Compatibility of Galileo E1 signals with the Radio-Astronomy band 9
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1. Introduction

The current Galileo E1 signal is located in the L-band at the central frequency 1575.42 MHz. Although the useful bandwidth is 32 MHz, its total spectral width is 40.92 MHz, including the secondary lobes, the upper part of its transmitting band being at 1595.88 MHz. The Galileo E1 signal has two useful components:

- the Open Service (OS) signal, and
- the Public Regulated Service (PRS) signal.

The Power Spectral Density (PSD) of the PRS component is located at the edge of the E1 band. This means that harsh constraints exist on the Galileo payload filter to ensure that the Galileo signal comply strictly with the ITU recommendation to protect the adjacent Radio Astronomy (RA) band.

The 1610.6-1613.8 MHz is an important RA band because it allows studying Hydroxyl (OH) radical transitions that are necessary to observe Maser effects in certain star forming regions, in giant red stars and in comets. The current ITU recommendation to protect this RA band is to ensure a Power-Flux Density (PFD) lower than -194 dBW/m²/20kHz. Radio Frequency Interference (RFI) has, however, been reported in this band that significantly disturbs exploitation of OH spectral lines. Examples of these interference are GLONASS L1 signal and Iridium [Monstein and Meyer, 2007; ERC, 1997; Galt, 1991]. In order to be immunized from these sources of disturbance, different antenna or signal processing techniques have been investigated and used. As an example, it is worth mentioning specific techniques designed to mitigate GLONASS C/A and P disturbances.

Although further away from the RA band than GLONASS, Galileo is still relatively close to it (15 MHz only). In particular, the ITU recommendation on the RA band implies a fairly steep slope to the Galileo E1 payload filter that can result in significant additional payload costs, and degradation of the group delay characteristics in the useful bandwidth. Consequently, it would be beneficial for Galileo users if that recommendation is only partly taken into account, while still ensuring that it does not disturb RA activities. One approach is to consider that the current ITU recommendation does not take into account current RFI mitigation techniques available to the RA community.

The goal of this paper is then to investigate a new processing technique that enables removing, at least partially, the Galileo E1 signal from the RA signal of interest. This method separately estimates the OS and the PRS components of the Galileo E1 signal. This is then used to subtract the Galileo E1 signal from the RA useful signal. The difference between the original PSD and the cleaned one would give an indication on how much could the ITU PFD constraint be loosened and thus, what would be the new constraints for the Galileo E1 payload filter. Obviously, this would have an impact on the Galileo user as well as on the cost.
The article will first introduce the Galileo E1 signal. The third part will then investigate the constraints put by the ITU recommendations for the RA band 9 on the Galileo E1 payload. Finally, the “estimate and remove” algorithm will be reviewed, explained and analyzed in the Galileo context. In particular, the algorithm will be tested and its initial implications on the Galileo payload filter will be stated.

2. The Galileo E1 Signal Structure

The Galileo E1 signal will be broadcast at a central frequency of 1575.42 MHz and within its allocated transmission bandwidth of 40.92 MHz. It will support 2 Galileo services:

- The OS that is accessible to any user that wants to receive Galileo signals. Two signals, known as OSA and OSB, are supporting this service:
  - The OSA signal is also referred to as the OS data component and carries the OS navigation message at a rate of 250 symbol/sec.
  - The OSB signal is also referred to as the OS pilot component and is just a ranging signal not modulated by any navigation message.

Both components are synchronized. They are both modulated by spreading codes of length 4092 with a rate of 1.023 chips/sec. The OSB also uses a secondary code of length 25 at a rate of 250 chips/s.

The overall OS signal uses a CBOC(6,1,1/11) modulation that is described in Hein et al (2007).

- The PRS that is restricted to authorized users such as public safety or emergency services. Only one signal is used on the Galileo E1 band to support this service. It uses a cosine-phased BOC(15,2.5) modulation. The spreading code used is encrypted and aperiodic.

The theoretical expression of the transmitted unfiltered Galileo E1 signal is given in [Rebeyrol 2007] by:

\[
\begin{align*}
    s_{\text{Gal}}(t) &= A_{E1} \left[ \frac{\sin(2\beta_1)}{2} \cdot [d_{\text{OS}}(t)c_{\text{OSA}}(t) + c_{\text{OSB}}(t)] \cdot x(t) \\
    &+ \frac{\sin(2\beta_2)}{2} \cdot [d_{\text{OS}}(t)c_{\text{OSA}}(t) - c_{\text{OSB}}(t)] \cdot y(t) \\
    &+ \frac{(\cos(2\beta_1) + \cos(2\beta_2))}{2} \cdot d_{\text{PRS}} \cdot c_{\text{PRS}} \cdot z(t) \\
    &+ \frac{(\cos(2\beta_1) - \cos(2\beta_2))}{2} \cdot d_{\text{OS}}(t)c_{\text{OSA}}(t)c_{\text{OSB}}(t) \cdot d_{\text{PRS}} \cdot c_{\text{PRS}} \cdot z(t) \right] \cdot \cos(2\pi f_1 t) \\
    &+ \sin(2\pi f_1 t)
\end{align*}
\]

where

- \( A_{E1} = \sqrt{2P_{E1}} \) is the amplitude of the overall Galileo E1 signal,
- \( P_{E1} \) is the power of the overall Galileo E1 signal,
- \( d_{\text{OS}} \) and \( d_{\text{PRS}} \) are the navigation messages carried by the OS and PRS signals,
- \( c_{\text{OSA}}, c_{\text{OSB}} \) and \( c_{\text{PRS}} \) are the spreading sequences carried by the OS and PRS signals (note that \( c_{\text{OSB}} \) actually contains a primary and a secondary code),

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\[ x, y \text{ and } z \text{ represent respectively the sine-phased BOC(1,1), the sine-phased BOC(6,1) and the cosine-phase BOC(15,2.5) sub-carriers}, \]
\[ \text{the coefficients } \beta_1 \text{ and } \beta_3 \text{ are a function of the relative power of each component of the Galileo E1 signal. In particular, the respective power of the PRS signal with respect to the other signals is not publicly available, and} \]
\[ f_{L_1} \text{ is the L1 frequency.} \]

It can be noticed in the previous equation that the fourth term does not represent any of the useful signals. It is known as Inter-Modulation (IM) signal and it is used to create a signal with constant envelope. As it can be seen, it is a mix of the PRS and the OS components. It has a cosine-phased BOC(15,2.5) modulation.

As already mentioned, the relative power of each component is not fully known yet and thus only assumptions can be made to determine the value of the parameters \( \beta_1 \) and \( \beta_3 \). In this paper, it will be assumed that the PRS signal has the same power as the OS signal. This assumption leads to:

\[ \beta_1 = 0.5788858; \quad \beta_3 = 0.1469235 \]

Assuming uncorrelated spreading codes between components, it can then be deduced that the Galileo E1 normalized (and unfiltered) Power Spectral Density (PSD) is:

\[ S_{\text{Gal}}(f) = \frac{\sin^2(2\beta_1)}{2} S_{\text{BOC(1,1)}}(f) + \frac{\sin^2(2\beta_3)}{2} S_{\text{BOC(6,1)}}(f) + \frac{\cos^2(2\beta_1) + \cos^2(2\beta_3)}{2} S_{\text{BOC(15,2.5)}}(f) \]

The shape of each individual PSD can be found in Rebeyrol (2007).

Before being transmitted, the Galileo E1 signal is filtered by the Galileo satellite payload so that it complies with its transmitted bandwidth assignment. This filtering also intends to reduce the level interference in adjacent bands.

### 3. Galileo E1 and the Radio Astronomy Band 9

#### a. Presentation of the Radio Astronomy Band 9

The 1610.6-1613.8 MHz is an important Radio Astronomy (RA) band because it allows studying Hydroxyl (OH) radical transitions that are necessary to observe Maser effects in certain star forming regions, in giant red stars and in comets. The observation of the OH radical transition is done at extremely low power levels, and is thus very sensitive to the interference environment present in the band. RA is a primary user of this band and thus, to protect the RA observations, the ITU has made different recommendations, including some dedicated to RNSS systems:

- Recommendation ITU-R RA.1513 indicates that the percentage of data loss due to interference from one single RNSS system should not exceed 2% at any RA station. The percentage of data loss is defined as the percentage of integration periods of 2000s in which the average aggregate spectral Power Flux Density (PFD) at the radio telescope exceeds -194dBW/m²/20KHz.
- Recommendation ITU-R M.1583 provides a methodology to evaluate the PFD level per satellite from this percentage of data loss and aggregate PFD:
  - The sky is divided into cells of nearly equal solid angle

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A statistical analysis is performed with 2 random variables:

- The direction of the pointing antenna
- The starting time of the satellite constellation

For each trial, the unwanted emission level is averaged over 2000s.

ITU-R RA.1631 provides the antenna pattern and maximum antenna gain to be used in compatibility studies involving the RAS. RA sites use parabolic antennas with a typical diameter between 13 and 100 meters. Such radio-telescopes provide a highly directional antenna with low side-lobes. Typical antenna gain patterns are reproduced in Figure 1. It can be seen that, for the range of antenna considered, the main lobe of the antenna is very sharp with a very high maximum gain (47 to 64 dBi).

![Antenna Gain Pattern of a Typical Radio-Telescope](image)

**b. Constraints on Galileo E1 Payload induced by the Protection of the Radio Astronomy Band 9**

It is thus important to ensure that Galileo E1 will meet the above-mentioned ITU recommendations in order not to harm the RA users. Figure 2 (top) shows the unfiltered Galileo E1 PSD with respect to the RA band 9. Two important facts can be noted in Figure 2:

- The Galileo E1 transmitted band ends very close from the PRS side-lobes. This means that the filtering done in the Galileo E1 payload to limit the transmitted band has to be carefully done in order not to deteriorate the PRS signal and thus the user.
- There is a far side-lobe of the Galileo E1 signal in the middle of the RA band.

Figure 2 (bottom) zooms on the normalized (and unfiltered) PSD of the Galileo E1 signal (and its different components) within the RA band 9. It can be seen that it is highly dominated by far secondary side-lobes of the PRS and IM signals. This means that it is mainly the PRS component that might affect the RA band. Comparatively, the PSD of the BOC(1,1) component is more than 5 dBs below, while the PSD of the BOC(6,1) component is more than 15 dB below.

Based on the ITU recommendations, the Agence Nationale des Fréquences (ANFR) wrote a program in order to compute the maximum PFD per Galileo satellite. A description of the program is given in [ANFR]. The following settings were selected:
- A constellation of 27 satellites,
- A 100-metre diameter radio-telescope located in Effelsberg in Germany,
- The sky is split in 2334 cells of 9 square degrees.

Using this configuration, the maximum PFD per satellite to ensure the recommendation ITU-R M.1583 was assessed to be $-212 \text{ dBW/m}^2/20\text{kHz}$ in the RA band 9.

![Graph showing normalized PSD and frequency bands](image)

**Figure 2 – Unfiltered Galileo E1 PSD in E1 Band and RA Band (Top) and Zoom on the RA Band (Bottom)**

According to [Galileo OS ICD], the maximum received power for the Galileo E1 OS component at the output of a 0 dBi antenna is -154 dBW. Following our assumptions, this means that the maximum received E1 power from 1 Galileo satellite is approximately -150.64 dBW.

The ratio between the total unfiltered Galileo E1 signal and the power contained in a 20 KHz bandwidth is plotted in Figure 3 for the RA band 9. It can be seen that the maximum value is -49.8 dB.

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Moreover, the effective area of a 0 dBi antenna at 1 593.834 MHz is equal to 25.5 dB.m².
Consequently, the unfiltered PFD of a single Galileo satellite in the RA band equals -150.64-49.8+25.5=-174.94 dB/m²/20KHz.

![Graph](attachment:image.png)

**Figure 3 - ratio between the power contained within the transmitted bandwidth and the power contained in a 20 KHz band in the RA band**

This means that the required attenuation provided by the payload filter in the RA band should be equal to -174.94+212=37.1 dBs. In order to meet the ITU recommendations linked to the RA band 9.

Such a rejection (37.1 dBs in 15 MHz) implies a high constraint on the Galileo payload filter. This is particularly true since the PRS signal that has its main lobes at the edge of the Galileo E1 transmitted band. Consequently, a high filtering constraint on the payload means a potentially significant added cost (mostly to improve the equalization). Moreover, it might also result in the degradation of the group delay and bandwidth characteristics. This will thus also be detrimental for the end user.

It would thus be interesting to find a way to reduce this payload filter constraint for Galileo without harming the RA community.

## 4. Proposed Strategy to Reduce Galileo E1 Impact on the RA Band

### a. The GLONASS Case

The RA band 9 has a long record of RF interference presence. The most important reported ones are Iridium and GLONASS [Monstein and Meyer, 2007; ERC, 1997; Galt, 1991]. The case of GLONASS is interesting since it is also an RNSS and, as Galileo E1, it has two components, an open one and an encrypted one. However, GLONASS has the disadvantage of occupying a band even closer to the RA band 9. An example of such interference is shown in Figure 4. Because of that, GLONASS has not been able to respect the ITU recommendation, and a series of agreements were reached with the RA community to reduce the constraints on GLONASS, while allowing acceptable levels to the RA community. In particular, the closest GLONASS frequencies from the RA band were abandoned during a first agreement. Also, recent analysis of GLONASS broadcast signals showed a notch in the RA band that
proved a certain level of implication of the GLONASS team. Still the signal in the RA band is still well above the ITU recommendation level.

![Image of PSD of an Observed RA Signal with an Apparent GLONASS Interference Located at 1609 MHz]

Figure 4 – PSD of an Observed RA Signal with an Apparent GLONASS Interference Located at 1609 MHz [Bunton, 2001]

There have been many publications by the RA community on interference removal. Several antenna or signal processing techniques have been investigated and used with success. In particular, specific techniques have been designed to mitigate the GLONASS C/A disturbances. One of them consisted, using the high gain radio-telescope, in tracking accurately the GLONASS signal in order for the RA receiver to estimate the signal’s amplitude, code delay and phase. This meant that the GLONASS C/A signal could then be reproduced accurately (the spreading sequence is known) and subtracted from the incoming signal of interest. This process is described in Ellingson et al. (2001).

In the case of the encrypted signal, the results were not as successful since only authorized receiver can track the P code. Some estimation techniques were tried [Bunton, 2001], but with the drawback of increasing the background noise.

The method developed by Ellingson et al (2001) could be used to track the Galileo OS signal, however, as already mentioned, this is not the Galileo component that creates the biggest interference in the RA band 9. Moreover, one has to keep in mind that the Galileo signal is further away spectrally from the RA band. It is then interesting to develop a new signal processing technique that allows the reliable estimation of the PRS component.

b. Overview of the Proposed Technique

The idea behind the proposed method is to take advantage of the high radio-telescope antenna gain to try to estimate each chip of the Galileo PRS signal. Assuming that behind the radio-telescope, there is a wide-band Low Noise Amplifier (LNA), the signal can then be split in 2:

- A channel that leads to a GNSS processing block which goal is the estimation of the Galileo E1 baseband signal.
- A channel that leads to the RA signal processing block. This block should also have, as input, the estimated Galileo E1 signal.
The GNSS processing block can be activated when a Galileo satellite is entering the second or third side-lobe of the radio-telescope gain pattern. This can be done either through signal processing/detection, or through the simple use of the Galileo satellites ephemeris.

At this stage, given the sharpness of the radio-telescope gain pattern, it seems appropriate to consider that only 1 Galileo satellite will be present in the main lobe. The probability of having several satellites from different GNSS within the first two lobes of the radio-telescope has to be assessed, but is supposed very low.

The proposed algorithm has three steps:

- The first step of the GNSS processing block is to get synchronized with the Galileo E1 signal of interest. This means implementing a method similar to Ellingson et al. (2001). This should be easily achieved using the known Galileo E1 OS component. This synchronization has three objectives: (1) since the PRS signal and the OS signal are synchronized, it also provides synchronization with the PRS signal; (2) a fine tracking of the carrier phase will allow splitting the OS and the PRS signal since they are in phase opposition; and (3) a fine estimation of the OS signal amplitude will also provide the PRS signal amplitude.

- The second step of the GNSS processing block is to estimate the value of each PRS samples based on estimation theory.

- The third step is to transform the estimated Galileo E1 baseband signal into the signal perturbing the RA receiver. This means passing the signal through a filter equivalent to the satellite payload filter and the receiver filter. The resulting signal is then subtracted to the signal entering the RA processing block.

Note that to achieve good results, it is tremendously important for the RA block and the GNSS to be synchronized. Thus it is ideal if the same oscillator could be used for both blocks.

c. Tracking the Galileo E1 OS Signal

Because RA observes very low-power signals, the RA reception chain is of very high quality and made to minimize the noise factor. In the following, it will be assumed that the noise PSD level at the entrance of the GNSS block equals -205dBW/Hz.

Moreover, the GNSS processing block will need to process the PRS signal. Thus, it is assumed that the equivalent front-end filter of this block has a one-sided bandwidth B of 17.9 MHz (although it could also be interesting to look only at the side-lobe itself, this option is not considered herein). This means that the noise power at the GNSS signal processing block input will be -132.5 dBW.

The assumed maximum received Galileo E1 OS signal power equals -154 dBW. In the following, the case of a 13-meter radio-telescope will be taken. Extension to an other radio-telescope diameter is straightforward. As already mentioned, its maximum gain is approximately 47 dBi. Thus, the received Galileo signal of interest is amplified by 47-\(L\) dBs where \(L\) represents the antenna gain with respect to the maximum antenna gain.

The correlation losses affecting Galileo E1 signal, due to the front-end filtering, are assessed to be lower than 0.15 dBs.

Obviously, the reception chain will bring some losses (antenna cables, quantization, etc...). However, it is difficult to quantify these at this stage, and thus they will be included in the parameter \(L\).
Consequently, the minimum C/N\(_0\) to consider for Galileo E1 tracking is approximately \((98-L)\) dB-Hz. Even for high values of L, this means that Galileo E1 tracking will be extremely accurate. In particular it means that power estimation should be very accurate. Moreover, precise carrier removal leads to the almost perfect split between the PRS+IM and the OS signal.

d. Estimating the Galileo E1 PRS+IM Signal

After carrier removal and low-pass filtering, it appears that:

\[
Q(t) = s_{Gal}(t) \sin(2\pi f_{1c} t) = \left[ \frac{A_E \sqrt{L_{F-BOC_{\text{os}}}(15.2,5)}}{4} \right] \frac{d_{\text{PRS}}(t) c_{\text{PRS}}(t) z(t) \times}{(\cos(2\beta_1) + \cos(2\beta_3)) + \left(\cos(2\beta_1) - \cos(2\beta_3)\right) \cdot W(t)}
\]

where

\[L_{F-BOC_{\text{os}}}(15.2,5)\] represents the filtering losses due to the front-end filter. With a one-sided bandwidth of 17.9 MHz, the loss is approximately 1.25 dBs, and

\[W(t) = d_{\text{OSA}} c_{\text{OSA}}^T c_{\text{OSA}}.\]

Note that \(W\) is known from the OS processing. Indeed, the spreading sequences are known, and it can be assumed that data demodulation is almost error-free due to the high C/N\(_0\) values.

Then,

\[
\text{if } W(t) = 1: \quad Q(t) = q_1(t) = \begin{cases} 
  \frac{A_E \sqrt{L_{F-BOC_{\text{os}}}(15.2,5)}}{2} \cos(2\beta_1) & \text{if } d_{\text{PRS}}(t) c_{\text{PRS}}(t) z(t) = 1 \\
  -\frac{A_E \sqrt{L_{F-BOC_{\text{os}}}(15.2,5)}}{2} \cos(2\beta_1) & \text{if } d_{\text{PRS}}(t) c_{\text{PRS}}(t) z(t) = -1 
\end{cases}
\]

\[
\text{if } W(t) = -1: \quad Q(t) = q_{-1}(t) = \begin{cases} 
  \frac{A_E \sqrt{L_{F-BOC_{\text{os}}}(15.2,5)}}{2} \cos(2\beta_3) & \text{if } d_{\text{PRS}}(t) c_{\text{PRS}}(t) z(t) = 1 \\
  -\frac{A_E \sqrt{L_{F-BOC_{\text{os}}}(15.2,5)}}{2} \cos(2\beta_3) & \text{if } d_{\text{PRS}}(t) c_{\text{PRS}}(t) z(t) = -1 
\end{cases}
\]

Since the sign of \(W(t)\) is supposed known, the sign of \(d_{\text{PRS}}(t) c_{\text{PRS}}(t) z(t)\) is directly given by the sign of \(Q(t)\). Because the magnitude of \(Q\) can have 2 different values, it means that the Bit Error Rate (BER) will depend on the sign of \(W(t)\).

The theoretical BER is given by:

\[
\text{BER} = \frac{1}{2} \text{erfc}\left( \sqrt{\frac{E}{2N_0}} \right)
\]
With $erfc(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^2} dt$ and $E = \frac{2|Q|^2}{B}$.

The probability that $W(t) = 1$ is equal to the probability that $W(t) = -1$, thus,

$$BER_{tot} = \frac{1}{4} \left( erfc \left( \sqrt{\frac{2|q_1|^2}{2BN_0}} \right) + erfc \left( \sqrt{\frac{2|q_2|^2}{2BN_0}} \right) \right)$$

The first two rows of Table 1 summarize theoretical and simulated results for the BER. The simulations were done at baseband using:

- a $4^{th}$-order Butterworth filter of width 20 MHz (single sided) to represent the payload filter, and
- a $3^{rd}$-order Butterworth filter of width 17.9 MHz (single sided) to represent the receiver filter

It can be seen that there is a gap between the theoretical and simulated results. This is mostly due to the filtering effect (the PRS side-lobes are very close to the filter bandwidth edge), since the theoretical result assume perfectly squared chips. It can also be noticed that for values of $L$ as small as 20 dBs, the BER starts to increase significantly.

One way to improve the method is to take advantage of the knowledge of the PRS and IM sub-carrier. Indeed, it is known that the PRS+IM signal will use a cosine-phased BOC(15,2.5) sub-carrier. It is then interesting to force the estimated signal to have such a sub-carrier. In particular, it is possible, in places where the PRS code chip remains unchanged, to compare the set of estimated samples with a pure cosine-phased BOC(15,2.5) sub-carrier. This, for instance can be done through a correlation process. The sign of the resulting correlation value would then give the sign of the current PRS code chip. This means that it is then possible to reproduce the PRS signal.

Table 1 – Improved BER for Estimating the Sign of $d_{prs}(t)c_{prs}(t)z(t)$

<table>
<thead>
<tr>
<th>Losses $L$ (dB) for an Radio-Telescope Diameter of 13 m</th>
<th>40</th>
<th>37.5</th>
<th>35</th>
<th>32.5</th>
<th>30</th>
<th>27.5</th>
<th>25</th>
<th>22.5</th>
<th>20</th>
<th>17.5</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>BER before correlation (Theoretical)</td>
<td>43.6</td>
<td>41.6</td>
<td>38.9</td>
<td>35.4</td>
<td>31.0</td>
<td>25.7</td>
<td>19.9</td>
<td>14.0</td>
<td>9.0</td>
<td>5.0</td>
<td>2.1</td>
</tr>
<tr>
<td>BER before correlation (Simulation)</td>
<td>46.2</td>
<td>44.9</td>
<td>43.1</td>
<td>41.2</td>
<td>38.4</td>
<td>35.1</td>
<td>31.0</td>
<td>26.5</td>
<td>21.8</td>
<td>17.2</td>
<td>13.3</td>
</tr>
<tr>
<td>BER after correction and oversampling (Simulation)</td>
<td>35.1</td>
<td>29.2</td>
<td>23.5</td>
<td>18.6</td>
<td>11.8</td>
<td>7.6</td>
<td>4.1</td>
<td>1.5</td>
<td>0.4</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
This is somehow equivalent to assess the sign of the PRS code chip without the sub-carrier. Moreover, it is possible, in order to improve the estimation process, to use oversampling to compensate the limitation of the bandwidth. This was the option chosen herein, using a sampling frequency of 89 MHz (to be compared with the minimum 36 MHz allowed by the Nyquist criteria). The results are shown in the last row of Table 1. It can be seen that a significant improvement (of approximately 5 dBs) is reached.

Since the power of the Galileo E1 PRS signal can be assessed through the OS tracking, the estimation process of the unfiltered Galileo E1 baseband signal can be considered as completed.

It is then necessary to pass this signal through a filter equivalent to the Galileo payload filter and the RA filter, to match the process RA signal. The corrective signal can then be subtracted from the signal received by the RA block.

It is now important to see the performances of the method in reducing the actual required PFD per satellite.

**e. Performance of the Proposed Method**

In this section, it is assumed that the Galileo E1 OS is perfectly tracked and thus it is assumed that the PRS and IM signals have been perfectly isolated. Only these 2 signals are thus investigated.

After the PRS+IM estimation, the reproduced signal (normalized with respect to the incoming Galileo E1 PRS+IM signal) at baseband can be written as:

\[
Q_R = \alpha (Q + \varepsilon_Q)
\]

where

\[Q = \hat{Q} - \varepsilon_Q\]

\[\varepsilon_Q\]

represents the estimation error of the normalized PRS+IM signal, and

\[\alpha\]

represents the amplitude estimation error.

It is important to remember that, even though there might be errors, \(\hat{Q}\) will still have a cosine-based BOC(15,2.5) sub-carrier. Indeed, using the correlation method, it is possible to make an error in the sign of the PRS code chip, but the sub-carrier will always be a cosine-based BOC(15,2.5). Thus, if there is an estimation error, \(\hat{Q}\) will just be using a different spreading code.

Let’s denote

\[h\]

the impulse response of the true filter equivalent to the succession of the satellite payload, the propagation channel, and the RA front-end filter, and

\[h_2\]

the estimation of the impulse response of the true filter equivalent to the succession of the satellite payload, the propagation, and the RA front-end filter, and

After subtraction of the estimated PRS+IM signal, considered at baseband herein, the error can be written as:

\[
\Delta = Q \ast h - Q_R \ast h_2 = Q \ast h - \alpha (Q + \varepsilon_Q) \ast h_2 = Q \ast (h - \alpha h_2) - \alpha \varepsilon_Q \ast h_2
\]
where the operator \( \ast \) represents the convolution operation.

The Fourier transform of these terms equal:

\[
F_\Lambda = F_Q (H - \alpha H_2) - \alpha F_{e_0} H_2
\]

where

\( F_X \) represents the Fourier Transform of the signal \( X \), and

\( H \) and \( H_2 \) represent the frequency response of the filters \( h \) and \( h_2 \).

Finally, the PSD of the error signal is:

\[
G_\Lambda = F_\Lambda F_\Lambda^* = \left( F_Q (H - \alpha H_2) - \alpha F_{e_0} H_2 \right) \left( F_Q (H - \alpha H_2) - \alpha F_{e_0} H_2 \right)
\]

\[
G_\Lambda = G_Q \left[ |H|^2 + \alpha^2 |H_2|^2 - 2\alpha \text{Re}(H H_2^*) + \alpha^2 G_{e_0} |H_2|^2 - 2\alpha \text{Re}\left( F_{e_0} F_{e_0}^* (H H_2^* - \alpha |H_2|^2) \right) \right]
\]

Noting that \( F_{e_0} = F_{\hat{Q}} - F_Q \) and \( G_{e_0} = G_Q + G_{\hat{Q}} - 2 \text{Re}(F_{e_0} F_{e_0}^*) \), then

\[
G_\Lambda = |H|^2 G_Q + \alpha^2 |H_2|^2 G_{\hat{Q}} - 2\alpha \text{Re}\left( F_{e_0} F_{e_0}^* H H_2^* \right)
\]

Consequently, the PSD of the remaining signal has a PSD depending upon 3 parameters:

- The knowledge of the equivalent filter response,
- The estimation of the incoming signal amplitude, and
- The estimation of the baseband signal.

The PSD of the original Galileo E1 signal entering the RA processing block is \( G_Q |H|^2 \). Consequently, the gain over the original signal can be quantified as:

\[
G_{DSP} = \frac{G_\Lambda}{|H|^2 G_Q} = 1 + \alpha \frac{|H_2|^2 G_{\hat{Q}} - 2\text{Re}(F_{e_0} F_{e_0}^* H H_2^*)}{|H|^2 G_Q}
\]

The PRS spreading code will be a very long aperiodic spreading code. Thus, it is possible to estimate its PSD by the PSD envelope (by opposition to the peak spectrum of GPS C/A, for instance, due to the short periodic spreading sequence used). In this case, the \( \hat{Q} \) will also have a very long aperiodic spreading code (with or without estimation errors). Consequently, the normalized PSDs of \( Q \) and \( \hat{Q} \) are the same.

The correlation between \( Q \) and \( \hat{Q} \) can be written as:

\[
E[Q \hat{Q}^*] = \int_{-\infty}^{+\infty} F_\hat{Q} F_Q^* df
\]

However, because both \( Q \) and \( \hat{Q} \) have the same modulation, it means that their cross-spectrum will have the same PSD envelope. Consequently, for infinite random codes, \( F_{e_0} F_{e_0}^* = K G_{e_0} \). And,

\[
E[Q \hat{Q}^*] = \int_{-\infty}^{+\infty} F_\hat{Q} F_Q^* df = K \int_{-\infty}^{+\infty} G_Q df = K E[Q Q^*]
\]
Thus \( K = \frac{E[Q\hat{Q}^*]}{E[QQ]} \)

\( E[Q\hat{Q}^*] \) will depend upon the probably that the spreading chips of \( \hat{Q} \) were correctly estimated.

It has been seen that the probably of correctly estimating \( Q \) was depending upon the sign of \( W \).

If \( W = 1 \), then

\[ E[Q\hat{Q}^*]_{W=1} = P(Q(i) = \hat{Q}(i))q_i^2 - P(Q(i) \neq \hat{Q}(i))q_i^2 = (1 - P(Q(i) \neq \hat{Q}(i)))q_i^2 - P(Q(i) \neq \hat{Q}(i))q_i^2 \]

\[ E[Q\hat{Q}^*]_{W=1} = q_i^2(1 - 2P_1) \]

where \( P_1 \) is the probability that the sign of the sample \( c(i) \) is well estimated when \( W = 1 \).

If \( W = -1 \), then

\[ E[Q\hat{Q}^*]_{W=-1} = q_{-1}^2(1 - 2P_{-1}) \]

Since the OS spreading sequence is a random sequence, \( P[W = 1] = P[W = -1] = \frac{1}{2} \). Thus,

\[ E[Q\hat{Q}^*] = \frac{1}{2} q_i^2(1 - 2P_1) + \frac{1}{2} q_{-1}^2(1 - 2P_{-1}) \]

And finally,

\[ K = \frac{q_i^2(1 - 2P_1) + q_{-1}^2(1 - 2P_{-1})}{q_i^2 + q_{-1}^2} \]

This means that the reduction of the DSP level can be approximated by:

\[ G_{DSP} = 1 + \alpha \left| H_2 \right|^2 - 2K \text{ Re}\left[ H H_2^* \right] \]

Assuming that the filter response is perfectly known, it gives:

\[ G_{DSP} = \frac{G_{\delta}}{|H|^2 |H_2|} = 1 + \alpha(\alpha - 2K) \]

It is difficult to compute the theoretical value for \( P_1 \) and \( P_{-1} \). However, it is possible to determine their value by simulations. The simulation parameters chosen were the same as the ones used to obtain Table 1. Moreover, only 10 ms of signals were analyzed. This value was chosen to reduce the computational load, while ensuring that the underlying PRS spreading code was long enough (25575 chips) to mimic a smooth enough PSD. This was done extensively assuming that \( H = H_2 \). Several values were taken for the estimation error of the PRS amplitude (\( \alpha \)).

Table 2 shows the results. It can be seen that the improvement brought by the proposed technique is significant, assuming that the Galileo E1 payload filter response is perfectly known, and for different
values of the loss $L$ and of the bias of the Galileo E1 power estimate. In particular, simulation match very well the theory.

The results show that the higher the Galileo E1 signal strength, the better the proposed algorithm will be able to reduce the Galileo E1 PSD. This is important because, as already explained, it is the Galileo signals coming within the radio-telescope main lobes that will have a significant impact on the PFD values.

Table 2 – Example of PSD Gain Brought by the Proposed Technique for Different Losses and Different Biases for the Galileo E1 Power Estimate

| Losses $L$ (dB) for a Radio-Telescope of Diameter = 13 m (for 100m add 17.7 dB to $L$) | $P_1$ | $P_{-1}$ |
|---------------------------------|-----------------|
| 40                              | 40.0            | 28.3       |
| 37.5                            | 36.2            | 22.2       |
| 35                               | 31.8            | 15.2       |
| 32.5                            | 26.8            | 9.7        |
| 30                               | 20.5            | 3.68       |
| 27.5                            | 14.4            | 0.97       |
| 25                               | 8.2             | 0.1        |
| 22.5                            | 3.0             | ~0         |
| 20                               | 0.6             | ~0         |
| 17.5                            | ~0              | 0.0        |
| 15                               | ~0              | 0.0        |

Reducing the PSD level of the interfering signal is equivalent to a reduction of the antenna gain in the direction of that satellite (Indeed, the PSD of the remaining signal is also a PRS PSD). Thus, this would mean that the method is equivalent to receiving the Galileo signal with a lower power. Figure 5 shows the resulting equivalent antenna gain taking into account the proposed method for the case where $\alpha^2 = 0.5$ dB.

Using this new equivalent gain pattern, it is then possible to re-run the simulations based on the ANFR program. The new results give a margin of approximately 10 dBs. This would mean that the Galileo E1 payload filter constraint could be relaxed by 10 dBs, which would be a significant step.
5. Conclusions

Galileo E1 signal will be broadcast in a frequency band neighboring the RA band. As such, it is important to ensure that Galileo will respect the ITU recommendations set to protect the RA band. It has been seen that due to the use of a cosine-phased BOC(15,2.5) modulation by the PRS signal, these recommendations induce that the Galileo E1 payload should provide a 37 dBs filtering in the RA band. This constraint is very high, especially since the PRS signal occupies the edge of the transmitted Galileo E1 band. This could have a negative impact on the payload cost, as well as on the end user.

This paper investigated a technique meant at removing the Galileo signal present in the RA band that could help, if accepted by the Ra community, to reduce the constraint on the Galileo E1 payload filter. It is based on the use of a dual structure: a “GNSS processing block” concentrating on the Galileo band in order to estimate the baseband Galileo E1 signal; and a “RA processing block” that first removes the estimated Galileo signal from the incoming signal and then processes the ‘clean’ RA signal. The GNSS processing block first synchronizes itself with the public open signals. Once this synchronization is achieved, it is then possible to estimate each sample value of the PRS and the IM signals. The algorithm is further enhanced by forcing the estimated signal to have a cosine-phase BOC(15,2.5) modulation.

The analysis of the method showed that its success is based on many mainly 3 parameters:

- the accurate knowledge of the Galileo E1 payload filter (include outside the transmitted band), and the receiver filter,
- the accuracy of the estimation of the Galileo signal amplitude,
- the accuracy of the estimated PRS+IM signal.
As a first step, the performance results assumed that the Galileo E1 payload filter was perfectly known. With that in mind, it showed the proposed algorithm could potentially trigger a relaxation of the required filtering constraint in the RA band by approximately 10 dBs. This means that, still meeting the ITU recommendations for the RA band 9, Galileo satellites could use an E1 payload filter with a significantly lower slope and thus induce a more cost effective solution and a better service for the end user.

These results, though, should be looked at as preliminary results that will trigger further investigations. In particular, the RF design has to be studied. Also, it is crucial to know if it is possible to know well enough the Galileo E1 payload filter outside its useful bandwidth to remove accurately the Galileo PRS signal from the incoming RA signal. Finally, it is also necessary to estimate the losses occurring in the receiver front-end (cable losses, noise factor, etc…) in order to have a more accurate assessment of the method.

6. Bibliography

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