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RAIM Performance in Presence of Multiple Range Failures

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BIOGRAPHY

Christophe Macabiau graduated as an electronics engineer in 1992 from ENAC (Ecole Nationale de l'Aviation Civile) in Toulouse, France. Since 1994, he has been working on the application of satellite navigation techniques to civil aviation. He received his Ph.D. in 1997 and has been in charge of the signal processing lab of the ENAC since 2000.

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Lionel Fillatre received the Ph.D. degree in systems optimization from the UTT- the University of Technology of Troyes, France, in 2004. He is currently an assistant professor at the Institute of Computer Sciences and Engineering of Troyes (ISTIT), FRE CNRS 2732. His current research interests include decision theory, signal and image processing and non-destructive testing.

Benoit Roturier graduated as a CNS systems engineer from Ecole Nationale de l'Aviation Civile (ENAC), Toulouse in 1985 and obtained a PhD in Electronics from Institut National Polytechnique de Toulouse in 1995. He was successively in charge of Instrument Landing Systems at DGAC/STNA (Service Technique de la Navigation Aerienne), then of research activities on CNS systems at ENAC. He is now head of GNSS Navigation subdivision at STNA and is involved in the development of civil aviation applications based on GPS/ABAS, EGNOS and Galileo.

Eric Chatre graduated as an electronics engineer in 1992 from the ENAC (Ecole Nationale de l'Aviation Civile), Toulouse, France. From 1994 to 2001, he worked with the Service Technique de la Navigation Aérienne (STNA) in Toulouse on implementation of satellite navigation in civil aviation. He is now part of the Galileo Joint Undertaking where he is in charge of standardisation and certification matters. He is secretary of Eurocae WG 62.

Mathieu Raimondi will graduate in 2005 as an electronics engineer from the Ecole Nationale de l'Aviation Civile (ENAC) in Toulouse, France.

Anne-Christine Escher graduated as an electronics engineer in 1999 from ENAC (Ecole Nationale de l'Aviation Civile) in Toulouse, France. Since 1999, she has been working on the application of satellite navigation techniques to civil aviation at the signal processing lab of the ENAC.

ABSTRACT

It is currently stated and widely accepted by industry and users that the RAIM is designed to provide timely warnings in the situation where only one of the range measurements used at the current epoch is affected by an unacceptable bias. However, given the range of potential applications of RAIM in the future, in particular with the advent of Galileo and the generalization of Safety of Life applications, which should spread from the civil aviation community to many other professional sectors, it is very important to better understand the fundamental properties of RAIM, and in particular the potential of RAIM to detect multiple failures on range measurements.

The purpose of the study presented in this paper is to analyze the performance of a RAIM in the presence of multiple simultaneous range errors. In particular, we conduct a theoretical analysis to determine in which cases the Least Squares Residuals RAIM detection criterion is not affected, and analyzed results of Monte-Carlo simulations in presence of up to four range failures.
The theoretical analysis outlined above aims at determining in what conditions the Least Squares Residuals RAIM detection criterion remains unaffected by multiple range failures, searching for what we call criterion unaffecting range errors. We show that, provided the satellite constellation does not have any degenerated geometrical properties, the dimension of the vector sub-space of these criterion unaffecting errors is max(4-(N-p),0), where N is the number of tracked satellites and p is the number of faulty pseudorange measurements. The immediate conclusion is that if N-4 pseudo range measurements are affected, or less than that, by a large error, there exists no error that will not affect the RAIM detection criterion, and globally due to the negligible probability that unintentional interference lies in a small dimension sub-space, the RAIM detection criterion exhibits a natural detection capability even if up to N-2 pseudo-range measurements are faulty. This theoretical result defines the properties of the errors that lead to zero change in the detection criterion. To jump to a more operational conclusion, it remains to know what the possibility is for multiple range errors to induce a detection criterion that is below the detection threshold. So to complement the theoretical analysis outlined above, we ran Monte Carlo simulations inserting up to four range failures and analyzing the detection capacity. The capacity of RAIM detection is also analyzed in the presence of intentional jamming.

I. INTRODUCTION

The pseudorange measurements made by a Global Navigation Satellite System (GNSS) receiver are affected by residual atmospheric delays, multipath, noise and background interference. These range errors induce errors in the estimated position.

The International Civil Aviation Organization (ICAO) has stated the requirements that need to be fulfilled by a GNSS to be used as navigation means for specific phases of flight. These requirements are applicable to the GNSS receiver and to the GNSS signal, and are stated in terms of accuracy, integrity, continuity and availability. These requirements specify a nominal RF environment and feared events leading to abnormal situations.

In order to satisfy the ICAO GNSS requirements from the en-route to the Non Precision Approach phases of flight, a GPS receiver needs to include a Receiver Autonomous Integrity Monitoring (RAIM) module. This module is mainly designed to warn rapidly the user in the case of an unacceptable measurement error, such as the error caused by a failure of an atomic clock onboard a GPS satellite. It can also include an algorithm to exclude the faulty measurement from the navigation solution calculation. It must provide the user with a Horizontal Protection Level (HPL), which is the estimation of the impact in the horizontal plane of the smallest range bias that can be detected.

It is important to recall that the RAIM performance can only be guaranteed in the situation where only one of the range measurements used at the current epoch is affected by an unacceptable bias.

The purpose of the study presented in this paper is to analyze the performance of a RAIM in the presence of multiple simultaneous range errors. In particular, we conduct a theoretical analysis to determine in which cases the LSR RAIM detection criterion is not affected, and analyzed results of Monte-Carlo simulations in presence of up to four range failures.

The first part of this paper recalls the design parameters of a RAIM algorithm and the principle of classical Least Squares Residuals RAIM. On this basis, we then determine what the mathematical condition on multiple range measurement errors is for the LSR RAIM detection criterion to be equal to zero, defining what we call criterion unaffecting range errors. We show that, for non-degenerated visibility matrices, the dimension of the vector sub-space of these criterion unaffecting range errors is max (4-(N-p),0), where N is the number of tracked satellites and p is the number of faulty pseudorange measurements. The natural resistance of the detection criterion in presence of unintentional jamming is then very strong. This condition is then physically justified, and graphically illustrated in a two dimension example. We also check this particular RAIM detection criterion performance in several cases, using a real GPS receiver connected to a GPS signal generator. In order to move towards a more operational conclusion on this RAIM performance by analyzing the impact of errors that induce a detection criterion which is below the detection threshold, we then provide results of Monte-Carlo simulations showing that this robustness is indeed very large in presence of up to four faulty ranges. Then, detection capability in presence of intentional interference is discussed. A conclusion on this overall detection performance is then drawn, particularly in the context of future GNSS, comprising GPS and GALILEO, broadcasting signals on several frequency bands.

II. PSEUDORANGE MEASUREMENTS MODEL

Let us denote $y^i(k)$ the pseudo-range measurements made by the user receiver at each epoch k on the signal coming from satellite i, and corrected from the broadcast satellite clock offset, ionospheric delay, tropospheric delay.

These corrected measurements are modelled as:

$$y^i(k) = \rho^i(k) + c\Delta t_u(k) + e^i(k)$$

where

- ρ^i is the true geometrical distance between the satellite antenna and the user receiver antenna
- Δt_u is the user receiver clock offset w.r.t GPS time. We note $b_u = c\Delta t_u$ the receiver clock bias expressed in meters.
- e^i is the sum of the measurement errors due to multipath, background interference, noise, ionospheric and atmospheric propagation delay residuals, satellite clock residuals.

These measurements can also be expressed as a function of the receiver true position, and of the satellite position as follows:

$$y^i(k) = \sqrt{(x - x^i)^2 + (y - y^i)^2 + (z - z^i)^2} + b_u + e^i$$

where

- x, y, z are the cartesian coordinates of the receiver antenna at the time of signal reception expressed in an ECEF reference frame.
- x^i, y^i, z^i are the cartesian coordinates of the satellite antenna at the time of signal emission expressed in an ECEF reference frame.

We gather in a vector denoted $Y(k)$ the corrected pseudorange measurements made by the receiver at time k. These measurements are modelled as follows:

$Y(k) = \begin{bmatrix} y^1(k) & \cdots & y^N(k) \end{bmatrix}^t$ where N is the number of satellites used for positioning at the current epoch.

$$Y(k) = \begin{bmatrix} \sqrt{(x-x^1)^2+(y-y^1)^2+(z-z^1)^2}+b_u \\ \vdots \\ \sqrt{(x-x^N)^2+(y-y^N)^2+(z-z^N)^2}+b_u \end{bmatrix} + \begin{bmatrix} e^1 \\ \vdots \\ e^N \end{bmatrix}$$

We note

$$X(k) = \begin{bmatrix} x \\ y \\ z \\ b \end{bmatrix}, E(k) = \begin{bmatrix} e^1 \\ \vdots \\ e^N \end{bmatrix}, h(X(k)) = \begin{bmatrix} h^1(X) \\ \vdots \\ h^N(X) \end{bmatrix}$$

where

$$h^i(X(k)) = \sqrt{(x-x^i)^2+(y-y^i)^2+(z-z^i)^2} + b_u(k)$$

The measurement model is thus expressed as:

$$Y(k) = h(X(k)) + E(k).$$

III. LEAST SQUARES NAVIGATION SOLUTION

The measurement model is not linear because the measurements do not linearly depend on X. Therefore, we implement an iterative least squares estimation technique. This method uses the linearization of the measurement model around successive estimates of the receiver position.

Let us denote $\hat{X}_0(k)$ an initial estimate of $X(k)$. This initial estimate can be determined using past measurements or can be provided by other navigation means.

We then denote $X(k) = \hat{X}_0(k) + \Delta X(k)$. Therefore, we can rewrite the measurement model as follows:

$$Y(k) = h(\hat{X}_0(k) + \Delta X(k)) + E(k)$$

This model is linearized around $\hat{X}_0(k)$:

$$Y(k) \approx h(\hat{X}_0(k)) + \frac{\partial h}{\partial X}(\hat{X}_0(k)) \times \Delta X(k) + E(k)$$

The first order derivative that appears in this last equation is an $N \times 4$ matrix that can be expressed as:

$$H = \frac{\partial h}{\partial X}(\hat{X}_0(k))$$

$$H = \begin{bmatrix} \frac{\partial h^1}{\partial x}(\hat{X}_0) & \frac{\partial h^1}{\partial y}(\hat{X}_0) & \frac{\partial h^1}{\partial z}(\hat{X}_0) & \frac{\partial h^1}{\partial b}(\hat{X}_0) \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial h^N}{\partial x}(\hat{X}_0) & \frac{\partial h^N}{\partial y}(\hat{X}_0) & \frac{\partial h^N}{\partial z}(\hat{X}_0) & \frac{\partial h^1}{\partial b}(\hat{X}_0) \end{bmatrix}$$

It can be shown that these derivatives can be expressed as:

$$\frac{\partial h^i}{\partial x}(\hat{X}_0(k)) = \frac{\hat{x}_0 - x^i(k)}{\sqrt{(\hat{x}_0 - x^i(k))^2 + (\hat{y}_0 - y^i(k))^2 + (\hat{z}_0 - z^i(k))^2}}$$

and $\frac{\partial h^i}{\partial b}(\hat{X}_0(k)) = 1$.

The linearized model can also be rewritten as:

$$Y(k) - h(\hat{X}_0(k)) = H \times \Delta X(k) + E(k)$$

or

$$\Delta Y(k) = H \times \Delta X(k) + E(k)$$

if we note $\Delta Y(k) = Y(k) - h(\hat{X}_0(k))$ the deviation between the measurements made and the predicted noiseless measurements that the receiver would have made if its position and clock delay were $\hat{X}_0(k)$.

Considering this new linear model between $\Delta Y(k)$ and $\Delta X(k)$, we can compute a least squares estimate of $\Delta X(k)$.

This estimate is:

$$\Delta \hat{X}(k) = \left[H^t H\right]^{-1} H^t \times \Delta Y(k)$$

Let us denote that if the measurement error covariance matrix is known, then the weighted least squares estimate is:

$$\Delta \hat{X}(k) = \left[H^t R^{-1} H\right]^{-1} H^t R^{-1} \times \Delta Y(k)$$

where $R(k) = Cov(E(k))$.

This quantity $\Delta \hat{X}(k)$ is an estimate of $\Delta X(k)$, which is defined as the deviation between the initial estimate $\hat{X}_0(k)$ and $X(k)$.

We can therefore imagine the implementation of an iterative algorithm starting from an initial estimate $\hat{X}_0(k)$ and improving progressively this estimate through the comparison between the measurements and the predicted measurements for each estimated position. The iterative algorithm can be implemented to stop if $\Delta \hat{X}(k)$ is a vector that has a small norm.

Another possibility is to look at $\Delta Y(k)$, but then starts the RAIM.

IV. RAIM ALGORITHM BASICS

The norm of vector $\Delta Y(k)$ can be used as a quality test. Indeed, if the mathematical model of the measurements is correct, if the measurements are only affected by noise with a standard deviation adequately described by the model, and if the estimated position and receiver clock bias are close to reality, then the norm of this vector $\Delta Y(k)$ is the of the same order as the noise.

Indeed:

$$\Delta Y = Y - h(\hat{X})$$

This can be expressed as:

$$Y - h(\hat{X}) = h(X) - h(\hat{X}) + E$$
$$= h(\hat{X}_0 + \Delta X) - h(\hat{X}_0 + \Delta \hat{X}) + E$$

This can be linearized as:

$$Y - h(\hat{X}) \approx H\Delta X - H\Delta \hat{X} + E = H(\Delta X - \Delta \hat{X}) + E$$

But $\Delta \hat{X}(k) = \left[H^t R^{-1} H\right]^{-1} H^t R^{-1} \times \left[Y(k) - h(\hat{X}_0(k))\right]$

,

therefore

$$\Delta \hat{X}(k) = \left[H^t R^{-1} H\right]^{-1} H^t R^{-1} \times \left[H \times \Delta X(k) + E(k)\right]$$

This is equivalent to

$\Delta \hat{X}(k) = \Delta X(k) + \left[H^t R^{-1} H\right]^{-1} H^t R^{-1} \times E(k)$, so

$$\Delta X(k) - \Delta \hat{X}(k) = -\left[H^t R^{-1} H\right]^{-1} H^t R^{-1} \times E(k)$$

If we integrate this in the first equation, we get

$$Y - h(\hat{X}) \approx -H\left[H^t H\right]^{-1} H^t E + E$$

and finally, we have

$$\Delta Y = Y - h(\hat{X}) = \left(I - H\left[H^t H\right]^{-1} H^t\right) E$$

where E is the vector containing all the measurement errors.

Therefore, there is a linear relation ship between the measurement error vector E and the prediction error vector ΔY.

The RAIM test statistics is then computed using the sum of squared residuals which is defined by:

$$SSE = \Delta Y \cdot \Delta Y^T = \left\|\Delta Y\right\|^2$$

As we can see, if all elements of E have the same Gaussian distribution, are independent and with zero mean and standard deviation σ, then, the statistical distribution of SSE is independent from the geometry of the constellation, for any value of N.

With the previous assumptions on the distribution of E, SSE/σ^2 is centred and χ^2-distributed with N-4 degrees of freedom, provided the number of visible satellites N is at least 6.

Note that for N=5, it is shown that SSE/σ^2 has a Gaussian distribution: $\frac{SSE}{\sigma^2} \sim N(\mu',1)$.

The classical operational test statistic is $T = \sqrt{\frac{SSE}{N-4}}$.

V. RANGE FAILURES UNAFFECTING THE DETECTION CRITERION

To identify the errors that do not affect the test statistics T in the above mentioned RAIM, we look for the systematic errors E that induce no variation of ΔY. We call these errors the criterion unaffecting range errors.

The pseudorange measurement errors E can be modelled as the sum of the noise plus the range errors that do not originate from noise itself. So we can write

$$E = E_{bias} + E_{noise}$$

where:

- E_{noise} is the vector of the noise errors
- E_{bias} is the vector of the range faults not originating from the noise, but rather from satellite failures or interference. These errors are modeled as biases at each epoch.

Let us assume that the measurement error vector E_{bias} has *p* non-zero coordinates. This means we assume now that not all the GNSS pseudorange measurements are affected by unacceptable errors but only *p* GNSS channels. We denote the measurement error vector in that case E_{bias_p}.

It is assumed that the 4 columns of the matrix *H* are independent, i.e. the partial derivatives of the non linear function *h* w.r.t. *x*, *y*, *z* and *b* are an independent family of vectors. This implies that $rank\, H = 4$. Let us define the set of indexes $I = \{ i_1, i_2, \cdots, i_p \}$ such that $E_p(i_k) \neq 0$ and $F_I = span\{ e_{i_1}, e_{i_2}, \cdots, e_{i_p} \}$ the linear space spanned by the vectors $\{ e_{i_1}, e_{i_2}, \cdots, e_{i_p} \}$, where e_i is the *i-th* vector of the canonical basis of R^n.

Let us now denote $S = \left[H^t H\right]^{-1} H^t$ the pseudoinverse of *H*. The prediction error vector ΔY, which is the base for the RAIM test statistic, can be expressed as:

$$\Delta Y = (I - HS)E$$

where E is the vector of the measurement errors.

Therefore the prediction error vector can be expressed as:

$$\Delta Y = (I - HS)E_{bias} + (I - HS)E_{noise} = \Delta Y_{bias} + \Delta Y_{noise}$$

The prediction error has two contributors: the noise and the range errors not originating from the noise. Let us look at the impact of the range errors

$$\Delta Y_{bias} = (I - HS)E_{bias_p} = P_H E_{bias_p},$$

where $P_H = I - HS$ is the orthogonal projection onto the null space of H^t. Consequently, all error vectors E_{bias_p} affect the detection criterion, $P_H E_{bias_p} \neq 0$, if the intersection *J* between F_I and $\operatorname{Im} H$ is restricted to the null-vector. In [Fillatre and Nikiforov, 2005], it is shown that it is equivalent to assume that the set of rows $\{ H_{j_1}, H_{j_2}, \cdots, H_{j_{N-p}} \}$ is linearly independent for all possible set *I* by denoting

$$H_j = \left[\frac{\partial h^j}{\partial x}(\hat{X}_0) \quad \frac{\partial h^j}{\partial y}(\hat{X}_0) \quad \frac{\partial h^j}{\partial z}(\hat{X}_0) \quad \frac{\partial h^j}{\partial b}(\hat{X}_0) \right]$$

and

$$K = \{ j_1, j_2, \cdots, j_{N-p} \} = \{ 1,2,\cdots,N \} - \{ i_1, i_2, \cdots, i_p \}$$

the set of indexes associated to the rows of *H* which are not affected by the *p* pseudorange errors whose locations are defined by *I*.
In other words, an error vector which affects p pseudorange measurements does not affect the criterion if and only if the sub-matrix H_K, composed of the *N-p* other measures, is not full rank column.

Given only one satellite constellation like the GNSS, it is straightforward to verify that each row of the matrix *H* is linear independent from the others due to the different orientations and positions of each satellite composing the constellation. Hence, it is straightforward to verify that the extraction of *N-p* rows of *H* by an arbitrary set *I* still generates a sub-matrix H_K such that $rank\, H_K = \min\{4, N - p\}$.

Consequently, an error vector always affects the detection criterion when N-4, or less, pseudorange measurements are erroneous. On the contrary, it always exists errors that do not affect the detection criterion when more than N-4 pseudorange measurements are affected by errors. More specifically, it will be proved later that the intersection *J* between F_I and $\operatorname{Im} H$ has the dimension *max(*4-*(N-p),0)*.

This means that, noting N the numbers of visible satellites:

- if N-4 pseudorange measurements are faulty, or less than that, there are no errors that do not affect the RAIM criterion.
- if N-3 pseudorange measurements are faulty, the vector sub-space of the criterion unaffecting errors has a dimension 1. This means that these measurement errors for all the satellites are all linearly related to a unique arbitrary parameter, which seems very unlikely.
- if N-2 pseudorange measurements are faulty, the vector sub-space of the criterion unaffecting errors has a dimension 2. This means that these measurement errors for all the satellites are all linearly related to 2 arbitrary parameters, which seems very unlikely also.
- if N-1 or N pseudorange measurements are affected, the vector sub-space of the criterion unaffecting errors has a dimension 3 or 4. This means that these measurement errors for all the satellites are all linearly related to 3 or 4 arbitrary parameters, which reduces also the probability for this to occur.

It is important to note that some changes in error detectability may appear when two satellite constellations are used simultaneously due to the existence of two satellites which have the same elevation and azimuth with respect to the GNSS receiver. Hence, some linear dependencies may exist between the rows of the matrix *H* and it certainly becomes possible to extract 4 linearly dependant rows of *H*. In this case, it may exist some criterion unaffecting errors when only N-4 pseudorange measurements are affected by errors.

VI. POSITION ERRORS INDUCED BY CRITERION UNAFFECTING RANGE FAILURES

Let us now try to characterize the position errors induced by these measurement errors that do not affect the LSR RAIM detection criterion. This characteristic will be expressed as the linear relationship between these position errors and a basic vector that has dimension 4-(N-p). This expression will be in turn used to express the linear space of the criterion unaffecting measurement errors themselves.

We have seen in section IV that the position errors induced by measurement errors are defined as:

$$\Delta\hat{X} - \Delta X = (H^t H)^{-1} H^t E$$

If we denote again $S = (H^t H)^{-1} H^t$, then the position errors $\Delta\hat{X} - \Delta X$ induced by the criterion unaffecting errors are such that $SE_{bias_p} = \Delta\hat{X} - \Delta X$, and as we know $\Delta Y_{bias} = (I - HS)E_{bias_p}$, then

$\Delta Y_{bias} = 0$ is equivalent to $E_{bias_p} = HS.E_{bias_p}$, and this leads to the following relationship between the criterion unaffecting errors and the position error that they induce:

$$E_{bias_p} = H\left(\Delta\hat{X} - \Delta X\right)$$

This relation can be used to split the H matrix in two blocks $H_{1\to p}$ from lines 1 to p and $H_{p+1\to n}$ from lines p+1 to N:

$$\begin{bmatrix} e_1 \\ \vdots \\ e_p \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} H_{1\to p} \\ H_{p+1\to n} \end{bmatrix} . \left(\Delta\hat{X} - \Delta X\right) = \begin{bmatrix} H_{1\to p}(\Delta\hat{X} - \Delta X) \\ H_{p+1\to n}(\Delta\hat{X} - \Delta X) \end{bmatrix}$$

$$\begin{bmatrix} e_1 \\ \vdots \\ e_p \end{bmatrix} = H_{1\to p}\left(\Delta\hat{X} - \Delta X\right), \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} = H_{p+1\to N}\left(\Delta\hat{X} - \Delta X\right)$$

Therefore, we see that $\Delta\hat{X} - \Delta X \in \ker\left(H_{p+1\to N}\right)$.

Let us now try to define this linear space of the position errors induced by the criterion unaffecting errors.

Let us denote *q=N-p* and let us decompose the vector $\Delta\hat{X} - \Delta X$ as the collection of a vector with dimension *q*, and a vector with dimension *4-q*:

$$W = \left(\Delta\hat{X} - \Delta X\right) = \begin{pmatrix} W_q \\ W_{4-q} \end{pmatrix}$$

The Nx4 matrix *H* can be accordingly rewritten as the following 4 block matrix:

$$H = \begin{bmatrix} H_{p,N-p} & H_{p,4-(N-p)} \\ H_{N-p,N-p} & H_{N-p,4-(N-p)} \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$

Note that both decompositions only exist if *4-q>0*, or equivalently $4 - (N - p) > 0$.

The previous property $\Delta\hat{X} - \Delta X \in \ker\left(H_{p+1\to N}\right)$ can be expressed as:

$\begin{bmatrix} H_{21} & H_{22} \end{bmatrix}\left(\Delta\hat{X} - \Delta X\right) = 0$.

We have then $\begin{bmatrix} H_{21} & H_{22} \end{bmatrix}\begin{pmatrix} W_q \\ W_{4-q} \end{pmatrix} = 0$. This is equivalent to $H_{21}W_q + H_{22}W_{4-q} = 0$.

As the left corner block of H, H_{21} is a square matrix with full rank N-p, we can write:

$$W_q = -H_{21}^{-1} H_{22} W_{4-q}$$

So the position errors can be expressed as a linear combination of the same vector W_{4-q}:

$$\Delta\hat{X} - \Delta X = \begin{bmatrix} -H_{21}^{-1} H_{22} W_{4-q} \\ W_{4-q} \end{bmatrix} = \begin{bmatrix} -H_{21}^{-1} H_{22} \\ Id \end{bmatrix} W_{4-q}$$

Therefore, the dimension of the vector sub-space of the position errors induced by the measurement errors that do not affect the detection criterion is the dimension of W_{4-q}, which is $4 - q = 4 - (N - p)$.

Let us now express in the same way the linear space of the criterion unaffecting range measurement errors that do cause these position errors:

$$\begin{bmatrix} e_1 \\ \vdots \\ e_p \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \end{bmatrix}\left(\Delta\hat{X} - \Delta X\right) = \begin{bmatrix} H_{11} & H_{12} \end{bmatrix}\begin{pmatrix} -H_{21}^{-1} H_{22} \\ Id \end{pmatrix} W_{4-q}$$

$$\begin{bmatrix} e_1 \\ \vdots \\ e_p \end{bmatrix} = (-H_{11} H_{21}^{-1} H_{22} + H_{12}).W_{4-q}$$

We see again that we are dealing with a linear sub-space defined as all linear combinations of W_{4-q}, so this linear sub-space of the criterion unaffecting errors has a dimension $4-q=4-(N-p)$.

VII. ILLUSTRATION IN 2 DIMENSIONS

To illustrate this theory on criterion unaffecting biases, we consider examples in simplified conditions. First of all, in order to simplify the interpretation of the drawings, we assume the user clock bias is resolved, so the user clock bias is considered estimated. Therefore, in absence of error, the position solution is at the intersection of the iso-range surfaces. Then, we consider that only 4 satellites are used for positioning.

Let us first consider the case without failure, as illustrated in figure 1. The RAIM does not detect any failure.

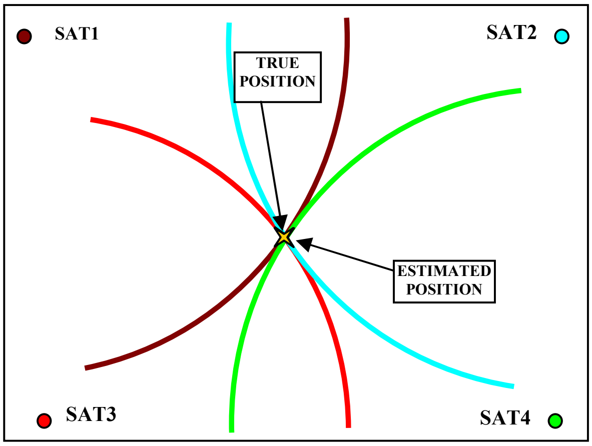

Figure 1: *Illustration in a 2 dimension case of an estimated position without range failure in the absence of noise.*

In the following, spheres are approximated as lines because the satellites are very far compared to the position errors illustrated in the figures shown.

We now consider in figure 2 the case where a single range error appears, on satellite 3. The estimated position is wrong. The measurement prediction error for each satellite can be illustrated as the projection of that estimated position on the satellite iso-range surface. As the deviation between the predicted range and the observed range is too large for satellites 2 and 3, we can anticipate that the RAIM criterion detection is affected by the error. This result is expected as the RAIM is designed to detect one range error among all the measurements.

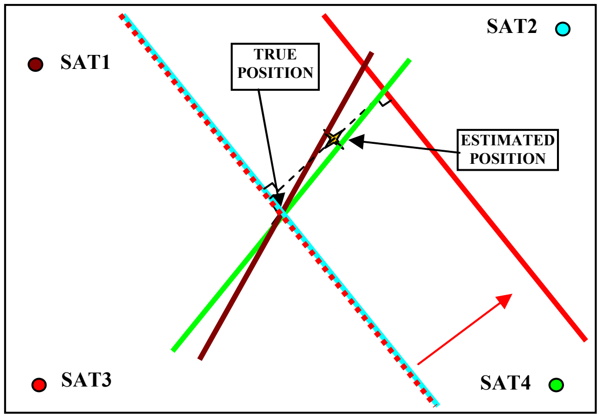

Figure 2: *Illustration in a 2 dimension case of an estimated position with one range failure.*

We can, on figure 2, discuss the conditions for range errors not to affect the detection criterion. We saw in the theoretical section that the detection criterion is based on the measurement prediction errors. For measurement errors not to affect the measurement prediction error, we see in this illustration that the estimated position must stay close to the iso-range surfaces of all satellites, including the unaffected satellites. For example, when only one range is affected by an error, for that error not to affect the detection criterion, the estimated position must stay close to the iso-range surfaces of the N-1 unaffected satellites.

If at least 5 satellites are used for positioning and one of them induces a faulty pseudo-range measurement, the iso-range surfaces of the 4 unaffected satellites already intersect in one unique point after clock resolution, so the estimated position will in any case move away from that point, therefore the detection criterion is always affected.

Also, in general, if more satellites are affected, then for the detection criterion not to be affected, the estimated position has to stay close to the iso-range surfaces of the unaffected satellites. For example, if only 1 satellite remains unaffected, the estimated position can be anywhere along the iso-range surface of that satellite, so there are 3 degrees of freedom for the estimated position if we include the clock, therefore 3 degrees of freedom for the affected ranges. If 2 satellites are unaffected, there are 2 degrees of freedom for the errors. If 3 satellites are unaffected, there is only one degree of freedom. If 4 satellites or more are unaffected, then there is no degree of freedom, no possibility for the criterion unaffecting errors to exist.

Figure 3 illustrates the case where two range failures occur: one on satellite 3 and one on satellite 4. The measurement prediction error for all 4 satellites is large, so the detection criterion is affected by the errors.

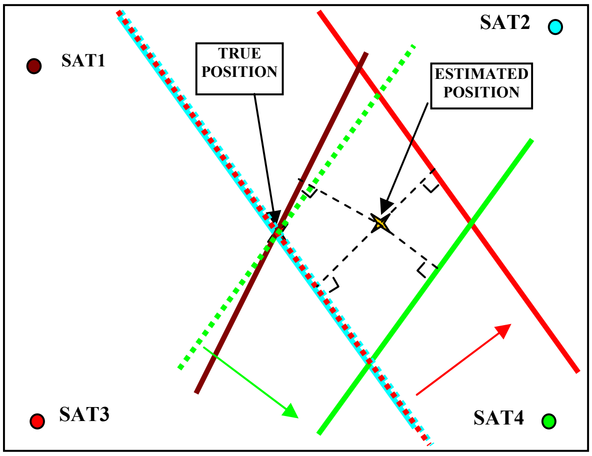

Figure 3: *Illustration in a 2 dimension case of an estimated position with 2 failures.*

Again, when 2 ranges are affected, for them not to affect the detection criterion, the estimated position must stay on the iso-range surfaces of the other N-2 sats. But if N-2 is greater or equal than 4, these iso-range surfaces already have a unique intersection, and the estimated position has to move away from this intersection, so the detection criterion sees the error in any case.

We now consider in figure 4 the case where three range errors appear, on satellites 1, 2 and 3.

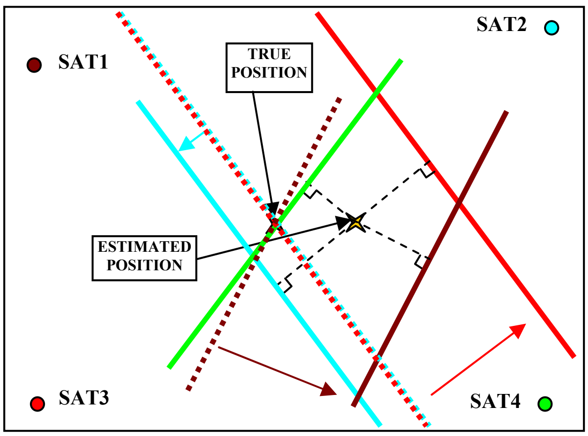

Figure 4: *Illustration in a 2 dimension case of an estimated position with 3 failures.*

In the case illustrated here, the measurement prediction error is large for all 4 satellites and the RAIM detection criterion is affected. For the detection criterion not to be affected, the estimated position has to stay on the iso-range surface of the unaffected satellite (sat 4). This is illustrated in figure 5: the prediction error is 0 for all sat, so the detection criterion stays unaffected.

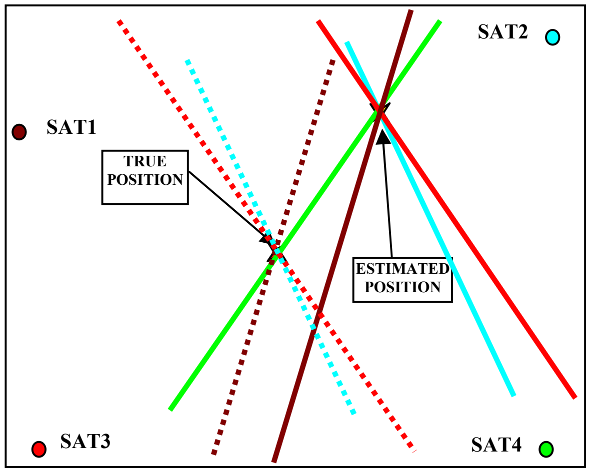

Figure 5: *Illustration in a 2 dimension case of an estimated position with 3 failures.*

As a conclusion, when several range failures occur, there are limited degrees of freedom for the pseudo-range errors to cause the estimated position to stay close to the iso-range surfaces of the unaffected satellites, except when all N or N-1 satellites are affected. Therefore, this leaves very limited occasions for the detection criterion not to be affected by the pseudo-range errors.

One question that now remains to be answered is: when is the impact on the detection criterion large enough for that criterion to go over the threshold ?

To test this and the theoretical results illustrated here, we conduct tests with a real receiver and a RAIM implemented with a threshold conform to DO229 specs. In a further section, we test the performance of the RAIM in presence of up to four range failures.

VIII. PRESENTATION OF TESTS

In order to check the definition of the linear sub-space of the criterion unaffecting errors, we have tested the performance of a LSR RAIM algorithm using pseudo-range measurements from a NovAtel OEM3 receiver connected to a GSS signal generator.

As our main goal is to test the existence of these criterion unaffecting errors in the linear sub-space identified earlier, we place the receiver in a situation where these errors exist.

As the dimension of the linear space of criterion unaffecting errors, is in the general case, 4-(N-p), then if p=2, the condition for criterion unaffecting errors to exist is N<6. If 5 satellites are tracked, the space of the undetectable errors has dimension 1.

We conduct two simulations: one with two identical range errors, one with ranges errors linearly related by a pre-computed value. This value is the coefficient of the unit vector of the solution sub-space. We know that this linear space is defined as:

$(I - H \times S)E_p = 0$ with $S = (H^t \times H)^{-1} \times H^t$. If we denote $G = H \times S$ this is equivalent to:

$$\begin{bmatrix} e_1 \\ e_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} & G_{15} \\ G_{21} & G_{22} & G_{23} & G_{24} & G_{25} \\ G_{31} & G_{32} & G_{33} & G_{34} & G_{35} \\ G_{41} & G_{42} & G_{43} & G_{44} & G_{45} \\ G_{51} & G_{52} & G_{53} & G_{54} & G_{55} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$E = \begin{pmatrix} 1 \\ \frac{1-G_{11}}{G_{12}} \\ 0 \\ 0 \\ 0 \end{pmatrix} e_1.$$

To insert errors e_1 and e_2, we determine first the matrix G in advance, and we instruct the GSS signal generator to generate signals with these errors.

In the same way, if we consider a case where there are 3 range failures (p=3), the dimension of the linear space of the criterion unaffecting errors is 4-(N-3)=7-N. With 5 satellites it is a 2 dimension sub-space. In the same way as above, we find a unit vector with 3 non zero components from equation $(I - H \times S)E_p = 0$.

IX. TESTS RESULTS

First of all, we consider the design case of the RAIM with one failure in figure 6. A ramp error (5 m/s) is applied on 1 among the 10 tracked satellites, and we observe here the detection occurs 20 seconds after the initiation of the failure, which is quite early because the threshold is low.

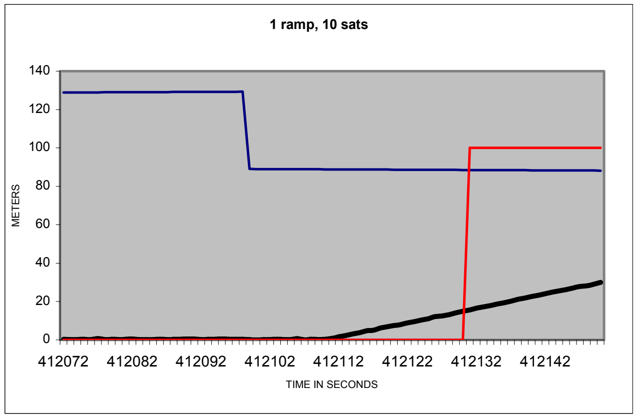

Figure 6: *Ramp of 5 m/s on 1 range among 10 (blue: HPL, black: horizontal position error, red: detection flag).*

Now, with 5 satellites in view, a 5 m/s ramp error is applied on 2 satellites, at GPS time 412 110 seconds. According to the values of G, the ramp on the 1st satellite must be equal to the ramp on the 2nd satellite multiplied by -2.44. Here, the ramps are identical, and as we can see in figure 7, the failure is detected only 2 s after its initiation.

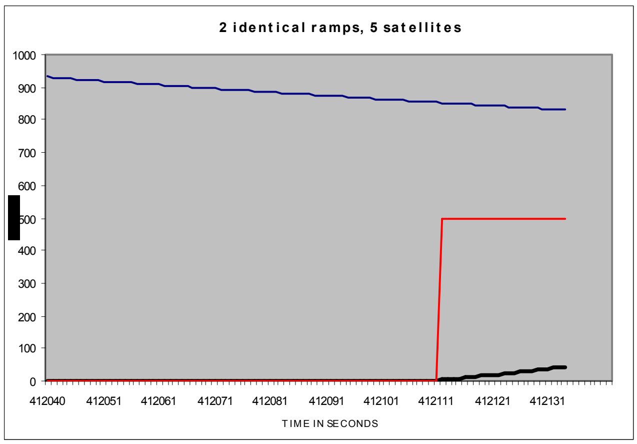

Figure 7: *2 identical ramps of 5 m/s (blue: HPL, black: horizontal position error, red: detection flag).*

It must be noted that the same thing happens when the identical ramps are applied on two different satellites.

It must be noted that the HPL is very high as the number of satellites is low (5 ou 6), but we are only interested here in the behaviour of the detection criterion.

Now the ramp failures are applied on 3 satellites while using 6 of them for positioning, as shown in figure 8. These range errors are identical on all three satellites and the detection occurs 10 seconds after the initiation of the failure (3 ramps at 5m/s). Note the horizontal position error does not go over the HPL.

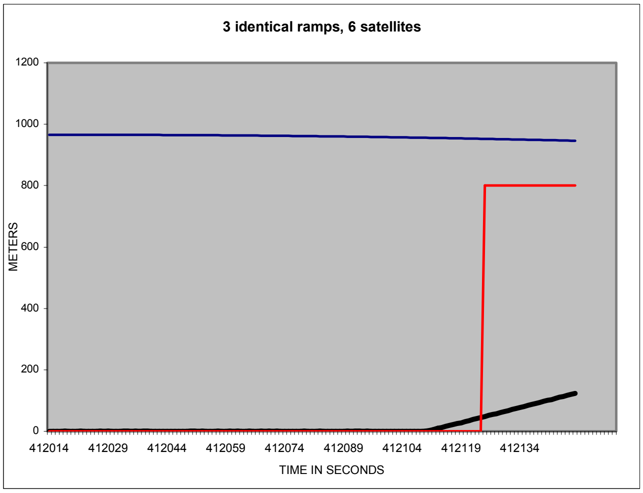

Figure 8: *3 identical ramps of 5 m/s (blue: HPL, black: horizontal position error, red: detection flag).*

The performance is the same for the same constellation but this time with ramps applied on 3 different satellites (5 m/s). The detection occurs 15 seconds after the initiation of the errors.

So the presence of these multiple range failures does affect the detection criterion. We now try to check the predicted performance in presence of undetectable errors.

The ramps are applied on 3 satellites and they are related by the coefficients calculated previously. The position error is very large. We can clearly see in figure 9 that the failures are not detected. The horizontal position error grows up to 400m, and there is no indication of detection.

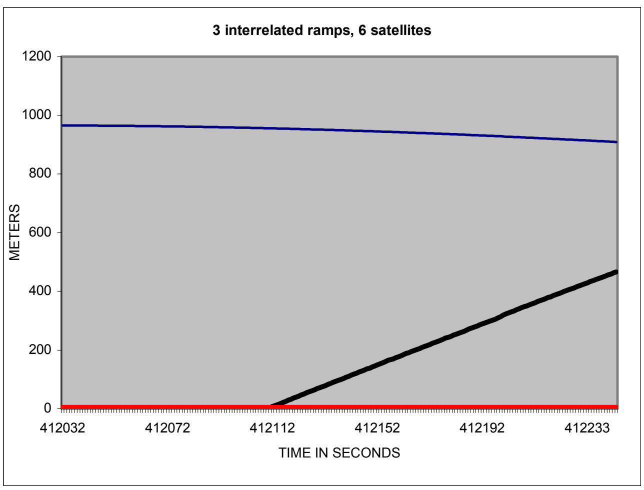

Figure 9: *3 ramps inter-related (blue: HPL, black: horizontal position error, red: detection flag).*

For the previous examples, the tests were made with simple configurations including few satellites (5 or 6) to simplify the calculation of the coefficients. But in the real case of GPS use for navigation, the number of satellites varies between 8 and 10. We show in figure 10 results of a simulation with a configuration of 10 satellites, 6 of them being affected by a ramp of 5 m/s at time 412 110. We can see that the detection is very quick, less than 15 seconds after the initiation of the failure.

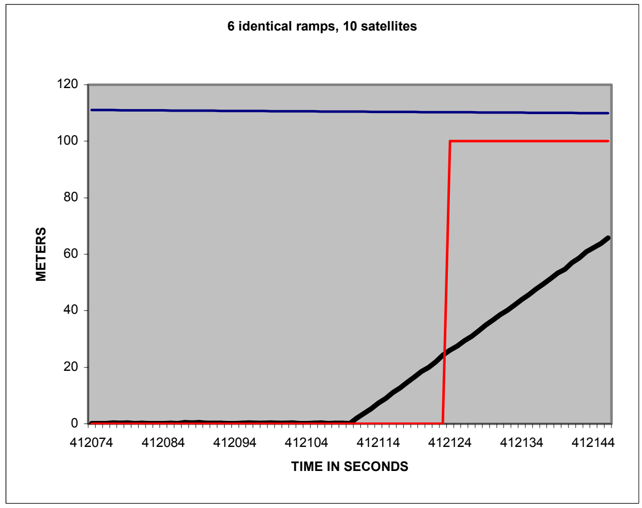

Figure 10: *6 identical ramps of 5 m/s and 10 satellites in view (blue : HPL, black: horizontal position error, red: detection flag).*

Indeed, with 10 satellites and 6 failures, the set of the undetectable errors has a dimension of 0.

X. MONTE CARLO ANALYSIS OF RAIM PERFORMANCE

The main objective of this analysis is to make a statistical assessment of the behaviour of RAIM facing several range errors. During those simulations, we introduce up to four simultaneous range errors with various amplitudes.

The simulation assumptions are :

- Constellation : ‘Optimized 24 sat’ (RTCA DO 229) with no satellite failure
- False alarm probability P_{fa}=3.333.10-7 per test (specification MOPS DO 229) corresponding to 10^{-5} / hour in presence of SA
- Missed detection probability $P_{md}=10^{-3}$
- Noise affecting the pseudoranges: σ=8m
- Simulation duration: 86400 s with a 1s step

The RAIM detection threshold is determined in order to insure a given false alarm rate and is computed as follows:

$$r_o = \sqrt{\frac{a \times \sigma^2}{n-4}} \qquad where\ a = \chi^2_{CDF\ INV}(1-Pfa, n-4)$$

The thresholds used for RAIM detection are :

n	Normalized threshold (a)	Threshold (r_o) for sigma 8m
5	26.046	40.828
6	29.828	30.895
7	32.929	26.504
8	35.701	23.900
9	38.267	22.132
10	40.689	20.833
11	43.001	19.828
12	45.226	19.021

Table 1: *Thresholds used for detection.*

The threshold used to determine the availability of the RAIM algorithm is determined as a function of the missed probability, of the considered alarm limit and the detection threshold r_o presented above. The criterion used in our case is the $\Delta HDOP_{ceil} = \frac{AL}{\sigma\sqrt{\lambda}}$ where λ is the normalized threshold computed as $\chi^2_{CDF}(a, n-4, \lambda) = P_{md} = 10^{-3}$.

The thresholds used for availability decision are:

Table 2: *Thresholds used for availability decision.*

The measurement error is assumed white and gaussian with identical power (σ =8 m) on all visible satellites. The failures are simulated as biases affecting a number k of measurements. The failure amplitude is uniformly distributed between $5^* \sigma_{noise}$ and A_{max} which can be preset.

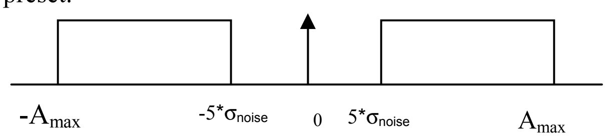

Figure 11: *Distribution of failures inserted in observations.*

All results presented below are average results on the complete duration of the simulation. The total number of samples is 8640000 (100 failures by constellation, 1 constellation per second over 24 h). Instant percentages can significantly vary as a function of the number of satellites used and the corresponding geometry.

The first important result is the rate of detection of the RAIM for all the simulations run, that is the percentage of runs where the RAIM detection criterion is above the threshold r_o. This is shown in figure 12.

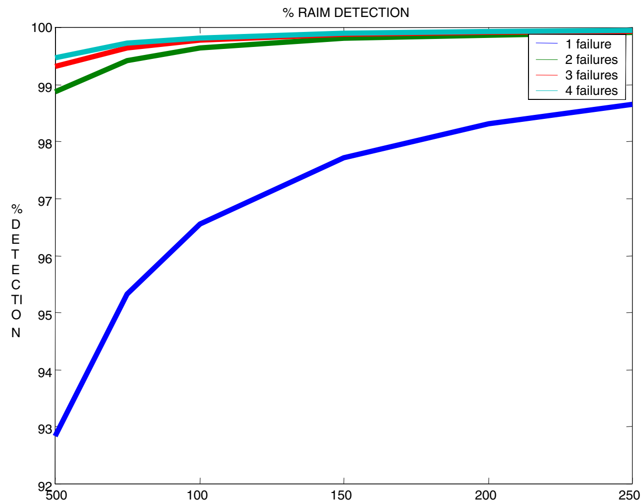

Figure 12: *Rate of RAIM detection.*

Then, figure 13 shows the rate of non integrity events, declared as the percentage of runs where the horizontal position error is larger than the horizontal alert limit while the RAIM detection criterion is below the threshold. This definition does not include the time to alert, as failures and their consequence are only analyzed

n	Normalized thresholds (√λ)	Threshold for sigma 8m
5	8.19	8.485
6	8.48	8.195
7	8.69	7.997
8	8.86	7.844
9	9.01	7.713
10	9.14	7.603
11	9.26	7.505
12	9.38	7.409

at the current epoch.

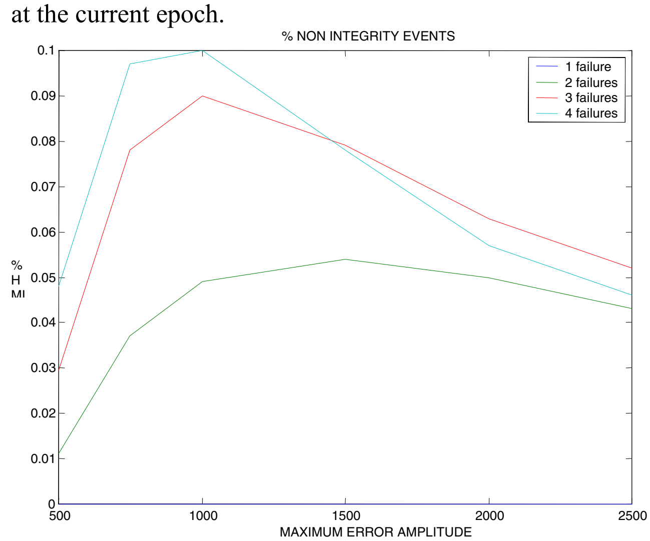

Figure 13: *Percentage of non integrity events.*

In order to test the statistical validity of the results, two cases were simulated on a double number of random samples (200 outdraws per constellation). The observed results are very close to the results obtained with 100 outdraw/constellation, so the Monte Carlo methodology used seems appropriate.

The results obtained show that the RAIM algorithms, although designed to guarantee integrity performance when facing a single failure on a single pseudo-range, allow to detect multiple failures. In presence of multiple failures, the detection performance varies as a function of amplitude and number of failures.

The worst integrity results are obtained for failures with average amplitude, around 1000 m, as the position error can be large enough to be above the alert limit while the RAIM does not detect. For a large number of failures or strong failures amplitudes, the integrity improves considering the better probability of going over the RAIM threshold.

In all simulated cases, the probability of non integrity (knowing a failure is present) is lower than 10^{-3}. To get an absolute probability of non integrity, it remains to multiply that conditional probability by the probability of occurrence of the worst case of failure.

These results are based on statistical averages and do not describe the worst detection performance that can be encountered by a specific user at a given epoch.

XI. SENSITIVITY OF CRITERION UNAFFECTING ERRORS TO POSITION UNCERTAINTIES

Let us assume now that the RAIM test statistics *T* is computed by using the matrix *H* and the decision unaffecting error E_p is computed by using the matrix $\hat{H}$ which is different from the matrix *H*. A question which naturally arises concerns the sensitivity of the decision function with respect to this error. In particular, the following scenario is assumed: the decision unaffecting errors E_p are generated at the geographical position which is different from the geographical position of the GNSS receiver. The distance between two positions varies from 0 to 150 km. The common orientation of these geographical positions is fixed. This scenario is relevant to examine the robustness of the decision unaffecting errors with respect to some lack of a priori information about the matrix *H*.

Pseudorange errors are generated to induce an error in the estimated altitude of the GNSS receiver. This result is presented in figure 15: the induced vertical position error is about 1090 meters. As it is shown in figure 16, these errors are not detectable by the receiver's RAIM.

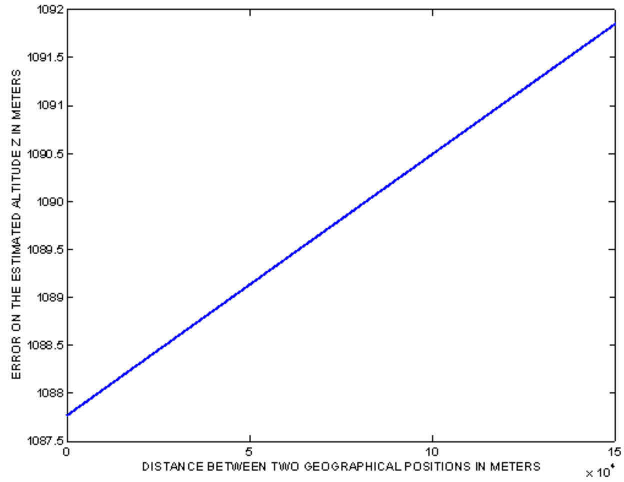

Figure 15: *Error on the estimated altitude as a function of the distance between two geographical positions.*

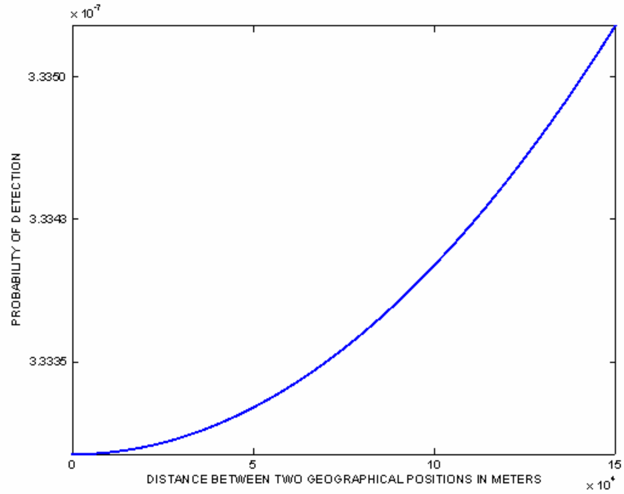

Figure 16: *Probability of detection as a function of the distance between two geographical positions*

This numerical illustration shows that the decision unaffecting errors generation method is low sensitive with respect to the distance between the GNSS receiver position and the location of the errors E_p calculation.

CONCLUSIONS

This paper has presented the results of a study that aims at characterizing the performance of RAIM in presence of multiple range failures.

The study was conducted in two steps: first we have made a theoretical analysis of the linear sub-space of the pseudo-range measurement errors that have no effect on the LSR RAIM detection criterion, and then we have run simulations to observe the conditional rate of non-integrity events induced by up to four simultaneous range failures.

From the theoretical analysis presented here, we have shown that, provided the satellite constellation does not have any degenerated geometrical properties, the dimension of the linear sub-space of the errors that do not affect the decision criterion is max(4-(N-p),0), where N is the number of tracked satellites and p is the number of faulty pseudorange measurements. The immediate conclusion is that if N-4 pseudo range measurements are affected, or less than that, by a large error, there exists no error that will not affect the RAIM detection criterion, and globally due to the negligible probability that unintentional interference lies in a small dimension sub-space, the RAIM detection criterion exhibits a natural detection capability even if up to N-2 pseudo-range measurements are faulty.

The level of impact of the errors on the decision criterion has been further assessed through Monte-Carlo simulations over 24 hrs, inserting up to four simultaneous range errors. It is mainly shown that in that case, the conditional probability of non integrity events knowing a failure is present is limited to 10^{-3}, disregarding of the time to alert.

In addition, it has been shown that, even though the exact position of the user receiver may not be known accurately to within tens of kilometres, it would be possible to determine what errors on the pseudorange measurements would go undetected by the RAIM.

A more complete analysis could be now conducted to characterize the theoretical definition of the linear sub-space of the errors inducing a detection criterion below the detection threshold, conducting simulations incorporating more failures, and taking into account the time-to-alert.

ACKNOWLEDGMENTS

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