]$ candidate for which one that prediction has been constant for 2 epochs.

Note that is is important for the validity of assumption (18) that the primary satellites do not have too high a PDOP factor. Indeed, if the estimate (7) is not accurate enough for the true solution, the ambiguity prediction (10) may be biased by a value of one full cycle. In that case, the error prediction will be accidentally large, and the a posteriori probability of the true solution will be abnormally low.

As assumption (18) is made, (17) becomes :

$$z_{abc}(k) \mid N_P = [a\,b\,c] = -C_S(k)S(k)b_P(k) + b_S(k) \quad (19)$$

Since the additive phase noise measurement assumed in (1) is a white noise process, expression (19) shows that the successive random vectors $z_{abc}(k) \mid N_P = [a\,b\,c]$ are independent over time. Thus ${z_{abc}}_1^k$ is composed of k independent vectors. The value of the a priori probability density function (15) can therefore be computed as

$$f\left({z_{abc}}_1^k \mid N_P = [a\,b\,c]\right) = \prod_{i=1}^k f\left(z_{abc}(i) \mid N_P = [a\,b\,c]\right) \quad (20)$$

Still using (19), as the phase measurement noise process is assumed to be a white gaussian noise with zero mean, we see that $z_{abc}(k) \mid N_P = [a\,b\,c]$ is a gaussian vector with zero mean. Its covariance matrix is

$$\begin{aligned}\Sigma(k) &= Cov(z_{abc}(k) \mid N_P = [a\,b\,c])) \\ &= C_S(k)S(k)\Sigma_{PP}(k)S(k)^\top C_S(k)^\top \\ &\quad + \Sigma_{SS}(k) - C_S(k)S(k)\Sigma_{PS}(k) \\ &\quad - \Sigma_{PS}(k)^\top S(k)^\top C_S(k)^\top\end{aligned}$$

where

- $\Sigma_{PP}(k)$ is the covariance matrix of the primary observations. Its expression is given in (8).
- $\Sigma_{SS}(k)$ is the covariance matrix of the secondary

observations. Its expression is :

$$\Sigma_{SS}(k) = \sigma^2 \begin{bmatrix} 4 & 2 & \dots & \dots & 2 \\ 2 & 4 & 2 & \dots & \vdots \\ \vdots & 2 & \ddots & 2 & \vdots \\ \vdots & \dots & 2 & 4 & \vdots \\ 2 & \dots & \dots & 2 & 4 \end{bmatrix}$$

- $\Sigma_{PS}(k) = \Sigma_{SP}(k)^\top$ is the cross-covariance matrix between the primary observations and the secondary observations. Its expression is :

$$\Sigma_{SP}(k) = \sigma^2 \begin{bmatrix} 2 & 2 & 2 \\ \vdots & \vdots & \vdots \\ 2 & 2 & 2 \end{bmatrix}$$

Thus, $f\left(z_{abc}{}_1^k \mid N_P = [a\,b\,c]\right)$ can be recursively computed by multiplying each of the successive values

$$f\left(z_{abc}(k)\,|N_P = [a\,b\,c]\right) \tag{21}$$
$$= \frac{1}{2\pi^{\frac{N_k-4}{2}}\sqrt{\det \Sigma(k)}} \times \exp -\frac{1}{2} z_{abc}(k)^\top \Sigma(k)^{-1} z_{abc}(k)$$

The normalization factor $\frac{1}{2\pi^{\frac{N_k-4}{2}}\sqrt{\det \Sigma(k)}}$ does not need to be computed, as the value given in (21) is only used to update (15) through (20), which is in turn used to calculate the a posteriori probability (16), where it cancels.

The algorithm of the procedure is shown in figure 2. The symbol $P(k)$ is used to designate the a posteriori probability of a candidate at epoch k, $f(k)$ designates the corresponding value of the a priori probability density function, and $s(k)$ is the sum of the prior probabilities of all the candidates in the set. P_0 is the upper decision threshold and P_{min} is the rejection threshold.

The algorithm shown in figure 2 evaluates $f(k)$ for the current epoch k, and $P(k-1)$ for the previous epoch $k-1$ using the sum $s(k-1)$. This is done to avoid a second scan of the set to calculate $P(k)$ with the sum of the current $f(k)$ in the big $while$ loop. This causes a non-sensitive delay of one epoch in the instant of decision.

If a cycle slip is experienced on the signal received from any primary satellite, the procedure must be restarted.

```
selection of primary satellites
read data
construction of 𝒩
while best P(k) < P_0
   read data
   compute Σ(k)
   for each N_P = [a b c] ∈ 𝒩(k)
      if f(k − 1) is available then
         compute P(k − 1)
         reject N_P if P(k − 1) < P_min
      end if
      if N_P ∈ 𝒩(k) then
         update best P(k − 1)
         compute N̂(k)
         compute z_abc(k)
         compute f(k)
         update s(k) = Σf(k)
      end if
   end for
end while
```

Figure 2: *Main steps of the MAPAS algorithm.*

4. SIMULATION RESULTS

The raw phase data used for the simulations is the double differenced phase data that could have been formed using two receivers in the city of Valence, on October 7^{th}, 1994.

In these simulations, one reference station is considered, and the moving receiver is assumed to be moving along a certain simulated trajectory.

The simulations were run using the satellite constellation visible from 0:00 a.m. till 12:00 a.m. on October 7^{th}, 1994. The procedure ignored the satellites of elevation less than 5 deg. Thus, the number of considered satellites during the various ambiguity resolution trials ranged from 6 to 11.

Phase measurements for these satellites were generated every second. The measurements were considered as affected by the same tropospheric and ionospheric propagation terms, and by a discrete white gaussian noise. The standard deviation of this noise was set to $\sigma = 1\,mm$.

The ambiguity resolution trials were performed one after the other over the 24 hours. For each trial, the number of measurement epochs and the computation time required for the algorithm to be able to make a decision, as well as the truthfulness of the value of the ambiguities selected were recorded. The size of the search ellipsoid was set with the following values of semi-axes : $(a_x = 2m, a_y = 2m, a_z = 2.5m)$.

The design parameters of the procedure are the rejection criterion P_{min} and the upper decision threshold P_0.

The values assigned to these parameters affect the error probability, the Average Sample Number (ASN), and the computation time of the procedure. The value of the true primary ambiguities does not influence the performance of the procedure, as long it is included in the initial set. Thus only one estimated ASN and one estimated error probability will be given.

A subtile tradeoff must be achieved when specifying the two thresholds P_{min} and P_0. Both of them must be adjusted so that false solutions are quickly rejected from the set, while the correct combination is kept. Assign-

Presented at ION GPS 95, Palm Springs, 1995

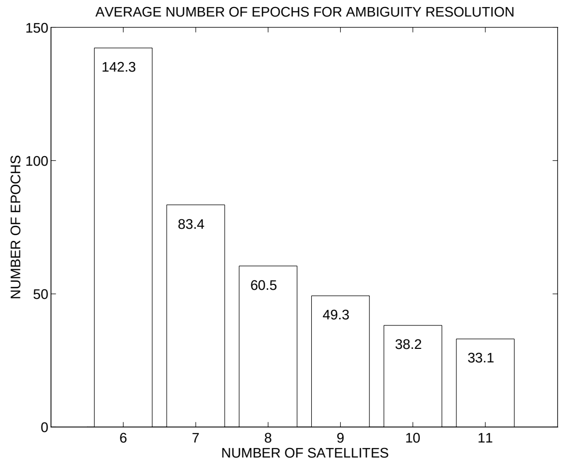

Figure 3: *Average duration of the trials in epochs as a function of the number of tracked satellites.*

ing a high value to P_{min} will make the algorithm reject quickly the false solutions. However, it will increase the chances for the true combination to be accidentally eliminated as well, and the error probability will be larger. If that value is set too low, then the ASN will be slightly higher, and a prohibitive number of operations will have to be performed by the processor. Similarly, setting P_0 with too low a value may enable a false solution to be elected, and the error probability will increase. On the other hand, setting it with a high value will increase the ASN.

It appears that for reasonable values of P_0 and P_{min}, P_0 has a strong influence on the ASN while P_{min} directly affects the error probability.

The influence of P_{min} is illustrated in table (4). As the abnormal transient values of posterior probability are mostly observed during the first ten epochs, when all the candidates share the unit probability, it happens very often that the a posteriori probability of the true solution reaches a low value. But it is very rare to see a false solution pass the upper acceptance threshold P_0 when the true solution is still in the set.

Thus, if a low error probability is to be reached, it is important that the rejection threshold P_{min} be set to a very low value. However, as the quantity of operations depends strongly on the number of candidates handled by the procedure, specifying too low a value for P_{min} may prevent the use of the algorithm for real time applications.

The adjustment of these different parameters required a lot of simulations, and only the first satisfying results obtained are presented here. The results of 2493 trials are presented in figures 3, 5, 6, 7 and 8. For these trials, the rejection criterion was set to $P_{min} = 10^{-5}$ and the decision threshold was set to $P_0 = 0.9999$. The observed global success rate for all the simulation runs

P_{min}	Error rate in %	Average computation time in s	ASN
10^{-5}	99.12	34.17	63.3
10^{-6}	99.34	36.25	63.4
10^{-8}	99.56	41.51	63.5

Figure 4: *Evolution of the performance parameters when P_{min} decreases, for $P_0 = 0.9999$.*

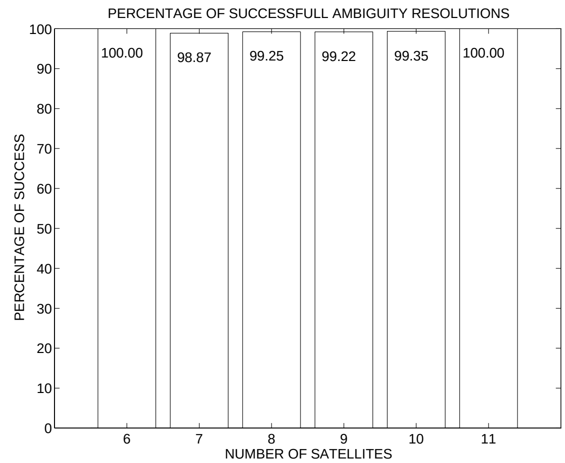

Figure 5: *Percentage of successful trials as a function of the number of tracked satellites.*

is 0.9912. The ASN is 63.

These performance parameters were estimated for the different numbers of tracked satellites, and are presented in figures 3, 5 and 8.

As the rate of acquisition of the information increases, that is as the number of satellites increases, both the rejection and the selection procedures are more efficient and this results in an improvement of the ASN, as it is shown in figure 3.

As seen in figure 6 and 7, the values selected for the design parameters enable the correct combination to be kept inside the search set despite its erratic initial path, and to be eventually isolated.

The average computation times required to perform the search on an HP 712/80 workstation are shown on figure 8. These durations represent the whole execution time of the entire resolution. The times presented here are to be used as rough indications only, as no particular effort was made to speed up the execution of the procedure so far. The simulation software was only designed to study the validity of the concept of the MAPAS method. However, the trend of the evolution of these figures with the number of satellites can be analysed. As seen in figure 8, the benefit of a larger number of observations per epoch offered by additional satellites is important only when 10 satellites are used. Before that, the gain in the ASN is not big enough to compensate the heavy volume of data processed by the computer.

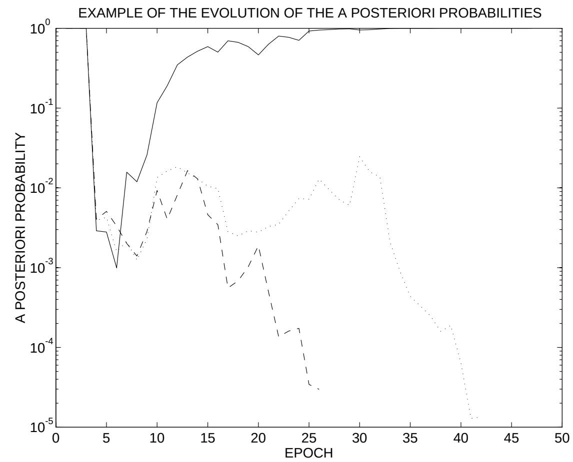

Figure 6: *Example of the evolution of the a posteriori probabilities over time. The solid line corresponds to the correct combination. The dashed and the dotted lines correspond to two wrong solutions that are rejected sooner or later.*

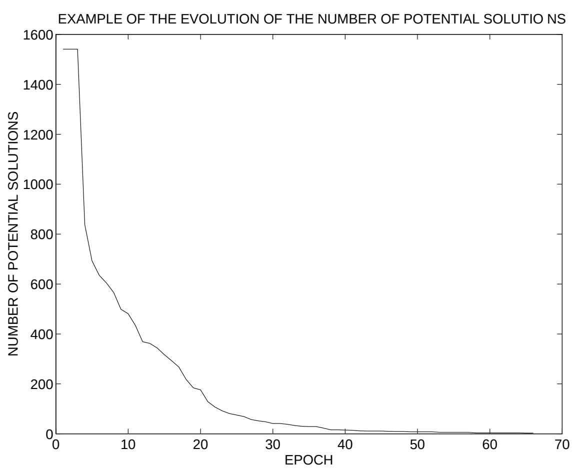

Figure 7: *Example of the evolution of the number of potential solutions in the set. The set contains 1541 initial potential solutions.*

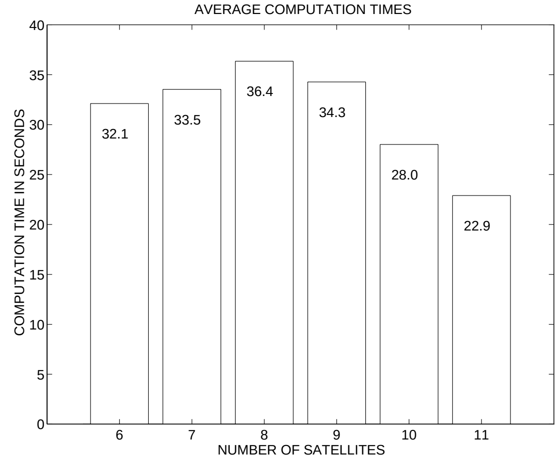

Figure 8: *Average execution time of the simulation software as a function of the number of tracked satellites.*

5. CONCLUSION

The MAPAS method makes an optimal use of all the current and past code and phase data available to the receiver, as the final decision is taken considering all the acquired data.

This method also provides a means to quantify the confidence that can be made in each potential solution. At each step of the resolution, the a posteriori probability of a particular combination can be delivered by the procedure.

As seen from the first simulation results presented here, over all the possible satellite configurations, the MAPAS method raised the correct ambiguities with a success rate of 99.1 %, and an average number of epochs of 63. This encouraging result demonstrates the validity of the concept of an ambiguity searching procedure based on the a posteriori probability.

Ongoing investigations aim to achieve a better adjustment of the design thresholds as well as an optimization of the algorithm, so that the real time performances of the method can be improved.

ACKNOWLEDGMENTS

The author wishes to thank the company SEXTANT AVIONIQUE for helping him in the development of the MAPAS method. He also wishes to thank the Service Technique de la Navigation Aérienne for enabling him to perform this research by providing funds and precious informations.

REFERENCES

Abidin H. (1991) *"New Strategy for 'On the fly' Ambiguity Resolution"*, proceedings of ION GPS-91, Albuquerque, September 11-13, The Institute Of Navigation, pages 875-886

Blewitt G. (1989) *"Carrier Phase Ambiguity Resolution for the Global Positioning System Applied to Geodetic Baselines up to 2000 Km"*, Journal of Geophysical Research, Volume 94 Number B8, pages 10187-10203

Brown R.G. and Hwang P.Y.C. (1983) *"A Kalman Filter Approach to Precision GPS Geodesy"*, Journal of the Institute of Navigation, Volume 30 Number 4, pages 155-166

Chen D. (1993) *"Fast Ambiguity Search Filter : A Noval Concept for GPS Ambiguity Resolution"*, proceedings of ION GPS-93, The Institute Of Navigation, Alexandria VA, pages 781-787

Frei E. and Beutler G. (1990) *"Rapid Static Positioning based on the Fast Ambiguity Resolution Approach : The Alternative to Kinematic Positioning"*, proceedings of Second International Symposium on Precise Positioning with GPS, Canadian Institute of Geomatics, Ottawa, Canada. September 3-7, pages 1197-1216

Hatch R. (1991) *"Instantaneous Ambiguity Resolution"*, proceedings of International Association of Geodesy Symposia 107 on Kinematic Systems in Geodesy, Surveying and Remote Sensing, New York, Springer-Verlag, pages 299-308

Hofman-Wellenhof B. and Lichtenegger H. and Collins J. (1993) *"Global Positioning System : Theory and Practice"*, Springer-Verlag

Lachapelle G. and Cannon M.E. and Lu G. (1992) *"High-Precision GPS Navigation with Emphasis on Carrier-Phase Ambiguity Resolution"*, Marine Geodesy, Volume 15 Number 4, pages 253-269

Landau H. and Euler H.J. (1992) *"On-The-Fly Ambiguity Resolution for Precise Differential Positioning"*, proceedings of ION GPS-92, The Institute of Navigation, Alexandria, September 16-18, Albuquerque, New Mexico, pages 607-613

Leick A. (1990) *"GPS Satellite Surveying"*, Wiley-Interscience

Mader G. (1992) *"Kinematic GPS Phase Initialization using the Ambiguity Function"*, proceedings of Sixth International Geodetic Symposium on Satellite Positioning, Columbus, Ohio, March 17-20, pages 712-719

Remondi B.W. (1984) *"Using the Global Positioning System GPS phase observable for relative geodesy : modeling, processing and results"*, Ph.D. Dissertation, University of Texas at Austin, May 1984

Rocken C. (1988) *"The Global Positioning System : A New Tool for Tectonic Studies"*, Ph.D. Dissertation, Universitat zu Koln, Cologne, West Germany.