A model of airport slot allocation with posted prices*

Nicolas Gruyer† Kévin Guittet‡

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Abstract

In this paper, we study the impact of the introduction of posted prices in the slot allocation process currently in use at congested airports in most European countries. In particular, we show that if the airport is initially saturated, while low level of slot prices entail no response from the airlines, requests for slots "suddenly and violently" drop when the price reaches a certain threshold. In general, there is therefore no market clearing price for airport slots. We also present a dynamic model which highlights how the current grandfather rule - stating that slots used today are kept in the future - generates baby-sitting, that is airlines requiring and using slots today just because they expect them to be profitable in the future.

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†ENAC-LEEA. Email gruyer@recherche.enac.fr. Tel +33 5 62 17 40 29. Fax +33 5 62 17 40 17.

‡Corresponding author. DSNA/DTI/R&D. 7, avenue Edouard Belin, BP 54005, Toulouse Cedex 4, France. Email guittet@tls.cena.fr. Tel +33 5 62 17 41 83. Fax +33 5 62 25 95 99.

1 Introduction

The slot allocation process currently in use in a number of congested airports worldwide[1] is often pointed out as a major source of inefficiency in the air transport industry. This process heavily relies on the use of grandfather rights, according to which airlines may keep their slots from one aeronautical season to another provided they have used them at least 80% of the time. Slots that are free of grandfather rights (the slot pool) may be claimed by airlines and their allocation is decided by a *coordinator* using a purely administrative procedure. A final stage in which slots may be exchanged one for one on a strictly non-monetary basis is provided to ensure that airlines are able to complete their schedule. Inefficiency in the current system are outlined, among others, in NERA (2004). Part of it stems from the fact that historic rights inherited from the grandfather rule act as a barrier to entry on the airport. Also, airlines do not pay the opportunity cost of "owning" their slots and have therefore little (if any) incentive to release excess capacity[2]. Finally, available slots (from the slot pool) are not necessarily allocated to the airlines that would make the most efficient use of them.

For these reasons, alternative modes for dealing with airport capacity have been suggested, studied and, more rarely, implemented. Congestion charging, for example, suggests that efficiency could be restaured by setting the cost of using airport capacity equal to the marginal social cost of using the service (see e.g. Carlin and Park (1970)). However, theoretical difficulties arise in the air transport industry, since airlines that have significant market power internalize part of the externality they impose on the system (see for instance Brueckner (2002a), Brueckner (2002b)). More concretely, airport congestion pricing has, up to now, failed to be efficiently implemented Schank (2005). Another approach proposes to use market based mechanisms to allocate slots. This idea heads back to the early eighties (Rassenti et al. (1982)) but is now gaining credit, with recent studies commisioned by the DGTREN (NERA (2004)) or the Department for Transport in UK (DotEcon (2006)). Among the mechanisms studied in NERA (2004) lies the introduction of "posted prices" and the use of slot auctions, along with the creation of a secondary market for slots. Interestingly enough, while some recent economic results (Eso et al. (2006), Ranger (2006)) may help us to understand how

[1]"EU Slot regulation applies at 60% of all capacity constraint airports worldwide" (2004, source IATA).

[2]NERA (2004) puts forward that in Düsseldorf and Paris-Orly, 2 airports that are congested throughout the day, under-utilization of capacity was larger than 10% in 2002. Hence, this inefficiency may be substantive even at very congested airports.

downstream competition is likely to impact outcomes in a capacity auction, only little as been done that can be used to acknowledge its effect if posted prices are introduced under the current allocation procedure. In the case of auctions, Eso et al. (2006) shows that Vickrey-Clarke-Groves mechanisms (that lead to an efficient allocation when one only takes the seller and the buyers' surplus into account) may lead to a very inefficient allocation of capacity where a firm buys a large share of the sold capacity before using it inefficiently on the market. In Ranger (2006), a sophisticated mechanism inspired from Ausubel and Milgrom (2002) is proposed that corrects for the externality created by the downstream competition and allows to implement general preferences of the seller. Hence, it seems that auctions could at least theoretically be used to solve efficiently the problem of slot allocation at congested airports. NERA (2004) however warns that practical difficulties of designing and participating in a slot auction should not be underestimated, especially because of the complex interdependencies between airport slots. Also, NERA (2004) stresses that the implementation cost of auctions will be higher than those of the other market mechanisms suggested in the study. In contrast, posted prices is an easily implementable method that presents the great advantage of encouraging certainty in airline route planning and continuity of scheduled services Reynolds-Feighan and Button (1999)[3].

In this paper, we study a Cournot-type model of airport slot allocation that mimics the current slot allocation process but allows posted prices on slots to be introduced: In a first stage, firms express capacity (slots) requests and capacity is allocated by the coordinator. In a second stage, firms compete "a la Cournot" under their production capacities. We show that, the introduction of posted prices usually does not succeed in clearing the market: First, for low posted prices, airlines' capacity requests appear not to be impacted by the posted price. Then, above a certain threshold, the global slot requests suddenly and discontinuously drop to a level that is strictly below the overall capacity. If, rather than posted prices, the regulator decides to introduce limited fines on unused slots, we show that excess capacity is actually reduced for limited fines and that prices fall. However, if fines are large enough, it happens

[3] During the concertation with the Commission that followed the publication of the NERA study NERA (2004), these reasons were also given by the French regulator to justify its preference for posted prices over all other proposed mechanisms for the primary allocation of slots, the preferred option for primary allocation being an improvement of the existing administrative procedure. See for example the "Outcome of study on slot allocation procedures", ECAC report DGCA/124-DP/6, issued in December 2005.

that they succeed in eliminating excess-capacity at the cost of a reduced produced quantity, or equivalently at the cost of a higher price (as compared with the fine free situation). We also show that posted prices may lead to a situation in which an increase in overall airport capacity results in a drop in produced quantity, which may seem somewhat paradoxal. Finally, we also present a dynamic model of slot allocation, in which we recover the well-known phenomenon of "baby-sitting of slots" and show that making the 80-20 rule more severe to avoid excess-capacity can sometimes be counterproductive, when it results in lower quantities being put on the market.

The first part of the paper, Section 2, presents the static model of slot allocation and introduces the technical assumptions used throughout the paper. Section 3 then presents the equilibrium structure of the game and discusses how it may be used to study the impact of posted prices when there is competition on the downstream market. In Section 4, we extend the static model to a two-period game to capture the strategic use of the grandfather rule as a barrier to expand and show how this may actually benefit consumers in the short run. Section 5 concludes.

2 A static model of slot allocation

In this section, we describe a general static model of airport slot allocation. In this model, we assume that capacity (airport slots) is fixed and we denote by $\overline{K}$ the total available capacity. The problem is modeled with a two-stage game. In the first stage, airlines are allocated production capacities via an allocation process: Airlines provide the coordinator with their capacity request. The coordinator then processes the requests and comes out with a capacity allocation. In the second stage of the game, airlines compete in quantities under their capacity constraints on the downstream market. In this game, we assume perfect information: All parameters are common knowledge, as well as the allocation rule (conditional on the airlines requests).

2.1 The allocation rule

The coordinator sets a procedure to allocate capacity on the basis of demands expressed by the firms. This procedure is described through an allocation rule (K_1^{all}, K_2^{all}): if firm 1 requires a capacity φ_1 and firm 2 a capacity φ_2, then firm 1 is granted a capacity $K_1^{all}(\varphi_1, \varphi_2)$, while firm 2 gets $K_2^{all}(\varphi_1, \varphi_2)$.

For exposition clarity, we define the airlines **capacity rights** as the capacities that are allocated when both airlines claim the overall capacity. These capacity rights are thus defined as $\left(K_1^{all}\left(\overline{K}, \overline{K}\right), K_2^{all}\left(\overline{K}, \overline{K}\right)\right)$.

We assume that the allocation rule $K^{all}(.,.)$ satisfies the following properties:

1. It matches demands if they are compatible with the maximum available capacity:

$$\varphi_1 + \varphi_2 \leq \overline{K} \Rightarrow K_i^{all}(\varphi_1, \varphi_2) = \varphi_i, \; i = 1, 2 \; . \tag{1}$$

2. The total available airport capacity $\overline{K}$ is allocated when both firms claim the whole capacity:

$$K_1^{all}\left(\overline{K}, \overline{K}\right) + K_2^{all}\left(\overline{K}, \overline{K}\right) = \overline{K} \; . \tag{2}$$

3. If both airlines request more than their capacity rights, then each airline gets its capacity rights:

$$\begin{cases} \varphi_1 \geq K_1^{all}\left(\overline{K}, \overline{K}\right) \\ \varphi_2 \geq K_2^{all}\left(\overline{K}, \overline{K}\right) \end{cases} \Rightarrow K_i^{all}(\varphi_1, \varphi_2) = K_i^{all}\left(\overline{K}, \overline{K}\right), \; i = 1..2 \; . \tag{3}$$

4. If firm i claims more than its capacity rights while firm j claims less than its capacity rights, then firm j's request is satisfied, while firm i gets the remaining capacity. Thus

$$\begin{cases} \varphi_i \geq K_i^{all}\left(\overline{K}, \overline{K}\right) \\ \varphi_j < K_j^{all}\left(\overline{K}, \overline{K}\right) \\ \varphi_1 + \varphi_2 > \overline{K} \end{cases} \Rightarrow \begin{cases} K_i^{all}(\varphi_1, \varphi_2) = \overline{K} - \varphi_j \\ K_j^{all}(\varphi_1, \varphi_2) = \varphi_j \end{cases} \tag{4}$$

Note that properties (1) to (4) imply that the capacity rights are sufficient to define uniquely the allocation rule over the set of possible capacity requests (φ_1, φ_2). Basically, they mean that each airline is given rights over a certain amount of capacity and can get more than this capacity only if its competitor requires less than its own rights.

2.2 Notations and technical assumptions

The market served by the airlines is characterized by the inverse demand $P(q)$. We assume that capacity and production are costly, with respective costs $c_i(q_i)$ and $k_i(K_i)$ for firm i.

Thus, if K_i stands for the capacity allocated to firm i and q_i for the quantity it produces, firm i incurs a cost equal to $C_i(q_i, K_i) = c_i(q_i) + k_i(K_i)$. We follow Kreps and Scheinkman (1983) on the assumptions on the costs and inverse demand, that is we assume that there exists some $X > 0$ such that

- $P(q)$ is strictly positive over $(0, X)$, $\mathcal{C}^2$ over $(0, X)$, concave and strictly decreasing.
- c_i and k_i are $\mathcal{C}^2$ over $[0, \infty)$, convex and satisfy $c_i(0) = k_i(0) = 0$ and $c_i'(0), k_i'(0) > 0$ (for $i \in \{1, 2\}$).

We also define best-responses for the unconstrained Cournot game :

$$r(q_j, c) = \underset{q_i}{\operatorname{argmax}} \; q_i P(q_i + q_j) - c(q_i) \; . \tag{5}$$

For a given airline (say 1), we will refer to $r(q_2, c_1)$ as the short-run Cournot best response to quantity q_2 and to $r(q_2, c_1 + k_1)$ as the long-run Cournot best response. Accordingly, related Nash equilibria will be referred to as, respectively, the short-run and long-run Cournot equilibria and will be denoted respectively $\left(K_1^{SR}, K_2^{SR}\right)$ and $\left(K_1^{LR}, K_2^{LR}\right)$. Under the assumptions on P, c and k, these equilibria always exist and are unique.

2.3 The firms problems

Once the coordinator has committed to an allocation rule, the airlines play a two-stage game.

- Stage 1: The two airlines simultaneously express capacity demands φ_1 and φ_2. According to these demands, firm i receives capacity $K_i^{all}(\varphi_1, \varphi_2)$.
- Stage 2: The two airlines compete *à la* Cournot with quantities constrained by their production capacities ($q_i \leq K_i^{all}(\varphi_1, \varphi_2)$).

Airline i's profit is therefore given by

$$\pi(q_i, q_j, K_i) = q_i P(q_i + q_j) - (c_i(q_i) + k_i(K_i)) \; . \tag{6}$$

Note that the current airport charging system is likely to correspond to the case of no capacity cost ($k_i \equiv 0$), as airport slots are allocated freely by coordinators. However, one could argue that the coordinator, often facing a large excess demand with respect to the number of available slots on highly congested airports[4] and being granted access to airlines information, might allocate the capacity to the ones most likely to operate the schedule, e.g. because they have the fleet it calls for. Hence, part of the capacity cost k_i might also relate to the airlines fleet. Finally, one of the goals of this paper is precisely to study the consequences of the introduction of positive posted prices on capacity, that is of costly capacity.

2.4 The Second Stage Cournot Subgame

In order to determine the airlines' claims over capacity, we first need to express the profits they realize in the Cournot competition stage as a function of their production capacities. This is a standard problem and is treated, among others, by Gabszewicz and Poddar (1997). The expression of the equilibrium quantities depends on how the capacities compare with the short-run Cournot best response. Proposition 2 sums up the results, while Figure 1 gives an illustration.

Proposition 1 *The constrained Cournot Nash equilibrium outputs levels (q_1^*, q_2^*) are determined by the following conditions:*

- *If (K_1, K_2) belongs to Region A (i.e. $K_1 \leq r(K_2, c_1)$ and $K_2 \leq r(K_1, c_2)$), both firms produce up to capacity, that is $(q_1^*, q_2^*) = (K_1, K_2)$.*
- *If (K_1, K_2) belongs to Region B (i.e. $K_1 > r(K_2, c_1)$ and $K_2 \leq K_2^{SR}$), firm 2 produces up to capacity K_2 and firm 1 produces the short-run Cournot best response to K_2. Hence $(q_1^*, q_2^*) = (r(K_2, c_1), K_2))$.*
- *If (K_1, K_2) belongs to Region C (i.e. $K_1 \leq K_1^{SR}$ and $K_2 > r(K_1, c_2)$), firm 1 produces up to capacity K_1 and firm 2 produces the short-run Cournot best response to K_2. Hence $(q_1^*, q_2^*) = (K_1, r(K_1, c_2))$.*
- *If (K_1, K_2) belongs to Region D (i.e. $K_1 > K_1^{SR}$and $K_2 > K_2^{SR}$), both firms produce the short-run Cournot quantities. Hence $(q_1^*, q_2^*) = (K_1^{SR}, K_2^{SR})$*

Proof: See e.g. Gabszewicz and Poddar (1997). To see the intuition behind this result, note that under under our assumptions, each firms' profit as a function of the quantity it

[4]As an example, 251.000 slot requests issued by as many as 43 airlines followed the bankruptcy of Air Lib at ORY, in which 35.658 slots became available (see NERA (2004)).

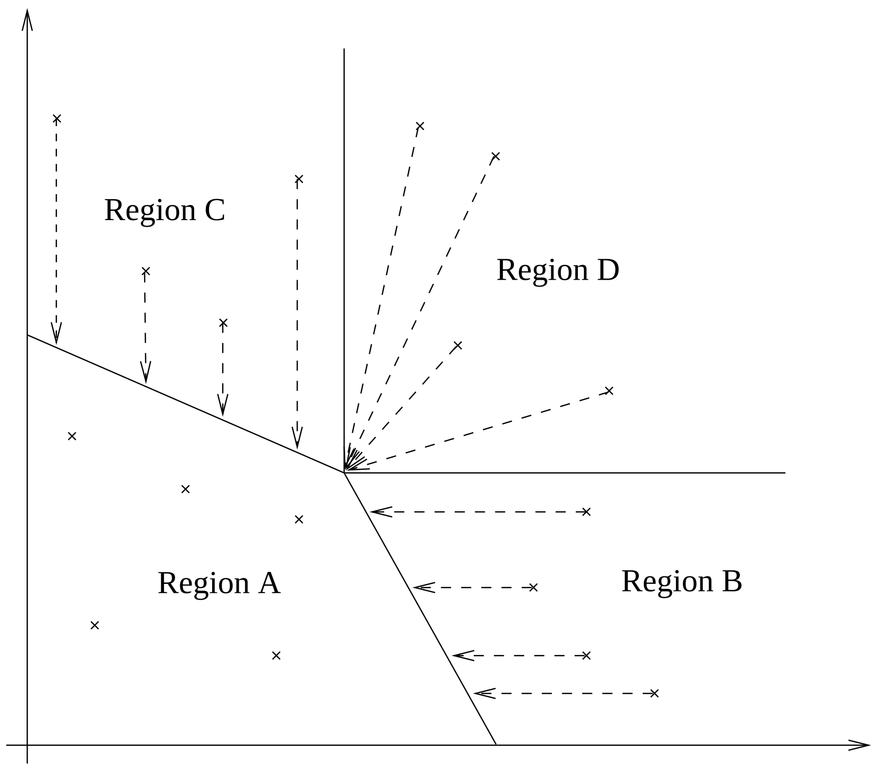


Figure 1: Quantities put on the market for given allocated production capacities.

puts on the market is quasi-concave when it has no capacity-constraint. Hence, when firm i is constrained by a production capacity K_i, its constrained best response to a quantity q_j put on the market by its competitor is simply the maximum of its unconstrained best response $r(q_j, c_i)$ and its production capacity K_i. These best responses then yield the equilibrium quantities announced in the proposition. ■

3 The Nash equilibria of the capacity allocation game

If airlines could get as much capacity as they wanted in the first stage, they would choose capacities equal to the long-run Cournot equilibrium quantities,[5] $(K_1, K_2) = (K_1^{LR}, K_2^{LR})$, and would therefore produce up to their capacities in the second stage. The allocation process, however, allows airlines to prevent, to some extent, their competitor to acquire capacity that could be used "against them". In the extreme case where capacity is free, airlines do not have any incentive to restrain their requests and would typically end up granted with their capacity

[5]See, for example, Gabzewicz and Poddar (1997).

rights. When capacity is costly, the situation is somewhat less simple and the advantages of limiting one's competitor capacity have to be balanced with the costs incurred for doing so. Precisely, whether or not an airline will decide to ask for more capacity than what would appear optimal in an unconstrained setting will depend on whether $\overline{K}$ is large enough, and on how the long-run Cournot profits $\Pi_1(K_1^{LR}, K_2^{LR})$ and $\Pi_2(K_1^{LR}, K_2^{LR})$ compare with the profits airlines would make if granted with their capacity rights. Indeed, if the capacity rights are asymmetric enough to ensure a profit above the long-run Cournot profit for one of the airlines, this airline might find in its own interest to "block" the Cournot outcome. To this end, the airline claims more capacity than what its long-run Cournot quantity would require, thus reducing its competitor's production capacity. The formal result is stated in the next section.

3.1 The equilibrium result

We will need to distinguish two Regions to describe firm demands (see figure 2).

- Region 1 is the set of capacities (K_1, K_2) which are either strictly below the long-run Cournot best-response curves or such that one of the firm's profit is strictly superior to Cournot long-run profit.

$$\begin{cases} K_1 < r(K_2, c_1 + k_1) \text{ and } K_2 < r(K_1, c_2 + k_2) \ , \\ \text{or} \\ \Pi_1(K_1, K_2) > \Pi_1(K_1^{LR}, K_2^{LR}) \text{ or } \Pi_2(K_1, K_2) > \Pi_2(K_1^{LR}, K_2^{LR}) \ . \end{cases} \tag{7}$$

- Region 2 is the set of capacities (K_1, K_2) which are above at least one of the long-run Cournot best-response curves and such that both firm's profits are inferior to Cournot long-run profit

$$\begin{cases} K_1 \geq r(K_2, c_1 + k_1) \text{ or } K_2 \geq r(K_1, c_2 + k_2) \ , \\ \Pi_1(K_1, K_2) \leq \Pi_1(K_1^{LR}, K_2^{LR}) \ , \\ \Pi_2(K_1, K_2) \leq \Pi_2(K_1^{LR}, K_2^{LR}) \ . \end{cases} \tag{8}$$

These regions are sketched in figure 2, Region 1 being the grayed area and Region 2 the other one (including the frontier).

We also need to introduce a technical assumption.

Technical assumption A1 Assumption **A1** is satisfied if and only if the long-run capacity

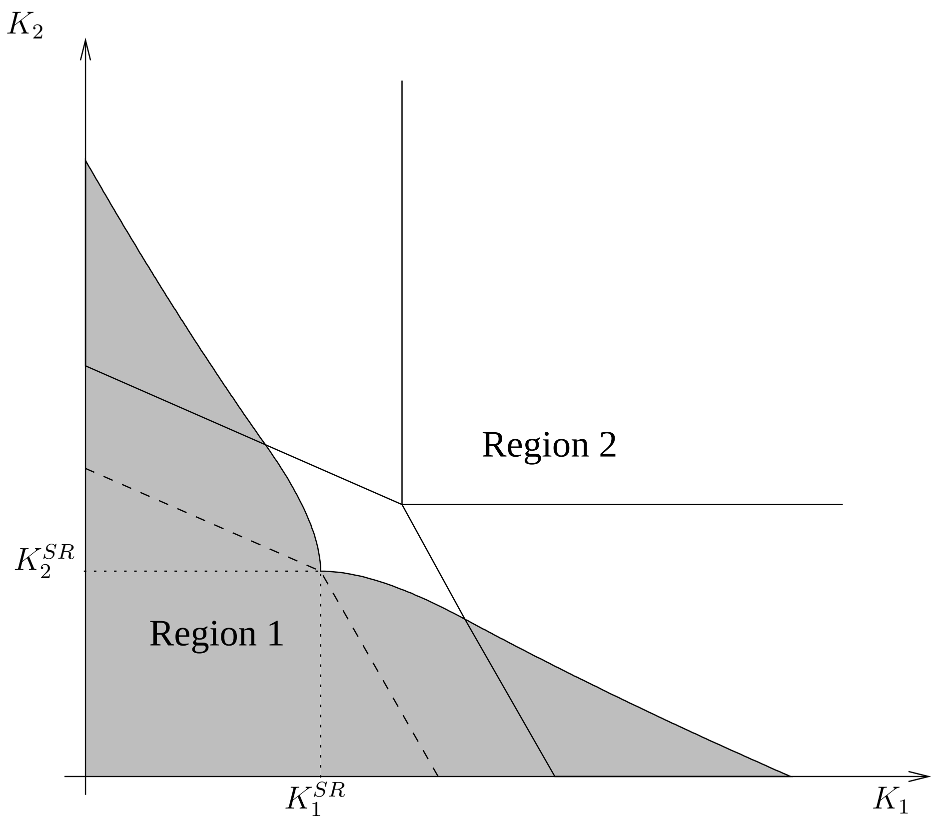


Figure 2: Regions 1 and 2

constrained Cournot profits of the firms, $\Pi_1(K_1, \overline{K} - K_1)$ and $\Pi_2(\overline{K} - K_2, K_2)$, are increasing in their own capacity on *Region* $1 \cap \{K_1 + K_2 = \overline{K}\}$.

This assumption formally states that airlines, if they know for sure that any capacity they do not get will be given to their competitor, never want to release any capacity (in Region 1). Assumption **A1** thus in particular implies that in Region 1 it is marginally better for an airline with excess capacity rights (that is, an airline with capacity rights such that it would eventually not find interesting to produce up to capacity if it received its capacity rights) to pay for unused capacity rather than to loose market power by allowing its competitor to produce more on the final market. Though this assumption may seem strong, it is noticeable that is always satisfied in the simple symmetric case where demand is linear and the capacity and production marginal costs are constant (see Proposition 2). In general, it is sufficient for this assumption to be satisfied that the marginal cost of capacity remains small compared to the markup at the long-run Cournot equilibrium.[6]

Proposition 2 *When firms costs are linear and symmetric and demand is linear, the capacity constrained Cournot Game satisfy Assumption **A1**.*

Proof: See in Appendix. ■

We can now characterize the set of Nash equilibria of the capacity allocation game that are not Pareto-dominated. The formal result is presented in proposition 3.

Proposition 3 *Under the technical assumption **A1**, we have*

- *If $K^{all}\left(\overline{K},\overline{K}\right)$ belongs to Region 2, capacity demands equal to the long-run Cournot quantities (K_1^{LR}, K_2^{LR}) constitute a Nash equilibrium of the capacity allocation game. If some other Nash equilibria exist, they are Pareto-dominated by (K_1^{LR}, K_2^{LR}).*

- *If $K^{all}\left(\overline{K},\overline{K}\right)$ belongs to Region 1, maximal capacity demands $\left(\overline{K},\overline{K}\right)$ constitute a Nash equilibrium of the capacity allocation game. If some other Nash equilibria exist, they all imply the same capacity allocation $K^{all}\left(\overline{K},\overline{K}\right)$.*

Proof: First, we can note that, if there exists a Nash equilibrium of the capacity allocation game (φ_1, φ_2) such that the available production capacities are not completely allocated ($K_1^{all}(\varphi_1,\varphi_2)+K_2^{all}(\varphi_1,\varphi_2) < \overline{K}$), then the granted capacities must be equal to the requested ones. Each firm could therefore obtain less or slightly more production capacity without modifying its competitors production capacity. It comes then as a consequence of the standard model (for $\overline{K} = +\infty$) that (φ_1, φ_2) would have to be equal to the long-run Cournot quantities. Indeed, otherwise at least one firm would find an interest in deviating slightly.

The proof then rely on several lemmas that are presented in appendix and can be decomposed in three steps:

- Step 1: From Lemma 1, if there exists a Nash equilibrium $(\varphi_1^*, \varphi_2^*)$ of the capacity game such that allocated capacities ($K_1^{all}(\varphi_1^*,\varphi_2^*), K_2^{all}(\varphi_1^*,\varphi_2^*))$ belong to Region 1, and such that all the available capacity is allocated $(K_1^{all}(\varphi_1^*,\varphi_2^*)+K_2^{all}(\varphi_1^*,\varphi_2^*) = \overline{K})$,

[6]Indeed, for symmetric firms, **A1** is satisfied whenever $k'\left(\overline{K}\right) \leq P\left(2K^{LR}\right) - c'\left(K^{LR}\right) - k'\left(K^{LR}\right)$, or, equivalently, if $k'\left(\overline{K}\right) \leq \frac{P^{LR}}{2\epsilon^{LR}}$, where ϵ^{LR} (resp. P^{LR}) stands for the demand-price elasticity (resp. price) at the long-run Cournot equilibrium.

then maximal capacity demands $\left(\overline{K},\overline{K}\right)$ also are a Nash equilibrium and yield the same capacity allocation: $K_1^{all}\left(\varphi_1^*,\varphi_2^*\right) = K_1^{all}\left(\overline{K},\overline{K}\right)$.

- Step 2: Suppose that allocated capacities for maximal demands, $K^{all}\left(\overline{K},\overline{K}\right)$, belong to Region 2. From Step 1, there can not be any Nash equilibrium yielding allocated capacities in Region 1 (otherwise $K^{all}\left(\overline{K},\overline{K}\right)$ would belong to Region 1). Moreover, from Lemma 3, the demands (K_1^{LR},K_2^{LR}) constitute a Nash equilibrium. The only remaining potential Nash equilibria require an allocation in Region 2, and are consequently Pareto-dominated by (K_1^{LR},K_2^{LR}). This completes the first part of the proof.

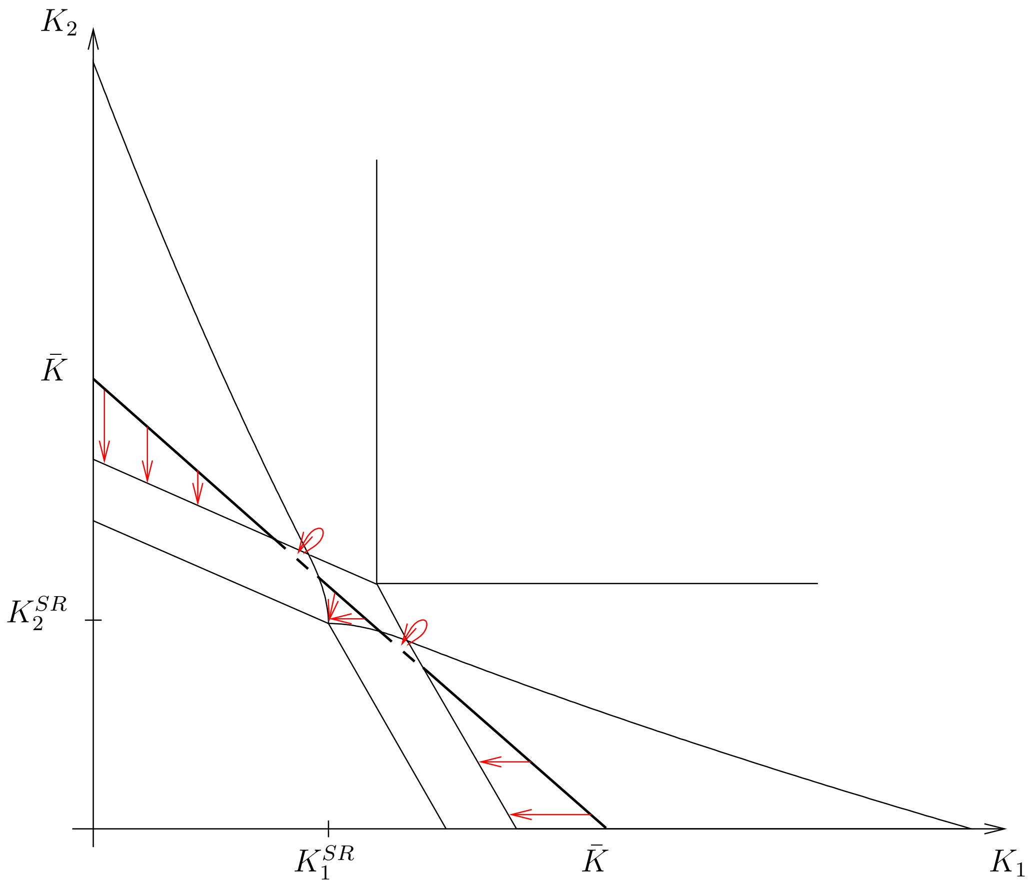
- Step 3: Suppose that allocated capacities for maximal demands, $K^{all}\left(\overline{K},\overline{K}\right)$, belong to Region 1. Then, from Lemma 4, $\left(\overline{K},\overline{K}\right)$ is a Nash equilibrium, and there is no Nash equilibrium yielding an allocation (K_1^{LR},K_2^{LR}). From Step 1, if there is a Nash equilibrium $(\varphi_1^*,\varphi_2^*)$, such that the capacity allocation $(K_1^{all}\left(\varphi_1^*,\varphi_2^*\right), K_2^{all}\left(\varphi_1^*,\varphi_2^*\right))$ belongs to Region 1, then it yields the same allocation as $\left(\overline{K},\overline{K}\right)$. To complete the second part of the proof, we use Lemma 5: There is no Nash equilibrium $(\varphi_1^*,\varphi_2^*)$ such that all the available capacity is allocated $(K_1^{all}\left(\varphi_1^*,\varphi_2^*\right) + K_2^{all}\left(\varphi_1^*,\varphi_2^*\right) = \overline{K})$ and such that the allocation $(K_1^{all}\left(\varphi_1^*,\varphi_2^*\right), K_2^{all}\left(\varphi_1^*,\varphi_2^*\right))$ belongs to Region 2.

This achieves the proof of the proposition. ■

3.2 Implications

First, we want to analyze how capacity rights impact on the price paid by consumers. When the overall capacity is small ($\overline{K} \leq K_1^{LR} + K_2^{LR}$), airlines end up granted with their capacity rights. In this case, the price paid by consumers is $P(\overline{K})$ as long as the capacity rights lie below the short-run Cournot curves but may rise above this level if the slot allocation is too asymmetric. In this case, the "most favored" airline will not produce up to capacity. Now consider overall capacities such that $\overline{K} > K_1^{LR} + K_2^{LR}$. In this case, the behavior of the airlines and the final quantity and price on the market depend on how balanced the capacity rights are[7]. Typically, starting from balanced capacity rights, the possible situations that may arise are depicted on Figure 3 and described below.

[7]Note here the use of the term *balanced* as a loose substitute for *symmetric*, which would actually be inappropriate here. Indeed, we do not impose that firms are symmetric, so that the Cournot quantities, for example, are not *symmetric* while they are *balanced*.



In this figure, arrows link capacity rights to the produced quantity on the final market.

Figure 3: An illustration of the equilibrium outcomes

1. if capacity rights are well balanced, airlines ask for the long-run Cournot capacities, receive these capacities, and produce up to capacity. The price on the market is given by $P(K_1^{LR} + K_2^{LR})$.

2. with a limited imbalance, capacity rights belong to Region 1 and Region A. Airlines therefore request and receive their capacity rights, and thereafter produce up to capacity. In this case, the price on the market, $P(\overline{K})$, is lower than if no capacity constraint was imposed. If the overall capacity is too large, this case may not exist.

3. when the imbalance becomes more significant, capacity rights still belong to Region 1 but are not anymore located in Region A. Accordingly, even if both airlines still request and get their capacity rights, one of them (the "favored" airline) prefers to produce less than its allocated capacity. The resulting price on the market is then increasing in

the imbalance, but may still be lower than the long-term Cournot price. Again, if the overall capacity is large enough, this case does not exist.

This analysis suggests that the design of the capacity rights may have an impact on efficiency for intermediate values of the available capacity. Noticeably, consumers are better off when the capacity rights are slightly less balanced than (capacities equal to) the long-run Cournot outcome. In this case, the positive effect linked to the firms not taking into account the capacity costs, which are sunk during the Cournot competition stage, is more important than the negative effect linked to the higher market power of the favored firm.

Note that the asymmetry in capacity rights is exogenous here. For the particular forms of capacity auction (such as a Vickrey-Clarke-Groves or a uniform-price share auction) considered in Eso et al. (2006), it may happen that asymmetry in the capacity allocation arise endogenously. This happens when the overall capacity is above a capacity threshold that is strictly lower than the unconstrained Cournot quantity. Accordingly, in Eso et al. (2006) the social surplus never benefits from the fact that the large firm prevents its competitors to access capacity.

3.3 Clearing the market with posted prices?

In the last section we discussed how, in the presence of capacity costs, asymmetric capacity rights may make consumers better off than they would be in an unconstrained setting. In this section we discuss the introduction of posted prices under the current slot allocation process. Note that posted prices, being defined as a "rate per slot per season", may be modeled as as tax on capacity. The idea behind posted prices is that they would provide incentives for airlines to use slots efficiently, as they would typically make use of slots by inefficient airlines not profitable and reduce the incentives to make strategic use of excess capacity.

In NERA (2004), it is suggested that a main difficulty for the implementation of posted prices stems from the fact that "airport operators might have little information about the way that airlines would be likely to respond to higher prices, at least during the early stages of adjusting from current charges toward *market clearing levels*". Our model predicts that, in the presence of competition on the airport and an initial imbalance of capacity, airlines responses are flat for low values of posted prices as long as capacity rights belong to Region 1. Then, if the posted price reaches a certain level τ_k^*, such that capacity rights belong to the frontier between Region 1 and Region 2, airlines requests for capacity encounter a discontinuity, negative for the "favored" airline, positive for the other one, and total allocated capacity decreases. Above this threshold, airlines requests for capacity decrease when posted prices increase and airlines produce up to capacity. At the regime switch, the global quantity put on the market "usually" decreases but may also increase. This mostly depends on the level of "excess-capacity" of the favored airline when there is no tax on capacity : if the excess-capacity is large, the final quantity put on the market will increase at τ_k^*, while it will decrease for limited excess-capacity.

Figure 4 provides an example where the final produced quantity is larger when no posted prices are imposed (in this case the final quantity is $K_1^{all} + q_2^{all}$) than with posted prices just large enough to make airlines request capacities equal to the long-term Cournot quantities (in this case the final quantity is $K_1^{LR} + K_2^{LR}$).

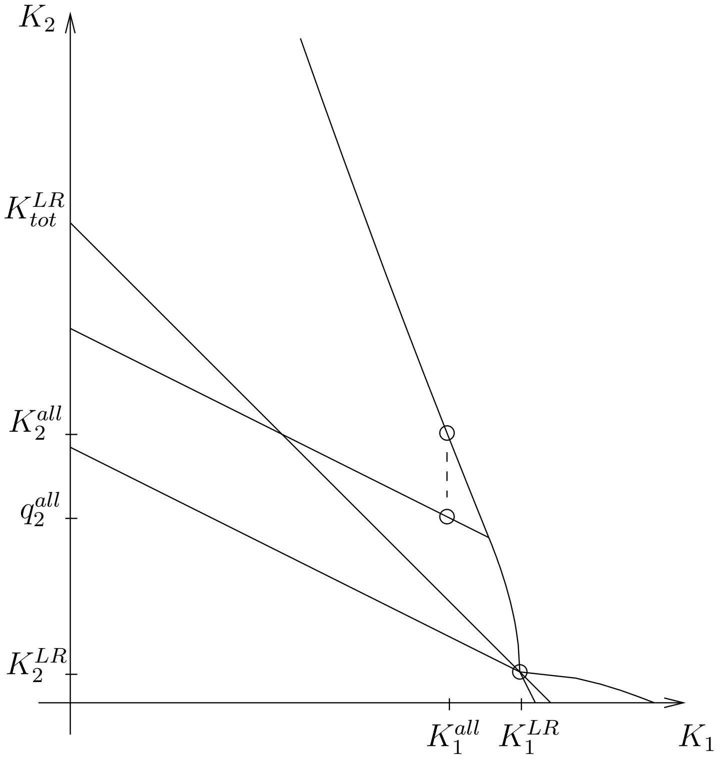


Figure 4: Introduction of posted prices may induce a fall in produced quantities

The intuition behind this result is simple given the equilibrium structure of the game. In this discussion we denote by τ_k the posted price. Assume that excess capacity initially (for $\tau_k = 0$) exists on the market. Then the capacity rights belong to Region 1. If τ_k increases, Region 1 shrinks but the short-run Cournot best responses remain unchanged. Accordingly, the capacities requested by the airlines and the produced quantities remain unchanged as long as the capacity rights remain in Region 1. The regime change occurs when τ_k is such that the capacity rights belong to the boundary between Region 1 and Region 2 (this posted price corresponds to the τ_k^* we have introduced above). For posted prices larger than τ_k^*, airlines request the long-run Cournot equilibrium quantities and produce up to capacity. Since the equilibrium quantity accounts for capacity costs when capacity rights belong to Region 2, the global quantity on the market decreases when τ_k rises further.

At τ_k^*, the allocated capacity is discontinuous and decreases, while the discontinuity in the overall produced quantity may be positive or negative. The idea is that if the excess capacity is large, then the cost incurred by the "favored" airline for preempting slots rises steeply when posted prices increase and the regime switch occurs with a limited capacity tax. The resulting long-run Cournot quantity may not be much lower than the initial long-run Cournot quantity, which may in turn be significantly larger than the initial total quantity. Conversely, if excess capacity is limited, then a large capacity tax is required to convince the "favored" airline not to preempt capacity. In this case (see Figure 4), it may happen that the resulting long-run Cournot quantity, strongly impacted by the capacity tax, is lower than the quantity produced before the introduction of posted prices.

Now it is worth noting that if by "market clearing price" one means the price that would make the airlines capacity requests match the available capacity, such a price usually does not exist. Indeed, starting from capacity rights such that excess capacity is present for limited posted prices, the sum of the airlines capacity requests will exceed $\overline{K}$ until the posted price reaches τ_k^*. Then quantity requests will drop, with their sum eventually falling lower than $\overline{K}$ (the case depicted in Figure 4 fall within this category). We therefore conclude that no market clearing price exists.

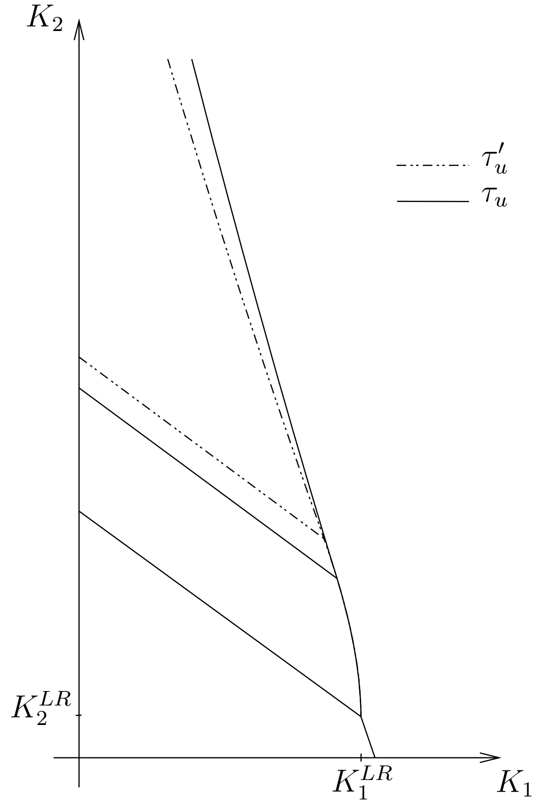
As mentioned above, in the present situation airport slots are allocated freely to airlines. Moreover the grandfather rule ensures that asymmetry in capacity endowments will persist over aeronautical seasons as incumbents will face no cost in preventing competitors to enter their market. In this context, it is commonly admitted that the introduction of posted prices will result in "excess" capacity being released, which in turn will foster competition on the market. Our model shows that this effect is likely to be very limited for low posted prices. Introduction of "conservative" posted prices, as advocated e.g. in DotEcon (2006), may therefore only have a limited impact on the capacity allocation and the competitivity of the market. In contrast, introduction of high posted prices may induce a drastic change of structure on the market, as dominant carriers could eventually be incited to give up their preemptive power on airport capacity. In theory, we showed that the impact of this regime switch on welfare can be either positive or negative. In practice, one could expect that further informations on the considered markets would help draw a conclusion. This is not clear, however. Indeed, although we emphasized the impact of the consumer welfare (through the produced quantity) in the discussion on the welfare, cost considerations are important as well if airlines do not share a common cost structure. In the present context where low cost airlines develop much faster than major airlines, this speaks in favor of a welfare improvement at the regime switch. However, incumbents are often former state owned monopolies that did not aim at maximizing profits and served much larger networks than a private monopolist would have. Since the grandfather rule makes it possible to prevent entry of competitors at the cost of using "unprofitable" capacity, this could support the assumption that incumbents produce enough to induce a decrease in welfare at the regime switch[8]...

3.4 The impact of the introduction of a tax on unused slots

It is worth noting that the introduction of posted prices, implying a monetary transfer from airlines to airports, is not painless to airlines. An alternative to posted prices has therefore been proposed that focuses on the inefficient use of "owned" slots: a tax on unused slots. The introduction of such a tax t changes the cost function of airline i from $C_i(q_i, K_i) = c_i(q_i) + k_i(K_i)$ to $C_i^*(q_i, K_i) = [c_i(q_i) - tq_i] + [k_i(K_i) + tK_i]$. This makes Region 1 shrink (as illustrated in Figure 5) but in the same time it makes the short-run best responses increase. Accordingly, if initially an airline is dominant on the airport and does not produce up to capacity, the introduction of a limited tax on the unused slots will actually make it produce more than it would have without the tax. However, in some cases the tax may induce a regime switch and results in airline requesting their long-run Cournot quantities. In this

[8] At this point of the article, the possibly positive impact of the grandfather rule on the welfare may not be obvious. It should become much clearer in the section devoted to the dynamic model of slot coordination.

case, it may happen that the produced quantity falls beyond its initial level.[9]



In this figure, Regions are sketched for different values of the tax on unused slots, τ_u and τ'_u, with $\tau_u < \tau'_u$. The long-run best responses remain unchanged while the short-run best responses are lifted. Moreover, the dashed line standing for the isoprofit $\Pi_2(K_1, K_2) = \Pi_2(K_1^{LR}, K_2^{LR})$ related to τ'_u is below the plain line standing for the one related to τ_u, so that Region 1 shrinks when t increases.

Figure 5: Impact of a tax on unused slots on Region 1

3.5 Capacity expansion under slot allocation

Now assume that an airport is able to increase its runway capacity while posted prices remain constant. Our equilibrium result implies that capacity expansion may result in a lower quantity being produced on the market. Indeed, if initially an airline is dominant on the market, the new airport capacity may push "capacity rights" away from Region 1. In this case, the dominating airline will not find profitable anymore to prevent its competitor to

[9]Note that, as the long-run Cournot equilibrium is by construction not impacted by a tax on unused capacity, this supposes that, initially, capacity costs are strictly positive.

acquire capacity and allocated capacity will decrease to the long-run Cournot quantity. Just as in the previous section, the quantity out on the market may then decrease or increase, since the preemptive behavior of the dominant airline may or may not be advantageous to consumers.

Note that a similar result (an increase in overall capacity inducing a drop in production) is also observed in Eso et al. (2006) in the case where capacity is sold through "efficient"[10] auctions and where marginal production costs are strictly convex. The authors find that, if the total available capacity is large enough, firms eventually buy asymmetric amounts of capacity, the big firm then exercising its market power to reduce the quantity put on the market and increase the price. This creates a discontinuity of the price in the total available capacity, the price abruptly rising when this capacity exceeds a certain threshold. The authors then advocate that the regulator should sometimes limit the amount of capacity to be sold, and show that other types of auctions can perform better in terms of social surplus. Interestingly, though the similar result is observed in both settings, underlying phenomena strongly differ. Indeed, in Eso et al. (2006) where the firms are initially identical and the capacity provision is endogenous, the produced quantities are symmetric below the capacity threshold. Instead, in our model where capacity provision is mostly exogenous, airlines need to be initially asymmetric (through capacity rights) and the outcomes become symmetric above the capacity threshold.

4 A dynamic model of slot allocation

In this section, our aim is to study the impact of the grandfather rule when slots are allocated though an administrative process. To this end, we present a dynamic model of slot allocation. This model is defined as follows: airlines play a first game of slot allocation (as described in the first part of this paper). The quantities and capacities chosen in this first round allow to define the new allocation rule, and a second slot allocation game is played. The timing of this dynamic game is sketched in Figure 6.

This setting allows to consider the issue of grandfather rights, according to which airport slots owned by an airline in the last related aeronautical season are automatically renewed if they have been sufficiently used, while slots whose utilization rate lies below 80% are put

[10]Here, "efficient" is to be taken as "maximizing the firms profits out of the payments for the goods."

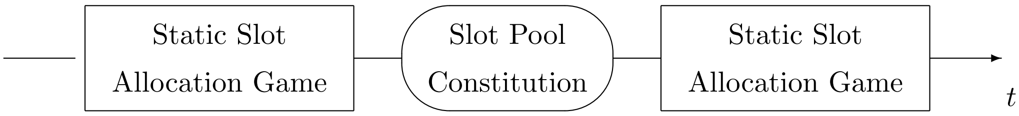


Figure 6: Timing of the dynamic game.

back to the slot pool. For the sake of simplicity[11], the usual grand-father rule described above is slightly modified here: If $K_i^{(1)}$ and $q_i^{(1)}$ stand for the capacity and quantity of airline i in the first period, then the inherited capacity $K_i^{I,2}$ is given by

$$\begin{aligned} K_i^{I,2} &= K_i^{(1)} - \rho\left(K_i^{(1)} - q_i^{(1)}\right) , \\ &= \rho\, q_i^{(1)} + (1-\rho)\, K_i^{(1)} . \end{aligned} \tag{9}$$

where ρ is a parameter (located between 0 and 1) chosen by the regulator (that would be the analogous of the 80% in the usual grand-father rule setting). As stated, ρ describes to what extend the over-provision of capacity is punished. If ρ is equal to 0, then the capacity owned in the first period is fully renewed to the airline, and there is no incentive to produce up to capacity. On the contrary, if ρ is equal to 1, unused slots are fully lost. Note that the capacity described in equation (9) stands for historical rights as introduced in the first part of the paper.

4.1 Technical assumptions

In the dynamic setting, unused capacity may be released after the first round to feed the slot pool. Because it seems too restrictive to impose that second round's capacity rights do not depend on the first round, we will assume here that the remaining capacity (say, the slot pool) is allocated evenly between the airlines. The capacity rights in the second round are therefore given by

$$K_i^{all,2} = K_i^{I,2} + \frac{1}{2}\left(\overline{K} - \left(K_1^{I,2} + K_2^{I,2}\right)\right) . \tag{10}$$

Combining (10) with the grand-father rule (9), we can write the capacity rights in the second round as a function of the capacities (resp. quantities) obtained (resp. produced) in the first

[11]This rule is smoother than the usual one, as its derivatives are continuous (and even constant). Also, airlines may manipulate the 80-20 rule to limit the release of under-used slots (see NERA (2004)). We therefore feel confident that providing a slightly modified grandfather rule that exhibits the same qualitative features as the theoretical one will not much change the picture.

round. For airline i (j stands for i's competitor) it writes

$$K_i^{all,2} = \frac{1}{2}\left(\overline{K} + \rho\,(q_i - q_j) + (1-\rho)\,(K_i - K_j)\right)\ . \tag{11}$$

To allow for explicit computations, we also make technical assumptions on the form of the costs and demands. Production and capacity costs are supposed to be linear and equal among periods, with marginal costs respectively equal to c and c_k. We assume further that the inverse demands $P^{(1)}$ and $P^{(2)}$ are linear and that demand is increasing from round 1 to round 2. Precisely, we write $P^{(1)}(Q) = a^{(1)} - b\,Q$ and $P^{(2)}(Q) = a^{(2)} - b\,Q$ with $a^{(2)} > a^{(1)}$. Finally, we assume that the traffic growth is such that in the second round, irrespective of the first round outcomes, airlines will request the whole capacity and thereafter produce up to capacity. This strong assumption reflects the fact that airlines are confident in the growth of air traffic demand and that demand in the second round is *larger* than the airport capacity.

4.2 Actualized profit of firm 1 in the first stage

Under the assumptions introduced in the previous section, the profit of airline i in the second round is given by

$$\Pi_i^{(2)}\,(q_i, K_i, q_j, K_j) = \frac{\eta}{2}\left(\overline{K} + \rho\,(q_i - q_j) + (1-\rho)\,(K_i - K_j)\right)\ , \tag{12}$$

where η is defined as

$$\eta = a^{(2)} - c - c_K - b\,\overline{K}\ . \tag{13}$$

The actualized profit of airline i is therefore given by

$$\begin{aligned}\Pi_i\,(q_i, K_i, q_j, K_j) \quad = \quad & \left(a^{(1)} - \left(c - \frac{\rho}{2}\eta\right) - b\,(q_i + q_j)\right) q_i - \left(c_K - \frac{1-\rho}{2}\eta\right) K_i \\ & + \frac{\eta}{2}\left(\overline{K} - (1-\rho)\ K_j - \rho\, q_j\right)\ .\end{aligned} \tag{14}$$

This expression of the actualized profit may be decomposed in the sum of a "first round like" profit (first line of (14)) and additional terms that do not involve airline i's decision variables q_i and K_i (second line of (14)). The two-stage game is therefore very similar to the static game studied in the first part of the paper, with the exception that the expectation of future profits impacts on the airlines marginal costs. In particular, the short-run and long-run production cost decreases respectively from c to $c - \frac{\eta}{2}\rho$ and from $c + c_k$ to $c + c_k - \frac{\eta}{2}$, so that the expectation of future profits makes airlines want to produce more than they would be prone to in the static game. The fact that airlines may request and use slots that are not immediately profitable, just because they expect them to become so in the near future is well-known in the airline industry, and often referred to as the "baby-sitting of slots". Note further that the "actualized" short-run production cost is increasing in ρ. This implies, as expected, that airlines are prone to produce more when the grandfather rule becomes more severe. In contrast, the long-run cost is independent of ρ. Thus, we see that a reinforcement of the grandfather rule is formally identical to the tax on unused slots described in the last section.

Though the expression for the actualized profit of the airlines involves more terms than the standard profit, the equilibrium result proved in the static case still holds in the dynamic case. This observation, together with the analogy between a decrease of ρ and the introduction of a tax on unused slots, fully characterizes the impact of an increase in the severity of the grandfather rule: if limited, it will increase production on the market, but if set too high, it will be counterproductive. These results are summarized in Proposition 4.

Proposition 4 *Under the technical assumptions introduced in Section 4.1, the dynamic slot allocation game is equivalent to the static slot allocation game. Moreover, the distortion induced by the grandfather rule on the first round of the game is formally identical to the one induced by a tax on unused slots.*

5 Summary and Conclusion

In this article we studied a model of airport slot allocation that mimics the slot allocation process currently in use at congested airports in most European Countries but allows capacity to be costly. Introduction of posted prices for slots is indeed one of the market-based mechanisms suggested by the EU Commission to improve efficiency of runway capacity allocations at congested airports.

In our model, airport capacity is first allocated by the *coordinator* through an administrative procedure before a constrained Cournot market game takes place. The equilibrium structure of the game is rather simple, with airlines either claiming their capacity rights or their long-run Cournot equilibrium quantities. Concretely, this implies that, if initially a carrier is dominant in the considered airport, competing airlines will not respond to the introduction of limited posted prices but may in turn cut production for high posted prices. Indeed, under the current slot allocation process a dominant carrier would not take capacity costs into account unless these are large. This has several implications. First, it implies that the introduction of a limited tax on unused capacity, which would convert production cost into capacity cost, would typically lead to an increase in the produced quantity. Also, it implies that asymmetry in airport slot allocations may benefit consumers even in the absence of network externalities or economies of scale. In this case, an increase in airport capacity may result in a price increase if it makes expansion deterrence from the dominant carrier unprofitable. Finally, the discontinuity of the airlines strategies supports the common view that a market clearing level of price for slots may be difficult to establish. It is indeed symptomatic that, in the static model presented in the paper, such a level does not even exists. In practice, the equilibrium of the game suggests that, if posted prices were to be introduced gradually, the regulator would initially face essentially unresponsive airlines, so that only little information could be gained on the (properly defined) market clearing level of price. Then, if slot prices were set high enough to induce the regime switch, the requested capacity would fail below the available capacity and call for subsequent adjustments in the posted prices. Only these could make at last the requested capacity match the airport capacity. Notice however that this reasoning is based on a straightforward dynamic extension of the model in which myopic airlines repeatedly engage in a static game. The adjustment process and the associated behaviors of the airlines are actually beyond the scope of the present paper and remain to be investigated.

In the final part of the paper, a natural two-period extension of the static model is presented. This models allows to study the impact of the grandfather rule on the behavior of the airlines. In particular, we observe the well-known phenomenon of "baby-sitting of slots", according to which airlines that expect demand for air transport to rise in the near future may exploit slots that are not immediately profitable. When demand is assumed to rise steeply between the two periods, as in the particular case presented in this article, the impact of the grandfather rule on the consumer welfare is positive. This contrasts with the general belief according to which baby-sitting is the sign of an inefficient use of runway capacity. We also show that a reinforcement of the 80-20 rule is formally equivalent to a tax on unused slots. Accordingly, it is shown to be counterproductive if set too constraining.

Now they are several directions that could be of interest for future research. It could indeed be interesting to allow new competitors to enter the market and see how results presented in this paper are impacted. Also, the study of the aforementioned dynamical adjustment of the slot level of price could give additional insight on what could be gained if posted prices were implemented.

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Appendix

Proof of Proposition 2

We recall that this proposition states that technical assumption **A1** is satisfied in the symmetric case when the demand is linear and the capacity and production marginal costs are constant.

Proof: In this proof, the inverse demand is given by $P(Q) = a - bQ$ and the cost of firm i are given by $C(q_i, K_i) = cq_i + c_K K_i$. Firm i's profit is therefore given by

$$\pi(q_i, q_j, K_i) = q_i(a - c - b(q_i + q_j)) - c_K K_i \; . \tag{15}$$

We first consider the case where (K_1, K_2) belong to Region A. In this region, firms choose to produce quantities equal to their capacities so that as long as $K_1 + K_2 = \overline{K}$, the final price is constant and given by $a - b\overline{K}$. Firm i's profit is therefore given by $K_i\left(a - c - c_k - b\overline{K}\right)$. Now remind that (K_1, K_2) belong to Region 1. Thus, either one of the firms profits is strictly greater than its long-run Cournot profit, or $\overline{K} \leq \frac{a-c-c_K}{3b}$. In both cases, we get that $a - c - c_k - b\overline{K}$ is positive, which imply that firm i's profit is increasing in K_i.

We now consider the case where (K_1, K_2) belong to Region 1 but not to Region A[12] and saturate the allocation constraint $K_1 + K_2 = \overline{K}$. Without loss of generality, we can assume that (K_1, K_2) belong to Region 1 and to Region B. In this region, the firm's profits are given by

$$\Pi_1(K_1, K_2) = \frac{(a - c - bK_2)^2}{4b} - c_K K_1,$$

and

$$\Pi_2(K_1, K_2) = K_2 \frac{(a - c - bK_2)}{2} - c_K K_2.$$

This implies that

$$\begin{aligned} \frac{\partial \Pi_1}{\partial K_2}(K_1, K_2) - \frac{\partial \Pi_1}{\partial K_1}(K_1, K_2) &= -\frac{(a - c - bK_2)}{2} + c_K \\ &= -\frac{a - c - 2c_K - bK_2}{2}, \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \Pi_2}{\partial K_2}(K_1, K_2) - \frac{\partial \Pi_2}{\partial K_1}(K_1, K_2) &= \frac{(a - c)}{2b} - bK_2 - c_K \\ &= \frac{a - c - 2c_K - 2bK_2}{2}. \end{aligned}$$

[12] According to the parameters values, such (K_1, K_2) may or may not exist.

We therefore need to show that $K_2 < \frac{a-c-2c_K}{2b}$.

Note that in Region B, firm 2's capacity is lower that its long-run Cournot capacity $K_2^{LR} = \frac{a-c-c_K}{3b}$. For parameter values such that $\frac{a-c-c_K}{3b} < \frac{a-c-2c_K}{2b}$, this inequality is sufficient to prove the proposition.

Assume now that $\frac{a-c-c_K}{3b} \geq \frac{a-c-2c_K}{2b}$. Notice that for (K_1, K_2) in Region 1 and Region B, firm 1's profit is strictly greater that its long-run Cournot profit. Since Π_1 is decreasing in K_2, we get that K_2 as to be lower that the capacity $\widetilde{K}_2(K_1)$ that solves

$$\Pi_1(K_1, \widetilde{K}(K_1)) = \Pi_1(K_1^{LR}, K_2^{LR}) \ . \tag{16}$$

An application of the Implicit Function Theorem shows that the function $\widetilde{K}_2$ defined implicitly in equation (16) is non-increasing. The maximum of the function $\widetilde{K}_2(K_1)$ for values of K_1 corresponding to couples (K_1, K_2) belonging to Region 1 and to region B is reached when its graph intercepts firm 1's short-run cournot reaction curve. We only need to prove that for $K_2^* = \frac{a-c-2c_K}{2b}$ and $K_1^* = r(K_2^*, c) = \frac{a-c-bK_2^*}{2b}$, the couple (K_1^*, K_2^*) does not belong to Region 1, that is $\Pi_1(K_1^*, K_2^*) < \Pi_1(K_1^{LR}, K_2^{LR}) = \frac{(a-c-c_K)^2}{9b}$. This is true if and only if

$$\begin{aligned}
& \frac{(a-c-bK_2^*)^2}{4b} - c_K K_1^* < \frac{(a-c-c_K)^2}{9b} \\
\Leftrightarrow \quad & \frac{(a-c-bK_2^*)^2}{4b} - c_K \frac{a-c-bK_2^*}{2b} < \frac{(a-c-c_K)^2}{9b} \\
\Leftrightarrow \quad & (\frac{a-c-bK_2^*}{2})(\frac{a-c-bK_2^*}{2} - c_K) < \frac{(a-c-c_K)^2}{9} \\
\Leftrightarrow \quad & (\frac{a-c+2c_K}{4})(\frac{a-c-2c_K}{4}) < \frac{(a-c-c_K)^2}{9} \\
\Leftrightarrow \quad & 9\left[(a-c)^2 - 4(c_K)^2\right] < 16\,(a-c)^2 - 32c_K\,(a-c) + 16(c_K)^2 \\
\Leftrightarrow \quad & 0 < 7\,(a-c)^2 - 32c_K\,(a-c) + 52(c_K)^2.
\end{aligned}$$

This is true for whatever values of the parameters. This achieves the proof of the proposition.

■

Proof of Proposition 3

In this section we state and prove formally the lemmas used in the proof of Proposition 3.

Lemma 1 *Assume that both firms profits satisfy* ***A1****. Then if there exists a Nash equilibrium* $(\varphi_1^*, \varphi_2^*)$ *of the capacity game such that allocated capacities* $(K_1^{all}(\varphi_1^*, \varphi_2^*), K_2^{all}(\varphi_1^*, \varphi_2^*))$ *belong* *to Region 1, and such that all the available capacity is allocated ($K_1^{all}(\varphi_1^*, \varphi_2^*) + K_2^{all}(\varphi_1^*, \varphi_2^*) = \overline{K}$), then maximum capacity demands $\left(\overline{K}, \overline{K}\right)$ also are a Nash equilibrium and yield the same capacity allocation: $K_1^{all}(\varphi_1^*, \varphi_2^*) = K_1^{all}\left(\overline{K}, \overline{K}\right)$.*

Proof: Assume that there exists a Nash equilibrium $(\varphi_1^*, \varphi_2^*)$ of the capacity game such that allocated capacities $(K_1^{all}(\varphi_1^*, \varphi_2^*), K_2^{all}(\varphi_1^*, \varphi_2^*))$ belong to Region 1, and such that all the available capacity is allocated $(K_1^{all}(\varphi_1^*, \varphi_2^*) + K_2^{all}(\varphi_1^*, \varphi_2^*) = \overline{K})$. Technical assumption **A1** implies that firm 1 must not be able to get more capacity by expressing a higher demand, so that[13] $K_1^{all}(\varphi_1^*, \varphi_2^*) = K_1^{all}\left(\overline{K}, \varphi_2^*\right)$. So the demands $\left(\overline{K}, \varphi_2^*\right)$ also constitute a Nash equilibrium and yield the same capacity allocation. Indeed, if φ_1^* is a best response to φ_2^* for firm 1, then $\overline{K}$ is also a best response to φ_2^* since it yields the same outcome. Moreover if φ_2^* is a best response to φ_1^* for firm 2, then it is also a best response to $\overline{K}$ (if φ_2^* is a best response to φ_1^* for firm 2, it must be true that firm 2 can not get more capacity than $K_2^{all}(\varphi_1^*, \varphi_2^*) = K_2^{all}\left(\overline{K}, \varphi_2^*\right)$ by choosing a higher demand than φ_2^* when firm 1 chooses demand φ_1^*. From the monotonicity of K_1^{all} in φ_1, this is also true when firm 1 chooses demand $\overline{K}$. Moreover for a given quantity K_2 granted to firm 2, firm 2's profit $\Pi_2(K_1, K_2)$ is non increasing in K_1, whatever the region (K_1, K_2) belong to. So if firm 2 has no interest is choosing a lesser capacity demand than φ_2^* when firm 1 chooses capacity demand φ_1^*, this is also true when firm 1 chooses capacity demand $\overline{K}$).
Applying the same logic to the Nash equilibrium $\left(\overline{K}, \varphi_2^*\right)$ shows that $\left(\overline{K}, \overline{K}\right)$ is also a Nash equilibrium and yields the same allocated capacities. ■

Lemma 2 *Assume that both firms profits satisfy **A1**. Then the set of capacity pairs (K_1, K_2) such that $K_1 + K_2 < K_1^{LR} + K_2^{LR}$ is included in Region 1.*

Proof: Assume that $\bar{K} < K_1^{LR} + K_2^{LR}$ and consider the capacity pair defined as $(K_1^0, K_2^0) = (K_1^{LR} - 1/2(K_1^{LR} + K_2^{LR} - \bar{K}), K_2^{LR} - 1/2(K_1^{LR} + K_2^{LR} - \bar{K}))$. By construction, these capacities are strictly below the long-run Cournot responses, so that (K_1^0, K_2^0) belongs to Region 1.

Now assume that $\Pi_1(K_1^0, K_2^0) \leq \Pi_1(K_1^{LR}, K_2^{LR})$. By construction (K_1^0, K_2^0) is in the interior of Region A, so that the function $\delta \mapsto \Pi_1(K_1^0 + \delta, K_2^0 - \delta)$ is locally strictly increasing. It is actually strictly increasing as long as the firms produce up to capacity, that is until $K_1^0 + \delta$ becomes large enough to cross the long-run Cournot best response. This happens for

[13] Because K_1^{all} is continuous and Region 1 is an open set.

$K_1^0 + \delta = r(K_2^0 - \delta, c_1 + k_1)$. Now remind that on firm 1's Cournot long run best response curve, firm 1's profit is given by

$$\Pi_1(r(K_2, c_1 + k_1), K_2)) = \max_q P(K_2 + q)q - c_1(q) - k_1(q) \ . \qquad (17)$$

The Envelop Theorem therefore implies that $\frac{\partial \Pi_1(r(K_2,c_1+k_1),K_2)}{\partial K_2} = P'(K_2 + r(K_2, c_1 + k_1))r(K_2, c_1 + k_1) < 0$. As a consequence, firm 1's profit is decreasing in K_2 on its long run best response curve. Since, $K_2^0 - \delta < K_2^0 < K_2^{LR}$, this proves that that firm 1's profit is larger than its long run Cournot profit when $K_1^0 + \delta$ crosses firm 1's long-run Cournot best response. Technical assumption **A1** then implies that the set $\{K_1 \geq K_1^0\} \cap \{K_1 + K_2 = \overline{K}\}$ belongs to Region 1.

Alternatively, if $\Pi_1(K_1^0, K_2^0) > \Pi_1(K_1^{LR}, K_2^{LR})$ technical assumption **A1** implies that the set $\{K_1 \geq K_1^0\} \cap \{K_1 + K_2 = \overline{K}\}$ belongs to Region 1. By symmetry, we get that the whole set $\{K_1 + K_2 = \overline{K} < K_1^{LR} + K_2^{LR}\}$ belongs to Region 1. This completes the proof of the lemma. ■

Lemma 3 *Assume that both firms profits satisfy **A1**. If allocated capacities for maximum demands, $K^{all}\left(\overline{K}, \overline{K}\right)$, belong to Region 2, then the demands (K_1^{LR}, K_2^{LR}) constitute a Nash equilibrium.*

Proof: First note that lemma 2 implies that if $K^{all}\left(\overline{K}, \overline{K}\right)$ belongs to Region 2, then the available capacity $\overline{K} = K_1^{all}\left(\overline{K}, \overline{K}\right) + K_2^{all}\left(\overline{K}, \overline{K}\right)$ must exceed $K_1^{LR} + K_2^{LR}$.

Suppose that firm 1 chooses a capacity demand K_1^{LR}.

If firm 2 demands any capacity φ_2 between 0 and $\overline{K} - K_1^{LR}$, then the allocated capacities are $K^{all}\left(K_1^{LR}, \varphi_2\right) = \left(K_1^{LR}, \varphi_2\right)$. Among these capacities demands φ_2, the long-run Cournot quantity K_2^{LR} is the best. This comes from the standard result of the capacity-unconstrained model ($\overline{K} = +\infty$) stating that (K_1^{LR}, K_2^{LR}) is a Nash equilibrium.

If firm 2 demands a capacity φ_2 above $\overline{K} - K_1^{LR}$, then we need to distinguish two cases:

- If $K_1^{all}\left(\overline{K}, \overline{K}\right) \geq K_1^{LR}$, the allocated capacities are $K^{all}\left(K_1^{LR}, \varphi_2\right) = \left(K_1^{LR}, \overline{K} - K_1^{LR}\right)$. So, if $K_1^{all}\left(\overline{K}, \overline{K}\right) \geq K_1^{LR}$, a demand $\varphi_2 = K_2^{LR}$ is a best response to a demand $\varphi_1 = K_1^{LR}$.

- If $K_1^{all}\left(\overline{K}, \overline{K}\right) < K_1^{LR}$, the the allocated capacities are

$$K^{all}\left(K_1^{LR}, \varphi_2\right) = \left(\overline{K} - \min\left(\varphi_2, K_2^{all}\left(\overline{K}, \overline{K}\right)\right), \min\left(\varphi_2, K_2^{all}\left(\overline{K}, \overline{K}\right)\right)\right),$$

and saturate the allocation constraint.

By assumption, firm 2's profit satisfies **A1**. This implies that if for any value of φ_2, the allocation $\left(\overline{K} - \min\left(\varphi_2, K_2^{all}\left(\overline{K}, \overline{K}\right)\right), \min\left(\varphi_2, K_2^{all}\left(\overline{K}, \overline{K}\right)\right)\right)$ belongs to Region 1, then it must also be true for higher values of φ_2, and in particular for the allocation $K^{all}\left(\overline{K}, \overline{K}\right)$ obtained with $\varphi_2 = \overline{K}$. As we assumed that $K^{all}\left(\overline{K}, \overline{K}\right)$ belongs to Region 2, we know that whatever φ_2 above $\overline{K} - K_1^{LR}$ leads to an allocation in Region 2, and to a firm 2's profit inferior to the long-run Cournot profit. This proves that a demand $\varphi_2 = K_2^{LR}$ is a best response to a demand $\varphi_1 = K_1^{LR}$.

Symmetry completes the proof. ■

Lemma 4 *If allocated capacities for maximum demands, $K^{all}\left(\overline{K}, \overline{K}\right)$, belong to Region 1, then $\left(\overline{K}, \overline{K}\right)$ is a Nash equilibrium, and there is no Nash equilibrium yielding an allocation (K_1^{LR}, K_2^{LR}).*

Proof: Suppose that allocated capacities for maximum demands, $K^{all}\left(\overline{K}, \overline{K}\right)$, belong to Region 1.

- From Lemma 1, we know that for (K_1, K_2) belonging to Region 1, such that $K_1 + K_2 = \overline{K}$, firm i's profit is all the higher as K_i is high. So, if firm 1 expresses a maximum demand, firm 2 prefers expressing a maximum demand rather than any other demand such that the allocation stays in Region 1. The only case where one of the firms, say firm 2, can express a demand yielding an allocation in Region 2 (by reducing firm 2's allocation), is when $\overline{K} \geq K_1^{LR} + K_2^{LR}$ and $K^{all}\left(\overline{K}, \overline{K}\right)$ gives firm 2 a profit which is superior to the long-run Cournot profit. But it has no interest for firm 2 as it would finally enjoy a profit inferior to the long-run Cournot profit.
  Using symmetry, this shows that $\left(\overline{K}, \overline{K}\right)$ is a Nash equilibrium.

- Moreover there is no Nash equilibrium yielding an allocation (K_1^{LR}, K_2^{LR}). This is clear when $\overline{K} < K_1^{LR} + K_2^{LR}$. Suppose now $\overline{K} \geq K_1^{LR} + K_2^{LR}$. If $K^{all}\left(\overline{K}, \overline{K}\right)$ is in Region 1, it can't be "below" both long-run Cournot reaction curves. So, for the allocation $K^{all}\left(\overline{K}, \overline{K}\right)$, one of the firms, say firm 1, has a profit strictly higher than the long-run Cournot profit. In this case, it must also be true[14] that $K_2^{all}\left(\overline{K}, \overline{K}\right) \leq K_2^{LR}$. Thus, if

[14]Because whatever K_2 above K_2^{LR}, and whatever K_1, firm 1's profit $\Pi_1(K_1, K_2)$ is inferior to $\Pi_1(K_1, K_2^{LR})$ which is inferior to $\Pi_1(K_1^{LR}, K_2^{LR})$.

firm 2 expresses a demand $\varphi_2 = K_2^{LR}$, firm 1's best reaction will be to choose a demand $\varphi_1 = \overline{K}$, giving the allocation $K^{all}\left(\overline{K}, \overline{K}\right)$.

This completes the proof. ■

Lemma 5 *If allocated capacities for maximum demands, $K^{all}\left(\overline{K}, \overline{K}\right)$, belong to Region 1, there is no Nash equilibrium $(\varphi_1^*, \varphi_2^*)$ such that all the capacity available is allocated (that is $K_1^{all}(\varphi_1^*, \varphi_2^*) + K_2^{all}(\varphi_1^*, \varphi_2^*) = \overline{K}$) and such that the allocation $(K_1^{all}(\varphi_1^*, \varphi_2^*), K_2^{all}(\varphi_1^*, \varphi_2^*))$ belongs to Region 2.*

Proof: Suppose allocated capacities for maximum demands, $K^{all}\left(\overline{K}, \overline{K}\right)$, belong to Region 1, and consider any demands $(\varphi_1^*, \varphi_2^*)$ such that all the capacity available is allocated $(K_1^{all}(\varphi_1^*, \varphi_2^*) + K_2^{all}(\varphi_1^*, \varphi_2^*) = \overline{K})$ and allocation $(K_1^{all}(\varphi_1^*, \varphi_2^*), K_2^{all}(\varphi_1^*, \varphi_2^*))$ belongs to Region 2. As noted in Lemma 3, this would require $\overline{K} \geq K_1^{LR} + K_2^{LR}$.
Moreover, as noted in Lemma 4, if $K^{all}\left(\overline{K}, \overline{K}\right)$ is in Region 1, for the allocation $K^{all}\left(\overline{K}, \overline{K}\right)$, one of the firms, say firm 1, has a profit strictly higher than the long-run Cournot profit, and it also requires that $K_2^{all}\left(\overline{K}, \overline{K}\right) \leq K_2^{LR}$. So, if firm 2 expresses a demand φ_2^*, firm 1 would prefer choosing demand $\overline{K}$, which would yield an allocation $K^{all}\left(\overline{K}, \overline{K}\right)$ and give it a profit strictly superior to the long-run Cournot profit (while choosing φ_1^* give it a profit inferior to the long-run Cournot profit). So $(\varphi_1^*, \varphi_2^*)$ is not a Nash equilibrium, which completes the proof. ■