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# BOC Power Spectrum Densities

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## BIOGRAPHY

Emilie Rebeyrol graduated as a telecommunications engineer from the INT (Institut National des Télécommunications) in 2003. She is now a Ph.D student at the satellite navigation lab of the ENAC. Currently she carries out research on Galileo signals and their generation in the satellite payload in collaboration with the CNES (Centre National d'Etudes Spatiales), in Toulouse.

Christophe Macabiau graduated as an electronics engineer in 1992 from the ENAC (Ecole Nationale de l'Aviation Civile) in Toulouse, France. Since 1994, he has been working on the application of satellite navigation techniques to civil aviation. He received his Ph.D. in 1997 and has been in charge of the signal processing lab of the ENAC since 2000.

Laurent Lestarquit graduated from the Ecole Polytechnique de Paris in 1994, and then specialized in space telecommunication systems at the Ecole Nationale de l'Aéronautique et de l'Espace (SUPAERO) in Toulouse, France. Since 1996, he has been collaborating to several projects related to GPS space receivers (HETE2 and STENTOR). He is now involved in the GALILEO program and supports EC and ESA through the GALILEO Signal Task Force. He invented the ALTBOC 8PSK signal proposed for GALILEO in E5. From now on he will be involved in GALILEO orbit determination and time synchronisation.

Lionel Ries is a navigation engineer in the Transmission Techniques and signal processing department, at CNES since June 2000. He is responsible of research activities on GNSS2 signal, including BOC modulations and GPS IIF L5. He is involved in the GALILEO program, in which he supports ESA, EC and GJU, through the GALILEO Signal Task Force. He graduated in 1997 from the Ecole Polytechnique de Bruxelles, at Brussels Free University (Belgium), in 1997, and received a M.S. degree from the Ecole Nationale Supérieure de l'Aéronautique et de l'Espace (SUPAERO) in Toulouse (France) in 1998.

Jean-Luc Issler is head of the Transmission Techniques and signal processing department of CNES, whose main tasks are signal processing, air interfaces and equipments in Radionavigation, Telecommunication, TT&C, High Data Rate TeleMetry, propagation and spectrum survey. He is involved in the development of several spaceborne receivers in Europe, as well as in studies on the European RadioNavigation projects, like GALILEO and the Pseudolite Network. With DRAST, he represents France in the GALILEO Signal Task Force of the European Commission.

Marie-Laure Boucheret is graduated from the ENST Bretagne in 1985 (Engineering degree in Electrical Engineering) and from Telecom Paris in 1997 (PhD degree). She worked as an engineer in Alcatel Space from 1986 to 1991 then moved to ENST as an Associated Professor then a Professor. Her fields of interest are digital communications (modulation/coding, digital receivers, multicarrier communications ...), satellite on-board processing (filter banks, DBFN ...) and navigation system.

Michel Bousquet is a Professor at SUPAERO (French Aerospace Engineering Institute of Higher Education), in charge of graduate and post-graduate programs in aerospace electronics and communications. He has over twenty five years of teaching and research experience, related to many aspects of satellite systems (modulation and coding, access techniques, onboard processing, system studies...). He has authored or co-authored many papers in the areas of digital communications and satellite communications and navigation systems, and textbooks, such as "Satellite Communications Systems" published by Wiley.

## ABSTRACT

The GPS and GALILEO systems will take advantage of new signal modulations such as Binary Offset Carrier (BOC) that uses a square wave sub-carrier to create separate spectra on each side of the transmitted carrier. That signal could share the existing frequency bands with each other while reserving the spectrum, it provides spectral isolation and leads to significant improvements in

terms of tracking, interference and multipath mitigation. As this BOC modulation, along with a modified version called alternate BOC (ALTBOC), is a serious candidate for GALILEO and GPS, it is important to understand all characteristics of this signal, in order to conduct several studies like payload design, receiver implementation, performance and robustness evaluation. One of the aspects of this signal is the power spectrum density that impacts all characteristics presented above.

The aim of this paper is, in a first part, to review the currently admitted BOC power spectrum density theoretical expressions, and to discuss a deviation of the assumptions necessary for the derivation of these classical theoretical expressions w.r.t to the reality of these signals. Then, a second objective is to present theoretical expressions of ALTBOC and constant envelope ALTBOC power spectrum densities.

First the paper recalls the formal expressions of the different possible offset carrier modulations: BOC, ALTBOC and constant envelop ALTBOC signals in the time domain. Then, the assumptions for the development of the classical power spectrum density calculation are clearly laid down. Next, it is shown that in the case of odd ratio ( $2 \cdot f_s / f_c$ ) BOC signals, these assumptions, and particularly the fact that the BOC square sub-carrier can be incorporated in the chip waveform, are not met. Then, we show that, for these calculation assumptions to be met in that case of odd ratio BOC, it is equivalent to modify the code sequence. Finally, with this view, the obtained power spectrum density is the same as the one obtained with the classical theory, due to the negligible effect of the code sequence correlation values. Finally, we present the theoretical expressions of the ALTBOC and constant envelope ALTBOC power spectrum densities.

## I. INTRODUCTION

The modernization of the Global Positioning System (GPS) and the development of the Galileo System have led to design new signals to provide different services to a variety of users.

The most important innovation used for these signals is the Binary Offset Carrier (BOC) modulation because it significantly improves the performance obtained until then in the radionavigation system. This modulation is candidate for the new GPS military signal, the M-code. But it is also candidate for many signals in the future Galileo system:

- in L1 band (1559-1592 MHz) a BOC signal would transmit the Open Service signal and the Public Regulated Services signal.
- in E6 band (1215-1300 MHz) the Public Regulated Services signal would also be transmitted with a BOC.

In E5a/E5b frequency (1164-1215 MHz) a modified BOC signal, called Alternate BOC (ALTBOC), is proposed. This signal is interesting because it provides spectral isolation between components of a same composite signal. Moreover with this signal, it is also possible to track the upper and the lower signals as independent BPSK signals.

The study of the performance of these two waveforms (BOC and ALTBOC) is based on the study of their power spectrum densities which are the subject of this paper.

The first part describes the BOC, ALTBOC and constant envelope ALTBOC signals. The odd case ( $n=2f_s/f_c$  is odd) is carefully examined because in this case the sub-carrier is not considered included in the chip waveform. Then a discussion is conducted on the assumptions made for the development of the classical power spectrum density expressions. It is shown that these assumptions are correct if a modification is made on the code sequence for the odd case. Finally, while taking into account this point, the expressions of the power spectrum densities of the BOC, ALTBOC and constant envelope ALTBOC signals are given.

## II. OFFSET CARRIER SIGNALS

### BOC Signal Definition

In the literature two notations are used to define the BOC signal.

The first model defines the BOC signal as the product of a materialized code with a sub-carrier which is equal to the sign of a sine or a cosine waveform. It is presented in [Betz, 2001], [Ries and al., 2003] and [Godet, 2001]. In this case if  $c(t)$  is the code sequence waveform and  $f_s$  the sub-carrier frequency, the expression of the sine-phased BOC signal is:

$$x(t) = c(t) \cdot \text{sign}(\sin(2\pi f_s t)) \quad (1)$$

with

$$c(t) = \sum_k c_k h(t - kT_c)$$

$h(t)$  is the code materialization, it is a NRZ materialization equal to 1 over  $[0, T_c]$  and 0 everywhere else.

The second model defines the BOC signal by the following equation:

$$x(t) = \sum_k c_k p(t - kT_c) \quad (2)$$

with  $p(t)$  broken up into  $n$  rectangular pulses of duration  $T_c/n$  with amplitudes of  $\pm 1$ . In this case the sine-phased or cosine-phased sub-carrier is considered as a part of the chip waveform. It is presented in [Pratt and Owen, 2003] and [Betz, 2001].

In the two cases BOC signals are commonly referred to BOC(p,q). The first parameter p defines the sub-carrier rate and the second parameter q defines the spreading code rate:

$$f_s = p \cdot 1.023 \text{ MHz and } f_c = q \cdot 1.023 \text{ MHz}$$

The ratio  $n = 2 \frac{f_s}{f_c} = 2 \frac{p}{q}$  is the number of half periods of the sub-carrier during one code chip. This ratio can be odd or even.

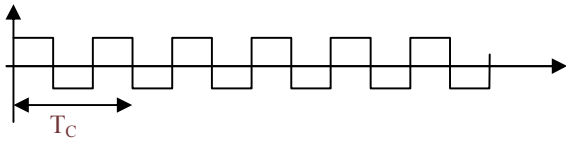
The two models presented above are identical if the ratio n is even. In fact if n is even it is true to consider that the sub-carrier is included in the chip waveform. However it is false to consider such a thing if n is odd. The example presented below shows clearly that these two conventions lead to different time series if n is odd.

#### Example:

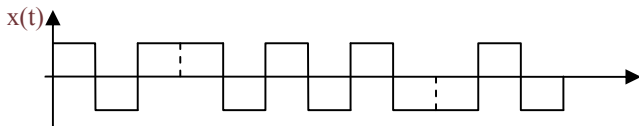
We consider:

- a code sequence equal to : {1,-1,-1,1}
- a square sub-carrier which is sine-phased and with  $2f_s/f_c = 3$ .

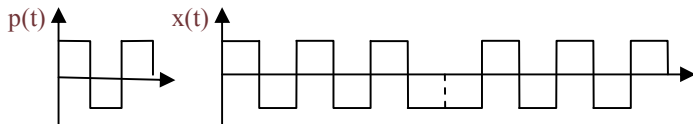
It is represented by this scheme:



If the BOC signal is written as (1), the scheme represented this signal is:



On the other hand, if we consider that the sub-carrier is included in the chip waveform and that the BOC signal can be written as (2), we have:



Both signals obtained are different. Indeed, there is a polarity inversion every 2 bits.

If n is odd a modification must be made on the time series if we want to consider that the sub-carrier is included in the code materialization. We have, in [Betz, 2001]:

$$x(t) = \sum_k c_k h(t - kT_c) \cdot \text{sign}(\sin(2\pi f_s t))$$

⇕

$$x(t) = \sum_k (-1)^k c_k p_{T_c}(t - kT_c) \quad n \text{ odd} \quad (3)$$

$$\text{with } p_{T_c} = \sum_{m=0}^{n-1} (-1)^m h_{\frac{T_c}{2}}(t - m \frac{T_c}{2})$$

A  $(-1)^k$  term is introduced. This introduction is in fact equivalent to a modification of the PRN code sequence, becoming now  $(-1)^k c_k$  and not  $c_k$ . Consequently in the case n odd if we want to consider that the sub-carrier is included in the chip waveform a modification must be made on the code sequence to obtain the same time domain expression as (1). It is quite easy to go from one convention to the other but this point is quite important because a receiver adapted to one convention would suffer large losses in receiving signal using the other convention.

After discussions with the recognized experts in the domain and according to us, the first convention is the most suitable definition if we want to stick to the original BOC definition. Afterwards we assume that the BOC signal is defined by the first notation whatever the parity of n.

#### Alternate BOC Signal Definition

Contrary to the BOC signal the ALTBOC signal provides spectral isolation between the two upper and lower components of a same composite signal. This signal allows keeping the BOC implementation simplicity while permitting to differentiate the lobes.

The idea of Alternate BOC modulation is to perform the same process as BOC modulation but the sub-carrier used is a “complex” sub-carrier. In that way, the signal spectrum is not split up, but only shifted to higher frequencies. Shifting to lower frequencies is obviously also possible.

As for the BOC signal the problem of notation could exist but in [Ries and al., 2003] the ALTBOC signal is clearly defined as the product of a PRN code sequence with a “complex” sub-carrier. The ALTBOC signal can be composed of two or four codes. If there are two codes there is no pilot component and the expression of the signal is:

$$x_{ALT\_BOC}(t) = c_u(t) \cdot er(t) + c_L(t) \cdot er^*(t)$$

with

$$er(t) = \text{sign}[\cos(2\pi f_s t)] + j \cdot \text{sign}[\sin(2\pi f_s t)] = c_r(t) + j \cdot s_r(t)$$

$c_u$  is the upper code and  $c_L$  the lower code.

If a pilot channel is introduced, four codes are needed and the expression of the ALTBOC signal is:

$$x_{ALT\_BOC}(t) = (c_u + j \cdot c'_u) \cdot er(t) + (c_L + j \cdot c'_L) \cdot er^*(t)$$

with

$$er(t) = \text{sign}[\cos(2\pi f_s t)] + j \cdot \text{sign}[\sin(2\pi f_s t)] = c_r(t) + j \cdot s_r(t)$$

$c_u$  is the data upper code,  $c'_u$  the pilot upper code,  $c_L$  the data lower code and  $c'_L$  the pilot lower code.

But in this case the signal doesn't have a constant envelope and thus may be distorted within the satellite payload due to non-linear amplification. That's why an innovation was proposed in [Godet, 2001] in order to create a constant envelope signal which is as close as possible to the ALTBOC signal. The innovation introduces new terms which can be compared to intermodulation products. The expression of the new signal obtained, called constant envelope ALTBOC, is presented in [Soellner and Erhard, 2003]:

$$x_{ALT\_BOC}(t) = \begin{cases} (c_L + j \cdot c'_L) \cdot \left[ sc_{as}(t) - j \cdot sc_{as}\left(t - \frac{T_s}{4}\right) \right] + \\ (c_U + j \cdot c'_U) \cdot \left[ sc_{as}(t) + j \cdot sc_{as}\left(t - \frac{T_s}{4}\right) \right] + \\ (\overline{c_L} + j \cdot \overline{c'_L}) \cdot \left[ sc_{ap}(t) - j \cdot sc_{ap}\left(t - \frac{T_s}{4}\right) \right] + \\ (\overline{c_U} + j \cdot \overline{c'_U}) \cdot \left[ sc_{ap}(t) + j \cdot sc_{ap}\left(t - \frac{T_s}{4}\right) \right] \end{cases}$$

with

$$\overline{c_L} = c_U c'_U c'_L \quad \overline{c'_L} = c_U c'_U c_L \quad \overline{c_U} = c_L c'_L c'_L \quad \overline{c'_U} = c_U c_L c'_L$$

and

$$sc_{as}(t) = \begin{cases} \frac{\sqrt{2}}{4} \text{sign}\left(\cos\left(2\pi f_s t - \frac{\pi}{4}\right)\right) + \frac{1}{2} \text{sign}(\cos(2\pi f_s t)) + \\ \frac{\sqrt{2}}{4} \text{sign}\left(\cos\left(2\pi f_s t + \frac{\pi}{4}\right)\right) \end{cases}$$

$$sc_{ap}(t) = \begin{cases} -\frac{\sqrt{2}}{4} \text{sign}\left(\cos\left(2\pi f_s t - \frac{\pi}{4}\right)\right) + \frac{1}{2} \text{sign}(\cos(2\pi f_s t)) - \\ \frac{\sqrt{2}}{4} \text{sign}\left(\cos\left(2\pi f_s t + \frac{\pi}{4}\right)\right) \end{cases}$$

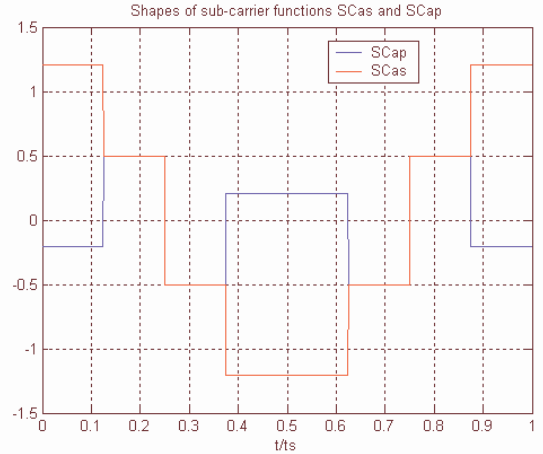


Fig. 1: Shapes of sub-carriers SCas and SCap

Finally the constant envelope ALTBOC signal is a classical 8-PSK modulation with a non-constant allocation of the 8 phase-states.

As the BOC signal, the ALTBOC is generally referred to an ALTBOC(p,q) with:

$$f_s = p \cdot 1.023 \text{ MHz and } f_c = q \cdot 1.023 \text{ MHz}$$

### III. POWER SPECTRUM DENSITY OF THE BOC SIGNAL

#### Assumptions

If we consider a stationary signal:

$$s(t) = \sum_k c_k \cdot p(t - kT_c - \theta) \quad (4)$$

with  $c_k$  the digital code sequence considered as random and non-periodic,  $p(t)$  the materialization waveform,  $T_c$  the code period and  $\theta$  a variable which is uniformly distributed on  $T_c$ .

The autocorrelation function of this signal is:

$$E[s(t)s^*(t-\tau)] = \sum_k \sum_n E[c_k c_n^*] \times E \left[ \begin{matrix} p(t - kT_c - \theta) \\ p^*(t - nT_c - \theta - \tau) \end{matrix} \right]$$

$$E[s(t)s^*(t-\tau)] = \sum_m \mathfrak{R}_c(m) \times \frac{1}{T_c} \sum_k \int_0^{T_c} p(t - kT_c - \theta) p^*(t - \tau - kT_c + mT_c - \theta) dt$$

$$E[s(t)s^*(t-\tau)] = \frac{1}{T_c} \sum_m \mathfrak{R}_c(m) \times \mathfrak{R}_p(\tau - mT_c)$$

The power spectrum density of  $s(t)$  is the Fourier Transform of the autocorrelation of  $s(t)$ :

$$G_s(f) = FT \left[ \frac{1}{T_c} \sum_m \mathfrak{R}_c(m) \times \mathfrak{R}_p(\tau - mT_c) \right]$$

$$G_s(f) = \frac{1}{T_c} \sum_m \mathfrak{R}_c(m) \cdot |P(f)|^2 e^{-2j\pi f m T_c}$$

$$G_s(f) = \frac{1}{T_c} |P(f)|^2 S_c(e^{2j\pi f T_c})$$

with  $P(f)$  the Fourier Transform of the waveform  $p(t)$ .

If the code sequence is binary, equiprobable and independent, the power spectrum density is equal to:

$$G_s(f) = \frac{|P(f)|^2}{T_c}$$

The BOC signal is regarded as stationary signal. The PRN code sequence is considered random, non periodic, identically distributed, binary, equiprobable and independent. Consequently this formula can be used to calculate the power spectrum density of the BOC signal if the BOC time domain expression is similar to (4). To have a signal similar to (4), the sub-carrier must be included in the chip waveform.

We have seen in the previous section that if  $n$  is even there is no problem because the materialization of the signal is equal to the sub-carrier over  $[0, T_c]$ . In this case the expression of the signal is:

$$s(t) = \sum_k c_k p_{T_c}(t - kT_c) \quad n \text{ even} \quad (5)$$

$$\text{with } p_{T_c} = \sum_{m=0}^{n-1} (-1)^m h_{\frac{T_c}{2}}(t - m \frac{T_c}{2})$$

$h$  is the code materialization, it is equal to 1 over  $[0, T_c/2]$ .

But as shown before if  $n$  is odd a modification on the PRN code sequence must be made if we want to consider that the sub-carrier is included in the code materialization. Nevertheless this PRN code modification has no impact on the final expression of the power spectrum density of the BOC signal if the new PRN code can be assumed to be an independent sequence over time. This assumption is true for most of the PRN sequences used in GNSS up to now. So this theory can be used to calculate the power spectrum density of the BOC signal. However it would be wrong to use this theory with short codes.

## Expressions of the Power Spectrum Densities

If the BOC signal is sine-phased, the normalized power spectrum densities are equal to:

$$G_{BOC}(f) = \frac{1}{T_c} \left( \frac{\sin(\frac{\pi f T_c}{n}) \sin(\pi f T_c)}{\pi f \cos(\frac{\pi f T_c}{n})} \right)^2 \quad \text{for } n \text{ even}$$

$$G_{BOC}(f) = \frac{1}{T_c} \left( \frac{\sin(\frac{\pi f T_c}{n}) \cos(\pi f T_c)}{\pi f \cos(\frac{\pi f T_c}{n})} \right)^2 \quad \text{for } n \text{ odd}$$

And if the BOC signal is cosine-phased the expressions are:

$$G_{BOC}(f) = \frac{1}{T_c} \cdot \left( \frac{\sin(\pi f T_c)}{\pi f \cos(\frac{\pi f T_c}{n})} \left\{ \cos\left(\pi f \frac{T_c}{n}\right) - 1 \right\} \right)^2 \quad \text{for } n \text{ even}$$

$$G_{BOC}(f) = \frac{1}{T_c} \cdot \left( \frac{\cos(\pi f T_c)}{\pi f \cos(\frac{\pi f T_c}{n})} \left\{ \cos\left(\pi f \frac{T_c}{n}\right) - 1 \right\} \right)^2 \quad \text{for } n \text{ odd}$$

## IV. POWER SPECTRUM DENSITY OF THE ALTBOC SIGNAL

The study is made on the ALTBOC signal with four codes and the power spectrum densities are calculated for the ALTBOC with a non constant envelope and for the ALTBOC with a constant envelope.

### Assumptions

The assumptions are the same as these made to calculate the power spectrum density of the BOC signal. The ALTBOC signals are regarded as stationary signals. The different PRN code sequences are considered identically distributed and independent. The most interesting ALTBOC signal is the ALTBOC(15,10) because it is proposed for the Galileo E5a/E5b frequency. Consequently the calculation of the power spectrum density will be made considering  $n$  odd and  $n$  even but the curves will be only plotted for  $n$  odd. The theory used to calculate the power spectrum density of the ALTBOC signal will be the same as the one used to calculate the power spectral density of the BOC signal. So, a modification will be made on each code sequence in order

to be able to consider that the sub-carrier is included in the chip waveform in the case n odd.

### Power Spectrum Density of the ALTBOC signal

The ALTBOC is equal to:

$$x_{ALT\_BOC}(t) = C_U(t) \cdot m(t) + C_L(t) \cdot n(t)$$

with  $C_U(t) = c_u(t) + j \cdot c_u'(t)$   
 $C_L(t) = c_L(t) + j \cdot c_L'(t)$

$$m(t) = a(t) + j \cdot b(t) = \text{sign}[\cos(2\pi f_s t)] + j \cdot \text{sign}[\sin(2\pi f_s t)]$$

$$n(t) = a(t) - j \cdot b(t) = \text{sign}[\cos(2\pi f_s t)] - j \cdot \text{sign}[\sin(2\pi f_s t)]$$

The autocorrelation of the ALTBOC is equal to:

$$K_{x_{ALT\_BOC}}(\tau) = \begin{cases} K_{c_u}(\tau) \cdot K_a(\tau) + K_{c_u}(\tau) \cdot K_b(\tau) + \\ K_{c_u'}(\tau) \cdot K_a(\tau) + K_{c_u'}(\tau) \cdot K_b(\tau) + \\ K_{c_L}(\tau) \cdot K_a(\tau) + K_{c_L}(\tau) \cdot K_b(\tau) + \\ K_{c_L'}(\tau) \cdot K_a(\tau) + K_{c_L'}(\tau) \cdot K_b(\tau) \end{cases}$$

assuming that the crosscorrelation between the different codes is equal to zero. Moreover the complex crosscorrelation cancel each other out.

So, the power spectrum density of the signal is equal to:

$$G_{ALTBOC}(f) = \frac{4}{T_c} |A(f)|^2 + \frac{4}{T_c} |B(f)|^2$$

with A(f) and B(f) the Fourier Transforms of  $\text{sign}[\cos(2\pi f_s t)]$  and  $\text{sign}[\sin(2\pi f_s t)]$ .

The first case studied is the case n even. In [Pratt and Owen, 2003] and [Betz, 2001] it is demonstrated that:

$$|A(f)|^2 = |FT[\text{sign}(\cos(2\pi f_s t))]|^2$$

$$|A(f)|^2 = \left(\frac{1}{\pi f}\right)^2 \frac{\sin(\pi f T_c)^2}{\cos\left(\pi f \frac{T_c}{n}\right)^2} \left\{ \cos\left(\pi f \frac{T_c}{n}\right) - 1 \right\}^2$$

and

$$|B(f)|^2 = |FT[\text{sign}(\sin(2\pi f_s t))]|^2$$

$$|B(f)|^2 = \left(\frac{1}{\pi f}\right)^2 \frac{\sin(\pi f T_c)^2 \sin\left(\pi f \frac{T_c}{n}\right)^2}{\cos\left(\pi f \frac{T_c}{n}\right)^2}$$

Consequently, for n even:

$$G_{ALTBOC}(f) = \frac{8}{\pi^2 f^2 T_c} \frac{\sin(\pi f T_c)^2}{\cos\left(\pi f \frac{T_c}{n}\right)^2} \left\{ 1 - \cos\left(\pi f \frac{T_c}{n}\right) \right\}$$

If n is odd, we have:

$$|A(f)|^2 = |FT[\text{sign}(\cos(2\pi f_s t))]|^2$$

$$|A(f)|^2 = \left(\frac{1}{\pi f}\right)^2 \frac{\cos(\pi f T_c)^2}{\cos\left(\pi f \frac{T_c}{n}\right)^2} \left\{ \cos\left(\pi f \frac{T_c}{n}\right) - 1 \right\}^2$$

and

$$|B(f)|^2 = |FT[\text{sign}(\sin(2\pi f_s t))]|^2$$

$$|B(f)|^2 = \left(\frac{1}{\pi f}\right)^2 \frac{\cos(\pi f T_c)^2 \sin\left(\pi f \frac{T_c}{n}\right)^2}{\cos\left(\pi f \frac{T_c}{n}\right)^2}$$

Finally the normalized power spectrum density of the ALTBOC with a non constant envelope and with n odd is:

$$G_{ALTBOC}(f) = \frac{8}{T_c \pi^2 f^2} \frac{\cos(\pi f T_c)^2}{\cos\left(\pi f \frac{T_c}{n}\right)^2} \left\{ 1 - \cos\left(\pi f \frac{T_c}{n}\right) \right\}$$

The next graph represents the power spectrum densities of the ALTBOC(15,10) with a non constant envelope computed with the expression above and with MATLAB simulations.

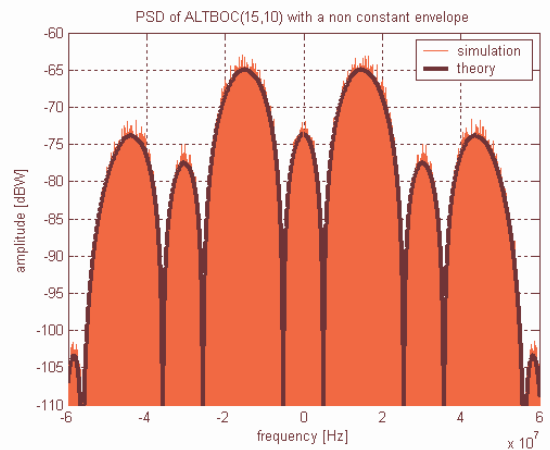


Fig. 2: Power Spectrum Density of the ALTBOC(15,10)

### Power Spectrum Density of the ALTBOC signal with a constant envelope

The calculation of the power spectrum density of the ALTBOC with a constant envelope is based on the calculation made in [Betz, 2001] to calculate the power spectrum density of the BOC signal.

The appendix shows that for n odd the normalized power spectrum density of the ALTBOC signal with a constant envelope is:

$$G_{ALTBOC}(f) = \frac{4}{\pi^2 f^2 T_c} \frac{\cos^2(\pi f T_c)}{\cos^2\left(\pi f \frac{T_c}{n}\right)} \left[ \begin{array}{l} \cos^2\left(\pi f \frac{T_s}{2}\right) - \cos\left(\pi f \frac{T_s}{2}\right) \\ -2 \cos\left(\pi f \frac{T_s}{2}\right) \cos\left(\pi f \frac{T_s}{4}\right) + 2 \end{array} \right]$$

The next graph represents the power spectrum densities of the ALTBOC(15,10) with a constant envelope calculated with the expression above and with MATLAB simulations.

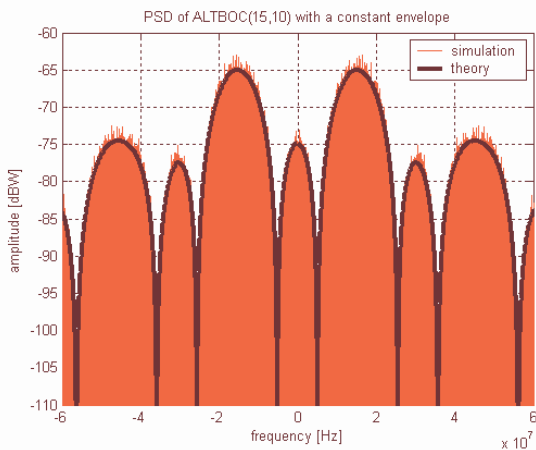


Fig. 3: Power Spectrum Density of the constant envelope ALTBOC(15,10)

A comparison can be made between the spectrum of the ALTBOC and the spectrum of the constant envelope ALTBOC.

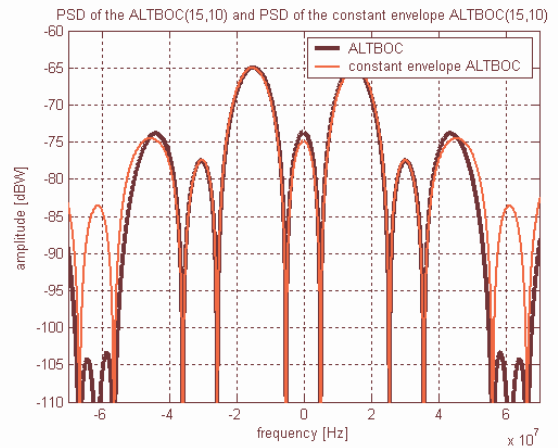


Fig. 4: Comparison of Power Spectrum Densities

The spectra of the ALTBOC signal and constant envelope ALTBOC signal have similar main-lobes and first side-lobes, even if the central lobe of the constant envelope ALTBOC is 1 dB weaker than this of the ALTBOC. However their further side-lobes are different. This difference is due to the modification made in order to obtain a constant envelope. Indeed the transformation made on the time domain ALTBOC signal expression added spectral lines.

In the Galileo system these spectral lines would be filtered by the RF filter after the amplifier and the two signals would, finally, have very close shapes.

### V. CONCLUSION

This paper has provided two main points. First it has underlined the problem of convention concerning the time domain expression of the BOC signal in the case  $n=2f_s/f_c$  odd. Indeed the two conventions are equivalent if n is even but if n is odd a modification must be made on the code sequence to consider that the sub-carrier is included in the chip waveform. Fortunately the properties of the PRN code sequence used in GNSS permit to neglect this modification on the code and the classical expressions of the power spectrum densities can be used.

Secondly the expressions of the power spectrum densities of the ALTBOC and the constant envelope ALTBOC signal have been presented. A particular attention is made on the ALTBOC(15,10) which is candidate for Galileo E5a/E5b frequency band. The constant envelope ALTBOC signal is very interesting because it provides spectral isolation between the two upper and lower components of a same composite signal. By this way it is possible to track each component separately or together.



## APPENDIX CALCULATION OF THE ALTBOC SIGNAL WITH A CONSTANT ENVELOPE

The ALTBOC signal with a constant envelope is expressed as:

$$x_{ALT\_BOC}(t) = \begin{cases} c_L sc_{as}(t) - j \cdot c_L sc_{as}\left(t - \frac{T_s}{4}\right) + j \cdot c_L' sc_{as}(t) + c_L' sc_{as}\left(t - \frac{T_s}{4}\right) + \\ c_U sc_{as}(t) + j \cdot c_U sc_{as}\left(t - \frac{T_s}{4}\right) + j \cdot c_U' sc_{as}(t) - c_U' sc_{as}\left(t - \frac{T_s}{4}\right) + \\ \overline{c_L sc_{ap}(t)} - j \cdot \overline{c_L sc_{ap}\left(t - \frac{T_s}{4}\right)} + j \cdot \overline{c_L' sc_{ap}(t)} + \overline{c_L' sc_{ap}\left(t - \frac{T_s}{4}\right)} + \\ \overline{c_U sc_{ap}(t)} + j \cdot \overline{c_U sc_{ap}\left(t - \frac{T_s}{4}\right)} + j \cdot \overline{c_U' sc_{ap}(t)} - \overline{c_U' sc_{ap}\left(t - \frac{T_s}{4}\right)} \end{cases}$$

We note  $sc_{as\_1} = sc_{as}(t)$ ,  $sc_{as\_2} = sc_{as}\left(t - \frac{T_s}{4}\right)$ ,

$sc_{ap\_1} = sc_{ap}(t)$  and  $sc_{ap\_2} = sc_{ap}\left(t - \frac{T_s}{4}\right)$ .

Different terms of crosscorrelation must be taken into account and analyzed. The code sequences are independent, so the crosscorrelation between two different codes is equal to zero. Consequently, all the crosscorrelation terms in which the crosscorrelation between two different codes appears are null. Other crosscorrelation terms are null because the crosscorrelation between the different sub-carrier ( $sc_{as}$  and  $sc_{ap}$ ) is equal to zero. The last crosscorrelation terms are complex. Fortunately they cancel each other out.

The correlation function of the constant envelope ALTBOC signal is equal to:

$$\Re(\tau) = \begin{cases} \Re_{c_L sc_{as\_1}}(\tau) + \Re_{c_L sc_{as\_2}}(\tau) + \Re_{c_L' sc_{as\_1}}(\tau) + \Re_{c_L' sc_{as\_2}}(\tau) + \\ \Re_{c_U sc_{as\_1}}(\tau) + \Re_{c_U sc_{as\_2}}(\tau) + \Re_{c_U' sc_{as\_1}}(\tau) + \Re_{c_U' sc_{as\_2}}(\tau) + \\ \Re_{\overline{c_L sc_{ap\_1}}}(\tau) + \Re_{\overline{c_L sc_{ap\_2}}}(\tau) + \Re_{\overline{c_L' sc_{ap\_1}}}(\tau) + \Re_{\overline{c_L' sc_{ap\_2}}}(\tau) + \\ \Re_{\overline{c_U sc_{ap\_1}}}(\tau) + \Re_{\overline{c_U sc_{ap\_2}}}(\tau) + \Re_{\overline{c_U' sc_{ap\_1}}}(\tau) + \Re_{\overline{c_U' sc_{ap\_2}}}(\tau) \end{cases}$$

As the autocorrelations of the different codes are equal, we obtained:

$$G_{ALT\_BOC}(f) = \begin{cases} \frac{4}{T_c} \cdot |SC_{as\_1}(f)|^2 + \frac{4}{T_c} \cdot |SC_{as\_2}(f)|^2 \\ + \frac{4}{T_c} \cdot |SC_{ap\_1}(f)|^2 + \frac{4}{T_c} \cdot |SC_{ap\_2}(f)|^2 \end{cases}$$

with  $SC_{as\_1}(f)$ ,  $SC_{as\_2}(f)$ ,  $SC_{ap\_1}(f)$  and  $SC_{ap\_2}(f)$  the Fourier Transform of  $sc_{as\_1}(t)$ ,  $sc_{as\_2}(t)$ ,  $sc_{ap\_1}(t)$  and  $sc_{ap\_2}(t)$  over  $[0, T_c]$ .

Beginning with the calculation of the Fourier Transform of  $sc_{as\_1}$  over  $[0, T_c]$ :

$$sc_{as\_1}(t) = \sum_{m=0}^{n-1} (-1)^m \mu_{T_s/2}\left(t - m \frac{T_s}{2}\right)$$

with

$$\mu_{T_s/2}(t) = \begin{cases} \frac{\sqrt{2}+1}{2} & \left[0, \frac{T_s}{8}\right] \\ \frac{1}{2} & \left[\frac{T_s}{8}, \frac{T_s}{4}\right] \\ -\frac{1}{2} & \left[\frac{T_s}{4}, \frac{3T_s}{8}\right] \\ -\frac{\sqrt{2}+1}{2} & \left[\frac{3T_s}{8}, \frac{T_s}{2}\right] \end{cases}$$

Before calculating the Fourier Transform of  $sc_{as\_1}(t)$  the calculation of the Fourier Transform of  $\mu_{T_s/2}(t)$  is necessary.

$$FT\left\{\mu_{T_s/2}\left(t - m \frac{T_s}{2}\right)\right\} = \int_{m \frac{T_s}{2}}^{(m+1) \frac{T_s}{2}} \mu_{T_s/2}\left(t - m \frac{T_s}{2}\right) \cdot e^{-2j\pi f t} dt$$

$$FT\left\{\mu_{T_s/2}\left(t - m \frac{T_s}{2}\right)\right\} = \begin{cases} \int_{m \frac{T_s}{2}}^{m \frac{T_s}{2} + \frac{T_s}{8}} \frac{\sqrt{2}+1}{2} \cdot e^{-2j\pi f t} dt + \int_{m \frac{T_s}{2} + \frac{T_s}{8}}^{m \frac{T_s}{2} + \frac{T_s}{4}} \frac{1}{2} \cdot e^{-2j\pi f t} dt \\ - \int_{m \frac{T_s}{2} + \frac{3T_s}{8}}^{m \frac{T_s}{2} + \frac{T_s}{2}} \frac{1}{2} \cdot e^{-2j\pi f t} dt - \int_{m \frac{T_s}{2} + \frac{T_s}{4}}^{m \frac{T_s}{2} + \frac{3T_s}{8}} \frac{\sqrt{2}+1}{2} \cdot e^{-2j\pi f t} dt \end{cases}$$

$$FT\left\{\mu_{T_s/2}\left(t - m \frac{T_s}{2}\right)\right\} = -\frac{e^{-2j\pi f m \frac{T_s}{2}}}{2j\pi f} \cdot e^{-j\pi f \frac{T_s}{2}} \begin{bmatrix} -(\sqrt{2}+1)\cos\left(\pi f \frac{T_s}{2}\right) \\ +\sqrt{2}\cos\left(\pi f \frac{T_s}{4}\right) + 1 \end{bmatrix}$$

So the Fourier Transform of  $sc_{as\_1}(t)$  is :

$$SC_{as\_1}(f) = -\frac{e^{-j\pi f \frac{T_s}{2}}}{2j\pi f} \begin{bmatrix} -(\sqrt{2}+1)\cos\left(\pi f \frac{T_s}{2}\right) \\ +\sqrt{2}\cos\left(\pi f \frac{T_s}{4}\right) + 1 \end{bmatrix} \cdot \sum_{m=0}^{n-1} (-1)^m e^{-2j\pi f m \frac{T_s}{2}}$$

For n odd we have:

$$\sum_{m=0}^{n-1} (-1)^m e^{-2j\pi f m \frac{T_s}{2}} = e^{-j(n-1)\pi f \frac{T_s}{2}} \frac{\cos\left(n\pi f \frac{T_s}{2}\right)}{\cos\left(\pi f \frac{T_s}{2}\right)}$$

So,

$$SC_{as\_1}(f) = -\frac{e^{-jn\pi\frac{T_s}{2}} \cos\left(n\pi f \frac{T_s}{2}\right) \left[ -(\sqrt{2}+1)\cos\left(\pi f \frac{T_s}{2}\right) \right]}{2j\pi f \cos\left(\pi f \frac{T_s}{2}\right) \left[ +\sqrt{2}\cos\left(\pi f \frac{T_s}{4}\right) + 1 \right]}$$

Finally if n is odd,

$$|SC_{as\_1}(f)|^2 = \frac{1}{4\pi^2 f^2} \frac{\cos^2\left(\pi f T_c\right)}{\cos^2\left(\pi f \frac{T_c}{n}\right)} \left[ \frac{-(\sqrt{2}+1)\cos\left(\pi f \frac{T_s}{2}\right)}{+\sqrt{2}\cos\left(\pi f \frac{T_s}{4}\right) + 1} \right]^2$$

The same calculation are made for  $SC_{as\_2}(f)$ ,  $SC_{ap\_1}(f)$  and  $SC_{as\_2}(f)$ .

$$sc_{as\_2}(t) = \sum_{m=0}^{n-1} (-1)^m \mu_{T_s/2} \left( t - m \frac{T_s}{2} \right)$$

with

$$\mu_{T_s/2}(t) = \begin{cases} \frac{1}{2} & \left[ 0, \frac{T_s}{8} \right] \\ \frac{\sqrt{2}+1}{2} & \left[ \frac{T_s}{8}, \frac{3T_s}{8} \right] \\ \frac{1}{2} & \left[ \frac{3T_s}{8}, \frac{T_s}{2} \right] \end{cases}$$

$$FT\left\{ \mu_{T_s/2} \left( t - m \frac{T_s}{2} \right) \right\} = \int_{\frac{mT_s}{2}}^{\frac{(m+1)T_s}{2}} \mu_{T_s/2} \left( t - m \frac{T_s}{2} \right) \cdot e^{-2j\pi ft} dt$$

$$FT\left\{ \mu_{T_s/2} \left( t - m \frac{T_s}{2} \right) \right\} = -\frac{e^{-2j\pi m \frac{T_s}{2}}}{2j\pi f} \cdot e^{-j\pi f \frac{T_s}{2}} \begin{bmatrix} -j \sin\left(\pi f \frac{T_s}{2}\right) \\ -j\sqrt{2} \sin\left(\pi f \frac{T_s}{4}\right) \end{bmatrix}$$

So the Fourier Transform of  $sc_{as\_2}(t)$  is :

$$SC_{as\_2}(f) = \frac{e^{-jn\pi\frac{T_s}{2}} \cos\left(n\pi f \frac{T_s}{2}\right)}{2j\pi f \cos\left(\pi f \frac{T_s}{2}\right)} \left[ \sin\left(\pi f \frac{T_s}{2}\right) + \sqrt{2} \sin\left(\pi f \frac{T_s}{4}\right) \right]$$

And if n is odd,

$$|SC_{as\_2}(f)|^2 = \frac{1}{4\pi^2 f^2} \frac{\cos^2\left(\pi f T_c\right)}{\cos^2\left(\pi f \frac{T_c}{n}\right)} \left[ \sin\left(\pi f \frac{T_s}{2}\right) + \sqrt{2} \sin\left(\pi f \frac{T_s}{4}\right) \right]^2$$

Continuing with the calculation of the Fourier Transform of  $SC_{ap\_1}(f)$ ,

$$sc_{ap\_1}(t) = \sum_{m=0}^{n-1} (-1)^m \mu_{T_s/2} \left( t - m \frac{T_s}{2} \right)$$

with

$$\mu_{T_s/2}(t) = \begin{cases} \frac{\sqrt{2}-1}{2} & \left[ 0, \frac{T_s}{8} \right] \\ \frac{1}{2} & \left[ \frac{T_s}{8}, \frac{T_s}{4} \right] \\ -\frac{1}{2} & \left[ \frac{T_s}{4}, \frac{3T_s}{8} \right] \\ \frac{\sqrt{2}-1}{2} & \left[ \frac{3T_s}{8}, \frac{T_s}{2} \right] \end{cases}$$

$$FT\left\{ \mu_{T_s/2} \left( t - m \frac{T_s}{2} \right) \right\} = \int_{\frac{mT_s}{2}}^{\frac{(m+1)T_s}{2}} \mu_{T_s/2} \left( t - m \frac{T_s}{2} \right) \cdot e^{-2j\pi ft} dt$$

$$FT\left\{ \mu_{T_s/2} \left( t - m \frac{T_s}{2} \right) \right\} = -\frac{e^{-2j\pi m \frac{T_s}{2}}}{2j\pi f} \cdot e^{-j\pi f \frac{T_s}{2}} \begin{bmatrix} (\sqrt{2}-1)\cos\left(\pi f \frac{T_s}{2}\right) \\ -\sqrt{2}\cos\left(\pi f \frac{T_s}{4}\right) + 1 \end{bmatrix}$$

So the Fourier Transform of  $sc_{ap\_1}(t)$  is :

$$SC_{ap\_1}(f) = -\frac{e^{-jn\pi\frac{T_s}{2}} \cos\left(n\pi f \frac{T_s}{2}\right) \left[ (\sqrt{2}-1)\cos\left(\pi f \frac{T_s}{2}\right) \right]}{2j\pi f \cos\left(\pi f \frac{T_s}{2}\right) \left[ -\sqrt{2}\cos\left(\pi f \frac{T_s}{4}\right) + 1 \right]}$$

And if n is odd,

$$|SC_{ap\_1}(f)|^2 = \frac{1}{4\pi^2 f^2} \frac{\cos^2\left(\pi f T_c\right)}{\cos^2\left(\pi f \frac{T_c}{n}\right)} \left[ \frac{(\sqrt{2}-1)\cos\left(\pi f \frac{T_s}{2}\right)}{-\sqrt{2}\cos\left(\pi f \frac{T_s}{4}\right) + 1} \right]^2$$

To finish with the calculation of the Fourier Transform,

$$sc_{ap\_2}(t) = \sum_{m=0}^{n-1} (-1)^m \mu_{T_s/2} \left( t - m \frac{T_s}{2} \right)$$

with

$$\mu_{T_s/2}(t) = \begin{cases} \frac{1}{2} & \left[0, \frac{T_s}{8}\right] \\ -\frac{\sqrt{2}-1}{2} & \left[\frac{T_s}{8}, \frac{3T_s}{8}\right] \\ \frac{1}{2} & \left[\frac{3T_s}{8}, \frac{T_s}{2}\right] \end{cases}$$

$$FT\left\{\mu_{T_s/2}\left(t - m\frac{T_s}{2}\right)\right\} = \int_{\frac{mT_s}{2}}^{\frac{(m+1)T_s}{2}} \mu_{T_s/2}\left(t - m\frac{T_s}{2}\right) \cdot e^{-2j\pi ft} dt$$

$$FT\left\{\mu_{T_s/2}\left(t - m\frac{T_s}{2}\right)\right\} = -\frac{e^{-2j\pi f\frac{T_s}{2}}}{2j\pi f} \cdot e^{-j\pi f\frac{T_s}{2}} \left[ -j\sin\left(\pi f\frac{T_s}{2}\right) + j\sqrt{2}\sin\left(\pi f\frac{T_s}{4}\right) \right]$$

So the Fourier Transform of  $sc_{ap\_2}(t)$  is :

$$SC_{as\_2}(f) = \frac{e^{-jn\pi f\frac{T_s}{2}} \cos\left(n\pi f\frac{T_s}{2}\right)}{2\pi f} \frac{1}{\cos\left(\pi f\frac{T_s}{2}\right)} \left[ -\sin\left(\pi f\frac{T_s}{2}\right) + \sqrt{2}\sin\left(\pi f\frac{T_s}{4}\right) \right]$$

And if n is odd,

$$|SC_{as\_2}(f)|^2 = \frac{1}{4\pi^2 f^2} \frac{\cos^2(\pi f T_c)}{\cos^2\left(\pi f\frac{T_c}{n}\right)} \left[ -\sin\left(\pi f\frac{T_s}{2}\right) + \sqrt{2}\sin\left(\pi f\frac{T_s}{4}\right) \right]^2$$

Finally, the normalized power spectrum density of the ALTBOC signal with a constant envelope is if n odd:

$$G_{ALTBOC}(f) = \frac{4}{\pi^2 f^2 T_c} \frac{\cos^2(\pi f T_c)}{\cos^2\left(\pi f\frac{T_c}{n}\right)} \left[ \begin{array}{l} \cos^2\left(\pi f\frac{T_s}{2}\right) - \cos\left(\pi f\frac{T_s}{2}\right) \\ -2\cos\left(\pi f\frac{T_s}{2}\right)\cos\left(\pi f\frac{T_s}{4}\right) + 2 \end{array} \right]$$

## REFERENCES

[Betz, 2001]: “Binary Offset Carrier Modulations for Radionavigation” – John W. Betz – Journal of the Institute of Navigation, Winter 2001-2002.

[Pratt and Owen, 2003]: “BOC Modulations Waveform” – Anthony R. Pratt & John I.R. Owen – ION GPS/GNSS, September 2003.

[Ries and al., 2003]: “Tracking and Multipath Performance Assessments of BOC Signals Using a Bit-Level Signal Processing Simulator” – L. Ries, F. Legrand, L. Lestarquit, W. Vigneau, J.L. Issler – ION GPS/GNSS, September 2003.

[Soellner and Erhard, 2003]: “Comparison of AWGN code tracking accuracy for Alternative-BOC, Complex-LOC and Complex-BOC modulation options in Galileo E5-Band” – M. Soellner & P. Erhard – GNSS Conference 2003, Graz, April 2003.

[Hein and al., 2001]: “The Galileo Frequency Structure and Signal Design” – G. W. Hein, J. Godet, J-L. Issler, J-C. Martin, R. Lucas-Rodriguez and T. Pratt – ION GPS/GNSS, Salt Lake City, September 2001.

[Ries and al., 2002]: “Software Simulation Tools for GNSS2 BOC Signal Analysis” – L. Ries, L. Lestarquit, P. Erhard, F. Legrand, C. Macabiau, Q. Jeandel and C. Bourgat – ION GPS/GNSS, Portland, September 2002.

[Godet, 2001]: “Technical Annex to Galileo SRD Signal Plans” – J. Godet – STF annexSRD 2001/2003 Draft 1, July 2003.