Efficiency characterization of vector-sensor antennas with distributed elements for 3D direction finding

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A. Chabory * C. Morlaas * B. Souny *

Abstract — **A general method is proposed to determine the ability of a vector-sensor antenna to estimate the electric and magnetic fields at one point. This approach works regardless of the positions and types of the antenna elements, and can take into account their mutual couplings. This method is based on a matrix, which computation involves reciprocity and two modal representations of the fields. Its condition number determines the capability of a configuration to estimate the direction of arrival in presence of spatial noise (multipath). Simulation results confirm the validity of this approach.**

1 INTRODUCTION

To determine the direction of arrival (DoA) of an incoming signal, several methods can be employed in radio direction finding. A solution is to consider an antenna array, which deduces the DoA via interferometry, i.e. from the phase differences between the signals reaching each element of the array. This method is widely employed in 2D direction finding. For example, the azimuth can be determined by means of a circular array. Another solution is to use a vector-sensor antenna, i.e. an antenna capable of measuring several components of the electric and magnetic fields. This corresponds to polarization diversity. The DoA can then be deduced from the direction of the electric and magnetic fields.

For 3D direction finding, all the field components are needed at one point. This can be achieved with an antenna constituted by 6 co-located elements [2]. For example, three of these elements can be electric short dipoles which orientations are orthogonal. They measure the components of the electric field. The three other elements can be three orthogonal magnetic short dipoles measuring the components of the magnetic field. However, this solution presents two important practical drawbacks. It is very difficult to co-locate the 6 elements, and even if it was possible, strong mutual couplings would exist between them. A way to overcome this limit is to spatially separate the elements of the antenna [1]. But in this case, the antenna does not simply measure all the field components at the same point. Thus we have to determine which spatial configuration of the vector-sensor antenna leads to an efficient DoA estimation.

In this article, we propose a general method to determine the ability of a vector sensor constituted by N_a elements to estimate the components of the electric and magnetic fields at one point in the vicinity of the antenna. This approach works regardless of the positions and types of elements, and takes into account their mutual couplings. This method is based on a matrix, which condition number determines the capability of the configuration to estimate the DoA in presence of spatial noise (multipath).

In Section 2, the configuration is exposed. In Section 3, we present the modal reprensations used to describe sources and fields. The matrix we propose is then introduced via reciprocity in Section 4. Finally, simulation results are discussed in Section 5.

2 CONFIGURATION

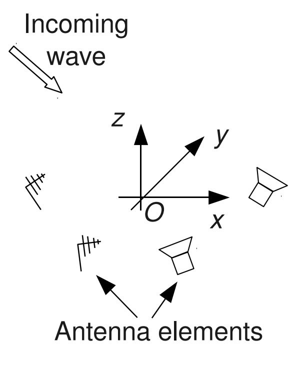

Figure 1: Antenna configuration .

We consider a vector-sensor antenna constituted by N_a elements as illustrated in Figure 1. These elements may not be localized at the same point. Besides, they can be of different types, and they can present mutual couplings. The vector-sensor

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antenna is illuminated by a wave, which direction of arrival is seeked from the signals measured at the port of each element. The method we propose can be divided in two steps. Firstly, from the measured signals, we determine the 6 components of the electric and magnetic fields at O. This part is based on spherical harmonics and on the reaction theorem. Secondly, the direction of arrival is obtained from the Poynting vector at O.

3 MODAL REPRESENTATIONS

3.1 Spherical harmonics

In our configuration, the antenna is receiving an incoming electromagnetic field, which can be represented by means of ingoing spherical harmonics. More generally, the represention of a field via ingoing/outgoing spherical harmonics can be written as [3]

$$\begin{aligned} \mathbf{E} &= \sum_p s_p^+ \mathbf{e}_p^{\mathrm{sph},+} + s_p^- \mathbf{e}_p^{\mathrm{sph},-}, \\ \mathbf{H} &= \sum_p s_p^+ \mathbf{h}_p^{\mathrm{sph},+} + s_p^- \mathbf{h}_p^{\mathrm{sph},-}, \end{aligned} \tag{1}$$

where $\left(\mathbf{e}_p^{\mathrm{sph},+}, \mathbf{h}_p^{\mathrm{sph},+}\right)$ and $\left(\mathbf{e}_p^{\mathrm{sph},-}, \mathbf{h}_p^{\mathrm{sph},-}\right)$ correspond to the electromagnetic fields of outgoing and ingoing spherical harmonics, respectively. Note that p is a multi-index employed here for the sake of clarity.

Spherical harmonics derive from electric and magnetic vector potentials oriented along the radial axis $\hat{\mathbf{r}}$, i.e. such that $\mathbf{A}_p^\pm = \psi_p^\pm \hat{\mathbf{r}}$, with

$$\begin{aligned} \psi_p^+(r,\theta,\phi) &= \psi_{m,n}^{0+} e^{jm\phi} P_n^{|m|}(\cos\theta) h_n^{(2)}(kr), \\ \psi_p^-(r,\theta,\phi) &= \psi_{m,n}^{0-} e^{-jm\phi} P_n^{|m|}(\cos\theta) j_n^{(2)}(kr), \end{aligned} \tag{2}$$

for $n > 0$ and $m \in \{-n, \cdots, +n\}$. Furthermore, k is the wavenumber, $(\psi_{m,n}^{0+}, \psi_{m,n}^{0-})$ are normalization coefficients, $P_n^{|m|}$ represent associated Legendre functions, $h_n^{(2)}$ and $j_n^{(2)}$ are spherical Bessel functions. The harmonic coefficients are obtained by means of the following integrals over a sphere S_R of radius R

$$\begin{aligned} s_p^+ &= \frac{1}{2}\int_{S_R} \left[\mathbf{E}.\left(\mathbf{h}_p^{\mathrm{sph},-} \times \hat{\mathbf{r}}\right) + \mathbf{H}.\left(\mathbf{e}_p^{\mathrm{sph},-} \times \hat{\mathbf{r}}\right)\right] dS, \\ s_p^- &= \frac{1}{2}\int_{S_R} \left[\mathbf{E}.\left(\mathbf{h}_p^{\mathrm{sph},+} \times \hat{\mathbf{r}}\right) + \mathbf{H}.\left(\mathbf{e}_p^{\mathrm{sph},+} \times \hat{\mathbf{r}}\right)\right] dS, \end{aligned} \tag{3}$$

with $\hat{\mathbf{r}}$ the outgoing normal to the sphere.

3.2 Ports of the antenna elements

In our configuration, the elements of the antenna are receiving signals. To characterize these signals, we assume that the element ports are constituted by waveguides, inside which the fields can be represented by fundamental modes. We note the outgoing/ingoing fundamental modes of the n-th port $(\mathbf{e}_n^+, \mathbf{h}_n^+)$ and $(\mathbf{e}_n^-, \mathbf{h}_n^-)$, respectively. The coefficients associated with these modes can be obtained via the following integral over the transverse surface S_n of the n-th waveguide

$$\begin{aligned} a_n^+ &= \frac{1}{2}\int_{S_n} \left[\mathbf{E}.\left(\mathbf{h}_n^- \times \hat{\mathbf{n}}\right) + \mathbf{H}.\left(\mathbf{e}_n^- \times \hat{\mathbf{n}}\right)\right] dS, \\ a_n^- &= \frac{1}{2}\int_{S_n} \left[\mathbf{E}.\left(\mathbf{h}_n^+ \times \hat{\mathbf{n}}\right) + \mathbf{H}.\left(\mathbf{e}_n^+ \times \hat{\mathbf{n}}\right)\right] dS, \end{aligned} \tag{4}$$

with $\hat{\mathbf{n}}$ the outgoing normal to the surface.

4 CHARACTERIZATION OF VECTOR-SENSOR ANTENNAS VIA RECIPROCITY

4.1 States of excitation

The method we propose is based on reciprocity. Thus, we define two states of excitations:

- State (a): The antenna is emitting. The ports of the N_a elements are excited by sources $(\mathbf{J}_e^{(b)}, \mathbf{J}_m^{(a)})$. The antenna radiates $(\mathbf{E}^{(a)}, \mathbf{H}^{(a)})$.
- State (b): The antenna is excited by a source to be localized $(\mathbf{J}_e^{(b)}, \mathbf{J}_m^{(b)})$. At the antenna ports, there are induced fields $(\mathbf{E}^{(b)}, \mathbf{H}^{(b)})$. This state corresponds to the initial configuration.

4.2 Analysis of state (a)

Using the modal representation for each antenna port, we write the sources (a) as

$$\begin{aligned} \mathbf{J}_e^{(a)} &= a_n^+ \hat{\mathbf{n}} \times \mathbf{h}_n^+, \\ \mathbf{J}_m^{(a)} &= -a_n^+ \hat{\mathbf{n}} \times \mathbf{e}_n^+, \end{aligned} \tag{5}$$

for $n \in \{1, \cdots, N_a\}$, and with $\mathbf{a}^+ = [a_1^+, \cdots, a_{N_a}^+]^T$ the vector of excitation.

We expand the fields associated with this excitation into outgoing spherical harmonics defined from a coordinate system centered at O. Theoretically, there is an infinite number of spherical harmonics. Nevertheless, since the elements of the antenna are distributed inside a limited area around O, only the lowest-order harmonics are radiated so that we can consider $s_p^+ = 0$ for $p > N_s$. Note that the number of significant harmonics N_s increases with the total size of the antenna. Finally, the radiated fields can be represented by the vector $\mathbf{s}^+ = [s_1^+, \cdots, s_{N_s}^+]^T$. Using the linearity of Maxwell equations, we define a matrix $\mathbf{M}^+$ that links the vector of excitation to the radiated spherical harmonics, i.e.

$$\mathbf{s}^+ = \mathbf{M}^+\mathbf{a}^+. \tag{6}$$

To compute the n-th column of this matrix, we can consider that only the n-th antenna element is excited by $a_n^+ = 1$. The other elements are assumed matched, i.e. $a_{n'}^+ = 0$ for $n \neq n'$. We compute the fields radiated by this excitation on a discretized sphere of radius R. This can be performed using any antenna computation software. On the sphere, the numerical integration of (3) yields the column elements.

4.3 Analysis of state (b)

As previously mentioned, the ingoing field can be represented by ingoing spherical harmonics. Thus, the sources (b) can be expressed by means of currents on a sphere of radius R, that are associated with ingoing harmonics. This leads to

$$\begin{aligned} \mathbf{J}_e^{(b)} &= -\sum_p s_p^- \hat{\mathbf{r}} \times \mathbf{h}_p^{\mathrm{sph},-}, \\ \mathbf{J}_m^{(b)} &= \sum_p s_p^- \hat{\mathbf{r}} \times \mathbf{e}_p^{\mathrm{sph},-}. \end{aligned} \tag{7}$$

Similarly to state (a), because of its limited size, the antenna is only capable of receiving the first N_s harmonics. We note $\mathbf{s}^- = [s_1^-, \cdots, s_{N_s}^-]^T$ the vector of excitation. The fields induced by these currents at each antenna port can be represented via fundamental modes of amplitudes a_n^- given by (4). Using linearity, we can define the matrix

$$\mathbf{a}^- = \mathbf{M}^-\mathbf{s}^-. \tag{8}$$

This matrix is of primary importance in our method. Indeed, if we determine $\mathbf{M}^-$ and invert this linear system, we can obtain the ingoing spherical harmonics from the measured signal. The field at O can then be obtained by summing the 6 lowest-order harmonics, i.e. for $n = 1$. Indeed, they are the only ingoing harmonics, that do not vanish at O.

4.4 Determination of $\mathbf{M}^-$

To determine $\mathbf{M}^-$ from $\mathbf{M}^+$, we make use of the reaction theorem. The reaction of the sources (a) with the field (b) can be formulated as

$$R_{a,b} = \sum_n \int_{S_n} \left(\mathbf{J}_e^{(a)}.\mathbf{E}^{(b)} - \mathbf{J}_m^{(a)}.\mathbf{H}^{(b)} \right) dS \tag{9}$$

Using (5) and (4), we find

$$R_{a,b} = -2\mathbf{a}^{-T}\mathbf{a}^+. \tag{10}$$

In the same way, the reaction of the sources (b) with the field (a) can be formulated as

$$R_{b,a} = 2\mathbf{s}^{-T}\mathbf{s}^+. \tag{11}$$

The reaction theorem imposes $R_{a,b} = R_{b,a}$. Taking into account (6) and (8), we end up with

$$\mathbf{s}^{-T}\mathbf{M}^+\mathbf{a}^+ = -\mathbf{s}^{-T}\mathbf{M}^{-T}\mathbf{a}^+, \tag{12}$$

which yields

$$\mathbf{M}^- = -\mathbf{M}^{+T}. \tag{13}$$

4.5 Solution of the linear system

Upon assuming $N_a \geq N_s$, i.e. the number of elements is greater than the number of significant harmonics, the linear system (8) can be solved in the least square sense. Thus, from the measured signals a_n^-, we can obtain the associated ingoing spherical harmonics s_p^-, from which we can deduce the electric and magnetic fields at O, and the DoA.

4.6 Efficiency of vector-antennas for 3D direction finding

The inversion of the linear system (8) is the main step to estimate the components of the electric and magnetic fields at O from the signals measured at output of the antenna. The condition number of $\mathbf{M}^-$ gives the sensitivy of the result to variations of $\mathbf{a}^-$. If the system is ill-conditioned, the determination of the ingoing harmonics, and consequently, the field at O, will lack of accuracy. Thus, the condition number of $\mathbf{M}^-$ is an estimation of the capability of the configuration to estimate the DoA in presence of spatial noise (multipath).

5 NUMERICAL EXPERIMENTS

5.1 Configuration

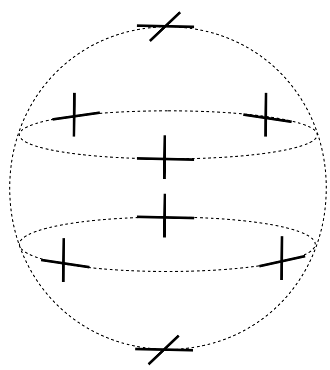

Figure 2: Antenna constituted by $N_a = 16$ electric short dipoles .

We consider the configuration presented in Figure 2. The antenna is constituted by $N_a = 16$ uncoupled electric short dipoles distributed over a sphere of diameter d. We assume that the number of significant spherical harmonics is of $N_s = 16$ so that (8) constitues a square linear system.

5.2 Simulation result

In Figure 3, we observe the condition number of $\mathbf{M}^-$ with respect to d. For small values of d, the system becomes ill conditioned. For such a small size, the antenna is not able to estimate all of the $N_s = 16$ first harmoncs. When d increases, the conditioned is improved. Nevertheless we note the presence of picks, at which the DoA esmitation will be sensitive to small variations of $\mathbf{a}^-$.

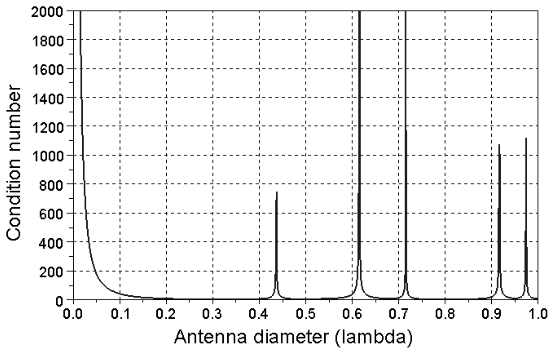

Figure 3: Condition number of $\mathbf{M}^+$ with respect to the antenna diameter d expressed in wavelegnth.

As an example of application, we consider the previous antenna illuminated by a plane wave of DoA $\theta = 80^o$ and $\phi = 46^o$, in the presence of one multipath of relative amplitude 0.1. Using classical formulaes about short dipoles, we obtain the values a_n^- that would be measured in such a configuration.

From these values, we estimate the DoA with our method. Firstly, we solve the linear system (8) to compute the ingoing harmonics. Then, we deduce the fields $\mathbf{E}_0, \mathbf{H}_0$ at O. Finally, we estimate the DoA from the direction of the Poynting vector $1/2\mathrm{Re}(\mathbf{E}_0 \times \mathbf{H}_0^*)$. For $d = 0.01\lambda$ and $d = 0.2\lambda$, with λ the wavelength, the errors in the estimation of the DoA are of 4.3^o and 0.8^o, respectively. This result is in good agreement with Figure 3. This confirms that for large condition numbers, multipath may yield significant errors in the DoA estimation.

6 CONCLUSION

We have proposed a method based on reciprocity and spherical harmonics to evaluate the efficiency of a vector-sensor antenna for 3D direction finding. We have introduced a matrix, which condition number measures the sensitivity of the antenna to multipath. This method has been successfully tested on an antenna constituted by 16 short dipoles. Thus, it can be used as a tool in the design of vector-sensor antennas.

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