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ENTRY MODELS OF MULTIPLE AGENTS : EMPIRICAL  
APPLICATION TO DOMESTIC AIR TRANSPORT WITHIN  
THE EUROPEAN UNION.

BY KARIM. ZBIDI<sup>1</sup>

## Abstract

This paper is composed of two parts. The first part of the paper deals with an example of two firms theory game of entry. After specifying, the set of firms, the firms space of pure strategies and the profit functions, we consider different types of games with respect to the information structure and the sequence of move of the two firms. We present the impact of different assumptions on the probability distribution of the outcomes of the game. With respect to the information structure, there exist two types of games: the complete information games where information about firms entry cost is perfectly known by both firms and asymmetric information games in which firms entry cost is a private information. So every firm knows its own cost of entry and have a partial information about its opponent entry cost (opponent's cumulative distribution function). Firms can participate either in a simultaneous move game or in a sequential move game. The combination of the nature of the structure of the information with the rule of move, gives arise to four games whose outcomes are different. We show that in some cases multiplicity of equilibria exists.

The second part of the paper begins by showing how previous literature have treated the problem of multiplicity of equilibria. We conclude that to come over this problem and to guarantee the uniqueness, like in einav(2003), one should use a sequential move asymmetric information game. Within the framework of domestic air transport within the European union, the sequential move doesn't appear to

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be a realistic assumption and an appropriate empirical model of oligopoly market structure must estimate simultaneously the decision of all companies. Due to the fact that the simultaneous move asymmetric information entry game becomes computationally intractable for more than 2 firms, we decide to opt for the simultaneous move complete information entry game. To deal with the problem of multiplicity of equilibria we adopt the same approach as berry (1992). In our empirical model the firms profits are specified as a function of the demand characteristics of the market, as well as the equilibrium number of companies present in the market. Like in berry (1992) the model specification supports both types of heterogeneity: observable and unobservable. Four models estimations are performed: three of them stem from specific constraints on parameters and the last one is the most general one. A comparison between models is built to find the best fit data model.

## Introduction

Entry models belongs to a more general class of models: the discrete choice models. Structural models of entry are a particular discrete choice models in which the agents decisions are binary (enter or not).

In this paper, we are interested in entry models of multiple agents where the entry decision of a particular firm depend on its information or anticipation about decision of its rivals. Games theory, supplies a natural structural framework to model decisions interdependency of economic agents. Discrete empirical games are useful tools to explain the individual agent's behavior in which strategic interactions play an important role.

In the economic literature, the entry have been modelled in both static and dynamic framework. Static entry models have been treated in several papers such as Bresnahan and Reiss [9], Berry[8], Mazzeo[5], Seim[3], Einav[4] and tamer[2].

One of the main critics addressed to these models, is the fact that agents decision process doesn't depend on dynamic consideration which could sometimes be unrealistic. The economic literature of dynamic entry models is not exhaustive. A few papers are related with, such as Posendorfer and Schmidt dengler[6], Aguirregabiria and Mira[10], Pakes and Berry[1] and Bajari, Benkard and Levin[7].

## 1 Static entry models: Example of a two-firms game

The purpose of this example is to show in one hand, the different outcomes according to the information structure and the sequence of move in the game and in the other hand the games structures that may give arise to multiplicity of equilibria. To illustrate this, we take the example of an entry game with two firms. It's a game specified under its strategic form. The different elements of the game are:

- The set of firms noted  $\{1,2\}$ .
- The space of firms pure strategies noted  $\mathcal{A}_i$ . Each firm decide to enter the market or not, so we have:  $\mathcal{A}_1 = \mathcal{A}_2 = \{0,1\}$ , where 1 codes the case when firm enters the market and 0 the case if not.

- The profit functions: If a firm  $i$  ( $i \in \{1,2\}$ ) stays out the market, it obtains a null payoffs. If it enters, it pays entry cost of  $\epsilon_i$  and collects a payoffs of  $\mu$  (here we consider for simplicity the same payoffs:  $\mu_i = \mu \forall i \in \{0,1\}$ ) if it is in monopoly position and  $\mu - \Delta$  if not ( $\Delta < \mu$ ).

All the parameters are positive and belongs to  $[0,1]$ .

For simplicity, and for the games with asymmetric information, we assume also that the  $\epsilon_i$  are both drawn from a uniform distribution over  $[0,1]$ .

The assumption of positive parameters, ensures profit decreasing with increasing competition.

After this, we consider different types of games according to information structure and the two-firms sequence of move. For all case of figure, we consider that  $\epsilon_1$  and  $\epsilon_2$  are unknown by the econometrician and  $\mu$  and  $\Delta$  are the parameters of interest to be estimated. We are interested in the impact of the different assumptions on the probability distribution of game outcomes.

According to the information structure there exist two types of games: a complete information game in which  $\epsilon_i$  are perfectly known for the two firms and the asymmetric information game where each firm  $i$  have a partial information about its opponent cost entry  $\epsilon_{-i}$  (commonly we suppose that each firm  $i$  knows the cumulative distribution function of  $\epsilon_{-i}$ ).

Firms decisions can be simultaneous or sequential. The combination of the nature of the structure of the information with the rule of move, gives arise to four games whose outcomes are different.

### 1.1 The simultaneous complete information game

Bresnahan and Reiss [9] and Berry[8] analyzed this type of games.

We note  $\pi_i^m (= \mu - \epsilon_i)$  respectively  $\pi_i^d (= \mu - \Delta - \epsilon_i)$  the monopoly profit of firm  $i$  respectively the duopoly profit. Firm  $i$ 's strategy is noted  $a_i$ . We have:

		<b>Firm 2</b>	
		(0,0)	(0, $\mu - \epsilon_2$ )
<b>Firm 1</b>	(0,0)	( $\mu - \epsilon_1, 0$ )	( $\mu - \Delta - \epsilon_1, \mu - \Delta - \epsilon_2$ )

TAB. 1 - Profit matrix of the two-firms entry game

$$\forall i \in \{1,2\}, a_i \in \mathcal{A}_i \text{ et } a_i = \begin{cases} 1 & \text{if firme } i \text{ enters} \\ 0 & \text{else} \end{cases}$$

The possible outcomes of this game are the different pure strategy Nash equilibria relative to combination of cost entry values.

1.  $a_1^* = a_2^* = 0$  : **No firm enters at the equilibrium**

This involves the following profit's restrictions :

$$\forall i \in \{1,2\}, \pi_i^m \leq 0$$

Each firm make negative profit even if its opponent stays out.

2.  $a_1^* = a_2^* = 1$  : **Both firms enter at the equilibrium**

This involves the following conditions:

$$\forall i \in \{1,2\}, \pi_i^d > 0$$

Both firms make positive profit in the duopoly case.

3.  $a_i^* = 1, a_{-i}^* = 0$  : **Only one firm enter at the equilibrium**

Two cases of figure are possible:

- *Nash equilibrium is unique*

This happened when this conditions are satisfied :

$$\exists i \in \{1,2\}, \pi_i^m > 0 \text{ and } \pi_{-i}^m \leq 0$$

or

$$\forall i \in \{1,2\}, \pi_i^m > 0 \text{ and } \exists i \in \{1,2\}, \pi_i^d > 0 \text{ and } \pi_{-i}^d \leq 0$$

In both cases, firm  $i$  enter the market and collects a monopoly profit  $\pi_i^m$ . The first condition means that at the equilibrium even if both firms have the guarantee to operate in monopoly position, only one firm enters. The second condition, means that even though the monopoly profit of the opponent firm is positive, it chooses to stay out because of its negative duopoly profit. Due to the complete structure of the information, the opponent firm knows that the first firm will enter in all cases because its duopoly profit is positive.

- *Nash equilibria are multiple*

Under this conditions we have multiple Nash equilibria:

$$\forall i \in \{1,2\}, \pi_i^m > 0 \text{ and } \pi_i^d \leq 0$$

both firms have a positive monopoly profit and a negative duopoly profit. In this case, there are two separate Nash equilibria in pure strategies. In each equilibrium, only one firm operates, but the identity of such firm is not identified without a further assumption in the order of entry.

The table 2, gives a numeric example of such a case. For both Nash equilibria, and taking in consideration the opponent decision, no firm is interested in changing its action.

The figure 1 summarized the different possible outcomes of this games. The central region corresponds to multiple Nash equilibria.

## 1.2 The sequential complete information game

In this game, we suppose firms sequential move action like in the stackelberg model of competition. In this case, equilibrium is unique. In fact, like in the simultaneous case, all the regions of the figure 1 where equilibrium is unique, stay identical. The central region of multiplicity is entirely assigned to the leader firm. Here the sequence of move is specified out of the game and so the multiplicity region is totally assigned to one firm.

Berry[8] and Mazzeo[5] propose another way to specify the sequence of move. The leader firm is the most profitable one. This case is presented in figure 2.

In the multiplicity region, firm 1 (respectively firm 2) enters if it is situated on the right (respectively left) of the line  $\epsilon_1 = \epsilon_2$ .

	<b>Firm 2</b>	
<b>Firm 1</b>	(0,0)	(0,4)
	(4,0)	(-1,-1)

TAB. 2 – *Case of multiple Nash equilibria* ( $\mu = 10, \Delta = 5, \epsilon_1 = \epsilon_2 = 6$ )

### 1.3 The simultaneous asymmetric information game

In this game both firms take simultaneous decisions. The information on entry costs is private such that each firm  $i$  knows its own cost entry  $\epsilon_i$  and has an incomplete information about its opponent cost entry  $\epsilon_{-i}$ . In fact each firm knows the opponent's cost entry distribution whose cumulative distribution is noted  $F$  (common for both firms).

For this type of games, Seim[3] shows that equilibrium strategy depends on threshold  $\epsilon_i^*$ . In fact, at the equilibrium, firms decide to enter if their cost entry type is lower than the indifference threshold. The indifference threshold is the cost entry type for which firms are indifferent between enter and stay out of the market.

Thus, we can compute the Bayesian Nash equilibrium by equalizing, for each firm, the expected profits of both types of firm decision: enter or stay out. Thus we have:

$$a_i^* = 1_{\{\epsilon_i < \epsilon_i^*\}} = \begin{cases} 1 & \text{if } \epsilon_i < \epsilon_i^* \\ 0 & \text{if } \epsilon_i \geq \epsilon_i^* \end{cases}$$

If firm  $i$  decides to stay out of the market, we have  $E(\pi_i) = 0$  because its profit is null independently of the opponent's decision.

Now, if firm  $i$  decide to enter, its expected profit can be written:

$$\begin{aligned} E(\pi_i) &= (\mu - \epsilon_i)P(\epsilon_{-i} \geq \epsilon_{-i}^*) + (\mu - \epsilon_i - \Delta)P(\epsilon_{-i} < \epsilon_{-i}^*) \\ &= (\mu - \epsilon_i)(1 - F(\epsilon_{-i}^*)) + (\mu - \epsilon_i - \Delta)F(\epsilon_{-i}^*) \\ &= \mu - \epsilon_i - \Delta F(\epsilon_{-i}^*) \end{aligned}$$

For  $\epsilon_i = \epsilon_i^*$  (the indifference threshold of firm  $i$ ), we obtain by equalizing both expected profits:

$$\forall i \in \{1, 2\}, E(\pi_i^*) = 0.$$

This allows us to obtain the following equation system:

$$\begin{cases} \mu - \epsilon_1^* - \Delta F(\epsilon_2^*) = 0 \\ \mu - \epsilon_2^* - \Delta F(\epsilon_1^*) = 0 \end{cases}$$

For the uniform distribution, the solution is:

$$\epsilon_1^* = \epsilon_2^* = \epsilon_{sim}^* = \frac{\mu}{1 + \Delta}$$



The figure 3 shows the identity of each firm at the equilibrium as function of the private information on cost entry.

Compared to the simultaneous case, and for this simple model, the Bayesian Nash equilibrium is unique and this occurs for every type of cost entry firm.

#### 1.4 Sequential asymmetric information game

As in the complete information game, the leader takes the entry decision first. The second firm, follows its complete information strategy conditionally to the leader action.

Let's suppose that firm 1 is the leader (the case where the firm 2 is the leader is perfectly symmetric). The firm 1 knows that, after it takes its decision, the game becomes of complete information for the second firm. In this case, the threshold  $\epsilon_1^*$  for which the firm 1 is indifferent between enter and stay out, corresponds to the following condition of the expected profit nullity:

$$0 = E(\pi_1^*) = (\mu - \epsilon_1^*)P(a_2^* = 0/a_1^* = 1) + (\mu - \epsilon_1 - \Delta)P(a_2^* = 1/a_1^* = 1)$$

but,

$$\begin{aligned} P(a_2^* = 0/a_1^* = 1) &= P(\mu - \Delta - \epsilon_2 \leq 0) \\ &= 1 - F(\mu - \Delta) \end{aligned}$$

similarly,

$$P(a_2^* = 1/a_1^* = 1) = F(\mu - \Delta)$$

finally, we get

$$\epsilon_1^* = \mu - \Delta F(\mu - \Delta)$$

For the uniform distribution, the solution is:

$$\epsilon_1^* = \epsilon_{seq}^* = \mu - \Delta(\mu - \Delta) \geq \epsilon_{sim}^*$$

Different comments can be done. In the case of the game of complete information, the leader has a certain advantage since he obtains all the multiplicity region of the simultaneous game. While, in the case of asymmetric information game, there exist some cases where the leader is disadvantaged. Suppose that  $\epsilon_1$  and  $\epsilon_2$  are just below  $\mu$  and higher than  $\epsilon_{seq}$ . The follower firm enters the market and gets a

monopoly profit. So the asymmetric information creates a tradeoff for the leader: it has a deterrence power but it faces by the same time an information uncertainty about his opponent.

Concerning the equilibria multiplicity, the asymmetric information framework can guarantee the unicity equilibrium. But compared to the complete information games, the likelihood of regret increases greatly. In fact, asymmetric information models, can give to a situation where entering firm makes a negative profit and regrets its action (for example in the simultaneous case, this happens when:  $\forall i \in \{1,2\}, \mu - \Delta < \epsilon_i < \epsilon_{sim}^*$ ). The non entering firm can also regret its action. For example, suppose that firms face this case of figure  $\epsilon_2 > \mu$  and  $\epsilon_{sim}^* < \epsilon_{seq}^* < \epsilon_1 < \mu$ . So, seeing the action of firm 2, firm 1 would have preferred inverting its action and enters the market.

Thus, information asymmetry could give rise to unsustainable outcomes in the long run given that firms would like to change their past actions. This regret feeling is on the definition limits of static game due to the fact that, regret is based on game's state describing the history of the game.

## 2 The literature of empirical games

The use of strategic discrete games in empirical models wasn't very developed. Agents decisions interdependency, was the main difficulty to implement empirical models. This interdependency causes often multiple equilibria in discrete games.

The literature treated the multiplicity problem in a different manners and this depending on the information structure of the game.

Within the framework of complete information games, Berry [8] and Bresnahan and Reiss[9] proposed to gather all multiple equilibria in a joint equilibrium uniquely predictable. For entry models, these authors, noticed that all multiple equilibria share a common feature: the equilibrium firms number. This aggregate manner to view equilibria, allows them to predict the number of equilibrium firms instead of their identity. To identify the entering firm, Berry [8] adopts a supplement assumption concerning the firms entry order. For example, he considers that the

more profitable firm moves first or the incumbent firm moves first. Other symmetry assumption is necessary to predict the unique equilibrium firms number. The firm's profit function are considered as invariant to opponent's entry decisions permutation, only the number of equilibrium entering firms is essential.

Two main critics have been addressed to this approach. First, this approach isn't efficient due to the fact that different observations are grouped and treated in the same way. In fact, this approach does not make the use of the entire available information in the data set. Second, the necessary symmetry assumptions on profit functions and output, does not seem to be always suitable. The firm's entry decisions could depend on both equilibrium number and the identity of firms (For example the profit of firm 1 decreases differently according to if firm 2 or firm 3 enters the market). Firms output is considered to be homogeneous while in some industry, product differentiation gives arise to competition's level varying with outputs types.

Mazzeo [5] relaxed partially symmetry assumptions by introducing different types of products and by conditioning the analysis to entry firms number of each type. The extension of mazzeo's model to more than three types becomes computationally intractable.

A recent alternative approach, developed by Tamer, shows that, in the presence of multiple equilibria, instead of estimating punctually parameters, one can estimate parameters limit bounds. This interesting approach authorizes a more flexible form for profit functions where asymmetry assumptions are not necessary. Ciliberto and Tamer[2], presented a first empirical application studying the competition structure of airlines markets.

A last approach consists on changing the order of firm's move such as the equilibrium unicity be guaranteed. Bresnahan and Reiss [9] proposed a Stackelberg game in which the sequential move structure ensure the perfect equilibrium unicity in under game (Notion introduced by Selten (1975)).

Due to computation difficulties and the complexity of integration regions, the game's complete information version becomes less interesting if the game's dimension increase or if we relax symmetry assumptions. Seim[3] changed the game's information structure and passed to asymmetry information games. The strategy of each

firm is more simple because it depends only on its private information and not on all the opponent's private information. Seim, found and estimated the unique Bayesian Nash equilibrium of his game and for his data set. In general, the equilibrium unicity is not guaranteed. The research of equilibrium strategies involves resolution of fixed point problem whose complexity increase with the number of firms. Finally, the symmetry assumptions is present and post-entry profits are symmetric.

Einav [4] tried to group the ideas above and proposed a sequential game structure with information's asymmetry. He obtained an empirical model, allowing both Bayesian Nash equilibrium unicity and direct computation of the different games outcomes probabilities. So the maximum likelihood estimation is possible. His model also allows to get rid of the symmetry assumption and to adopt a more flexible form for profits. No entry's order assumption is needed, the outcomes likelihood are performed conditionally to all possible firms permutations.

Nevertheless, the sequential move structure of the game is not always appropriate. In some industries of oligopoly structure (for example air transport industry), an appropriate empirical model should model simultaneously the decisions of all firms. Thus, there is a tradeoff between entry empirical models which are relatively simple to resolve (and allowing a more profit's flexible form) and entry empirical models which are more economically appropriate to describe inter-dependent firms decisions.

### **3 The empirical application**

Our empirical goal is to identify and measure the factors which have influenced the european airlines operating decisions within the intra-European market. The unit of observation is a non directional airport-pair market. Thus, we assume that passengers are in the same market regardless of which direction they are travelling (i.e passengers travelling from Toulouse to Paris-Orly are assumed to be in the same market as those travelling from Paris-Orly to Toulouse). It's important to define the market and the firms that operate within a market. So, we define a market as a market of air passenger travel between two airports, irrespective of

intermediate transfer points, so flights to different airports in the same city are in separate markets. The nature of the available data helped us to fix such a definition. Concerning the definition of firms operating such market, we will present this, in the next section. This section contains some descriptive results concerning entries and exits. To construct such variables, we focused on particular unit of time. We compared two time periods which are one year separated. The first period, is the year 1999 and the second period is the year 2000. As mentioned by Berry, the one year period is probably long enough for an airline to plan and execute an entry decision, but it is not so long that fundamental cost and demand factors are likely to change. This makes it possible to focus on strategic decisions that may affect decisions on the short run while abstracting from long-run factors such as price factors.

### 3.1 Data and some descriptive results

We use the OAG data, which contains intra-european flight data. OAG data allows to describe only the supply side of airlines services. Each line of the database describes the number of connection, the unicity code, the origin, the destination, the distance, the airline, the allied airline, the aircraft type, the aircraft capacity, the itinerary, the scheduled departure, the scheduled arrival, Available Seat Kilometer (ASK) and the agreement's type code.

The unicity code, is a code showing if the flight appears one time or more in the database. It takes the value of «U» when the flight is unique and the value of «D» otherwise. The existence of flight multiplicity in the database is due to commercial agreements signed between airlines. The agreement's type code specify the nature of such an agreement. It takes four possible value: «R» for flights with no agreement signed, «L» for leased space agreement, «J» for joint operation agreement and «S» for franchised flight agreement. The combination of these two codes gives arise to six types of airlines operation. The table 3 summarizes these different types.

These different types of operation can be defined as follows:

1. Direct operation flight: A flight where the operating airline owns all the seats/space of that flight.

	Agreement's type			
Unicity code	R	L	J	S
U	Direct operation	Total leased space	Joint operation	Franchised
D	-	Partial leased space	-	Code shared

TAB. 3 – *Different types of flight's operation*

2. Leased space flight: A flight where the operating airline leases some (Partial leased flight) or all (total leased flight) seats/space to one or more other airlines and all participants to such an agreement sell their seats/space on that flight under their own designator(s)
3. Joint operation flight: a flight on which more than one airline operates one or more of its legs.
4. Code shared flight: A flight where the operating airline allows seats/space to be sold by one or more than one airline and all participants to such an agreement sell their seats/space on that flight under their own Flight Designator. Operating airline pays monetary compensation to other airlines.
5. Franchised flight: A flight where the operating airline operate only under the designator of an other airline and pays much more monetary compensation.

One important issue is how to treat airlines operating through commercial agreements flights. We assume that franchised airlines and airlines that operate through code-sharing, take their own decision to serve a route independently. So we treat them separately and the total number of flights and capacity is uniquely assigned to the airline operating the route. Concerning total leased space flight, we consider that the airline which leases capacity is the only airline operating. Finally, and for partial leased flight and joint operation flight, we make the assumption that both allied airlines shared equally the capacity and operate the same number of flight. Under these assumptions, an airline is considered as operating a route if it operates directly or leases capacity (Totally or partially) or operates jointly with an allied airline.

Our Base sample consists of the 2581 intra-european markets which have been served by any airline as of 1999. We considered that markets with density less

than 20 available seat per day (AS/day)<sup>1</sup> are not commercially viable and so we excluded them from our base. We also excluded markets with distances less than 150 kilometers as they are not likely to be the targets for our airlines bundle retained. Our final sample consists on 2051 airport-pair markets combined with the 15 largest<sup>2</sup> airline within intra-european space.

Table[?] presents the 15 largest firms by number of markets served (a measure of the size of the network) in the first period (1999), and the number of markets newly entered and exited by each firm. Markets newly entered are defined as markets served in 2000 and not in 1999. Markets served in 1999 and 2000 are also markets entered but not newly entered. By the same way we define markets exited as markets operated in the first period but not in the second.

Table[?] shows that nearly all firms adopt simultaneously entry and exit behavior. This is consistent with the idea of important differences in airline profitability across markets, differences that can't be entirely explained by markets and demand characteristics. Airlines seem to be heterogeneously suited to serve different markets.

There are 2051 markets in the data set, of which 736 are not served by any airline of our sample. The maximum number of airlines serving a route is 4. We have classified markets by density (in AS/day) observed in 1999. One of the relevant issues in presenting the descriptive statistics is whether demand size alone determines market structure (Bresnahan and Reiss[9]).

Table[?] shows that the average number of carriers in each market is clearly an increasing function of demand size. This means that demand size is an important factor that determines the market structure. Within the same class of density, the variation in the number of airlines across markets tends to show that demand size is not the only factor that explains market structure.

Another analogue analysis is to see if there's a correlation between market structure and route distance.

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1. Density is commonly measured on total passenger carried per day (Pax/day). Due to the supply nature of OAG database, we opt to this factor which is highly correlated with pax/day.

2. The classification is made by considering the airlines shares in the intra-european total available seat kilometers (ASK).

Airline's code	Airline's name	Markets served	Markets entered	Markets exited
LH	Lhufthansa	307	86	34
DE	Condor	214	21	13
AF	Air France	169	54	13
SK	Scandinavian Airlines	162	17	15
AZ	Alitalia	161	15	20
BA	British Airways	147	48	15
IB	Iberia	143	22	22
AY	Finnair	80	3	8
OA	Olympic Airways	72	8	1
SN	Sabena	67	32	9
KL	KLM	62	2	7
TP	Tap air Portugal	54	8	6
EI	Air lingus	44	4	4
FR	Ryanair	34	1	0
OS	Austrian Airlines	31	23	1

TAB. 4 – Number of markets, markets entered and exited in the sample by airline

Number of airlines	Density (AS/day)			
	[20,75[	[75,225[	[225,625[	$\geq 625$
0	349	220	127	40
1	178	245	259	254
2	10	38	89	185
3	2	1	10	42
4	0	0	0	2
Average	0.38	0.64	0.96	1.45

TAB. 5 – Distribution of airline's number by density



Number of airlines	Distance (Kilometers)			
	[150,400[	[400,700[	[700,1300[	$\geq 1300$
0	200	199	134	203
1	245	268	208	215
2	30	77	113	102
3	10	14	16	15
4	1	1	0	0
Average	0.70	0.84	1.02	0.87

TAB. 6 – *Distribution of airline's number by distance*

Table[?] shows the average number of carriers in each market by distance, which is similar across airport-pair of different distance. Nevertheless, airlines seem to be relatively more interested in operating medium-haul markets than others.

Another relevant issue in presenting descriptive statistics is to see the distribution of newly entered and exited markets across demand size and distance.

	Distance (Kilometers)								Total	
	[150,400[		[400,700[		[700,1300[		$\geq 1300$			
Density	Entry	Exit	Entry	Exit	Entry	Exit	Entry	Exit	Entry	Exit
[20,75[	18	5	25	5	11	11	11	15	65	36
[75,225[	14	14	24	18	30	13	21	13	89	58
[225,625[	19	10	22	6	23	10	22	14	86	40
$\geq 625$	25	8	53	6	17	13	9	7	104	34
Total	76	37	124	35	81	47	63	49	344	168

TAB. 7 – *Distribution of markets newly entered and exited across density and distance*

Table[?] shows that in medium-haul markets with high density, airlines are relatively more dynamic. The number of airlines markets newly entered is maximum for the range of distance between 400 and 700 km with high demand size(53). For the same range of density, this number is relatively stable for other markets distances with a weakness for long-haul markets. Concerning the three first quartile of density, the number of markets newly entered is globally stable across distances

with relative airlines preference for medium-haul markets. If we look for the variation of this number across different demand size markets within a predefined class of distance, we see that market's density plays a positive role specially for medium-haul markets.

The number of exited markets remains globally stable across different markets distance within the same range of demand size except for low density markets. This could be explained by the fact that for long-haul markets, carriers prefer operate high capacity aircraft to benefit from economies of density and balance the increase in total cost of operating longer markets. Low density markets, involve in this case a high unit cost which could give rise to loss profit and firm's exit. If we fix distance and vary demand size, we observe that the number of markets exited remains stable with a small decrease for high density markets.

### 3.2 Model and Specifications

As mentioned in the section2, to come over the problem of multiplicity and to guarantee the equilibrium uniqueness , one should use a sequential move asymmetric information game. Within the framework of domestic air transport within the European union, the sequential move doesn't appear to be a realistic assumption. An appropriate empirical model of oligopoly market structure must estimate simultaneously the decision of all carriers. Due to the fact that the simultaneous move asymmetric information entry game becomes computationally intractable for more than 2 firms, we decide to opt for the simultaneous move complete information entry game. To deal with the problem of multiplicity of equilibria, we adopt the same approach as berry (1992).

The game is specified under a strategic form where the set of firms is noted  $S = \{1, \dots, K\}$ , the space of firm's pure strategies is noted  $\mathcal{A}_k$  and the profit function of firm  $k$  in market  $i$  is noted  $\pi_{ik}$ . We also note the total number of markets in the sample by  $M$ .

The game is played in two stage. During the first stage, each firm decides if it will enter the market  $i$  or not. So we have  $\forall k \in S, \mathcal{A}_k = \{0,1\}$  where 1 codes the firm's entry decision and 0 the decision of not serving the market. So specified as this, the

model doesn't, *a priori*, differentiate between new entry decision and entry decision or between exit decision and the decision to stay out of the market. In the second stage, firms play some game, for example here we consider a Cournot competition game, that yields to respective profits. For a market  $i$ , the equilibrium is a  $K$  by 1 vector of ones and zeros noted  $\mathbf{a}^* = (a_1^*, \dots, a_K^*)$ .

A pure strategy equilibrium is an equilibrium in which all firms that enter make positive profit and all firms that don't expect negative profit from entry. More formally and for a given market  $i$  this conditions can be written as follows:

$$\forall k \in S, a_k^* \pi_{ik}(\mathbf{a}^*) \geq 0 \text{ and } (1 - a_k^*) \pi_{ik}(\mathbf{a}^{*+k}) \leq 0$$

where  $\mathbf{a}^{*+k}$  is an identical vector as  $\mathbf{a}^*$  in which the  $k^{th}$  component  $a_k^{*+k} = 1$ .

To guarantee the existence of the equilibrium, some further assumptions must be done. First, profits are supposed to be a decreasing function of entry decisions of other firms. This assumption doesn't seem to be economically unrealistic since it supposes decreasing profits with competition intensification. Second, the firms are supposed to be ranked by profitability's level and this order of ranking is supposed to be exogenous and so independent of the actual taken entry decisions. To impose this latter condition, we assume like berry[?], that firms characteristics can be aggregated into a single index of profitability,  $\phi_{ik}$ , that varies across firm  $k$  and market  $i$ . Thus, at least one equilibrium will exist. If we order firms by decreasing profitability and let them enter the market in this order until the next firm entering makes loss, by construction all firms entering make positive profit and all other firms would not. This equilibrium is not unique, we can often skip the last entrant for example and go to the following firm making positive profit instead. Thus this model gives arise to multiplicity of equilibria.

To come over this indeterminacy, Berry proposes a structure on profit function that ensures a unique number of firms for each equilibrium. Thus and like in Bresnahan and Reiss [9], one can model and estimate the number of firms at the equilibrium instead of the identity of entrants. We adopt a profit functional form compatible with Cournot competition. So the post entry profit's variable component for a market  $i$  is identical across firms. Firms profits differ only by their fixed part  $\phi_{ik}$  so we have:

$$\pi_{ik}(\mathbf{a}) = v_i[N_i(\mathbf{a})] + \phi_{ik}$$

In this specification of profit function, the characteristics of other firms enter only indirectly through  $N_i$  but not directly. So, the firm's profit is invariant to opponents entry decision permutations.

Berry proved that all equilibria of such a model share the same number. The number of firms at the equilibrium can be obtained by ordering decreasingly  $\phi_{ik}$  and letting firms enter in this order until next firm non positive profit. Formally:

$$\text{order } \phi_{i1} > \phi_{i2} > \dots > \phi_{iK}$$

$$N_i^* = \max_{0 \leq n \leq K} \{n : v_i(n) + \phi_{in} \geq 0\}$$

.

### 3.3 Exogenous Variables

The exogenous variables in our analysis are chosen based on their capacity to impact the airlines post-entry profitability in each market. We characterize these exogenous variables into three different categories: market characteristics (i.e, market density and distance), airlines pre-existing airport and airline market presence and finally competition and concentration in the market and the endpoint airports. These variables are detailed below:

1. **Markets characteristics:** Low cost carriers are known for choosing short and medium haul markets with sufficient traffic density to support high frequency and point to point service. Due to the nature of their fleet, classical carriers, could be interesting in serving dense long-haul markets with high capacity aircraft to benefit from cost advantages. Thus, density and distance are important market profitability determinants. Our measure of density is the variable **dense** and is defined as the average number of daily available seat in the market carried by all carriers in 1999. Distance **dist** measures the non stop distance between endpoint airports of the market. The two measures **dense** and **dist** are expressed on 10 thousands unit.

2. **Airline's Market/Airport presence:** We believe that an airline might be more likely to enter markets if it already carries a high proportion of the passengers in that market. Conversely, airlines are more likely to exit markets where their market's share is too low to support viable commercial operations. That's why we include **carriershare**, the airline's share of the total available seats in each market. The estimated coefficient of this variable is expected to be positive.

The air transport's literature stressed the impact of network strategies on traditional hub and spoke carriers entry decisions. To consider this network externalities factor, we include several variables that reflect airlines pre-entry presence at endpoint airports of each market. **Max(carrierairport)** and **min(carrierairport)** are the numbers of destination airports served by the carrier from each of the endpoint airport of market, sorted from largest to smallest. If airlines are engaged in the formation of intra-european hub and spoke type network, we could expect a strong explanatory power of these variables. Another feature pointed by Berry[8] is that entry is more likely to occur when the carrier already serves one or both of the endpoint airports. To measure carrier's airport presence, we include **max(airportshare)** and **min(airportshare)** which are the carrier's share of the total available seats in each endpoint airport sorted from largest to smallest.

3. **Market/Airport competition and concentration:** In our models, the post-entry competition is captured through the decreasing profit function on the endogenous equilibrium number of entering firms. The pre-entry state of competition and concentration in each market is captured through several exogenous variables. Thus we include, **markethhi**, the herfindhal-Hirschman Index (HHI) of available seats for each market. A higher HHI index indicates a more concentrated market structure and a potentially less competitive environment. We also include **max(airporthhi)** and **min(airporthhi)**, which are concentration levels measured (in term of carrier's available seat at airport) at the endpoint airport, sorted from the largest to the smallest. For the computation of all hhi index, the airlines market share are excluded so as to measure the competitive pressure from rivals.

All these exogenous variables, have been measured for the first period of the games i.e 1999. To estimate the model below, exogenous variables must be differentiated into market specified variables and airlines observed heterogeneity variables. The market specified variables are identical to variables describing **markets characteristics**. Airlines observed heterogeneity variables are variables that belong to the following categories of exogenous variables: **Airline's Market/Airport presence** and **Market/Airport competition and concentration**.

Summary statistics for the variables described above for the full sample of markets is presented in table 8.

Variable's name	Variable's definition	Mean (Std.Dev)
<b>dense</b>	Market's density in 10 thousands available seats	0.0593 (0.106)
<b>dist</b>	Market's non stop distance in 10 thousands Km	0.0961 (0.0751)
<b>carriershare</b>	Airline's market share of available seats (between 0 and 1)	0.03 (0.17)
<b>markethhi</b>	HHI index of available seats of the market (between 0 and 1)	0.73 (0.28)
<b>max(airportshare)</b>	The largest carrier's share of total available seats at endpoint airports (between 0 and 1)	0.06 (0.14)
<b>min(airportshare)</b>	The lowest carrier's share of total available seats at endpoint airports (between 0 and 1)	0.01 (0.06)
<b>max(airporthhi)</b>	HHI index of available seats for the more concentrated of the endpoint airports (between 0 and 1)	0.39 (0.22)
<b>min(airporthhi)</b>	HHI index of available seats for the less concentrated of the endpoint airports (between 0 and 1)	0.2 (0.1)
<b>max(carrierairport)</b>	Maximum of the number of airports served by airlines at the endpoint airports of the market	4.8 (11.6)
<b>min(carrierairport)</b>	Minimum of the number of airports served by airlines at the endpoint airports of the market	0.63 (1.9)
<b>M</b>	Number of sample markets	2051

TAB. 8 – Variable definitions and descriptive statistics

### 3.4 Estimation and results

To estimate berry's entry models we use a parametric specification form for  $v_i(N)$  and  $\phi_{ik}$ . We write the profits portion that is common to all firms as:

$$v_i(N) = X_i\alpha - \delta \ln(N) + \rho u_{i0} \quad (1)$$

where  $X_i$  is a vector of market characteristics,  $N$  is the equilibrium number of firm, and the vector  $\alpha$ ,  $\delta$  and  $\rho$  are parameters to be estimated.  $u_{i0}$  represents characteristics of the market that are observed by the firms, but not by the econometrician.

The firm specific portion of profits is specified as:

$$\phi_{ik} = Z_{ik}\beta + \sqrt{1 - \rho^2} u_{ik} \quad (2)$$

where  $Z_{ik}$  is a vector of observed firm characteristics, the vector  $\beta$  and  $\sigma$  are parameters to be estimated.  $u_{ik}$  represents firm characteristics in market  $i$  observed by all the firms, but not by the econometrician.

Equation 2 assumes that fixed costs depend on both observed and unobserved firm characteristics.

Under these assumptions, the profits of firm  $k$  in market  $i$  are written:

$$\pi_{ik}(N) = X_i\alpha - \delta \ln(N) + Z_{ik}\beta + \rho u_{i0} + \sqrt{1 - \rho^2} u_{ik} \quad (3)$$

We assume that  $u_{ik}$  and  $u_{i0}$  are distributed i.i.d standard normal across firms and markets. For a given market, the correlation between firms is  $\rho^2$ . The term  $\sqrt{1 - \rho^2}$  ensures an unobservable variance of 1 which allows us to handle the problem of non identification of the units of profits.

Berry[8] noticed that in the general case, outcomes likelihood are difficult to calculate due to, first, the non rectangular region of integration and second to the increasing complexity in  $K$ . To manage these difficulties, one can either make restriction and estimate special models or build a simulator estimator to estimate the full model.



The different possible restrictions are:

1.  $\rho = 1$  and  $\beta = 0$ : it is a traditional entry model with large number of equally potential entrants. There is no heterogeneity in this model. This model gives an idea of the maximum number of firms the market can support
2.  $\rho = 1$  and  $\beta \neq 0$ : in this model there is no unobserved airline heterogeneity, all heterogeneity is observed.
3.  $\rho = 0$ : In this model there is no correlation between unobservables across firms. We are restricted to consider the outcomes  $N = 0$ ,  $N = 1$  and  $N = 2$ .

Table 9 presents results of these three special cases which are amenable to maximum likelihood estimation.

**The SMM estimation of the full model is in progress.**

## 4 Conclusion

In this paper, we estimated models of entry allowing firms heterogeneity and declining profit with entrant firms number. We built estimations to compare the impact of classical determinants as density and distance, to firm characteristics determinants. We find that classical determinants are powerful predictors of equilibrium number of firms but they do not explain all the firms decisions. All the exogeneous variables representing heterogeneity are significative, but their respective parameter level varies with the adopted model. The level of profit's decrease depends on the type of retained model. We tried to estimated the full model and compare results with the models above but the simulation procedure takes time and it is still in progress.

Variable's name	No heterogeneity	Only observed heterogeneity	No Correlation(Std.Dev)
<b>cste</b>	0.022 (0.50)	0.13 (1.60)	-1.39 (-11.50)
<b>dense</b>	4.61 (21.90)	2.16 (10.77)	0.77 (4.78)
<b>dist</b>	1.69 (5.00)	-0.90 (-11.19)	-0.68 (-2.40)
<b>carriershare</b>	–	1.41 (13.90)	1.89 (10.89)
<b>markethhi</b>	–	-1.26 (-15.31)	-0.95 (-10.29)
<b>max(airportshare)</b>	–	0.013 (3.44)	1.10 (5.24)
<b>min(airportshare)</b>	–	0.81 (6.92)	1.63 (4.05)
<b>max(airporthhi)</b>	–	-0.31 (-3.42)	-0.15 (-1.05)
<b>min(airporthhi)</b>	–	-1.10 (-14.51)	-1.29 (-2.46)
<b>max(carrierairport)</b>	–	0.013 (13.21)	0.02 (9.47)
<b>min(carrierairport)</b>	–	0.09 (12.84)	0.12 (9.88)
$\delta$	2.27 (48.92)	1.19 (27.84)	0.11 (1.96)

Standard deviation between parentheses

TAB. 9 – *Maximum likelihood results*

## List of figures

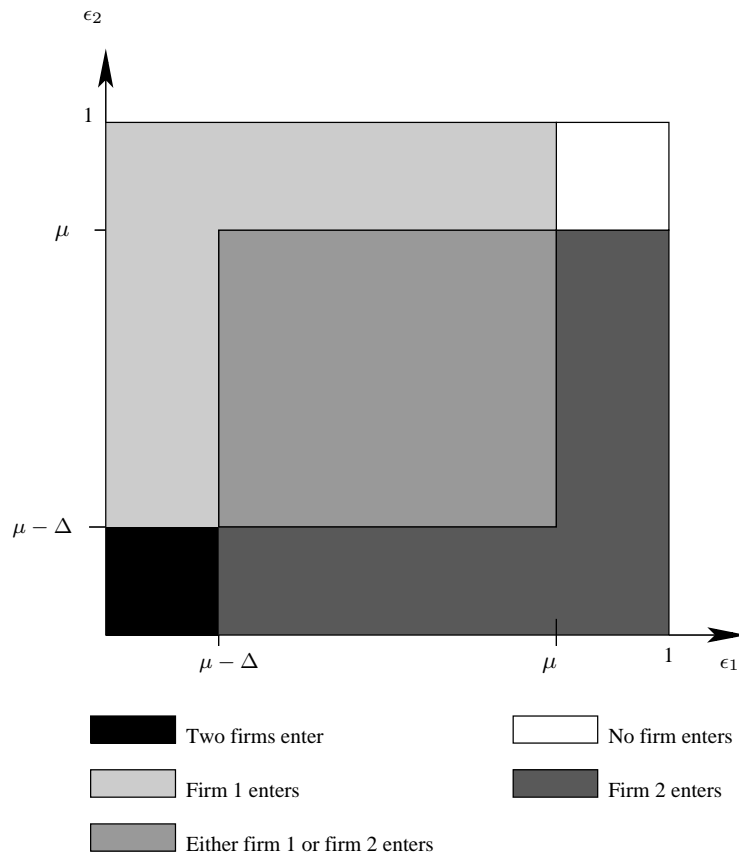


FIG. 1 – *Simultaneous complete information game*

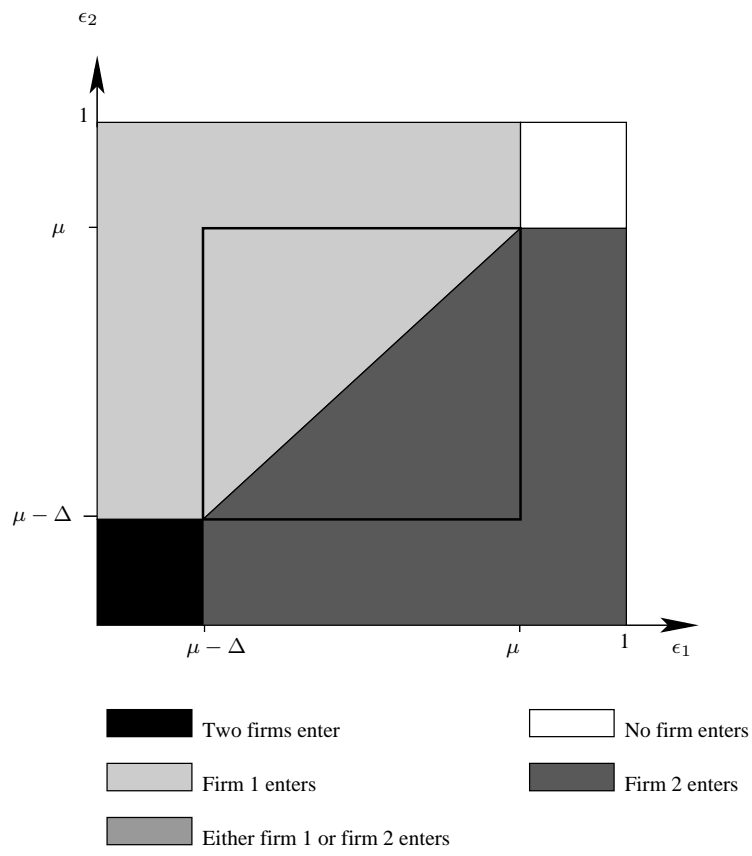


FIG. 2 – *Sequential complete information game*

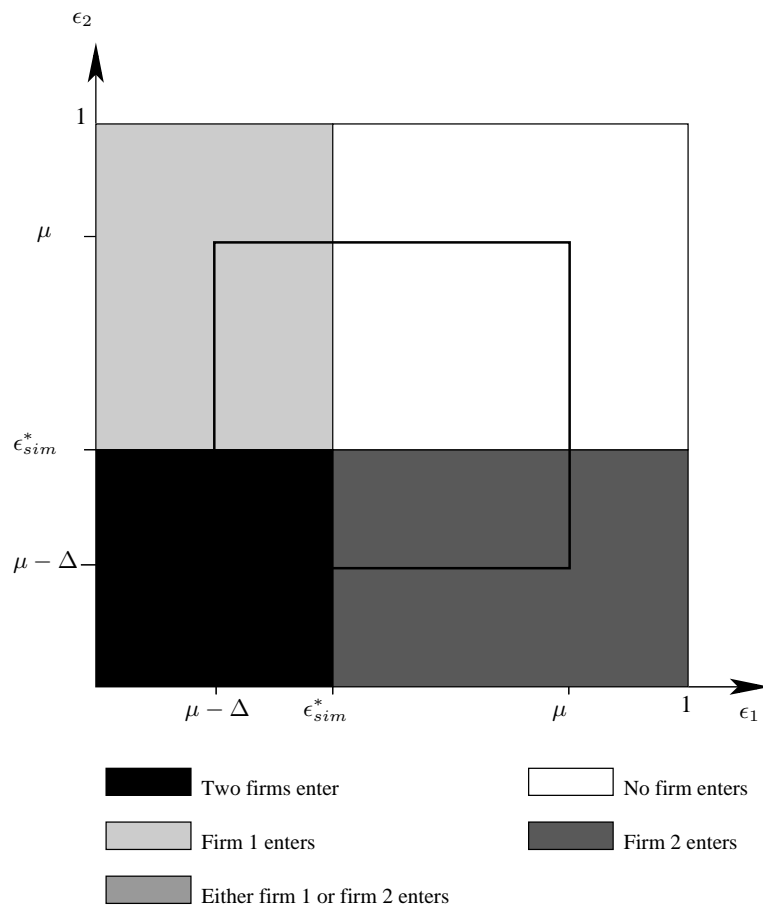


FIG. 3 – *Simultaneous asymmetric information game*

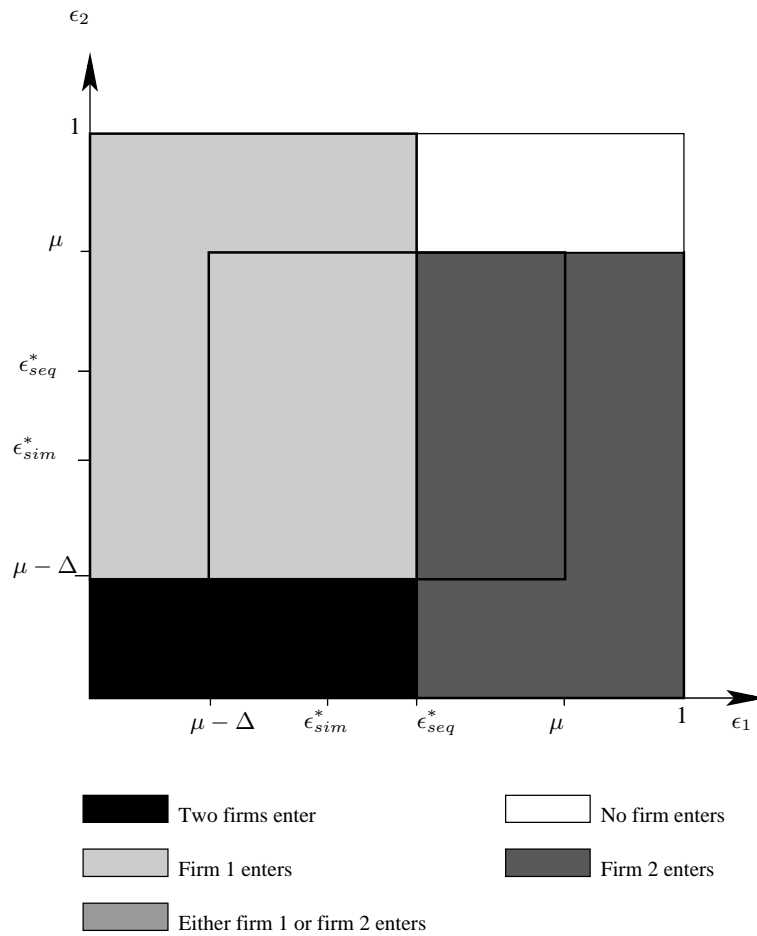


FIG. 4 – *Sequential asymmetric information game*

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