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# GNSS/IRS Hybridization: Fault Detection and Isolation of More than One Range Failure

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## BIOGRAPHY

Anne-Christine Escher graduated as an electronics engineer in 1999 from ENAC (Ecole Nationale de l'Aviation Civile) in Toulouse, France. Since 1999, she has been working on the application of satellite navigation techniques to civil aviation at the signal processing lab of the ENAC as a PhD. student.

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Nicolas Martin is graduated as an aeronautical engineer in 1994 from ENSAE (Ecole nationale Supérieure de l'Aéronautique et de l'Espace) in Toulouse, France. Since 1995 he has been working for THALES Avionics (in Valence) in inertial navigation, INS/GNSS hybridization and integrity fields. He is currently working on Galileo signal and receiver definition.

Benoit Roturier graduated as a CNS systems engineer from Ecole Nationale de l'Aviation Civile (ENAC), Toulouse in 1985 and obtained a PhD in Electronics from Institut National Polytechnique de Toulouse in 1995. He was successively in charge of Instrument Landing Systems at DGAC/STNA (Service Technique de la Navigation Aérienne), then of research activities on CNS systems at ENAC. He is now head of GNSS Navigation subdivision at STNA and is involved in the development of civil aviation applications based on GPS/ABAS, EGNOS and Galileo.

Vincent Vogel graduated in June 2002 as an electronics engineer from the ENAC.

## ABSTRACT

GPS/IRS hybridization is a good candidate to fulfill demanding civil aviation requirements. When the integrity of the GPS measurements is ensured they may be

used to calibrate inertial position and improve accuracy. This can be done in a tightly coupled manner by means of a Kalman filter. Calibrated IRS can ensure coasting while maintaining good short term accuracy and helps detect large GPS failures.

The hybridized system must also be able to detect slowly growing errors on GPS measurements that may affect inertial calibration. These errors may affect one or several GPS measurements.

Fault detection and exclusion capacity of Receiver Autonomous Integrity Monitoring is limited for GPS, as it is designed for one satellite failure only. Therefore other solutions have been proposed. Among these are the extrapolation method and the solution separation method that are also designed to detect one satellite failure at a time.

The study reported in this paper presents first results obtained with an extension of Mats Brenner's separation algorithm for fault detection. That extension concerns detection and isolation of more than one failure during operations ranging from en-route to NPA, with the objective of satisfying the availability requirement.

The paper starts with a brief review of the classical Solution Separation method for a single satellite failure detection and isolation. In this section the model of the tightly coupled GPS/IRS system involved in simulations is also presented. The third section is dedicated to theoretical description of the one failure Solution separation fault detection and exclusion algorithm.

The next section considers the behavior of the algorithm when two simultaneous range failures occur. In particular the impact of two faulty measurements on the test metrics in the former fault detection and exclusion algorithm is examined. These simulation results are used to discuss the contribution of a two range failure detection algorithm.

Afterwards, a two simultaneous range failure detection and exclusion algorithm based on an extension of the nominal Solution Separation method is introduced. Definitions of performance parameters (Horizontal Protection Level, Horizontal Exclusion Level, and Missed Detection Probability), when several pseudo range failures (ramp, bias, etc.) occur simultaneously, are discussed. Finally initial simulation results are presented.

## I. INTRODUCTION

In GPS/IRS hybridization, GPS measurements are used as observations of the integration processor which provides estimates of inertial errors. These estimates are on one hand added to IRS output to form the navigation solution to the pilot (position, velocity, Euler angles). They are on the other hand used to calibrate the inertial platform (nominal position, velocity and Euler angles, and inertial error sources compensation). Thus when GPS data are unavailable, the calibrated IRS can ensure the navigation continuity.

However, several types of perturbations can affect the signal processed by a GPS receiver and thus lead to a corrupted inertial calibration. Since inertial navigation systems greatly depend on initial conditions, this may have a great incidence on solution navigation accuracy.

GPS perturbations are thermal noise, atmospheric disturbances, multipath and interference. Interference remains the most feared perturbation for civil aviation users because it can affect several tracking channels at a time during a long period.

Efficient failure detection and isolation algorithms have been designed ensuring high availability and integrity monitoring of GNSS navigation means, but they have been implemented assuming that there is a unique satellite failure.

For this reason, there is a need to appreciate the ability of fault detection and exclusion algorithms to detect and isolate multiple range failures.

As a first step the following paper discusses the use of Solution Separation theory to achieve two-failure detection and isolation.

## II. SOLUTION SEPARATION METHOD

Fault detection and exclusion function of the Solution Separation algorithm consists in maintaining a primary Kalman filter which incorporates the  $N$  measurements of the whole system, as many Kalman sub-filters as the number of tracked satellites (i.e.  $N$  sub-filters), and for each of these sub-filters a bank of  $N-1$  sub-sub-filters using measurements of  $N-2$  satellites.

Initially, the primary filter  $F_{00}$  provides an estimate of IRS error in order to form the inertial corrected position at the output of the whole system.

Sub-filters which each of these incorporate the measurements from  $N-1$  satellites are dedicated to detection only. They are noted  $F_{0n}$ ,  $n = 1 \dots N$ .

Sub-sub-filters are dedicated to exclusion. Each of these is excluding the measurement excluded by its "parent" sub-filter, and the measurement from a different satellite. They are noted  $F_{nm}$  with  $n = 1 \dots N$ ,  $m = 1 \dots N$  and  $n \neq m$ .

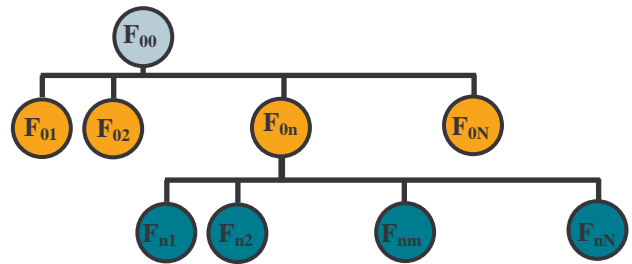


Figure 1. Detection and exclusion filters hierarchy

### Fault detection

Solution separation between  $F_{00}$  and each  $F_{0n}$  is estimated: it is the difference in the horizontal/vertical plane between the state vector estimated by  $F_{00}$  and the state vector estimated by  $F_{0n}$ .

Fault detection is performed by monitoring the separation between the main-solution and each of sub-solutions and comparing it to a computed detection threshold that depends on the separation statistics and the false detection probability  $P_{FD}$  which is deduced from false alarm rate.

When a failure occurs at least one solution separation will exceed this threshold. This method also guarantees a protection radius against any failure (even slow ramp) in straight relationship with the threshold and the value of the required missed detection probability  $P_{MD}$ .

### Fault exclusion

Fault exclusion assumes that there is only one satellite failure at a time.

When there is a detection, the separation between each sub-sub-filter and its parent sub-filter is computed and compared to an exclusion threshold that depends on the expected separation statistics and  $P_{FD}$ . If for sub-filter  $F_{0n}$ , there exists one solution separation such that one separation between it and sub-sub-filter  $F_{nm}$  exceeds the threshold, it can't be the faulty satellite. But if there is only one sub-filter  $F_{0n}$  for which all separations between its solution and its sub-sub-filters  $F_{nm}$  are under the threshold, satellite  $n$  is the faulty satellite.

After exclusion succeeds, the sub-filter which has not used measurement from the failed satellite becomes the primary filter. Its sub-sub-filters are the new bank of detection filters.

All of the Kalman filters are running in the same way.

## II. GPS/IRS KALMAN HYBRIDIZING FILTERS

All measurements and simulations have been made with Matlab.

This section presents the models we have implemented.

For any of the Kalman filters, the implemented system is composed of three units: an inertial unit – Inertial

Reference System - (Inertial measurement Unit + Strapdown calculator), a GNSS receiver (GPS measurements) and an integration process (Kalman filters) that also performed FDE function. Figure 2.

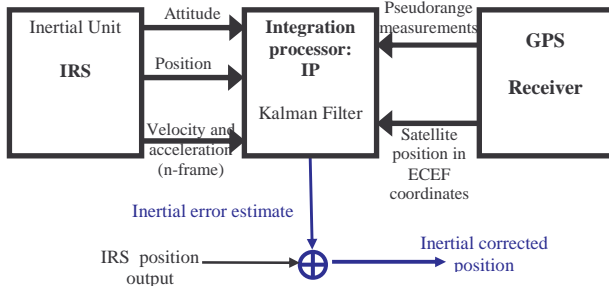


Figure 2. GPS/IRS architecture

### Inertial unit

It provides to the integration process Matlab-generated inertial position, velocity and attitude angles and gyrometric and accelerometric measurements.

The Inertial Measurement Unit generates realistic gyro and accelero measurements at 100 Hz rate from trajectory definition and attitude angles evolution of an aircraft. Sensor noises and biases have also been modelled.

IMU measurements are processed by Strapdown inertial navigation to provide inertial solution at 100 Hz: mobile positions and velocities relative to the earth frame in the navigation frame, and the attitude angles of the mobile body frame to the navigation frame.

### GPS Receiver

Pseudo range measurements (PRs) are generated at 1Hz. Random noise is added on each satellite range measurement and is composed of a white-Gaussian noise due to noise process ( $b_{PR}$ ).

### GPS/IRS Kalman filtering

As shown in figure 2, the position output by the whole system is the corrected inertial position. The role of Kalman filters is to estimate inertial errors using GPS measurements in order to correct inertial outputs.

### Kalman filtering state model

The dynamical evolution of the system is given by inertial error model equations. The measurement vector consists of the difference between two PRs to each satellite (GPS PR and predicted PR computed with inertial data).

Each component  $\delta x$  of the Kalman filter state vector stands for the difference between the true value  $x$  and the measured  $\tilde{x}$  (or computed  $\hat{x}$ ) value.

The state vector is a 17 error state variable

$$\Delta \mathbf{X} = [\boldsymbol{\rho}, \delta \mathbf{v}, \delta \mathbf{p}, \delta \boldsymbol{\omega}, \delta \mathbf{f}, \delta \mathbf{b}]^T$$

where

- $\boldsymbol{\rho}$  Attitude error vector- 3 states
- $\delta \mathbf{v}$  Inertial velocity error vector- 3 states
- $\delta \mathbf{r}$  Inertial position error:
  - $\delta L$  latitude error
  - $\delta G$  longitude error
  - $\delta h$  altitude error
- $\delta \boldsymbol{\omega}$  Gyro drift – 3 states
- $\delta \mathbf{f}$  Accelero bias – 3 states
- $\delta \mathbf{b}$  Receiver clock error vector- 2 states

Many different inertial error models are available in literature. They are actually equivalent [5]. In our system the inertial navigation error model applied is

$$\begin{aligned} \dot{\boldsymbol{\rho}} &= \delta \boldsymbol{\omega} + \boldsymbol{\omega}_{in}^n \times \boldsymbol{\rho} \\ \delta \dot{\mathbf{v}} &= \delta \mathbf{f} + (\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{in}^n) \times \delta \mathbf{v} + \boldsymbol{\rho} \times \mathbf{a}_{mt}^n \\ \delta \dot{\mathbf{r}} &= \delta \mathbf{v} + \boldsymbol{\omega}_{en}^n \times \delta \mathbf{r} \end{aligned} \quad (1)$$

where

- $\mathbf{a}_{mt}^n$  accelerometric measurement: inertial non-gravitational acceleration of the mobile in the mobile frame
- $\boldsymbol{\omega}_{in}^n$  rotation rate of the navigation frame relative to the inertial frame in the navigation frame
- $\boldsymbol{\omega}_{ie}^n$  Earth rotation rate in the navigation frame
- $\boldsymbol{\omega}_{en}^n$  rotation rate of the navigation frame relative to the Earth frame in the navigation frame

Using inertial data as nominal trajectory these non-linear equation are linearized and lead to the matrix presentation of the dynamical evolution equation of the linearized Kalman filter [6]:

$$\dot{\mathbf{X}} = \mathbf{F} \cdot \mathbf{X} + \mathbf{v} \quad (2)$$

where

- $\mathbf{F}$  is the state transition matrix
- $\mathbf{v}$  is the state noise vector. It is a zero-mean Gaussian white noise vector, whose components are all independent

Let  $\Phi$  be the discrete form of  $\mathbf{F}$ .

Let  $\mathbf{Q}$  be the state noise covariance matrix:

$$\mathbf{Q} = E[\mathbf{v} \cdot \mathbf{v}^T] \quad (3)$$

### Kalman filtering measurement model

Each component of the measurement vector  $\mathbf{z}$  at filter input is the difference between two PRs to each satellite. One is the measured PR input from the GPS receiver; the other is the predicted PR computed on the basis of the satellite positions obtained from the GPS receiver and the user location as calculated by the IRS.

The coefficients of the measurement matrix  $\mathbf{H}$  are direction cosines computed from the GPS navigation equation linearization. Let  $\mathbf{R}$  be the measurement noise covariance matrix: all measurement noises are independent.

The size of the  $\mathbf{z}$  – vector and of the  $\mathbf{R}$  and  $\mathbf{H}$  – matrices depends on the number of tracked satellite.

### Kalman filters implementation

Let  $ij$  be the subscript for the Kalman filter  $F_{ij}$ . The maximum number of tracked satellites is  $N = 10$ . There is one primary filter,  $N$  sub-filters, and  $N*(N-1)$  sub-sub-filters that are running in parallel.

The estimated state vector for each are

primary filter:  $\hat{\mathbf{x}}_{00}$

sub-filter:  $\hat{\mathbf{x}}_{0n}, n = 1, N$

sub-sub-filter:  $\hat{\mathbf{x}}_{nm}, n = 1, n; m = 1, N; m \neq n$

The estimation error covariance matrix for each is

primary filter:  $\mathbf{P}_{00}$

sub-filter:  $\mathbf{P}_{0n}, n = 1, N$

sub-sub-filter:  $\mathbf{P}_{nm}, n = 1, n; m = 1, N; m \neq n$

where

$$\mathbf{P}_{ij} = E[\delta\mathbf{x}_{ij} \cdot \delta\mathbf{x}_{ij}^T], \delta\mathbf{x}_{ij} = \hat{\mathbf{x}}_{ij} - \mathbf{x}$$

$\Delta\mathbf{x}$  is the true error between estimated IRS and true position:  $\Delta\mathbf{x} = \hat{X}_{IRS} - X$

Because GPS and IRS data are not available at the same rate, the error model is updated at 100Hz and Kalman filters measurements are updated at 1Hz. So the Kalman corrections are available every second.

Next section details implementation of FDE algorithm using Kalman filters parameters.

### III. FAULT DETECTION AND EXCLUSION FUNCTION WITH KALMAN FILTERS

#### Fault detection function

At each estimation time the discriminator for the  $n^{\text{th}}$  sub-filter is based on Solution Separation vector between the primary filter and the  $n^{\text{th}}$  sub-filter:

$$\begin{aligned} \mathbf{dx}_{0n}^+(k) &= \hat{\mathbf{x}}_{00}^+(k) - \hat{\mathbf{x}}_{0n}^+(k) \\ &= \delta\mathbf{x}_{00}^+(k) - \delta\mathbf{x}_{0n}^+(k), n = 1 \dots N \end{aligned} \quad (4)$$

whose statistics are described by the covariance matrix

$$d\mathbf{P}_{0n}^+(k) = E\left[\left(\mathbf{dx}_{0n}^+(k)\right) \cdot \left(\mathbf{dx}_{0n}^+(k)\right)^T\right] \quad (5)$$

with  $^+(k)$  stands for a posteriori state vector estimate at time  $k$ .

The statistic of the horizontal separation vector  $\mathbf{X}_H = \mathbf{dx}_{0n}^+[7:8]$ , is described by  $\mathbf{L}_{0n} = d\mathbf{P}_{0n}^+[7:8,7:8]$ .

Because separations on North and East axes are correlated,  $\mathbf{L}_{0n}$  is projected in an orthogonal plane so that,

$$\mathbf{L}_{0n} = \mathbf{P}_{\perp} \cdot \cdot \mathbf{P}_{\perp}^T \quad (6)$$

and  $\mathbf{X}_{\perp} = \mathbf{P}_{\perp}^T \cdot \mathbf{X}_H$  is a Gaussian vector whose covariance matrix is the diagonal matrix (eigenvalues matrix of  $\mathbf{L}_{0n}$ ).

One of the two eigenvalues is dominating. Let  $\lambda^{dp}$  be the variance ( (1,1) or (2,2) ) of the dominating error in the horizontal plane.

For each sub-filter the decision threshold -  $D_{0n}$  - is set so that the test metric  $d_{0n}$  will exceed  $D_{0n}$  with the probability  $P_{FD}$ . Since all sub-filters have the same chance of false alarm

$$D_{0n} = \sqrt{\lambda^{dp}} \cdot Q^{-1}\left(\frac{P_{FD}}{2N}\right) \quad (7)$$

with

$$Q^{-1}(u) = \text{erfc}(u)$$

$P_{FD}$  is the false detection probability [see Table 1].

$$\text{and } d_{0n} = |X_{\perp}(1)| \text{ or } |X_{\perp}(2)| \quad (8)$$

depending on  $\lambda^{dp} = (1,1)$  or  $\lambda^{dp} = (2,2)$ .

Indeed  $\mathbf{L}_{0n}$  is degenerated, the test metrics norm value is only expressed in  $|X_{\perp}(1)|$  or  $|X_{\perp}(2)|$ .

The test for detecting one satellite failure is

- $H_0$  no detection:  $d_{0n} < D_{0n}$
- $H_1$  detection:  $\exists n \in \{1, N\}; d_{0n} \geq D_{0n}$

[test 1]

#### Horizontal protection level HPL

By definition  $HPL$  is the error bound that contains the primary filter error with a probability of  $1 - p_{MD}$  when  $d_{0n} = D_{0n}$  [2].

Since for each sub-filter

$$\delta \mathbf{x}_{0n}^+(k) = \mathbf{d} \mathbf{x}_{0n}^+(k) + \delta \mathbf{x}_{0n}^+(k), n = 1 \dots N \quad (9)$$

at detection (for the  $n^{\text{th}}$  subset)

$$HPL_n = D_{0n} + a_{0n} \quad (10)$$

where  $a_{0n}$  is an upper bound of the  $n^{\text{th}}$  sub-filter horizontal error  $\Delta \mathbf{x}_{0n}^+[7:8]$  whose distribution is described by  $\mathbf{L} = \mathbf{P}_{0n}^+[7:8,7:8]$ .

Let  $\lambda^{P_{0n}}$  be the maximum eigenvalue of  $\mathbf{L}$

$$a_{0n} = \sqrt{\lambda^{P_{0n}}} \cdot Q^{-1}(1 - p_{MD}) \quad (11)$$

Besides we have to consider the rare normal performance that is to say the contribution of the primary filter  $HPL_0$

$$HPL_0 = \sqrt{\lambda^{P_{00}}} \cdot Q^{-1}\left(\frac{P_{ff}}{2}\right) \quad (12)$$

where  $P_{ff}$  is the rare normal performance rate and  $\lambda^{P_{00}}$  the maximum eigenvalue of  $\mathbf{P}_{00}^+[7:8,7:8]$ .

The horizontal protection level is

$$HPL = \max\{HPL_0, \max(HPL_n)\}, n = 1, N \quad (13)$$

### Fault exclusion function

Fault exclusion is accomplished in the same manner than detection but one layer down in the filters hierarchy.

The discriminator of each of the sub-sub-filter is the separation between each sub-sub-filter estimate solution and its parent's sub-filter estimate solution in the horizontal plane. For the  $n^{\text{th}}$  sub-filter they are

$$\begin{aligned} \mathbf{d} \mathbf{x}_{nm}^+(k) &= \hat{\mathbf{x}}_{0n}^+(k) - \hat{\mathbf{x}}_{nm}^+(k) \\ &= \delta \mathbf{x}_{0n}^+(k) - \delta \mathbf{x}_{nm}^+(k), m = 1 \dots N, m \neq n \end{aligned} \quad (14)$$

whose statistics are described by the covariance matrix  $\mathbf{d} \mathbf{P}_{nm}^+(k)$ .

Since the  $N$  sub-systems formed by the  $N$  sub-filters are tested in an independent manner, for each sub-sub-filter the decision threshold is set so that (assuming that each filter  $F_{nm}$  has the same chance for a false alarm)

$$D_{nm} = \sqrt{\lambda^{dP_{nm}}} \cdot Q^{-1}\left(\frac{P_{FD}}{2(N-1)}\right) \quad (15)$$

The test statistic associated to  $D_{nm}$  is

$$d_{nm} = |X_{m\perp}(1)| \text{ or } |X_{m\perp}(2)| \quad (16)$$

Satellite  $r$  is excluded as the failed satellite if and only if [test 2]

$$d_{rm} < D_{rm} \text{ for all } m \neq r$$

and

$$d_{nr} \geq D_{nr} \text{ for all } r \neq n$$

### Horizontal exclusion level HEL

*HEL* is the error bound that contains the primary filter error with a probability of  $1 - p_{FE}$  when the failure is excluded.

“For a failed exclusion to occur, the solution of one of the sub-filters  $\hat{\mathbf{x}}_{0n}$  which does contain the failed satellite must be separated from one of its sub-sub-filter's solution  $\hat{\mathbf{x}}_{nm}$  by less than the threshold  $D_{nm}$  plus the sub-sub-filter position error” [2]

Let  $a_{nm}$  be the sub-sub-filter horizontal estimation error bound and  $\lambda^{P_{nm}}$  be the maximum eigenvalue of  $\mathbf{P}_{nm}^+[7:8,7:8]$ :  $a_{nm} = \sqrt{\lambda^{P_{nm}}} \cdot Q^{-1}(1 - p_{FE})$ .

For the  $n^{\text{th}}$  sub-filter *HEL<sub>n</sub>* is given by

$$HEL_n = \max\{D_{nm} + a_{nm}\}, m = 1, N \quad (17)$$

The horizontal exclusion level is

$$HEL = \max\{HEL_n\}, n = 1, N \quad (18)$$

as defined in [9]

## IV. TWO-FAILURE DETECTION AND EXCLUSION WITH A ONE-FAILURE ASSUMPTION FDE ALGORITHM – SIMULATION RESULTS

### Failures impact on test metrics of row 2

Let  $b$  be the pseudo range bias vector when measurements on channel  $i$  and  $j$  are affected by bias of value  $b_i$  and bias  $b_j$  respectively

$$b = \begin{bmatrix} 0 & \dots & 0 & b_i & 0 & \dots & 0 & b_j & 0 & \dots & 0 \end{bmatrix}^T,$$

Since  $\mathbf{d} \mathbf{x}_{nm}$  and  $\delta \mathbf{x}_{0m}$  are statically dependent, let us form the 34x1 error vector

$$\delta \mathbf{x}^+(k) = [\delta \mathbf{x}_{0m}^+(k), \mathbf{x}_{nm}^+(k)]^T$$

whose statistics are described by  $\mathbf{d} \mathbf{P}_{0n}^{\text{dual}+}(k)$

One can show that the impact of  $b$  (at its first time of apparition) on the test metrics which links sub-filter  $F_{0m}$  to sub-sub-filter  $F_{mn}$ , with  $m = 1 \dots N, m \neq n$ , is

$$\delta x^+(k) = \Sigma(k) \cdot \delta x^-(k) + \Gamma(k) \cdot w_{0m}(k) + \Gamma(k) \cdot b(k)$$

with

$$\Sigma = \begin{bmatrix} I - \mathbf{K}_{0m} \cdot \mathbf{H}_{0m} & 0 \\ (\mathbf{K}'_{mn} - \mathbf{K}_{0m}) \cdot \mathbf{H}_{0m} & I - \mathbf{K}'_{mn} \cdot \mathbf{H}_{0m} \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} \mathbf{K}_{0m} \\ \mathbf{K}_{0m} - \mathbf{K}'_{mn} \end{bmatrix}$$

$\mathbf{K}_{mn}$  is the Kalman gain of filter  $F_{mn}$ .

$\mathbf{K}'_{mn}$  is constructed from  $\mathbf{K}_{mn}$  so that it operates on the full set of measurements by extending the  $17 \times (N-1)$  matrix  $\mathbf{K}_{mn}$  sub-solution matrix with a  $n^{\text{th}}$  zeroed column.

$$\text{Since } E[\delta x^+(k) \cdot (\delta x^+(k))^T] = P_{mn}^{dual}(k),$$

with,  $P_{mn}^{dual}(k) = \Sigma(k) \cdot (P_{mn}^{dual}(k)) \cdot (\Sigma(k))^T + \Gamma(k) \cdot R_{0m}(k) \cdot (\Gamma(k))^T$ , when the two simultaneous range failures assumption is made we may theoretically expect that there is no sub-filter  $F_{0m}$  for which there is no test metrics  $d_{mn}$  which won't exceed its threshold.

In other words we may expect that there is no sub-filter  $F_{0m}$  for which two simultaneous range failures have no impact at row 2 of Kalman filters.

For the following we denote this state as "simultaneous impact" [figure 3].

Let assume that SAT1 and SAT2 are at channels 2 and 4

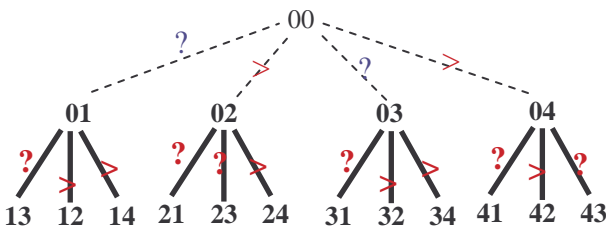


Figure 3. Test metrics vs decision threshold: simultaneous impact.

Yet simulations we have conducted have put the stress on the "transition states" which may exist.

A reason for this is that under no-failure condition, all test metrics involved at rows 1 and 2 in the Kalman filters hierarchy are lower than their respective decision threshold [see figure 4]. The presence of the two failures implied these test metrics to become progressively larger than their decision thresholds and this even for the sub-filters which exclude one of the two failures.

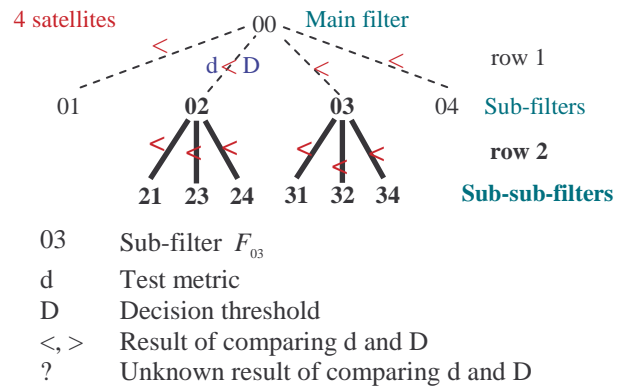


Figure 4. Test metrics vs decision threshold: no-failure case.

Depending on the constellation and the range failures size value, due to noise process in GPS measurements, the impact of one of the two failures may be delayed such as there is one of the sub-filters for which tests metric at row 2 are all inferior to their decision threshold.

In that case if condition of [test 2] is fulfilled, the satellite tested by this sub-filter may be excluded from the constellation [see figure 5].

It is similar to two successive one range failure assumptions.

For the following we denote this phenomenon as "delayed impact".

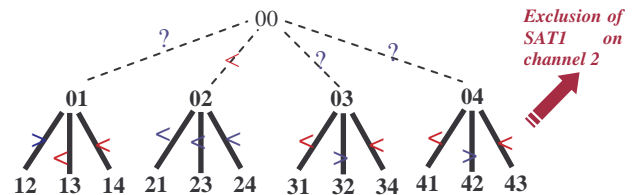


Figure 5. Test metrics vs decision threshold: delayed impact.

Yet, there is a risk to exclude a wrong satellite, but we may assume it is greatly reduced since in [test 2] we identify the faulty satellite for each of the subset which contains a detection alarm.

### Simulations background

#### Failures setting

Two ramp failures are added to the measurements of the worst two case satellites; that is to say the pair of tracking satellites for which the two ramps will produce the greatest SLOPE: SLOPEmaxmax. The calculation of SLOPEmaxmax is performed according [7].

Let SAT1 and SAT2 be the faulty satellites.

#### Civil aviation requirements

Appendix R to RTCA/DO 229C [3] clarifies FDE requirements to GPS/IRS application for en-route to non-precision approach. (Table 1)

Missed detection probability	0.001
False detection rate (SA off)	$1/3 \cdot 10^{-6}/hr$
Probability ( $p_{MI}$ ) of exceeding HPL	$10^{-7}/hr$
Rare normal performance rate	$<10^{-7}/sample$
Failed exclusion probability	0.001
Continuity	$1/3 \cdot 10^{-6}/hr$

Table 1. FDE requirements.

These requirements are defined assuming that there is only one satellite failure at a time.

*Simulation conditions*

- GPS constellation: mask angle  $5^\circ$  (no change)
- GPS receiver: total measurement noise  $12.5m$  ( $1\sigma$ )
- Inertial error sources

These errors are added on accelero and gyro measurements at the IMU output

Gyrometer constant bias	$0.01 \text{ }^\circ/hr$
Gyrometer white noise	$3 \cdot 10^{-5} \text{ }^\circ/s/\sqrt{Hz}$
Accelerometer constant bias	$50 \text{ } \mu g$
Accelerometer white noise	$6.4 \text{ } \mu g/\sqrt{Hz}$

Table 2. Good IRS error sources

- GPS/IRS hybridization is performed in an open-loop manner.

FDE algorithm

- No. of independent tests/hr=4
- $P_{FA} = 1/3 \cdot 10^{-6} / hr = 1/12 \cdot 10^{-6} / test$
- $P_{ff} = 1/3 \cdot 10^{-8} / test$
- ramp errors (m/s) are added simultaneously at time  $t_0$  on satellites SAT1 and SAT2.

We are interesting in the Horizontal error.

*Simulations results*

*Delayed failure impacts*

In the case in which failures impact is delayed, failures behave such as there were two successive failures. The classical Solution Separation FDE algorithm is able to detect and isolate them.

To illustrate this, the following simulation was performed:

- Table 3: simulation hypothesis
- Figure 6 and Table 4: simulation results on position error

Number of tracked satellites = 9, $t_0 = 300s$		
SAT1	channel: 3	error size: 1m/s
SAT2	channel: 9	error size: 0.7m/s

Table 3: Simulation hypothesis: delayed failure impacts

	delay after $t_0$	
	detection	isolation
SAT1	47 s	51 s
SAT2	69 s	74 s

Table 4: Time of detection and isolation

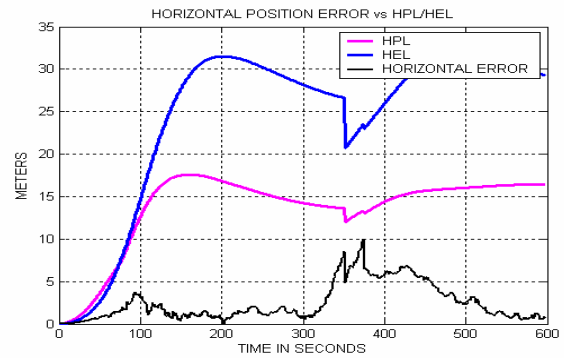


Figure 6: Horizontal position error vs HPL/HEL

Despite the success of isolating the two faulty satellites, the impact of the faulty measurements is not negligible on the horizontal position solution.

After isolation of SAT1, the error size is of 5 m and is getting growing due to a miss-initialization of Kalman filters. Indeed,  $F_{00}$  is initialized with  $F_{0SAT1}$  which is still running with SAT2 faulty measurements. This corrupted initialization is creating an additional error on position solution; this error is growing to 10m until isolation of SAT2.

At time 373, the horizontal error size is reduced to 6.5m. Despite no Monte-Carlo simulations have been conducted, simulations have shown that the error size after isolation greatly depends on the constellation. In fact long-duration simulations – with fixed-positioning trajectory assumptions – have shown that the delay after isolation for Kalman filter re-convergence depends on constellation.

*Simultaneous failure impacts*

In the more rare case in which failures impact is simultaneous, the classical Solution Separation FDE algorithm is no more able to isolate them. The first row of Kalman filters is effectively detecting failures, but the test at the second row can't find any sub-filter for which all the tests metric between it and its parents sub-sub-filters are under their decision threshold.

This is a failed exclusion case that leads to a true alert [8].

To illustrate this, the following simulation was performed:

- Table 5: simulation hypothesis
- Figure 7 and Table 6: simulation results on position error

Number of tracked satellites = 9, $t_0 = 300s$		
SAT1	channel: 3	error size: 0.7m/s
SAT2	channel: 9	error size: 1m/s

Table 5: Simulation hypothesis: simultaneous failure impacts

Delay after $t_0$	detection: 60s	isolation: none
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Table 6: Time of detection



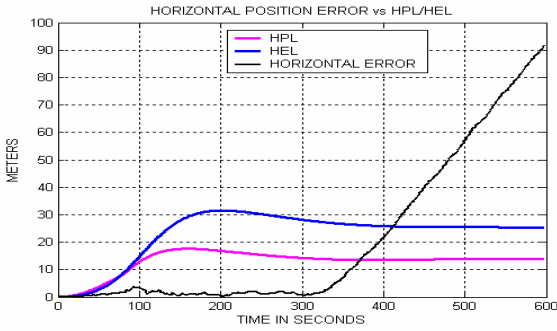


Figure 7: Horizontal position error vs HPL/HEL

This impossibility to isolate the two failures may be solved by adding one more Kalman filters layer: sub-sub-sub-filters which each one is excluding measurements from 3 satellites of the constellation [see figure 8]

## V. PRINCIPLE OF ISOLATING TWO FAILURES

### Principle

Isolation of two failures is performed by estimating the solution separations between sub-sub-filters and sub-sub-sub-filters -  $\mathbf{dx}_{mnp}$ , forming test metrics -  $d_{mnp}$ , and comparing them with their correspondent decision threshold -  $D_{mnp}$ .

In the nominal case, the entire tests results are negative. When there are two failures, there is no sub-filter for which all the tests metric between it and its parents' sub-sub-filters are under their decision threshold. But, there exists only two sub-sub-filters  $F_{nm}$  and  $F_{mn}$  for which all separations between their solution and their sub-sub-sub-filters solution are under the corresponding thresholds. Thus satellites  $n$  and  $m$  are claimed as the faulty satellites.

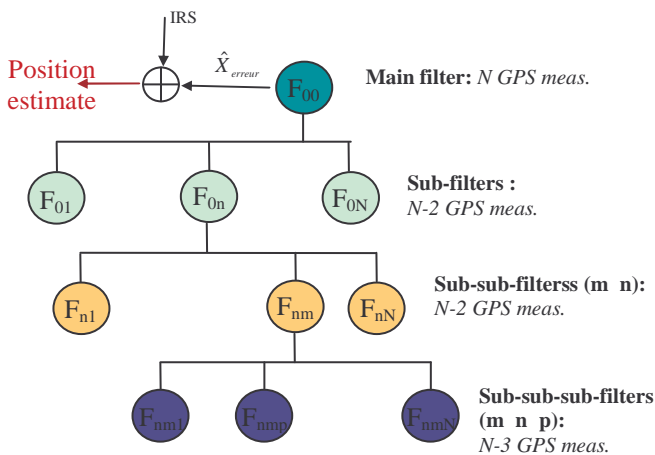


Figure 8. New Kalman filters hierarchy

### Test data definition

Let  $\hat{\mathbf{x}}_{mnp}$  be the state vector of sub-sub-sub-filter  $F_{mnp}$ , separation between  $F_{mn}$  and  $F_{mnp}$  is given by

$$\begin{aligned} \mathbf{dx}_{mnp}^+(k) &= \hat{\mathbf{x}}_{mn}^+(k) - \hat{\mathbf{x}}_{mnp}^+(k) \\ &= \delta\mathbf{x}_{mn}^+(k) - \delta\mathbf{x}_{mnp}^+(k), \end{aligned} \quad (19)$$

$$p = 1 \dots N, m \neq n, p \neq m, p \neq n$$

whose statistics are described by the covariance matrix  $\mathbf{dP}_{mnp}^+(k)$ .

For each sub-sub-sub-filter the decision threshold is set so that (assuming that each filter  $F_{mnp}$  has the same chance for a false alarm)

$$D_{mnp} = \sqrt{\lambda^{d_{mnp}}} \cdot Q^{-1} \left( \frac{P_{FD}}{2(N-2)} \right) \quad (20)$$

The next paragraph discuss with the use of  $P_{FD}$ .

The test statistic associated to  $D_{mnp}$  is

$$d_{mnp} = |X_{p\perp}(1)| \text{ or } |X_{p\perp}(2)| \quad (21)$$

Satellites  $r$  and  $s$  are excluded as the failed satellites if and only if [test3]

$$d_{rsm} < D_{rsm} \text{ for all } m \neq r, m \neq s$$

$$\text{and } \forall i \neq r, \forall j \neq s, \exists k / d_{i,j,k} \geq D_{i,j,k}$$

and

$$d_{srn} < D_{srn} \text{ for all } m \neq r, m \neq s$$

$$\text{and } \forall i \neq s, \forall j \neq r, \exists k / d_{i,j,k} \geq D_{i,j,k}$$

## VI. FAILURES DETECTION AND EXCLUSION WITH A TWO-FAILURE ASSUMPTION FDE ALGORITHM

### Process principle

As for one-failure FDE algorithm, the new algorithm discussed below is based on hypothesis testing

$$H_0 \quad \text{Normal case: no range fault}$$

$$H_1 \quad \text{Abnormal case: one range failure}$$

$$H_n, n > 1 \quad \text{Abnormal case: more than one range failure}$$

The following process is based on the experimental assumption that when there are two simultaneous range failures; filter solution which is using the two faulty measurements is far more affected by the failures than filter solution which is using only one faulty measurement. Thus the impact of the second failure is such that the test metric between a one-failure filter and a two-failure filter is always exceeding the threshold (in the same manner that, the test metric between a one-failure filter and a zero-failure filter is always exceeding the threshold).

(i) Failures detection

Detection occurs at the first row, but there is no decision taken whether there are one or two range failures. The hypothesis test is

$$\begin{aligned} H_0 & \text{no detection: } d_{0n} < D_{0n} \\ H_1, H_n, n > 1 & \text{detection: } \exists n \in \{1, N\}, d_{0n} \geq D_{0n} \end{aligned}$$

For the following points the assumption that detection occurs is made.

(ii) *One-failure  $H_1$  identification and exclusion*

The identification of  $H_1$  assumption and exclusion of the faulty satellite occurs at row 2. It is performed with the test described in section 3 of one-failure exclusion.

Satellite  $r$  is excluded as the faulty satellite if and only if

$$\begin{aligned} d_{rm} < D_{rm} & \text{ for all } m \neq r \\ \text{and} \\ d_{nr} \geq D_{nr} & \text{ for all } r \neq n \end{aligned}$$

[see figure 4]

(iii) *More than one failure  $H_n$  identification*

The identification of  $H_n$  assumption also occurs at row 2

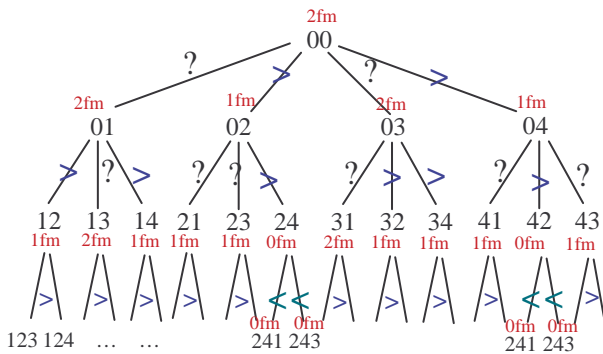
$$H_n, n > 1 \quad \forall n, \exists r \neq n / d_{nr} \geq D_{nr}$$

In the two-failure assumption, this test is used as identification of  $H_2$ .

(vi) *Two failures exclusion*

The exclusion of the two faulty satellites occurs at row 3 according [test3].

Figure 9 is illustrating what we theoretically expect when there are two failures (points (iii) and (iv)). Let assume that SAT1 and SAT2 are at channels 2 and 4.



**0fm** Kalman filter using no faulty measurement  
**1fm** Kalman filter using one faulty measurement  
**2fm** Kalman filter using two faulty measurements

Figure 9. Two failures theoretical impact on tests metric.

**Discussion on the use of the FDE requirements**

Setting the thresholds

*One-failure exclusion function*

In [2], the author defines exclusion threshold as a function of  $P_{FD}$  [see equation 14] since the detection function has to be available after exclusion.

Indeed, after exclusion, the main filter and the sub-filters are re-initialized with the safety sub-filter and sub-sub-filters respectively. These last sub-sub-filters are then dedicated to detection of a next failure.

*Two-failure exclusion function*

The same idea has been applied to set the thresholds for the tests at the third layer of Kalman filters [see equation 20]. After exclusion of the two faulty satellites, the main filter and the sub-filters are re-initialized with the safety sub-sub-filter and sub-sub-sub-filters respectively.

Definitions of  $HPL$

The  $HPL$  may be defined as a function of the Integrity Risk requirement ( $P_{MI}$ )

$$P[HE > HPL \text{ no alert}] < P_{MI}$$

where  $HE$  is the horizontal error; and “no alert” means “without an alert during time to alert”.

(for phases of flight from en-route to  $NPA$ ,  $P_{MI} = 10^{-7} / hr$  and the time to alert is 10s).

Using  $H_0, H_1, H_2$ , the former expression may be written

$$P \left\{ \begin{aligned} & (HE > HPL \text{ no alert} \cap H_0) \\ & OR (HE > HPL \text{ no alert} \cap H_1) \\ & OR (HE > HPL \text{ no alert} \cap H_2) \end{aligned} \right\} < P_{MI}$$

which is similar to

$$\begin{aligned} & P(HE > HPL \text{ no alert} | H_0) \cdot P(H_0) \\ & + P(HE > HPL \text{ no alert} | H_1) \cdot P(H_1) \\ & + P(HE > HPL \text{ no alert} | H_2) \cdot P(H_2) < P_{MI} \end{aligned}$$

As for the one-failure assumption the following allocation is made

- i.  $P(HE > HPL \text{ no alert} | H_0)$  is the rare normal performance rate
- ii.  $P(H_1)$  is the probability of having one GPS satellite failure per hour per aircraft
- iii.  $P(HE > HPL \text{ no alert} | H_1)$  is the missed detection probability in the case when there is only one satellite failure
- iv.  $P(H_2)$  is the probability of having two GPS range failures per hour per aircraft

v.  $P(HE > HPL_{no\ alert} | H_{21})$  is the missed detection probability in the case when there are two range failures

The difficulty is that we need to allocate values to these probabilities.

The acknowledgment of these may allow us to give a definition of the global  $HPL$ , which would be determined by the following ways

1. we calculate  $HPL_0$  value from (i)
2. we calculate  $HPL_1$  value from (iii)
3. we calculate  $HPL_2$  value from (v)

with

$HPL_0$  Is the value of  $HPL$  for which in the fault free case,

$$P(HE > HPL_0) = P(HE > HPL_{no\ alert} | H_0)$$

$HPL_1$  Is the value of  $HPL$  for which in the one range failure case,

$$P(HE > HPL_1) = P(HE > HPL_{no\ alert} | H_1)$$

$HPL_2$  Is the value of  $HPL$  for which in the two range failures case,

$$P(HE > HPL_2) = P(HE > HPL_{no\ alert} | H_2)$$

Since the global  $HPL$  has to satisfy these three conditions, in a first approach it would be defined as

$$HPL = \max(HPL_0, HPL_1, HPL_2)$$

$HPL_0, HPL_1, HPL_2$  can be analytically calculated using  $P_{00}^+ [7:8,7:8]$ ,  $P_{0n}^+ [7:8,7:8]$  and  $P_{nm}^+ [7:8,7:8]$  [see section 3].

At the current time, no value is calculated for  $HPL_2$  as  $P(H_2)$  is not allocated and requires further discussion.

### Definitions of $HEL$

In the one-failure assumption, by adding “the requirement that detection must be possible after exclusion has occurred [...]  $HEL$  is given by the largest  $HPL$  of the subsets of  $(N-1)$  satellites” [9] [see Chapter 3]. Let us denote it  $HEL_1$ .

In the case of multiple-failure assumption we could also calculate  $HEL_2$ .

$HEL_2$  would be given by the largest  $HPL$  of the subsets of  $(N-2)$  satellites under the condition that detection must be possible after exclusion has occurred.

And  $HEL$  would be the maximum of  $HEL_1, HEL_2$ .

The definition of  $HEL_2$  and so forth  $HEL$  is only assumption, and requires further analysis.

### Simulation results

With similar assumptions than those taken at chapter IV, we get the following result with the process presented above

- Table 7: simulation hypothesis
- Figure 10 and Table 8: simulation results on position error

The process succeeds in isolating the two faulty satellites and after isolation the main Kalman filter is re-initialized with  $F_{39}$ .

Number of tracked satellites = 9, $t_0 = 300s$		
SAT1	channel: 3	error size: 0.7m/s
SAT2	channel: 9	error size: 1m/s

Table 7: Simulation hypothesis

Delay after $t_0$	
Failures detection	60s
Identification of $H_2$	70s
Exclusion	118s

Table 8: Times of detection and isolation

We can notice that the identification of the presence of two failures occurs soon after the algorithm is detecting failures.

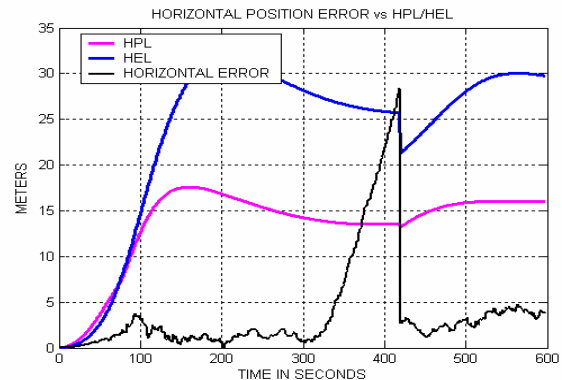


Figure 10: Horizontal position error vs HPL/HEL

$HPL$  and  $HEL$  which are presented on this figure are  $HPL$  computed at the first row of filters hierarchy and  $HEL$  calculated at the second row. They are standing for the one-failure FDE algorithm levels and are not representative of the whole process. As mentioned in chapter VI, we have not clearly defined a global  $HPL/HEL$ , which could be compared to  $HAL$  in order to appreciate the availability of the process.

Yet the horizontal error at the output of the system is very bellow the  $HAL$  defined for the non-precision approach phase of flight (556m), and thus it can't involve an alert.

### VII. SUMMARY AND CONCLUSION

Despite the fact that the occurrence of multiple simultaneous range failures is a roughly unlikely event when failures are independent, it can become more important in the case of jamming.

This paper presents a hypothesis test which is being investigated at the ENAC in order to use the GPS/IRS hybridization Solution Separation integrity monitoring method under the assumption that two simultaneous range failures can occur. The paper also proposes the integration of this test with the current one-failure FDE algorithm to form three test levels.

Practically, the detection of failures occurs at the first level.

In case of one failure, the exclusion of the faulty satellite - as well the identification of the one-failure hypothesis - is performed at the second level.

In case of two failures, due to the fact that the FDE process is based on hypothesis testing, the simulations which were conducted have shown that the failures may have simultaneous or delayed impact on the test metrics. In the first case, the tests allow identifying the two-failure hypothesis at the second level, and the two faulty measurements are excluded with the tests at the third level. In the second case, the tests at the second level allow identifying two successive failures and excluding them.

Thus the FDE function can be performed either by a one-failure assumption or by a two-failure assumption algorithm.

The definitions of global *HPL* and *HEL* are not clearly laid down and doesn't allow us to completely appreciate the performance of the process.

## REFERENCES

1. M. BRENNER, Integrated GPS/inertial detection availability, Journal of The Institute of Navigation, Sept. 1995.
2. K. VANDERWERF, FDE Using Multiple Integrated GPS/Inertial Kalman Filters in the Presence of Temporally and Spatially Correlated Ionospheric Errors, ION GPS 2001, 11-14 Sept. 2001, Salt Lake City
3. Requirements and Test Procedures for Tightly Integrated GPS/Inertial Systems Appendix R to DO-229C
4. J. C. RADIX, Systèmes Inertiels à Composants Liés "Strapdown", Cépadues Educus, 1991
5. R. DA, Investigation of a Low-Cost and High-Accuracy GPS/IMU System, Proceedings of ION GPS, Sept. 1996
6. FARRELL J. A., BARTH M., The Global Positioning System & Inertial navigation, McGraw-Hill, 1998
7. BROWN R. G., Solution of the Two-Failure GPS RAIM Problem Under Worst-Case Bias Conditions: Parity Space Approach, navigation Journal of the Institute of Navigation, Vol. 44, No. 4, Winter 1997-1998
8. RTCA/DO-229, Minimum operational performance standards for global positioning system / wide area

augmentation system airborne equipment RTCA SC-159, January 16, 1996

9. LEE Y., VAN DYKE K., DECLEENE B., STUDENNY J., BECKMAN M., Summary of RTCA SC-159 GPS Integrity Working Group Activities, Global Positioning System - Papers published Navigation - Redbook Volume V, 1998, p195-226