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BIOGRAPHY

Anaïs Clot graduated in July 2005 as an electronics engineer from the Ecole Nationale de l'Aviation Civile (ENAC) in Toulouse, France. She is now working as a Ph.D. student at the signal processing lab of the ENAC. Currently she carries out research on integrity monitoring techniques

Christophe Macabiau graduated as an electronics engineer in 1992 from the ENAC in Toulouse, France. Since 1994, he has been working on the application of satellite navigation techniques to civil aviation. He received his Ph.D. in 1997 and has been in charge of the signal processing lab of the ENAC since 2000.

Igor Nikiforov received his M.S. degree in automatic control from the Moscow Physical–Technical Institute in 1974, and the Ph.D. in automatic control from the Institute of Control Sciences (Russian Academy of Science), Moscow, in 1981. He joined the University of Technology of Troyes (UTT) in 1995, where he is Professor and Head of the Institute of Computer Sciences and Engineering of Troyes. His scientific interests include statistical decision theory, fault detection/ isolation/ reconfiguration, signal processing and navigation.

Benoît Roturier graduated as a CNS systems engineer from ENAC, Toulouse in 1985 and obtained a Ph.D. in Electronics from Institut National Polytechnique de Toulouse in 1995. He was successively in charge of Instrument Landing Systems at DGAC/STNA (Service Technique de la Navigation Aérienne), then of research activities on CNS systems at ENAC. He is now head of GNSS Navigation subdivision at DGAC/DTI (Direction de la Technique et de l'Innovation, formerly known as STNA) and is involved in the development of civil aviation applications based on GPS/ABAS, EGNOS and Galileo. He is also currently involved in standardization activities on future multiconstellation GNSS receivers within Eurocae WG62 and ICAO Navigation Systems Panel.

ABSTRACT

Conventional snapshot Receiver Autonomous Integrity Monitoring (RAIM) algorithms, which use a single set of measurements collected simultaneously, have limited performances to check the integrity of GNSS in safety-critical civil aviation applications when vertical guidance requirements are applied.

In this context, snapshot and sequential RAIM FDE algorithms based on the constrained Generalized Likelihood Ratio (GLR) test have been proposed. These techniques, described in this paper, are based on a formalized definition of a horizontal or vertical error that must be considered as a positioning failure and consist in computing for each epoch and for each satellite channel a minimal additional pseudo range bias that leads to such situations. Proceeding like this allows partially compensating the lack of availability of conventional methods. As such, sequential technique is particularly attractive. Indeed this algorithm takes into account history of measurements to make its decision and moreover the pseudo range correlation is directly integrated through an autoregressive (AR) model.

The purpose of our study is to benefit fully from these improvements by targeting more complex fault profiles. A sequential constrained GLR is designed to detect combined step ramp pseudo range errors. Our goal is to protect ourselves from faults that only depend on two parameters: initial position (amplitude of the step) and speed (rate of the slope) and that tend to lead to a positioning failure within an observation window Δt .

A first evaluation of this advanced RAIM performances is presented in this paper. The robustness of this method is assessed for several profiles of additional error through a comparison of detection rates. The impact of the

constellation geometry on our RAIM availability targeting at APV (APproach with Vertical guidance) requirements is also studied.

The results obtained show that the robustness of the existing techniques has been improved by our method. For a combined constellation GPS + Galileo, this first performance evaluation is quite encouraging, before a complete study on a world wide case.

INTRODUCTION

Receiver Autonomous Integrity Monitoring (RAIM) is nowadays a standard solution to check the integrity of GNSS in safety-critical civil aviation applications. Unfortunately, the conventional snapshot Least Square-based RAIM algorithms, which use a single set of GPS measurements collected simultaneously, have a limited performance when the civil aviation vertical guidance requirements are applied. This is why sequential techniques are attractive. Because they take into account the history of measurements, they should provide better detection availability. Currently, sequential RAIM algorithms are designed to detect step error profiles. But if to benefit fully from their performances and to address the largest class of error behaviours, it could be interesting to adapt sequential RAIM to combined step ramp detection.

New RAIM algorithms have been already built by using the constrained GLR test based on the current (snapshot) or all past and current sets of GPS measurements (sequential) and the LS-residual pre-filtering technique [1]. Their potential to detect stepwise fault has been already demonstrated [2]. The goal of this study is to present the adaptation of this sequential method in order to detect combined step-ramp pseudo range biases. This work focuses on design and performance evaluation of this algorithm. During all the study it will be assumed that only one individual satellite channel fault is possible at a time.

The paper is organized as follows: first International Civil Aviation Organization requirements concerning integrity of GNSS for various phases of flight are briefly recalled. Then, the principle of our advanced RAIM based on the constrained Generalised Likelihood Ratio Test, including the definition of the detection/exclusion criterion and the computation of the smallest bias that leads to a positioning failure, is described. Next the constrained GLRT based RAIM FDE algorithm is designed for a more complex fault profile including two parameters: initial position (amplitude of the step) and speed (rate of the slope). The concept of a minimal positioning failure, that is to say the “minimum speed” and the “minimum amplitude of the step” that lead to a positioning failure is redefined. Finally the robustness of the advanced RAIM FDE against these more complex range failures will be tested using a simulation model.

I- PRINCIPLE OF THE ALGORITHMS

According to [3], the detection function of a RAIM FDE algorithm is defined to be available when the constellation of satellites provides a geometry for which the missed alert and false alert requirement can be met on all satellites being used for the applicable alert limit and time to alert. Corresponding civil aviation requirements for different modes of flight are represented by those typical values:

Mode of flight	HAL / VAL	Integrity risk	Time to alert
Terminal	1 NM	10-7/h	15 s
NPA	0.3 NM	10-7/h	10 s
APV I	40m/50m	2.10-7/150s	10 s
APV II	40m/20m	2.10-7/150s	6 s

Our algorithm will be considered as available if it can detect/exclude for each pseudo range the smallest bias that will lead to a positioning failure, that is to say *Horizontal Error > HAL* or *Vertical Error > VAL*, with a probability equal to the integrity risk, during the time to alert. This is why the objective is first to define for each satellite channel these smallest biases which correspond to the worst case detection/exclusion situations.

In a few words, the steps of such an advanced RAIM are:

- For a given user position and a given moment, identification of visible satellites
- Computation for each single satellite channel of the smallest bias that leads to a positioning failure with a probability equal to the integrity risk
- Simulation of nominal measurments and injection of this eventual additional bias
- Calculation of detection rates

First, let us define our detection and exclusion criterion.

In contrast to the traditional (unconstrained) snapshot or sequential RAIM schemes, constrained GLR test directly analyses the impact of each single satellite channel fault on the positioning accuracy. They are designed to detect and/or exclude only such faults which lead to a positioning failure.

The navigation equation, relating range measurements from several satellites with known locations to a user, gives us: $Y = h(X) + \xi + B$

where B is an eventual bias and $\xi \sim N(0, \Sigma)$ with

$$\Sigma = \begin{bmatrix} \sigma_1^2 & & \\ & : & \\ & & \sigma_n^2 \end{bmatrix}$$

For $i \in [1, n]$ the variance $\sigma_i^{\ 2}$ are defined as function of the elevation angle.

By linearizing this pseudo range equation with respect to the vector X around the working point X_0:

$$Y = h(X_0) + H(X - X_0) + \xi + B$$

Defining $\Delta X = X - X_0$ and $\Delta Y = Y - h(X_0)$:

$$\Delta Y = H\Delta X + \xi + B$$

And by normalizing with $N = \Sigma^{-1/2}$:

$$\Delta Y_{norm} = H_{norm}\Delta X + \xi_{norm} + B_{norm}$$

The matrix W is built such as $WH_{norm} = 0$. This matrix $W^T = (w_1, ..., w_{n-4})$ of size $n \times (n-4)$ is composed of the eigenvectors $w_1, \ldots, w_{n-4}$ of the projection matrix P_H:

$$P_H = I_n - H_{norm}\left(H_{norm}{}^t H_{norm}\right)^{-1} H_{norm}{}^t$$

W satisfies the following conditions: $W^t W = P_H$ and $WW^t = I_{n-4}$.

The parity vector is defined by $Z = W\Delta Y_{norm}$.

Since W satisfies $WH_{norm} = 0$, transformation by W removes the interference of the parameter ΔX such as:

$$Z = W\Delta Y_{norm} = W\left(\xi_{norm} + (B_{norm})\right)$$

where $B_{norm} = [0, ...0, v_i / \sigma_i, 0, ..., 0]$, $i = 1, ..., n$ if there is an additional bias on the pseudo range i.. Different statistical tests will be applied on this parity vector.

Considering the residual vector $e = P_H \Delta Y$ and denoting for $i \in [1, n]$, W_i the i[th] column of W and $P_H(i,i) = P_{i,i}$: $W^t Z = W^t W \Delta Y_{norm} = P_H \Delta Y_{norm}$

$$W_i^t Z = \frac{P_{i,i} e_i}{\sigma_i}$$

In fact, the GLR tests are strongly based on the following idea: the magnitude of the fault $E[Z] = WB_{norm}$ in the parity space is unknown and has to be estimated by the detector.

II- SNAPSHOT RAIM ALGORITHM BASED ON THE CONSTRAINED GLR

The constrained GLR algorithm has to choose between different hypotheses:

- $H_0 = \left\{\bigcup_{j=1}^{n} H_{j,0}\right\}$

where $H_{j,0} = \left\{Z \sim N\left(W\widetilde{B}_j, I\right), |v_j| \le a_j\right\}$

- $H_l = \left\{Z \sim N\left(W\widetilde{B}_l, I_{n-4}\right), |v_l| \ge b_l\right\}$ for $l = 1, ..., n$

The parameters $0 \le a_l \le b_l$, $l = 1, ..., n$ define the selectivity of the test with respect to each pseudo range bias v_l. For $l = 1, ..., n$, b_l will be the smallest bias on the channel *l* that will lead to a positioning failure and a_l the smallest bias that has to be consider, this allows the algorithm to be robust against insignificant additional pseudo range biases. During all the study it is assumed that $a_l = 0$.

The test is given by the following equation:

$$\delta(\widetilde{Y}) = \begin{cases} H_0 \ \ if & \max\limits_{1 \le l \le n} \dfrac{f_l(Z)}{f_0(Z)} < h \\ H_l \ \ if & v = \arg\max\limits_{1 \le l \le n} \dfrac{f_l(Z)}{f_0(Z)} \ge h \end{cases}$$

If we have a geometric interpretation of the decision rule we can re-write the equation this way:

- $\delta(\widetilde{Y}) = H_0$ if :

$$\max_{1\le l \le n}\left[\min_{1 \le i \le n}\min_{|v_i| \le a_i}\left\|Z - W_i \frac{v_i}{\sigma_i}\right\|_2^2 - \min_{|v_l| \ge b_l}\left\|Z - W_l \frac{v_l}{\sigma_l}\right\|_2^2\right] < h$$

- $\delta(\widetilde{Y}) = H_v$ if

$$v = \arg\max_{1\le l \le n}\left[\min_{1 \le i \le n}\min_{|v_i| \le a_i}\left\|Z - W_i \frac{v_i}{\sigma_i}\right\|_2^2 - \min_{|v_l| \ge b_l}\left\|Z - W_l \frac{v_l}{\sigma_l}\right\|_2^2 \ge h\right]$$

where:

- $d(Z, H_{i,0}) = \min\limits_{|v_i| \le a_i}\left\|Z - W_i \dfrac{v_i}{\sigma_i}\right\|_2^2$ is the distance from the observation Z to the partial null hypothesis $H_{i,0}$
- $d(Z, H_0) = \min\limits_{1 \le i \le n}\min\limits_{|v_i| \le a_i}\left\|Z - W_i \dfrac{v_i}{\sigma_i}\right\|_2^2 = \min\limits_{1 \le i \le n} d(Z, H_{i,0})$ is the distance from Z to H_0

- $d(Z, H_l) = \min_{|v_l| \geq b_l} \left\| Z - W_l \frac{v_l}{\sigma_l} \right\|_2^2, l = 1, \ldots, n$ are the distance from Z to each alternatives hypothesis H_l

In order to test the alternatives H_l for $l = 1, \ldots, n$ against the null hypothesis H_0, the differences $d(Z, H_0) - d(Z, H_l)$ are computed and the index that maximise them is identified. Thus for $i = 1, \ldots, n$ two functions are defined:

- $S_0(e, i) = \min_{|v_i| \leq a_i} \left\| Z - W_i \frac{v_i}{\sigma_i} \right\|_2^2$ which represents the probability that there is no fault or no significant fault on the pseudo range *i*.
- $S_1(e, i) = \min_{|v_i| \geq b_i} \left\| Z - W_i \frac{v_i}{\sigma_i} \right\|_2^2$ which represents the probability that there is a bias on the channel *i* that will lead to a positioning failure.

$\left\| Z - W_i \frac{v_i}{\sigma_i} \right\|_2^2 = \|Z\|_2^2 - 2 \frac{v_i}{\sigma_i} W_i^t Z + \|W_i\|_2^2 \frac{v_i^2}{\sigma_i^2}$ and the function $f(x) = \|Z\|_2^2 - 2 \frac{W_i^t Z}{\sigma_i} x + \frac{\|W_i\|_2^2}{\sigma_i^2} x^2$ reaches its minimum for $\hat{v}_i = \frac{\sigma_i W_i^t Z}{\|W_i\|_2^2}$.

If $|\hat{v}_i| \leq a_i$, $S_0(e, i) = \min_{|v_i| \leq a_i} \left\| Z - W_i \frac{v_i}{\sigma_i} \right\|_2^2 = f(\hat{v}_i)$ and if $|\hat{v}_i| > a_i$, $S_0(e, i) = \min_{|v_i| \leq a_i} \left\| Z - W_i \frac{v_i}{\sigma_i} \right\|_2^2 = f(a_i)$

Thus,

$$S_0(e, i) = \begin{cases} \|Z\|_2^2 - \dfrac{(W_i^t Z)^2}{\|W_i\|_2^2} & if \ |\hat{v}_i| \leq a_i \\ \|Z\|_2^2 - 2 \dfrac{W_i^t Z}{\sigma_i} a_i + \dfrac{\|W_i\|_2^2}{\sigma_i^2} a_i^2 & if \ |\hat{v}_i| > a_i \end{cases}$$

Using the least square residual vector $e = PY$ to represent these values, $W^t W = P$, and for $i = 1, \ldots, n$:

$$S_0(e, i) = \begin{cases} \|Z\|_2^2 - \dfrac{e_i^2}{\sigma_i^2 p_{i,i}} & if \ |\hat{v}_i| \leq a_i \\ \|Z\|_2^2 - \dfrac{2a_i |e_i|}{\sigma_i^2} + \dfrac{p_{i,i} a_i^2}{\sigma_i^2} & if \ |\hat{v}_i| > a_i \end{cases}$$

Similarly, if $|\hat{v}_i| > b_i$, $S_1(e, i) = f(\hat{v}_i)$ and if $|\hat{v}_i| \leq b_i$, $S_1(e, i) = f(b_i)$.

$$S_1(e, i) = \begin{cases} \|Z\|_2^2 - \dfrac{e_i^2}{\sigma_i^2 p_{i,i}} & if \ |\hat{v}_i| > b_i \\ \|Z\|_2^2 - \dfrac{2b_i |e_i|}{\sigma_i^2} + \dfrac{p_{i,i} b_i^2}{\sigma_i^2} & if \ |\hat{v}_i| \leq b_i \end{cases}, \hat{v}_i = \frac{e_i}{p_{i,i}}$$

The alarm time and exclusion function of detection/exclusion algorithm is based on the decision rule given by the following equation:

$$N = \inf \left\{ t \geq 1 : \max_{1 \leq l \leq n} \left(\min_{1 \leq j \leq n} S_0(e_t, j) - S_1(e_t, l) \right) \geq h \right\}$$

and $\nu = \arg \max_{1 \leq l \leq n} \left(\min_{1 \leq j \leq n} S_0(e_t, j) - S_1(e_t, l) \right)$

This algorithm needs several parameters to be implemented: the threshold *h* that will be compared to the statistic test and two size n vectors *a* and *b*. *a* is the vector of maximum acceptable bias and *b* the vector of minimum biases which are considered as positioning failures. The computation of the smallest bias that will lead to a positioning failure and the choice of the threshold will be detailed later in appendices.

An example of the behaviour of the test when an additional bias that leads to a positioning failure is injected on a pseudo range (figure 1):

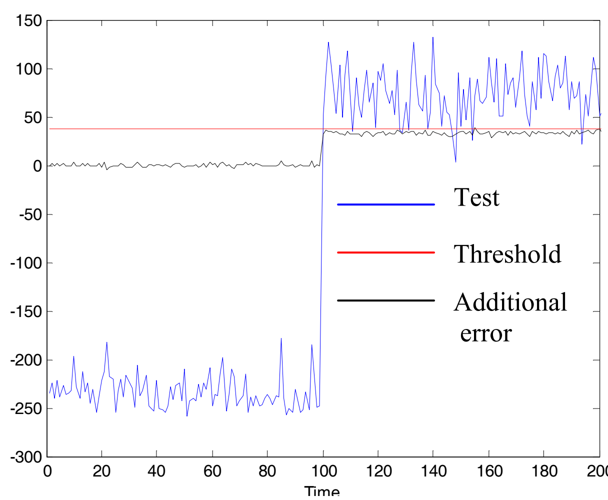


Figure1: Snapshot constrained GLR test

III- SEQUENTIAL RAIM ALGORITHM BASED ON THE CONSTRAINED GLR

Here the pseudo range correlation is directly integrated in the constrained GLR algorithm through an AR model and the last m observations $Z_1, \ldots, Z_m$ are considered.

The test is given by the following equation:

$$\delta(\widetilde{Y}) = \begin{cases} H_0 \quad if & \max\limits_{1\le l\le n} \dfrac{f_l(Z_1,...,Z_m)}{f_0(Z_1,...,Z_m)} < h \\ H_l \quad if & \nu = \arg\max\limits_{1\le l\le n} \dfrac{f_l(Z_1,...,Z_m)}{f_0(Z_1,...,Z_m)} \ge h \end{cases}$$

Assuming that the additive pseudo range noise ξ is represented by the following first autoregressive model: $\xi_{k+1} = \xi_k + \sqrt{1-a^2}\zeta_k$ with $\zeta_k \sim N(0,\Sigma)$ and $\xi_1 \sim N(0,\Sigma)$ or $\xi_1 \sim N(B,\Sigma)$ and considering the standardised vector of the pseudo range bias $\xi_{norm} = N\xi$:

$$\xi_{k+1,norm} = \xi_{k,norm} + \sqrt{1-a^2}\varsigma_{k,norm}$$

where $\zeta_{k,norm} \sim N(0,I_n)$ and $\xi_{1,norm} \sim N(0,I_n)$ or $\xi_{1,norm} \sim N(B_{norm},I_n)$ with an eventual bias.

If we want to have a geometric interpretation of the decision rule, $Z_1 \sim N(W\widetilde{B},I_{n-4})$ has to be to compared with $W_i \dfrac{v_i}{\sigma_i}$. But for $k \ge 2$ it is the random variable $\dfrac{1}{\sqrt{1-a^2}}[Z_k - aZ_{k-1}]$ which follows a normal distribution with the covariance matrix I_{n-4} and that is to be compared with $\dfrac{(1-a)}{\sqrt{1-a^2}} W_i \dfrac{v_i}{\sigma_i}$ for $i = 1,...,n$.

Thus considering the m last observations, the following expression has to be minimised:

$$\left\| Z_1 - W_i \frac{v_i}{\sigma_i} \right\|_2^2 + \frac{1}{1-a^2}\sum_{k=2}^{m}\left\| Z_k - aZ_{k-1} - (1-a)W_i \frac{v_i}{\sigma_i} \right\|_2^2 \text{ with}$$

respect to v_i .

Let us define the function:

$$f(x) = \|Z_1\|^2 + \frac{1}{(1-a^2)}\sum_{k=2}^{m}\|Z_k - aZ_{k-1}\|^2$$
$$-2\left(Z_1 + \frac{1-a}{1-a^2}\sum_{k=2}^{m} Z_k - aZ_{k-1}\right)^t \frac{W_i}{\sigma_i} x + \left(1 + \frac{(1-a)^2}{1-a^2}(m-1)\right)\frac{\|W_i\|^2}{\sigma_i^2} x^2$$

We derive it and obtain its minimum:

$$\hat{v}_i = \frac{\sigma_i \left(Z_1 + (1-a)\sum_{k=2}^{m-1} Z_k + Z_m \right)^t W_i}{\|W_i\|^2 (2a + m(1-a))}$$

If $|\hat{v}_i| \le a_i$, $S_0(e,i) = \min\limits_{|v_i|\le a_i} f(v_i) = f(\hat{v}_i)$ and if $|\hat{v}_i| > a_i$, $\min\limits_{|v_i|\le a_i} f(v_i) = f(a_i)$. Using the least square residual vector $e = PY$, $W^t W = P$, we get for $i = 1,...,n$:

$$S_0(\bar{e},i) = \begin{cases} -\dfrac{\left(e_{1,i} + (1-a)\sum\limits_{k=2}^{m-1} e_{k,i} + e_{m,i}\right)^2}{\sigma_i^2 p_{i,i}[(1-a)m + 2a](1+a)} & if \quad |\hat{v}_i| \le a_i \\ -\dfrac{2a_l \left| e_{1,i} + (1-a)\sum\limits_{k=2}^{m-1} e_{k,i} + e_{m,i} \right|}{(1+a)\sigma_l^2} & if \quad |\hat{v}_i| > a_i \\ +\dfrac{p_{i,i} a_i^2}{\sigma_i^2}\left[1 + \dfrac{1-a}{1+a}(m-1)\right] & \end{cases}$$

$$S_1(\bar{e},i) = \begin{cases} -\dfrac{\left(e_{1,i} + (1-a)\sum\limits_{k=2}^{m-1} e_{k,i} + e_{m,l}\right)^2}{\sigma_i^2 p_{i,i}[(1-a)m + 2a](1+a)} & if \quad |\hat{v}_i| > b_i \\ -\dfrac{2b_l \left| e_{1,i} + (1-a)\sum\limits_{k=2}^{t-1} e_{k,i} + e_{m,i} \right|}{(1+a)\sigma_i^2} & if \quad |\hat{v}_i| \le b_i \\ +\dfrac{p_{i,i} b_i^2}{\sigma_i^2}\left[1 + \dfrac{1-a}{1+a}(m-1)\right] & \end{cases}$$

where

$$\hat{v}_i = \frac{\lambda_t(i)}{[(1-a)m + 2a]p_{i,i}}, \lambda_t = e_{t-m+1} + (1-a)\sum_{u=t-m+2}^{t-1} e_u + e_t$$

$$g(t,l) = \left[\min_{1\le j\le n} S_0(e_{t-m+1},...,e_t,j) - S_1(e_{t-m+1},...,e_t,l)\right]^+$$

The stopping time for the channel l is:

$$N(l) = \inf\left\{k \ge 1 : \min_{1\le j\ne l\le n}[g_k(l) - g_k(j)] \ge h_{l,j}\right\}$$

and the channel to exclude is $\nu = \arg\min\limits_{1\le l\le n}\{N(l)\}$

Our statistical test for this algorithm will be:

$$T = \max_{1\le l\le n}\left[g_k(l) - \max_{1\le j\ne l\le n}(g_k(j)) \right]$$

m is chosen such that the satellites in view at the epochs *t-m+1*, …, *t* are the same and our technique comes down to work with a weighted mean of the last m observations:

$$\overline{Z} = \frac{Z_1 + (1-a)\sum_{t=2}^{m-1} Z_t + Z_m}{2a + (1-a)m}$$

and its associated distances for $i = 1,\ldots,n$:

$$d(\overline{Z}, H_{i,0}) = \min_{|v_i| \le a_i} \left\| \overline{Z} - W_i \frac{v_i}{\sigma_i} \right\|_2^2 .$$

Figure 2 shows an example of the behaviour of this test when the same additional bias as previously is injected on a pseudo range. The impact of this error on the sequential test has considerably increased comparing with the snapshot one

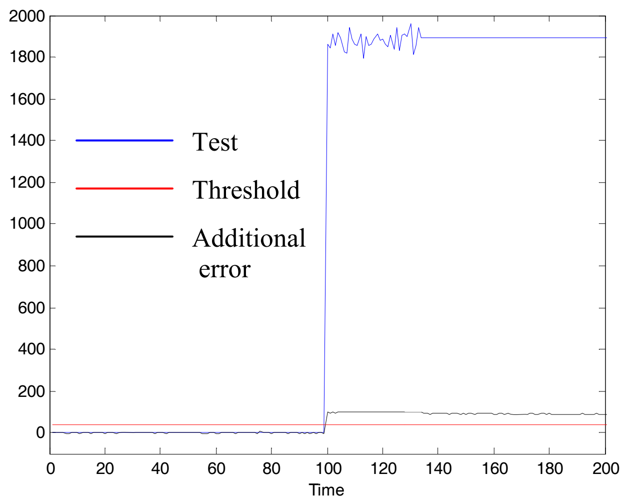


Figure2: Sequential constrained GLR test

IV- SEQUENTIAL RAIM ALGORITHM BASED ON THE CONSTRAINED GLR: ADAPTATION TO COMBINED STEP RAMP PSEUDO RANGE ERRORS

The objective here is to target more complex fault profiles but we must determine what kinds of error have to be detected.

The case where faults only depend on two parameters: the initial position (amplitude of the step) and the speed (rate of the slope) is considered. For this an observation window Δt, which can correspond to an approach duration for example, is set.

Let b_i be the minimal bias in meters on the i[th] pseudo range that leads to a positioning failure, b_i is assumed to be a constant on the observation window.

For each pseudo range, every "error couple" $(v_i, \dot{v}_i)$ with v_i and $\dot{v}_i$ constants such as $\exists t \in [0, \Delta t]$, $|v_i + t\dot{v}_i| \ge b_i$ have to be detected. In fact we want to protect ourselves from situations that can lead to a positioning in the 150 future seconds.

The criterion will be designed in the same way as previously, that is to say comparing a weighted mean of parity vectors with several hypothetic increasing errors on different channels.

Having a geometric interpretation of the decision rule, $Z_1 \sim N(W\widetilde{B}, I_{n-4})$ has to be compared to $W_i \frac{v_i}{\sigma_i}$ and for $k \ge 2$, Z_k has to be compared to $\frac{W_i}{\sigma_i}[v_i + (k-1)\dot{v}_i]$.

To work with the m last observations and the random variables $\frac{1}{\sqrt{1-a^2}}[Z_k - aZ_{k-1}]$ (which follow a normal distribution with the covariance matrix I_{n-4}), the following expression is to be minimised:

$$\left\| Z_1 - W_i \frac{v_i}{\sigma_i} \right\|_2^2 + \frac{1}{1-a^2} \times$$

$$\sum_{k=2}^{m} \left\| Z_k - aZ_{k-1} - [(1-a)v_i + [(1-a)k + 2a - 1]\dot{v}_i] \frac{W_i}{\sigma_i} \right\|_2^2$$

Let us define a function of two variables not considering constant terms and using the least square residual vector. As previously, two cumulative sums are used:

$$\lambda = e_1 + (1-a)\sum_{k=2}^{m-1} e_k + e_m \text{ and}$$

$$\lambda' = -ae_1 + (a^2 + 2a - 1)\sum_{k=2}^{m-1} (1-k)e_k + ((1-a)m + 2a - 1)e_m \cdot$$

After simplifications, we finally obtain:

$g(x, y) =$

$$-2\frac{1-a}{1-a^2}\lambda \frac{x}{\sigma_i^2} + \left(1 + \frac{(1-a)^2}{1-a^2}(m-1)\right)\frac{x^2 p_{i,i}}{\sigma_i^2} - \frac{2}{1-a^2}\lambda' \frac{y}{\sigma_i^2}$$

$$+\frac{1}{1-a^2}\left[\begin{array}{l}(1-a)^2\left[\frac{m(m+1)(2m+1)}{6} - 1\right] \\ +(2a-1)^2(m-1) + 2(1-a)(2a-1)\left[\frac{m(m+1)}{2} - 1\right]\end{array}\right] y^2 \frac{p_{i,i}}{\sigma_i^2}$$

$$+\frac{2}{1-a^2}\left[(1-a)^2\left[\frac{m(m+1)}{2} - 1\right] + (1-a)(2a-1)(m-1)\right] xy \frac{p_{i,i}}{\sigma_i^2}$$

And for $i = 1,\ldots,n$:

$$S_0(\bar{e},i) = g(\hat{x}_i,\hat{y}_i)$$

with $(\hat{x}_i,\hat{y}_i) = \arg\min\left\{\begin{matrix} g(x,y)/|x+t.y| \le a_i, \\ x\in\Re, y\in\Re, t\in[0,\Delta t]\end{matrix}\right\}$

and $S_1(e,i) = \min\left\{\begin{matrix} g(x,y)/|x+t.y| > b_i, \\ x\in\Re, y\in\Re, t\in[0,\Delta t]\end{matrix}\right\}$

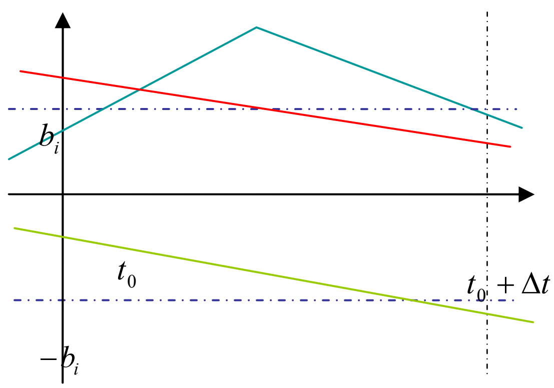
The method will be a little bit more complicated since a recursive function of two variables has to minimize under more complex constraints.

Simplification of the constraint criteria

Under the assumption $\dot{v}_i$ constant:

$$\begin{pmatrix}\exists t\in\,]0,\Delta t], |v_i + t\dot{v}_i| \ge b_i \\ |v_i| < b_i\end{pmatrix} \Rightarrow (|v_i + \Delta t\dot{v}_i| > b_i)$$

A couple (a constant step v_i and a constant slope $\dot{v}_i$) will be considered as faulty if $(|v_i + \Delta t\dot{v}_i| > b_i)$ or if $(|v_i| > b_i)$ as shows the following figure:



: $(|v_i| > b_i)$, faulty

: $(|v_i + \Delta t\dot{v}_i| > b_i)$, faulty

: $\begin{pmatrix}\exists t\in\,]0,\Delta t], |v_i + t\dot{v}_i| \ge b_i \\ |v_i| < b_i, |v_i + \Delta t\dot{v}_i| < b_i\end{pmatrix}$

but $\dot{v}_i$ is not a constant and this case is not taken into account

$$A = \{(v_i,\dot{v}_i)/|v_i| < b_i, \exists t\in\,]0,\Delta t], |v_i + t\dot{v}_i| \ge b_i\}$$
$$= \{(v_i,\dot{v}_i)/|v_i| < b_i, |v_i + \Delta t\dot{v}_i| \ge b_i\}$$

and the likelihood function has to be minimized on:

$$\left\{\begin{matrix}(v_i,\dot{v}_i) \\ \exists t\in[0,\Delta t], |v_i + t\dot{v}_i| \ge b_i\end{matrix}\right\} = A + \{|v_i| \ge b_i\}$$

Thus the likelihood function is first minimized under the constraint $(|v_i| > b_i)$ and then under the constraint $(|v_i + \Delta t\dot{v}_i| > b_i)$. The minimum of these two minimizations will be finally chosen. This way $(v_i,\dot{v}_i)$ the most likely couple considering the m last observations and under the constraint $\exists t\in[0,\Delta t], (|v_i + t\dot{v}_i| > b_i)$ is obtained.

Computation of the GLR test

The minimum of the function g can be found by computing its gradient and finding its zeros. Effectively, numerical values of the polynomial coefficients are such that this function reaches a minimum and not a maximum.

Re writing it this way:

$$g(x,y) = ax^2 + by^2 + 2cxy - 2dx - 2ey$$

then $\vec{\nabla} g(x,y) = \left|\begin{matrix}\partial g/\partial x \\ \partial g/\partial y\end{matrix}\right. = \left|\begin{matrix}-2d + 2ax + 2cy \\ -2e + 2by + 2cx\end{matrix}\right.$

which results in solving a simple linear system of two equations and two unknowns:

$$(\vec{\nabla} g(x,y) = 0) \Leftrightarrow \begin{pmatrix}ax + cy = d \\ cx + by = e\end{pmatrix} \Leftrightarrow \begin{pmatrix}x = \dfrac{ec - db}{c^2 - ab} \\ y = \dfrac{cd - ae}{c^2 - ab}\end{pmatrix}$$

under the constraint $c^2 - ab \neq 0$

Nevertheless, if the absolute minimum of this likelihood function is not in the constraint domain, we will carry out another way.

Minimizing $g(x,y) = ax^2 + by^2 + 2cxy - 2dx - 2ey$ under the constraint $|x + \Delta ty| \ge b_i$ results, considering function g properties (monotonous, regular), in minimizing under the constraint $|x + \Delta ty| = b_i$ that is to say to consider the limits of the constraint domain. This is due to the fact that $g(x,y)$ forms a paraboloid.

$x + \Delta ty = b_i$ and $x + \Delta ty = -b_i$ are successively set, which results in considering the functions:

$g(y) = a(b_i - \Delta ty)^2 + by^2 + 2c(b_i - \Delta ty)y - 2d(b_i - \Delta ty) + ey$ or
$g(y) = a(-b_i - \Delta ty)^2 + by^2 + 2c(-b_i - \Delta ty)y - 2d(-b_i - \Delta ty) + ey$

Thus two « constrained » minimums are obtained:

$$y_1 = \frac{\Delta t a b_i - c b_i - \Delta t d + e}{a\Delta t^2 + b - \Delta t c}, x_1 = b_i - \Delta t . y_1 \quad \text{and}$$

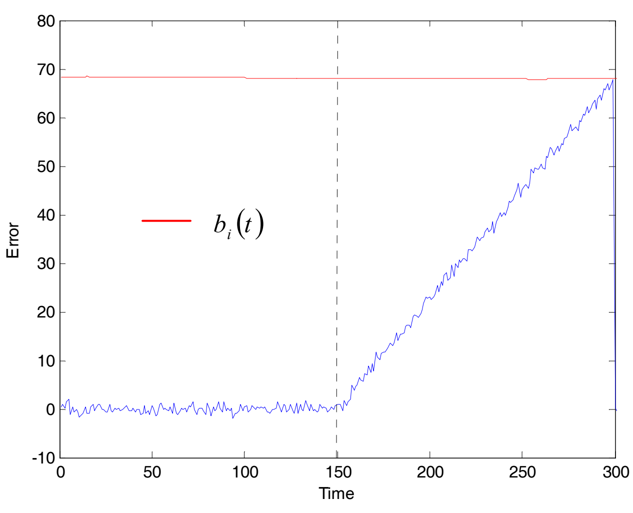
$$y_2 = \frac{-\Delta t a b_i + c b_i - \Delta t d + e}{a\Delta t^2 + b - \Delta t c}, x_1 = b_i - \Delta t . y_1$$

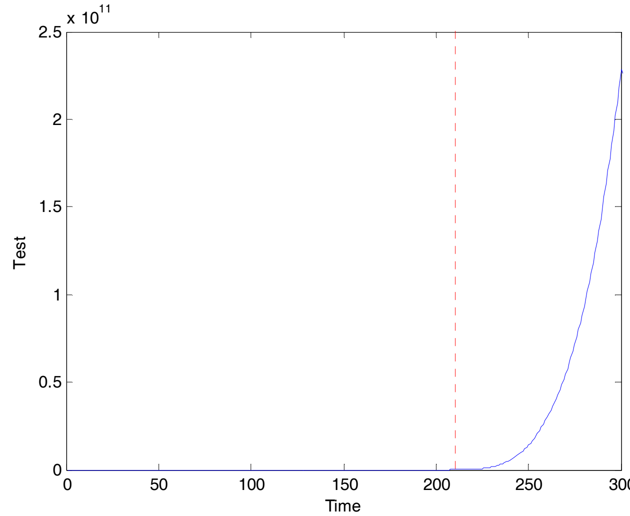
Likewise minimizing $g(x,y) = ax^2 + by^2 + 2cxy - 2dx - 2ey$ under the constraint $|x| \geq b_i$ results, considering function g properties (monotonous, regular), in minimizing under the constraints $|x| = b_i$, that is to say two cases: $x = b_i$ and $x = -b_i$.

If the memory has been reset $(m=1)$ the likelihood function is $g(x) = ax^2 - 2dx$ and its absolute minimum $x = \frac{d}{a}$. In the same way, if this absolute minimum does not respond to the constraints, the algorithm has to choose between the limits of the domain $x = -b_i$ or $x = b_i$ to find the "constrained" minimum.

Behaviour of the test facing different profiles of error

Let us visualise the effect on the statistical test of the injection at time $t = \Delta t = 150s$ of an error on the pseudo range corresponding to PRN 24 of an optimized GPS constellation.





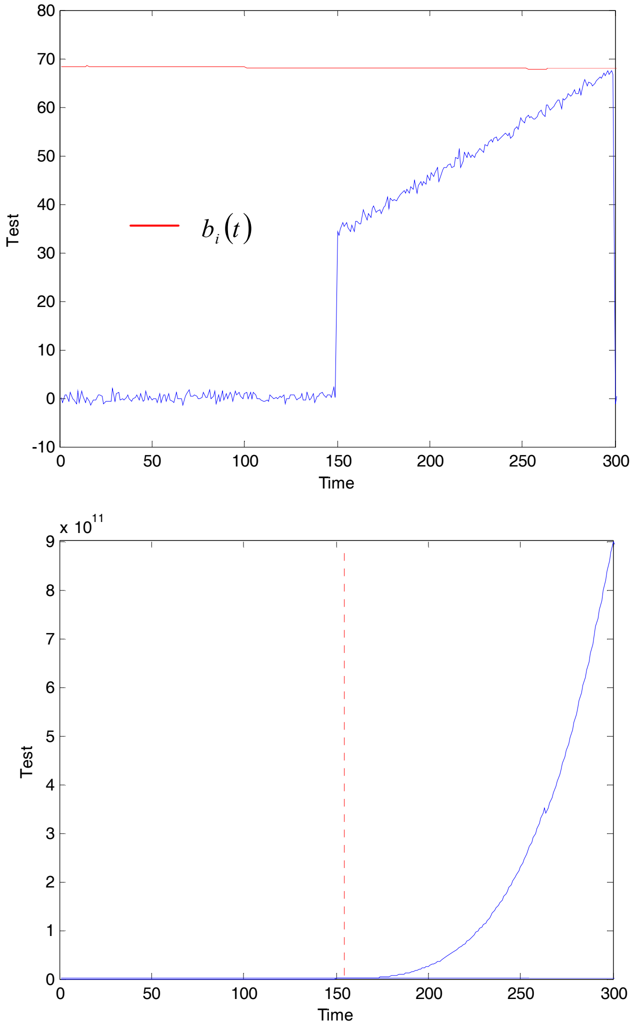
Figures 3 &4: Adaptated sequential constrained GLR test injection of a slope

Fist a bias is added on the i[th] pseudo range measure with a profile $b(t) = \begin{cases} 0 & if \quad t \in [0, \Delta t] \\ \frac{b_i(t)}{\Delta t} t & if \quad t \in [\Delta t, 2\Delta t] \end{cases}$ where $b_i(t)$ is at the instant t the smallest additional bias on the i[th] pseudo range (figure 3)

This additional error will become a "dangerous" one at $t = 2\Delta t$. The algorithm detects it in advance that is to say about 60 seconds after the beginning of the failure but 90 seconds before it becomes dangerous (figure 4). This can also be used to monitor the evolution of an error and provide warning when it becomes dangerous.

Then we consider an additional bias with a profile

$$b(t) = \begin{cases} 0 & if \quad t \in [0, \Delta t] \\ \frac{b_i(t)}{2} + \frac{b_i(t)}{2\Delta t} t & if \quad t \in [\Delta t, 2\Delta t] \end{cases}$$, see figure 5.



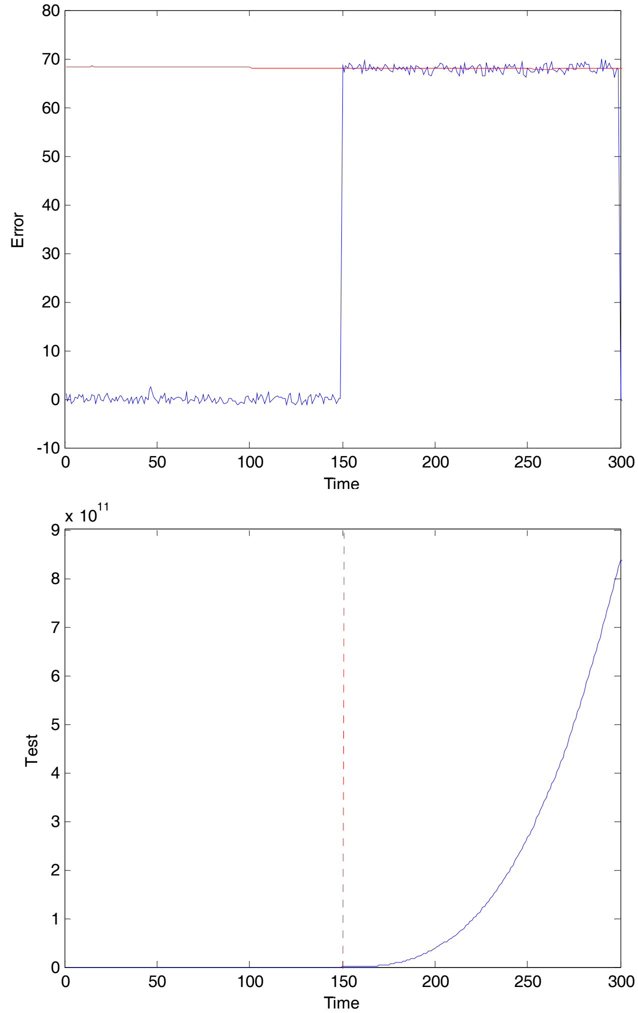
Figures 5 & 6: Adapted sequential constrained GLR test: injection of combined step-ramp pseudo-range bias

The studied algorithm performs better here since it detects this type of error quite instantaneously (figure 6).

Finally for an additional bias with a profile:

$$b(t) = \begin{cases} 0 & if \quad t \in [0, \Delta t] \\ b_i(t) & if \quad t \in [\Delta t, 2\Delta t] \end{cases}$$

the algorithm detects it immediately (figure 7 &8).



Figures 7 & 8: Adapted sequential constrained GLR test: injection of constant pseudo-range bias

V- PERFORMANCES EVALUATION

The snapshot and sequential RAIM based on the constrained GLRT detection and exclusion functions availabilities have been computed for a user grid points parameterized in latitude and longitude spread over the earth surface for all configurations of reference constellations. The improvement obtained due to the constrained GLR technique is about a few percent availability gain in most of the cases tested [2], in case where the feared event is a step error.

Here is presented a first performance evaluation of the sequential RAIM based on the constrained GLR test designed to detect step ramp pseudo range errors. First the robustness of this method has been tested for several profiles of additional error. A nominal GPS constellation has been considered. The measurements available from each satellite are dual frequency L1/L5 corrected by SBAS. The rate of detection of our algorithm has been computed through a 24 hours simulation by injecting successively on each available pseudo range measurements the kind of errors that have been described formerly. Requirements that have bee applied were APV I requirements. We have considered only one user position: Toulouse (France).

Denoting t_0 the instant when the error is added on the pseudo range and $t_{failure}$ the instant when this fault is larger than the corresponding smallest bias that lead to a positioning failure, the following profile of additional error, $b(t) = \frac{b_i(t)}{\Delta t}(t - t_0)$ if $t \in [t_0, t_0 + \Delta t]$, is injected successively on each available pseudo range i.

Figure 9 shows that the availability of the detection function of our algorithm logically depends on the amplitude of the average bias that has to be detected. This amplitude is also linked to the number of visible satellites.

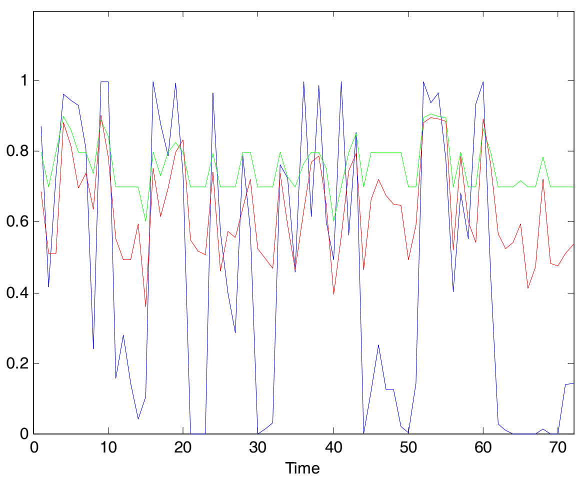


Figure 9: GPS (SBAS L1/L5) injection of a slope

Average number of visible satellites
Average smallest bias to detect
Rate of detection at $t_{failure}$ + 8 seconds

Now a profile of error supposed easier to detect: $b(t) = \frac{b_i(t)}{2} + \frac{b_i(t)}{2\Delta t}(t - t_0)$, if $t \in [t_0, t_0 + \Delta t]$ is injected on our different pseudo range. But in fact, as figure 10 shows, the average availability that it is computed from the detection rate at $t_{failure}$ + 8 seconds is about the same throught a day simulation. The algorithm will detect earlier this second type of failure but at the end the availability periods are the same.

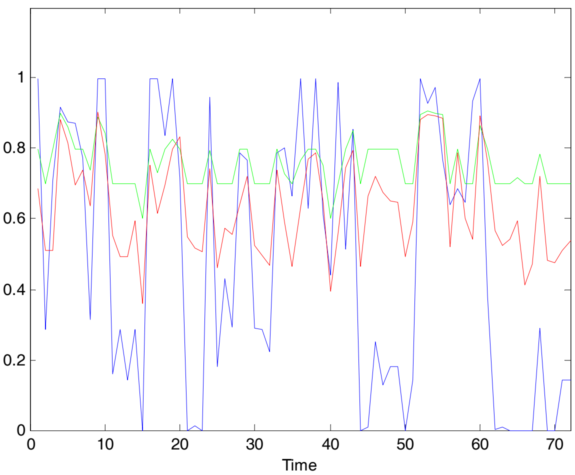


Figure10: GPS(SBASL1/L5), injection of step-ramp error

Figure 11 which represents the average availability for an error profile $b(t) = b_i(t)$ if $t \in [t_0, t_0 + \Delta t]$ shows better results. Nevertheless the detection rate is about the same for the three cases, that is positive for the robustness of our algorithm which at the end performs the same way for different profiles of errors.

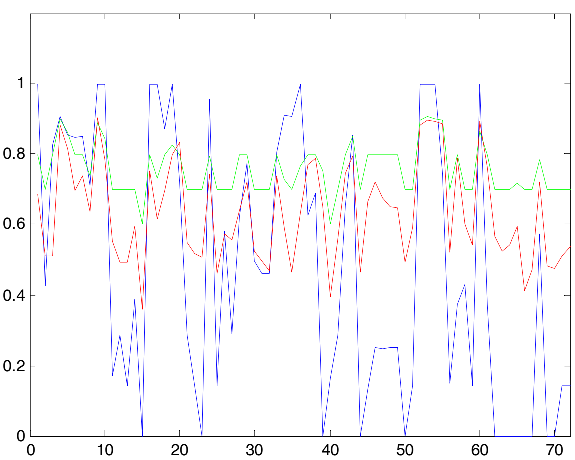


Figure 11: GPS/SBAS L1/L5, injection of a bias

The amplitude of the biases that have to be detected, which depends on the constellation geometry, has a strongest impact on our algorithm availability. It is logical since the more satellites we have the less they individually impact on the user position, the bigger their corresponding additional smallest bias that leads to a positioning failure is, the easier it is to detect. This is why we test our algorithm availability for a same period, a same profile of error but for different constellations: a nominal GPS (SBAS L1/L5) constellation (figure 10), a Galileo E1/E5 constellation (figure 12) and a combined GPS (SBAS L1/L5) + Galileo E1/E5 constellation (figure 13).

The availability of our algorithm to detect combined step ramp pseudo range errors has been seriously improved by the great number of available satellites, as figure 13 shows.

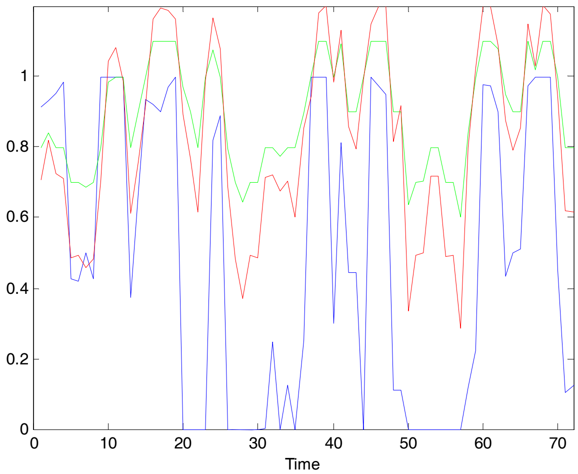


Figure 12: Galileo E1/E5, injection of step-ramp error

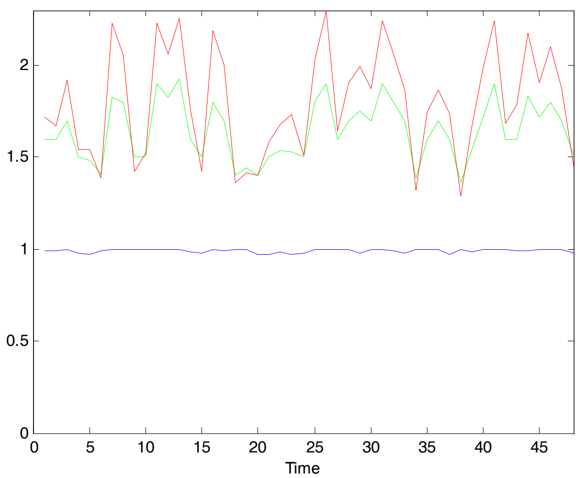


Figure13: Combined constellation GPS L1/L5 Galileo E1/E5, injection of step-ramp error

CONCLUSION

Thanks to a sequential RAIM built to detect only hazardous faults, we have managed to design a RAIM to detect step ramp errors. We have seen that the detection capability of this new algorithm is better in any case (step only, ramp only and combined step ramp) than the one designed to detect step errors and thus the robustness of the existing technique has been improved.

Detecting step ramp error for APV with GPS only or Galileo only is difficult but the results obtained for a combined constellation GPS + Galileo are quite encouraging. Now more analysis need to be made on a wide world basis. The high detection capability of this RAIM for APV I phases enables the algorithm to detect step ramp error that can be dangerous in the in the 150 future seconds. This can be used to monitor the evolution of an error and provide warning only when it is necessary.

During all the study we have supposed that only one individual satellite channel fault was possible at a time. This simplification can be optimistic for a double constellation and this is why the multiple failures case has to be investigated.

APPENDIX A: POSITIONING FAILURE

Definition of a positioning failure

A fault γ is considered as a horizontal positioning failure if its impact violates the integrity risk:

$$(1-p_f)\max P_0\begin{pmatrix}\exists t : k_0 \le t \le k_0+m-1, \\ \left\| X_{horiz,t} - \hat{X}_{horiz,t}\right\|_2 > HAL\end{pmatrix} + p_f P_\gamma\left(\left\| X_{horiz,t} - \hat{X}_{horiz,t}\right\|_2 > HAL\right) > P_{IntRisk}$$

A fault γ is considered as a vertical positioning failure if its impact violates the integrity risk:

$$(1-p_f)\max P_0\begin{pmatrix}\exists t : k_0 \le t \le k_0+m-1, \\ \left\| X_{vertical,t} - \hat{X}_{vertical,t}\right\|_2 > VAL\end{pmatrix} + p_f P_\gamma\left(\left\| X_{vertical,t} - \hat{X}_{vertical,t}\right\|_2 > VAL\right) > P_{IntRisk}$$

where p_f is the probability of failure of one satellite, P_0 designed the fault free case, P_γ the faulty case, HAL and VAL are the horizontal and vertical alert limits

Minimum biases that lead to a positioning failure

The purpose is to compute the parameters b_i for $i \in \{1, n\}$. For each satellite channel they represent the bias on the pseudo range that will lead to a positioning failure with a probability equal to the integrity risk.

The error in the position domain is:

$$\varepsilon_{pos,WGS84} = \left(H^t \Sigma^{-1} H\right)^{-1} H^t \Sigma^{-1}(\xi + B)$$

and projecting this error in thee local geographic frame:

$$\varepsilon_{pos,local} = n_{local}{}^t . \varepsilon_{pos,WGS84} \text{ where}$$

$$n_{local} = \begin{bmatrix} -\sin\varphi\cos\lambda & -\sin\varphi\sin\lambda & \cos\varphi & 0 \\ -\sin\lambda & \cos\lambda & 0 & 0 \\ \cos\varphi\cos\lambda & \cos\varphi\sin\lambda & \sin\varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let us define the matrix $M = n_{local}{}^t . \left(H^t \Sigma^{-1} H\right)^{-1} H^t \Sigma^{-1}$ in order to make the projection in the local geographic frame such as $\varepsilon_{pos,local} = M(\xi + B)$

In the fault free case $(B=0)$, the covariance matrix of the error such as $\varepsilon_{pos,local} \sim N(0_{4\times1}, C)$ is :

$$C = E\left[\varepsilon_{pos,local}.\varepsilon_{pos,local}{}^t\right] = n_{local}{}^t.\left(H^t\Sigma^{-1}H\right)^{-1} n_{local}$$

First we look at the computation of the probability that a given error in the horizontal plane leads to a positioning failure.

If we are not in the fault free case and thus in a more general way, the horizontal positioning error is a two dimensions vector which follows a gaussian bi-dimensional law of mean impact×r and of covariance matrix C_H. Its density function is:

$$f_0(X_H) = \frac{1}{2\pi\sqrt{\det C_H}} \exp\left(-\frac{1}{2} X_H^t C_H{}^{-1} X_H^t\right)$$

Considering this in the space of singular values decomposition of C_H and denoting λ_1 and λ_2 the two eigenvalues of this covariance matrix:

$$f_0(X_{H\perp}) = \frac{1}{2\pi\sqrt{\lambda_1\lambda_2}} \exp\left(-\frac{1}{2}\left(\frac{(x-\Omega_1)^2}{\lambda_1} + \frac{(y-\Omega_2)^2}{\lambda_2}\right)\right)$$

where $X_{H\perp} = \begin{bmatrix} x \\ y \end{bmatrix}$ and $\begin{bmatrix} \Omega_1 \\ \Omega_2 \end{bmatrix}$ is due to the change of coordinates.

The probability that a couple (x,y) be such that $x^2+y^2 \le HAL^2$ considering its distribution is:

$$P(X_H \in D) =$$
$$\iint_D \frac{1}{2\pi\sqrt{\lambda_1\lambda_2}} \exp\left(-\frac{1}{2}\left(\frac{(x-\Omega_1)^2}{\lambda_1} + \frac{(y-\Omega_2)^2}{\lambda_2}\right)\right) dxdy$$

denoting D the domain such as $x^2+y^2 \le HAL^2$.

Let's make a change of coordinates such as we could have $\frac{(x-\Omega_1)^2}{\lambda_1} + \frac{(y-\Omega_2)^2}{\lambda_2} = r^2$. We re-write (x,y) this way:

$$\begin{cases} x = \Omega_1 + r\sqrt{\lambda_1}\cos\theta \\ y = \Omega_2 + r\sqrt{\lambda_2}\sin\theta \end{cases}$$

The equation $x^2+y^2 = HAL^2$ that defines the boundaries of the integration domain becomes:

$$\begin{aligned} x^2+y^2 &= \left(\Omega_1 + r\sqrt{\lambda_1}\cos\theta\right)^2 + \left(\Omega_2 + r\sqrt{\lambda_2}\sin\theta\right)^2 \\ &= \Omega_1^2 + r^2\lambda_1\cos^2\theta + 2\Omega_1 r\lambda_1\cos\theta \\ &+ \Omega_2^2 + r^2\lambda_2\sin^2\theta + 2\Omega_2 r\lambda_2\sin\theta = HAL^2 \end{aligned}$$

Solving this equation, two roots $r_1(\theta)$ and $r_2(\theta)$ for $\theta \in [0,\pi]$ are obtained such as:

$\begin{cases} x = \Omega_1 + r_1(\theta)\sqrt{\lambda_1}\cos\theta \\ y = \Omega_2 + r_1(\theta)\sqrt{\lambda_2}\sin\theta \end{cases}, \theta \in [0,\pi]$ and

$\begin{cases} x = \Omega_1 + r_2(\theta)\sqrt{\lambda_1}\cos\theta \\ y = \Omega_2 + r_2(\theta)\sqrt{\lambda_2}\sin\theta \end{cases}, \theta \in [0,\pi]$ define the boundaries of the integration domain.

The jacobian of this transformation is computed to make our change of coordinates $J = |r|\sqrt{\lambda_1\lambda_2}$, and:

$$P(X_H \in D) = \iint_{D'} \frac{|r|}{2\pi} \exp\left(-r^2/2\right) drd\theta$$

where the new domain D' is defined by $\begin{cases} (r-r_1(\theta))(r-r_2(\theta)) \le 0 \\ \theta \in [0,\pi] \end{cases}$.

Considering properties of second order polynomials:

$$P(X_H \in D) = 1/2\pi \int_{\theta=0}^{\theta=\pi}\int_{r=r_1(\theta)}^{r=r_2(\theta)} |r| \exp\left(-r^2/2\right) drd\theta$$

Assuming for example that $r_1 \le 0 \le r_2$,

$$P(X_H \in D) = 1/2\pi \times$$
$$\int_{\theta=0}^{\theta=\pi}\left[-\int_{r=r_2}^{r=0} r\exp\left(-r^2/2\right)dz + \int_{r=0}^{r=r_1} r\exp\left(-r^2/2\right)dr\right]d\theta$$

$$P(X_H \in D) = 1 - 1/2\pi \int_{\theta=0}^{\theta=\pi}\left[\exp\left(-r_2^2/2\right) + \exp\left(-r_1^2/2\right)\right]d\theta$$

and this last integral is computed numerically.

Thus the probability that the point (x,y) is out of the circle of radius HAL is:

$$P(X_H \notin D) = 1/2\pi \int_{\theta=0}^{\theta=\pi}\left[\exp\left(-r_2^2/2\right) + \exp\left(-r_1^2/2\right)\right]d\theta$$

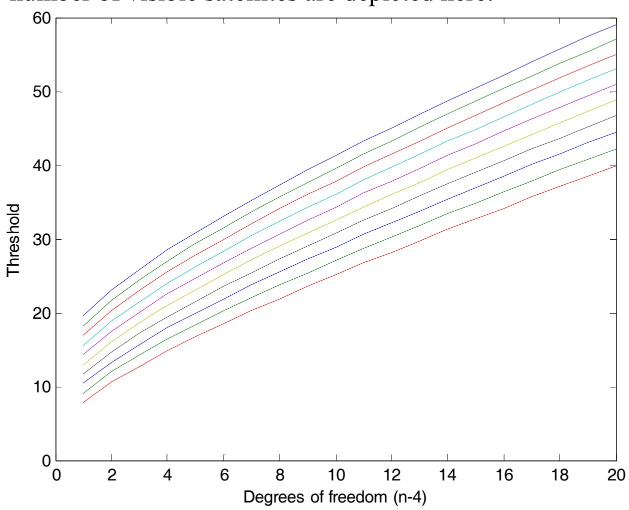
In order to pass from a bias b on a given pseudo range to an error vector in the local horizontal plane , projections are made using slopes variables. Denoting $A = \left(H^T H\right)^{-1} H^T$ and $B = H.A$, we define for $i \in [1,N]$: $HslopeN(i) = \sqrt{A_{1i}^2} / \sqrt{(1-B_{ii})}$

and $HslopeE(i) = \sqrt{A_{2i}^2} / \sqrt{(1-B_{ii})}$

An equivalent analysis of the vertical risk (which is easier in one dimension) must also be done. Then by comparing successively the obtained probabilities with the integrity risk for different bias amplitudes, the minimum bias which leads to a positioning failure with a probability equal to the integrity risk is finally obtained.

APPENDIX B: THRESHOLDS

The thresholds of RAIM algorithms as function of the number of visible satellites are depicted here:



A class of threshold h will correspond to a false alarm rate. Choosing an index is equal to tuning our detection/exclusion algorithm.

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