Non-Linear Filtering Approaches for INS/GPS Integration

Audrey Giremus, Arnaud Doucet, Anne-Christine Escher, Jean-Yves Tourneret

NONLINEAR FILTERING APPROACHES FOR INS/GPS INTEGRATION

Audrey Giremus*, Arnaud Doucet†, Anne-Christine Escher* and Jean-Yves Tourneret*

* IRIT/ENSEEIHT/TéSA, 2 rue Charles Camichel, BP 7122, 31071 Toulouse cedex 7, France
† Signal Processing Laboratory, University of Cambridge, Trumpington Street, CB2 1PZ, Cambridge, UK
* ENAC, 7 av Edouard Belin, BP 54005 Toulouse Cedex 4, France

ABSTRACT

Navigation with an integrated INS/GPS approach requires to solve a set of nonlinear equations. In this case, nonlinear filtering techniques such as Particle Filtering methods are expected to perform better than the classical, but suboptimal, Extended Kalman Filter. Besides, the INS/GPS model has a conditionally linear Gaussian structure. A Rao-Blackwellization procedure can then be applied to reduce the variance of the state estimates. This paper studies different algorithms combining Rao-Blackwellization and particle filtering for a specific INS/GPS scenario. Simulation results illustrate the performance of these algorithms. The variance of the estimates is also compared to the corresponding posterior Cramer-Rao bound.

1. INTRODUCTION

The global positioning system (GPS) has been extensively used in navigation because of its accuracy and worldwide coverage. GPS augmentations such as satellite-based augmentation systems (SBAS) and ground-based augmentation systems (GBAS) allow to improve the accuracy of the navigation solution. Indeed, ground stations estimate GPS measurement errors that are then transmitted to the GPS receiver thanks to a satellite constellation. However, many tracking channels can be affected simultaneously by interferences, which can result in a loss of the GPS signal. In this case, a self-contained system such as a calibrated inertial navigation system (INS) can ensure the continuity of navigation with a good accuracy.

The principles of the hybridization GPS/INS are recalled in section 1. Section 2 focuses on the nonlinear filtering model which is usually associated to satellite-based systems. Section 3 studies different appropriate estimation strategies for this nonlinear model. These strategies include the extended Kalman filter and particle filtering methods. Simulation results are presented in section 4. Conclusions are reported in the last section.

2. GPS/INS INTEGRATION

GPS is a satellite-based navigation system that provides precise positioning and timing information to any user properly equipped with a receiver. The basic principle is to estimate the user's position by processing distance measurements from the receiver to GPS satellites of known locations. The ranging information is determined from the propagation delay of a signal transmitted by the satellite and is therefore biased by the receiver clock offset with respect to the satellite time. Consequently, this measurement is called pseudorange (instead of range) to allow for this bias. Moreover, each satellite broadcasts useful information to solve the navigation problem: data relative to the satellite's current position and different parameters necessary to compensate for some GPS measurement errors (for instance, the satellite clock offset with respect to the GPS time and the atmospheric delays). Assuming these terms are reasonably well corrected, the user position can be obtained as the solution of the so-called pseudorange equations:

$$\rho_i = r_i + c\tau_r + w_i, \qquad 1 \leq i \leq n_s,$$

where ρ_i and r_i denote the pseudorange and the true geometric range from the receiver to the satellite, τ_r is the receiver clock offset from the GPS time, c is the speed of light (3×10^8m/s), w_i are the residual measurement errors and n_s is the number of visible satellites. Note that a minimum of four equations is required to estimate both the position in free space and the receiver clock offset.

Contrary to satellite-based navigation systems, INS are autonomous onboard navigation systems. They are based on inertial sensors (accelerometers and gyrometers) that measure the rectilinear acceleration and the inertial angular velocity of a vehicule according to the physical laws of motion. The sensor outputs are integrated to maintain an estimate of the vehicule dynamics, provided a precise knowledge of the initial position, velocity and attitude is available. Note that INS are robust to interferences because the inertial sensors do not depend on the reception of an exterior signal. INS consist of an inertial measurement unit containing the cluster of sensors, combined with a computer that determines the navigated trajectory. This paper focuses on strapdown systems, which are characterized by sensors rigidly attached to the host vehicule. The measurements are resolved in the vehicule frame. Consequently, coordinate transformations are required to express the measurements in a frame of interest for the user (usually the locally level frame with the X, Y and Z axis pointing respectively north, east and down).

The sensor outputs are sequentially processed to derive the navigation solution. The accelerometers deliver a non gravitational acceleration called specific force $\boldsymbol{f}_s$. This force is first transformed in an appropriate coordinate system, then compensated for the gravity, and finally double-integrated to yield the position. The gyrometers outputs are used to compute the attitude angles of the vehicule, hence to update the transformation between the vehicule frame and the desired reference frame. The equations relating the sensor outputs to the dynamic states estimated by the INS are referred to as *mechanization equations*. They will be denoted as $\dot{\boldsymbol{X}}_{\text{INS}} = f(\boldsymbol{X}_{\text{INS}}, \boldsymbol{U}_{\text{INS}})$, where $\boldsymbol{X}_{\text{INS}}$ is the vector of navigated states and $\boldsymbol{U}_{\text{INS}}$ is the vector of actual sensor outputs. In contrast, the *ideal equations* corresponding to the error-free sensor outputs and the actual dynamic states are written $\dot{\boldsymbol{X}} = f(\boldsymbol{X}, \boldsymbol{U})$. Due to the integration process, instrumentation and initialization errors result in a linear and parabolic growth of the velocity and position errors.

GPS and INS have complementary features. A data fusion approach can take full advantage of this synergism to improve the accuracy and reliability of the navigation solution. On the one hand, interferences do not affect INS sensors, but they may lead to a loss of GPS measurements. On the other hand, the navigation states are computed by a differentiation-based process for the GPS and an integrative-based process for the INS, which entails respectively high frequency and low frequency errors. Allowing for the distinct frequency content of GPS and INS errors, a complementary filter methodology can be applied advantageously: the GPS pseudoranges are used to estimate the inertial errors (rather than the *total* dynamic states). The system can still rely on the INS in case GPS measurements are lost.

3. THE GPS/INS NONLINEAR FILTERING MODEL

3.1 State model

In the proposed framework, the state vector is composed of the INS error that are defined as the deviation between the actual dynamic quantities and the INS computed values $\delta \boldsymbol{X} = \boldsymbol{X} - \boldsymbol{X}_{\text{INS}}$. The state model describes the INS error dynamic behavior depending on the instrumentation and initialization errors. It is obtained by linearizing the *ideal equations* around the INS estimates as follows:

$$\begin{aligned} \delta \dot{\boldsymbol{X}} &= f(\boldsymbol{X}, \boldsymbol{U}) - f(\boldsymbol{X}_{\text{INS}}, \boldsymbol{U}_{\text{INS}}) \\ \delta \dot{\boldsymbol{X}} &= \bigtriangledown f(\boldsymbol{X}_{\text{INS}}, \boldsymbol{U}_{\text{INS}}) \delta \boldsymbol{X}. \end{aligned}$$

The state vector is usually augmented with systematic sensor errors:

$$\boldsymbol{\delta X} = (\delta \boldsymbol{v}^n, \delta \boldsymbol{\rho}, \boldsymbol{b_a}, \boldsymbol{b_g}, \delta\lambda, \delta\phi, \delta h, b, d),$$

where

- $\boldsymbol{v}^n$ stands for the velocity relative to the earth centered/earth fixed frame and resolved in the locally level frame,
- $\boldsymbol{\rho}$ is the vector of attitude angles,
- (λ, ϕ, h) is the geodetic position in latitude, longitude, altitude,
- $\boldsymbol{b_a}$ and $\boldsymbol{b_g}$ represent the accelerometers and gyrometers biases,
- $b = c\tau_r$ and d are respectively the GPS clock offset and its drift.

A coarse analysis shows that the errors develop through a variety of sources. Horizontal position and velocity experience a pendulum-like motion called Schuler oscillation. In addition, the earth rotation introduces a cross-coupling between the horizontal dynamic components, hence a modulation of the previous oscillation (Coriolis effect at Foucault frequency). Supposing that the attitude angles are perfectly known, such a behavior is well described by:

$$\begin{aligned} \delta\ddot{\phi} &= -\Omega^2_{\text{Schuler}} \delta\phi + \Omega_{\text{Foucault}} \delta\dot{\lambda}, \\ \delta\ddot{\lambda} &= -\Omega^2_{\text{Schuler}} \delta\lambda. \end{aligned}$$

As for vertical channel, gravity compensation results in an unstable altitude error propagation of the form:

$$\delta h = \frac{1}{2}(e^{-kt} + e^{kt})\delta h_0,$$

where $k = \sqrt{\frac{2g}{R}}$, g being the gravity and R the earth radius. However, this paper assumes that the altitude h is known (thanks to an independent vertical reference such as a barometric altimeter).

Accurate models of the instrument biases are required to achieve a good localization performance. For short-term applications, the acceleremoters and gyrometers can be properly defined as random walk constants $\dot{\boldsymbol{b}}_a = \boldsymbol{w}_a$ and $\dot{\boldsymbol{b}}_g = \boldsymbol{w}_g$. Note that the standard deviations of the white noises $\boldsymbol{w}_a$ and $\boldsymbol{w}_g$ are related to the sensor quality. The navigation solution also depends on the receiver clock parameters b and d modeled as $\dot{b} = d + w_b$ and $\dot{d} = w_d$, where w_b and w_d are mutually independent zero-mean Gaussian random variables (whose variances can be determined by the Allan variance parameters [4]). For simplicity, denote as $\boldsymbol{X}$ (instead of $\delta\boldsymbol{X}$) the state vector. The discrete-time state model takes the following form:

$$\boldsymbol{X}_{t+1} = A_t \boldsymbol{X}_t + \boldsymbol{v}_t, \quad \boldsymbol{v}_t \sim \mathcal{N}(\boldsymbol{0}, \Sigma_v),$$

where $t = 1, \ldots, T$ and T is the number of samples. The coupling effects between the components of X_t results in a block diagonal matrix A_t whose elements are detailed in [5] or in many standard textbooks such as [4, p. 204]. Section 4 explains how this block diagonal structure for A_t can be used to reduce the computational complexity of the estimation algorithm.

3.2 Measurement model

The hybridization filter is driven by the GPS pseudoranges. Consequently, the observation equation associated to the *ith* satellite can be defined as:

$$\rho_i = \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} + b + w_i, \qquad (1)$$

where $i = 1, \ldots, n_s$ (recall that n_s is the number of visible satellites). The vectors $(x, y, z)^T$ and $(X_i, Y_i, Z_i)^T$ are the positions of the vehicule and the ith satellite expressed in the rectangular coordinate system WGS-84 [4]. However, these observations have to be expressed as functions of the state vector components to make the filtering problem tractable. Thus, the position is transformed from the geodetic to the rectangular coordinate system as follows:

$$\left\{ \begin{array}{lcl} x &=& (N + h_{\text{INS}} + \delta h)\cos(\lambda_{\text{INS}} + \delta\lambda)\cos(\phi_{\text{INS}} + \delta\phi) \\ y &=& (N + h_{\text{INS}} + \delta h)\cos(\lambda_{\text{INS}} + \delta\lambda)\sin(\phi_{\text{INS}} + \delta\phi) \\ z &=& (N + h_{\text{INS}} + \delta h)\sin(\lambda_{\text{INS}} + \delta\lambda), \end{array} \right.$$

where $N = \frac{a}{\sqrt{1 - e^2 \sin^2 \lambda}}$. The parameters a and e denote the semi-major axis length and the eccentricity of the earth's ellipsoid. These expressions have to be substituted in (1) to obtain the highly non-linear measurement equation:

$$\boldsymbol{Y}_t = h_t(\boldsymbol{X}_t) + C\boldsymbol{X}_t + \boldsymbol{w}_t, \qquad (2)$$

where $\boldsymbol{w}_t \sim \mathcal{N}(\boldsymbol{0}, \Sigma_w)$, $Y_t = (\rho_1, \ldots, \rho_{n_s})$ is the pseudorange vector and $C = [\ 0_{n_s \times 14} | 1_{n_s \times 1} | 0_{n_s \times 1}\ ]$ ($0_{p \times q}$ is the null matrix of size $p \times q$ and $1_{p \times q}$ is a matrix of ones of size $p \times q$).

4. FILTERING APPROACH

This section investigates some strategies to estimate recursively the inertial errors $\boldsymbol{X}_t$ conditioned upon the GPS measurements $\boldsymbol{Y}_t$. In a Bayesian setting, all inference about the unknown state parameters is constructed from the conditional probability density function (pdf) $p(\boldsymbol{X}_{0:t} | \boldsymbol{Y}_{1:t})$, where $\boldsymbol{X}_{0:t} \triangleq (\boldsymbol{X}_0, \ldots, \boldsymbol{X}_t)$ and $\boldsymbol{Y}_{1:t} \triangleq (\boldsymbol{Y}_1, \ldots, \boldsymbol{Y}_t)$. However, estimating the state vector via the minimum mean square estimator (MMSE) or the maximum *a posteriori* (MAP) generally leads to intractable integration or optimization procedures. A notable exception is the linear Gaussian state space model. Indeed, the MMSE for this model can be implemented easily by using a standard Kalman filter. In this case, the pdf $p(\boldsymbol{X}_t | \boldsymbol{Y}_{1:t})$ is Gaussian and thus characterized by its mean and covariance matrix that obey closed-form recursions. When the state-space model exhibits nonlinearities as in (2), the popular Extended Kalman Filter (EKF) can be used. This paper studies several alternatives based on Particle Filtering (PF) which approximate the posterior distribution of interest from a set of weighted particles. These methods are presented in the next sections.

4.1 The Extended Kalman Filter

The EKF proceeds by linearizing the model about the latest estimate to meet the Kalman Filter assumptions. The state space model described in section 3 can be classically approximated as follows:

$$\left\{ \begin{array}{lcl} \boldsymbol{X}_t &=& A_t \boldsymbol{X}_{t-1} + \boldsymbol{v}_t \\ \boldsymbol{Y}_t &\simeq& H_t(\boldsymbol{X}_t - \boldsymbol{m}_{t|t-1}) + h_t(\boldsymbol{m}_{t|t-1}) + C_t \boldsymbol{X}_t + \boldsymbol{w}_t, \end{array} \right. \qquad (3)$$

where $H_t = \frac{dh_t(x)}{dx}|_{x = \boldsymbol{m}_{t|t-1}}$. Consequently, the conditional probability of the state $p(\boldsymbol{X}_t | \boldsymbol{Y}_{1:t})$ can be estimated by a Gaussian pdf whose mean $\boldsymbol{m}_{t|t}$ and covariance $P_{t|t}$ can merely be computed by Kalman recursions.

4.2 Particle Filtering methods

PF methods refer to a class of recursive simulation-based estimation methods that can handle nonlinear possibly non Gaussian state space models. An increase in the computational power has recently drawn much attention on these techniques (see [2] and references therein). PF methods construct an empirical estimate of distributions such as $p(\boldsymbol{X}_{0:t}|\boldsymbol{Y}_{1:t})$ from weighted random support points (called particles):

$$p(\boldsymbol{X}_{0:t}|\boldsymbol{Y}_{1:t}) \simeq \sum_{i=1}^{N} w(\boldsymbol{X}_t^i)\delta(\boldsymbol{X}_{0:t} - \boldsymbol{X}_{0:t}^i).$$

$\boldsymbol{X}_{0:t}^{(i)}$ $(i = 1, \ldots, N)$ are the particles and $w(\boldsymbol{X}_{0:t}^i)$ the associated weights. The particles are propagated sequentially according to an *Importance sampling* (IS) method. Indeed, it is usually difficult to draw samples directly from the posterior distribution $p(\boldsymbol{X}_{0:t}|\boldsymbol{Y}_{1:t})$. IS consists of generating samples from an arbitrary proposal distribution $q(\boldsymbol{X}_{0:t}|\boldsymbol{Y}_{1:t})$. Weights are then assigned to these particles according to their relevance with respect to the distribution of interest. A simple rule for computing the weights is recalled below:

$$w(\boldsymbol{X}_{0:t}^i) \propto \frac{p(\boldsymbol{X}_{0:t}^i|\boldsymbol{Y}_{1:t})}{q(\boldsymbol{X}_{0:t}^i|\boldsymbol{Y}_{1:t})}, \qquad i = 1, \ldots, N.$$

An appropriate *importance function* allows to preserve the past trajectories ($\boldsymbol{X}_{0:t+1}^i = (\boldsymbol{X}_{0:t}^i, \boldsymbol{X}_{t+1}^i)$) and evaluate recursively the weights according to the following procedure:

$$\frac{w(\boldsymbol{X}_{0:t}^i)}{w(\boldsymbol{X}_{0:t-1}^i)} \propto \frac{p(\boldsymbol{Y}_t|\boldsymbol{X}_{0:t}^i, \boldsymbol{Y}_{1:t-1})p(\boldsymbol{X}_t^i|\boldsymbol{X}_{0:t-1}^i)}{q(\boldsymbol{X}_t^i|\boldsymbol{X}_{0:t-1}^i\boldsymbol{Y}_{1:t})}.$$

Unfortunately, the variance of the weights is bound to increase until all but one particle have negligible weight. This problem referred to as degeneracy is classically reduced by a *resampling* step. In order to concentrate the particules in regions of high probability, a good choice of the importance function is also decisive. This choice is crucial in our GPS/INS application since the small noise variance in the state equation prevents an appropriate exploration. The optimal importance function $p(\boldsymbol{X}_t|\boldsymbol{X}_{0:t-1}^i, \boldsymbol{Y}_{1:t})$ was derived in [3]. However, it is not straightforward to sample from this distribution. Some strategies overcoming this difficulty are briefly presented hereafter.

4.2.1 Local linearization

By linearizing the measurement equation, a Gaussian approximation of the optimal importance function denoted by $\mathcal{N}(\boldsymbol{m}_t, \Sigma_t)$ is obtained [3]. This linearization is performed for each particule and yields the following recursions:

$$\Sigma_t^{-1} = \Sigma_v^{-1} + H_t^T \Sigma_w^{-1} H_t,$$

$$m_t = \Sigma_t(\Sigma_v^{-1} A_t \boldsymbol{X}_{t-1} + H_t^T \Sigma_w^{-1}(\boldsymbol{Y}_t - h_t(A_t\boldsymbol{X}_{t-1}) - C_t A_t \boldsymbol{X}_{t-1} - H_t A_t \boldsymbol{X}_{t-1})),$$

where $H_t = \frac{dh_t(x)}{dx}|_{x=A_t\boldsymbol{X}_{t-1}}$.

4.2.2 Auxiliary particule filter

Another approach improving particle exploration is the auxiliary particule filter (APF) [7]. The APF only requires to simulate from the prior density $p(\boldsymbol{X}_t|\boldsymbol{X}_{t-1})$. The principle is to propagate the particles that are expected to yield a high value of the likelihood. The propagation of each particule one step ahead enables to compute the predictive density $p(\boldsymbol{Y}_t|\boldsymbol{\mu}_t)$, where $\boldsymbol{\mu}_t$ is related to $p(\boldsymbol{X}_t|\boldsymbol{X}_{t-1})$. The particules $\boldsymbol{X}_{t-1}^i$ $(i = 1, \ldots, N)$ that will actually evolve are then obtained by simulating from this predictive density. The standard steps of PF, i.e. *Importance sampling* and *Resampling*, are then applied to derive the state estimates.

4.3 Rao-Blackwellization

The performance of the previous PF algorithms can be improved when the state-space model has a block structure. Indeed, Rao-Blackwellization techniques allow to decrease the variance of the state estimates by solving analytically the linear part of conditionally linear Gaussian models ([1]). A convenient partition of the state vector allows to rewrite the INS-GPS system model as follows:

$$\boldsymbol{X}_{t+1} = \begin{pmatrix} A_t^1 & C_t^2 & [0] \\ C_t^1 & A_t^2 & [0] \\ [0] & [0] & A_t^3 \end{pmatrix} \boldsymbol{X}_t + \begin{pmatrix} B_t^1 & [0] \\ D_t^1 & [0] \\ [0] & B_t^3 \end{pmatrix} \begin{pmatrix} v_n^1 \\ v_t^3 \end{pmatrix},$$

$$\boldsymbol{Y}_t = h_t(\boldsymbol{X}_t^2) + C^3\boldsymbol{X}_t^3 + D_t^3\boldsymbol{W}_t,$$

where $\boldsymbol{X}_t^1 = (\delta\boldsymbol{v}^n, \delta\boldsymbol{\rho}, \boldsymbol{b}_a, \boldsymbol{b}_g,)^T$, $\boldsymbol{X}_t^2 = (\delta\lambda, \delta\phi, \delta h)^T$, $\boldsymbol{X}_t^3 = (b, d)^T$ and $\boldsymbol{X_t} = ((\boldsymbol{X}_t^1)^T, (\boldsymbol{X}_t^2)^T, (\boldsymbol{X}_t^3)^T)^T$. Note that the measurement equation does not depend on $\boldsymbol{X}_t^1$. Moreover, the observations depend non-linearly on $\boldsymbol{X}_t^2$ and linearly on $\boldsymbol{X}_t^3$. Consequently, the distributions $p(\boldsymbol{X}_{0:n}^1|\boldsymbol{X}_{0:n}^2)$ and $p(\boldsymbol{X}_{0:n}^3|\boldsymbol{Y}_{0:n}, \boldsymbol{X}_{0:n}^2)$ are Gaussian. The means and variances of these distributions, denoted $\boldsymbol{m}_{n|n,1}, \boldsymbol{m}_{n|n,3}, P_{n|n,1}$ and $P_{n|n,3}$ respectively, can be computed by standard recursions associated to the following Kalman state-space models:

$$\begin{cases} \boldsymbol{X}_t^1 &= A_t^1\boldsymbol{X}_{t-1}^1 + C_t^2\boldsymbol{X}_{t-1}^2 + B_t^1 v_t^1, \\ \boldsymbol{X}_t^2 &= C_t^1\boldsymbol{X}_{t-1}^1 + A_t^2\boldsymbol{X}_{t-1}^2 + D_t^1 v_t^1, \end{cases}$$

$$\begin{cases} \boldsymbol{X}_t^3 &= A_t^3\boldsymbol{X}_{t-1}^3 + B_t^3 v_t^3, \\ \boldsymbol{Y}_t &= h_t(\boldsymbol{X}_t^2) + C^3\boldsymbol{X}_t^3 + D_t^3 w_t, \end{cases}$$

The posterior distribution of the state $\boldsymbol{X}_t^2$ (non linearly related to the observations) is estimated by one of the PF methods described before:

$$p(\boldsymbol{X}_t^2|Y_t, \boldsymbol{X}_{0:t-1}^2) \simeq \sum_{i=1}^{N} w_t^{(i)}\delta(\boldsymbol{X}_t^2 - \boldsymbol{X}_t^{2,i}).$$

Note that the Kalman filters associated to each particle $\boldsymbol{X}_t^{2,i}$ provide both the importance distribution $p(\boldsymbol{X}_t^2|\boldsymbol{X}_{0:t-1}^2)$ and the likelihood $p(Y_t|\boldsymbol{X}_{0:t}^2)$. The pdfs $p(\boldsymbol{X}_t^k|Y_{1:t})$ $(k = 1, 3)$ are finally approximated by a mixture of Gaussian distributions:

$$p(\boldsymbol{X}_t^k|Y_t, \boldsymbol{X}_{0:t-1}^k) \simeq \sum_{i=1}^{N} w_t^{(i)}\mathcal{N}(\boldsymbol{X}_t^k, \boldsymbol{m}_{t|t,k}^i, P_{t|t,k}^i).$$

4.4 The posterior Cramer Rao Bound (PCRB)

The PCRB provides a lower bound on the mean square errors (MSEs) of the state estimates. It can be viewed as a reference to which the state MSEs of suboptimal algorithms can be compared. The PCRB is often referred to as the Bayesian version of the Cramer-Rao Bound. The recursive formula allowing to compute the PCRB are detailed in [1]. As the altitude is supposed to be known, this paper focuses on the PCRB for the horizontal INS position error denoted by HPCRB. We recall here that the horizontal position error HE can be defined as a function of the latitude $\delta\lambda$ and longitude $\delta\phi$ inertial errors by HE $= (R\delta\lambda)^2 + (R\delta\phi\cos\lambda)^2$, where R stands for the earth radius [6].

Figure 1 illustrates the intuitive dependence of the HPCRB on the number n_s of visible satellites. Indeed, the higher n_s, the lower the PCRB. Further investigations show that the number of visible satellites is not the only important feature. In navigation, a well-known parameter referred to as Horizontal Dilution Of Precision (HDOP) quantifies the effect of the GPS satellites configuration on the horizontal position estimation error. The HDOP represents the

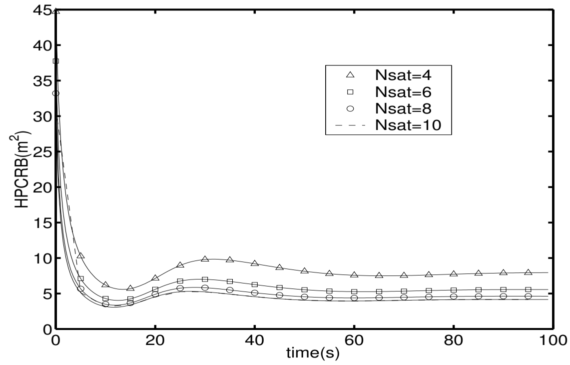

Fig. 1. HPCRBs en meters^2 for different values of n_s.

ratio of the positioning accuracy to the measurement accuracy. This factor can be computed as follows:

$$\begin{aligned} V &= (\widetilde{H}_t^T \widetilde{H}_t)^{-1}, \\ \text{HDOP} &= \sqrt{(V_{11} + V_{22})}, \end{aligned}$$

where $x = (X_l, Y_l, Z_l)$ denotes the mobile position in the locally level frame, b is the GPS receiver clock offset, $\widetilde{H}_t = \frac{dh_t(x)}{dx}|_{X_l,Y_l,Z_l,b}$ and $V = (V_{ij})_{i,j=1,\ldots,4}$. We invite the reader to consult [4, p. 160] for more details. The variations of HPCRBs for different values of HDOP and for the same number of visible satellites are depicted in figure 2. The lower the HDOP, the lower the HPCRB, as expected.

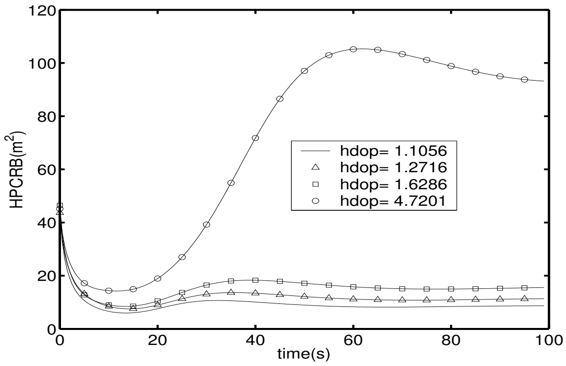

Fig. 2. HPCRBs for different values of HDOP.

5. SIMULATION RESULTS

The different estimation strategies detailed in the previous sections are applied to an INS/GPS scenario. First, the vehicule trajectory is generated according to the *position-velocity-acceleration* model (the acceleration is modeled as a Gauss-markov process). The INS estimates of the vehicule dynamics are then computed for low cost inertial sensors with an accelerometer bias of $500\mu g$ and a gyrometer bias of $1deg/h$. The observation model is finally used to compute the GPS pseudoranges that are associated to the simulated trajectory. The standard deviation of the GPS measurement noise is chosen as $\sigma_w = 10$ meters and the number of visible satellites ensures a good observability (i.e. $n_s \geq 4$).

The results obtained with the EKF and the PF (with 5000 particles) are compared for a simulation duration of 500s. For each method, the horizontal positioning root mean square error HRMSE is computed from 100 Monte carlo runs. The RMSEs are also compared to the corresponding posterior bound $\sqrt{\text{HPCRB}}$. Figure 3 shows the results obtained with the EKF and the local linearization (the APF approach has not been plotted since it performs very similarly to the APF). After 50s, the HRMSEs of both state estimates converge to a constant which is close to $\sqrt{\text{HPCRB}}$ ($\simeq 2.3$ meters). Figure 3 shows that the EKF and the PF perform quite similarly in nominal situations where the error dynamics are reasonable (i.e. when the local linearization is appropriate).

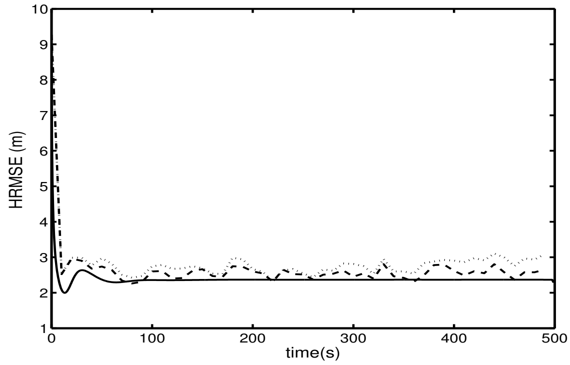

Fig. 3. HRMS estimation errors for the EKF (dashed), the Rao-Blackwellized PF (dotted), and $\sqrt{\text{HPCRB}}$ (solid line).

6. CONCLUSIONS

This paper has studied alternatives to the classical EKF to solve the nonlinear INS/GPS filtering problem. PF methods do not require any approximation on the state-space model, contrary to the EKF. A Rao-Blackwellization procedure was applied to decrease the variance of the estimates. However, simulation results conducted in nominal situations (with a good observability) show that quite similar performance is achieved with the different filtering strategies. A comparison in critical situations (such as loss of observability or presence of multipath) is currently under investigation.

REFERENCES

[1] N. Bergman, A. Doucet, and N. Gordon. Optimal estimation and Cramér-Rao bounds for partial non-Gaussian state space models. *Ann. Inst. Statist. Math.*, 53(1):97–112, 2001.

[2] A. Doucet, N. de Freitas, and N. Gordon. An introduction to sequential Monte Carlo methods. In A. Doucet, N. de Freitas, and N. Gordon, editors, *Sequential Monte Carlo methods in practice*, pages 3–14. Springer Verlag, New York, 2001.

[3] A. Doucet, S. J. Godsill, and C. Andrieu. On sequential Monte Carlo sampling methods for Bayesian filtering. *Statistics and Computing*, 10(3):197–208, 2000.

[4] J. A. Farrell and M. Barth. *The Global Positioning System and Inertial Navigation*. McGraw–Hill, New York, 1999.

[5] A. Giremus, A. Doucet, V. Calmettes, and J. Y. Tourneret. A rao-blackwellized particle filter for INS/GPS integration. In *Proc. IEEE ICASSP-04*, Montreal, Canada, May 2004.

[6] P. J. Nordlund. *Sequential Monte Carlo filters and integrated navigation*. PhD thesis, Linkping University, Linkping, Sweden, 2002.

[7] M. K. Pitt and N. Shephard. Auxiliary variable based particle filters. In A. Doucet, N. de Freitas, and N. Gordon, editors, *Sequential Monte Carlo methods in practice*, pages 273–293. Springer Verlag, New York, 2001.