Airport Investment Risk Assessment under Uncertainty

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***Abstract*—**The construction of a new airport or the extension of an existing one requires massive investments and many times public private partnerships were considered in order to make feasible such projects. One characteristic of these projects is uncertainty with respect to financial and environmental impacts on the medium to long term. Another one is the multistage nature of these types of projects. While many airport development projects have been a success, some others have turned into a nightmare for their promoters.

This communication puts forward a new approach for airport investment risk assessment. The approach takes explicitly into account the degree of uncertainty in activity levels prediction and proposes milestones for the different stages of the project for minimizing risk. Uncertainty is represented through fuzzy dual theory and risk management is performed using dynamic programming. An illustration of the proposed approach is provided.

***Keywords*—**Airports, fuzzy logic, risk, uncertainty.

I. Introduction

AIRPORTS are a paramount piece of the global infrastructure puzzle, with a multiplier economic, social and environmental impact at national, regional and international level. In a highly volatile and uncertain economic environment, airports must be capable to attract sufficient revenues to finance their operations and investments while maintaining a satisfactory quality of service for both their primary clients: airlines and passengers, and also maintaining its role of economic driver supporting in a sustainable manner its local community.

Airports are asset-intensive businesses that require extensive amount of time to recover the significant financial investments in the specific infrastructure, like runways, terminals. This aspect forces airports investors to make strategic moves and to carefully calculate the risks before taking investment decisions. The highly deregulated and liberalized air transportation market determined airports to adopt a more business like operational approach, focusing on non-aeronautical activities as a strategy to achieve self-reliance and financial independence which will allow them to develop in accordance with the market needs. This process of airport commercialization shifted the focus towards the passenger as the ultimate beneficiary of airport infrastructure.

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In the last decades, airports evolved from being simply infrastructure elements to business oriented service providers, pressured to operate in an optimal manner. They proved to be flexible in turbulent economic times, proving they had the capability to meet the needs of the air transportation industry, sector that has known a sustained high rate of growth of approximately 5% annually in the last decades even through global economic disturbances, with more than 3 billion passengers transported in 2013 [1].

The structure of the article is as follows: Section II gives a concise formulation of the long term airport planning problem with emphasis of the financial aspects and uncertainty degree, Section III details the risks airports are exposed with particular interest on their financial impact, in Section IV is presented the adopted airport planning context, in section V is proposed a mathematical model to address airport investment risk assessment and in Section VI a fuzzy dual dynamic programming approach is discussed to tackle the considered airport case study. Final conclusions are presented in Section VII.

II. The Long Term Airport Planning Problem

As the world economy is slowly recovering from the most powerful economic downturn, the air transport industry will continue to grow steadily on the long run. Since demand in air transportation sector is highly impacted by economic activity it is expected that the industry will recover its sustainable growth.

Airport long term planning has at its core the following objectives: optimized infrastructure development costs and functionality, optimized economic and operational performance and a high degree of flexibility in order to integrate all the shifts in demand and potential disturbances accordingly to the airport future needs and level of growth. The new business culture concepts that airports need to embrace include strong air service competitor advantages, capability of taking long term risks, adopting the stakeholder collaborative decision making culture, diversifying the revenues sources and most of all putting the passenger at the core of the business.

The construction of a new airport or the extension of an existing one requires huge investments and many times public private partnerships were considered in order to make feasible such projects. One characteristic of these projects is uncertainty with respect to the financial and environmental impacts on the medium to long term. Another one is the multistage nature of these types of projects. While many airport development projects have been a success, like Munich Airport [2], some others have turned into a nightmare for their promoters like the ghost airport of Ciudad Real, Spain.

Airports were traditionally seen as the responsibility of governments to manage and operate, typically in line with strategic economic and defense policies [3]. In the more recent economic environment, a paradigm shift occurred were private stakeholders emerged as investors evolving from decision makers in airport planning and development to full owners and operators. Privatization of airports emerged as the tool "to go to" for governments looking for strategies to make the local aviation market more dynamic and to achieve their long term planning goals when the costs of funding new infrastructure or maintaining the existing one exceeds their resources. The privatization of airports makes for a "fuzzy" governance "space" where different governance modes intersect and overlap as noted in [3].

The long term airport planning process is a complex endeavor due to the intricacies of the airport system, stakeholders involved and the significant degree of uncertainty. In a highly volatile economic context the planning process needs to be constantly adjusted to the realities of the market the airport will serve. Notions like "demand" and "capacity" need to be rethought in order to accurately compute the operational parameters of the future airport. Overall, we need to acknowledge the fact that long term airport planning is a multibillion business investment requiring a systemic and flexible approach.

The demand for air transport services has risen much faster than demand for most other goods and services in the world economy. Since 1970, air travel demand, measured by Revenue Passenger Kilometers flown (RPKs) has risen 10 fold compared to a 3-4 fold expansion of the world economy. Air cargo demand, both reflecting and facilitating the globalization of business supply chains and economies generally, raised 14 fold [4].

An economically sustainable industry has to cover the costs of operations and provide a reasonable return on investment so that capital can be renewed [5]. The airport sector had a substantial growth in annual investment, from USD 308 billion in 2009 to USD 463 billion in 2011, representing an important 36% of the total investment in the aviation value chain for 2011[4]. According to Airports Council International 2011 Annual Report, total revenue for airports worldwide was $102 billion in 2011 [6].

III. Airport Risk Analysis

The risks airports are continuously facing due to the highly dynamic environment they are exposed to, can be categorized as exo-industry and endo-industry risks.

The main exo-industry risks are:

1) Volatility of the economic environment with major market shifts: The traditionally strong and robust North American and European markets have become stagnant while emergent Asian and Latin American markets are soaring. Air traffic evolution follows economic trends.
2) Political policy and regulation regarding environment, taxation, security regulations, and bilateral and open skies agreements, all have the potential to be a major constraint for future airport development.
3) "Black swans" are events or occurrences that deviate beyond what is normally expected of a situation and that would be extremely difficult to predict. This term was popularized in [7]. The following events can be categorized as such: the terrorist attacks of September 2011, the SARS outbreak (2003), the Indian Ocean tsunami (2004), Hurricane Katrina (2005), the global financial crisis (2008), the volcanic eruption of Eyjafjallajökull (2010).
4) Social and cultural aspects have a powerful impact on local communities. Public awareness on aviation environmental impact, the prevalence of Internet video conferencing over business travel, the living standard, all these factors impact decisively the propensity to fly.

The main endo-industry risks are:

1) The airport performance is strongly dependent on airline operations. Airports are impacted by the operational, financial and overall business models of airlines (legacy, low-cost, start-up). To all these aspects the trending airlines alliance model can rapidly turn from an opportunity or strength, to a weakness or a threat, depending on the context the airport finds itself in. Powerful alliances offer to the airport the opportunity to reach a larger and more diverse market but also internal instability within an alliance can significantly complicate airport future development plans. In conclusion, airports should take all the necessary steps to minimize the disruptions to which the airline industry is exposed to.
2) The emergence of private investors in the airport market, ranging from partial privatization to full ownership and operation, brings a new degree of uncertainty and risk to the system due to the complexity of investor variety and to the fact they no longer see airports as a very secure and profitable endeavor, compared with the pre-financial crisis era.
3) Airport competition is emerging as a serious pressure point in the industry with more visibility between primary and secondary airports and even more pronounced for cargo airports;
4) Technological advancements determine airports to adjust their infrastructure in order to keep up with the new aircrafts which gain popularity in a far more accelerated pace than the specific airport infrastructure (Airbus A380, Airbus A350, Boeing 787). Also major operational improvements like A-CDM (Airport – Collaborative Decision Making), SESAR (Single European Sky – ATM Research) or NextGen are pushing airports forward in terms of infrastructure and operational advancements.
5) Forecasting errors, statistical and modeling errors, misinterpretation of data, errors in the data, are adding to the overall error margin for mid and long term forecasting.

In this context, airport development projects are exposed to a very complex and dynamic environment, characterized by a significant degree of uncertainty and risk.

IV. Adopted Airport Planning Context

The starting point of any airport planning project and its financing is the potential demand forecast and its evolution. The forecast generally covers the time horizon of the project and includes potential demands for the annual volumes of international and domestic scheduled and non-scheduled passenger, freight and aircraft movements. Also, daily and monthly traffic distributions are required in order to identify traffic trends and peaking patterns along with the fleet mix. Of paramount importance is the integration of uncertainty in demand forecasting since the decisions taken at a specific step of the development plan can have a long term impact over the general outcome of the project.

Long term airport planning can expand up to 20 years as a time horizon with a proposed six months incremental milestone in order to accurately monitor the progress of the development project. In this way, an important degree of adaptability is insured which will allow airport planers to take better informed decisions over a more controllable time frame with far more reduced uncertainty degree.

Let the level of predicted potential demand for traffic type i along the planning horizon k be given by: $D_k^i, i \in I, k \in \{1,2,...,K\}$, where I is the set of traffic activities.

The necessary aircraft traffic to cope with a demand level is given by:

$$T_k^i = D_k^i / (S_k^i \alpha_k^i) \qquad (1),$$

where T_k^i, D_k^i, α_k^i are real numbers. Here S_k^i is the mean capacity of aircraft type i at time k corrected by the mean load factor α_k^i. The rates of return, r_k^i associated with the traffic of type i at time k, depend on the investments made until that period. The potential airport passenger processing capacity is written C_k^{Pi} and the available potential aircraft movements processing capacity is written C_k^{Ti}. Then the actual level of demand of type i at period k:

$$\bar{D}_k^i = \min\{D_k^i, C_k^{Pi}, S_k^i \alpha_k^i C_k^{T_i}\} \qquad (2)$$

Let L_i be the number of candidate upgrades which can be performed for traffic type i at the considered airport. Let θ_l^i be the period at which upgrade l for traffic type i is planned to be done. When a project is retained, the corresponding value θ_l^i is within the set {1, 2,…,K} and when it is not retained θ_l^i =K+1, $l \in$ {1,2,…, L_i }.

Technical considerations impose, in general, sequence constraints, so it is supposed that constraints such as:

$$\exists l, l' \in \{1, \cdots, L_i - 1\}, i \in I : \theta_l^i \le \theta_{l'}^i \qquad (3)$$

can be encountered. Also there are exclusion constraints such as if project l is retained. A set of concurrent or contradictory projects will be dismissed:

$$\theta_l^i \in \{1, 2, ..., K\} \Rightarrow \theta_{l'}^i = K + 1, l' \in \Lambda_i \subset \{1, \cdots, L_i\} \qquad (4)$$

Since the different types of traffic make use of common resources in the airport, global capacity constraints must be satisfied. Let Δ_k be the set of projects which have been retained until period k so the corresponding capacities are: $C_k^{Pi}(\Delta_k)$ and $C_k^{T_i}(\Delta_k)$.

$c_l^{ik}(\Delta_k)$ is defined as the cost of upgrade l when performed at period k and $r_k^i(\Delta_k)$ represents the rate of return at period k for traffic type i.

V. Mathematical Formulation and Solution Approach

The adopted strategy develops at first a deterministic approach which leads to the formulation of an optimization problem. Then the parameters and variables subject to uncertainty are pointed out and a fuzzy-dual based model of their uncertainty is established. Finally a fuzzy dual formulation of the airport planning problem is proposed.

A deterministic formulation of the optimal programming problem associated to airport planning can be such as:

$$\max_{\theta_l^i} \quad \pi([\theta_l^i], l \in \{1, \cdots, L_i\}, i \in I \qquad (5)$$

under constraints (3) and (4).

Here the current return is given by:

$$\pi([\theta_l^i], l \in \{1, \cdots, L_i\}, i \in I = \sum_{i \in I} \left(\sum_{k=1}^{K} \left(\frac{r_k^i(\Delta_k)}{(1+\rho)^k} \bar{D}_k^i \right) - \sum_{\substack{l=1 \\ \theta_l^i \le K}}^{L_i} \left(\frac{c_l^{ik}(\Delta_k)}{(1+\rho)^{\theta_\rho^i}} \right) \right) \qquad (6)$$

where ρ is the financial rate of actualization.

Here it is considered that uncertainty regarding the actual levels of demand, the rates of return and the upgrade costs can be represented by fuzzy dual numbers. A fuzzy dual number $a + \varepsilon.b$ is composed of a likely value a and a degree of uncertainty b, ε representing the pure dual number such that $\varepsilon^2 = 0$ [8]. Then let the fuzzy dual representations of the actual levels of demand, the rates of return and the upgrade costs be given by:

$$r_k^i(\Delta_k) = r_k^{iL}(\Delta_k) + \varepsilon\, r_k^{iD}(\Delta_k) \qquad (7)$$

$$\bar{D}_k^i = \bar{D}_k^{iL} + \varepsilon\, \bar{D}_k^{iD} \qquad (8)$$

$$c_l^{ik}(\Delta_k) = c_l^{ikL}(\Delta_k) + \varepsilon\, c_l^{ikD}(\Delta_k) \qquad (9)$$

where the likely components are indexed by L and the dual components are indexed by D. In many situations, the likely components can be associated with mean estimated values while the dual components can be associated with the corresponding standard deviations.

Then the expression of the fuzzy dual return is given by:

$$\pi([\theta_l^i], l \in \{1,\cdots,L_i\}, i \in I) = \pi^L([\theta_l^i], l \in \{1,\cdots,L_i\}, i \in I) + \varepsilon\, \pi^D([\theta_l^i], l \in \{1,\cdots,L_i\}, i \in I) \quad (10)$$

where

$$\pi^L([\theta_l^i], l \in \{1,\cdots,L_i\}, i \in I) = \sum_{i\in I}\left(\sum_{k=1}^{K}\left(\frac{r_k^{iL}(\Delta_k)}{(1+\rho)^k}\bar{D}_k^{iL}\right) - \sum_{\substack{l=1\\ \theta_l^i \le K}}^{L_i}\left(\frac{c_l^{ikL}(\Delta_k)}{(1+\rho)^{\theta_\rho^i}}\right)\right) \quad (11)$$

and

$$\pi^D([\theta_l^i], l \in \{1,\cdots,L_i\}, i \in I) = \sum_{i\in I}\left(\sum_{k=1}^{K}\left(\frac{r_k^{iL}(\Delta_k)\bar{D}_k^{iD} + r_k^{iD}(\Delta_k)\bar{D}_k^{iL}}{(1+\rho)^k}\right) - \sum_{\substack{l=1\\ \theta_l^i \le K}}^{L_i}\left(\frac{c_l^{ikD}(\Delta_k)}{(1+\rho)^{\theta_\rho^i}}\right)\right) \quad (12)$$

Now, the optimal programming problem associated to airport planning which takes into account the level of uncertainty can be formulated as:

$$\max_{\theta_l^i} \quad \pi^L([\theta_l^i], l \in \{1,\cdots,L_i\}, i \in I) \quad (13)$$

under constraints (3) and a global uncertainty level constraint such as:

$$\pi^D([\theta_l^i], l \in \{1,\cdots,L_i\}, i \in I) \le \Delta\pi_{\max} \quad (14)$$

where $\Delta\pi_{\max}$ represents the maximum level of uncertainty.

Observe here that the solution of problem (13) with (3), (4), and (14) is not straightforward since the actual levels of demand and their associated degree of uncertainty are dependent of the timing and size of investment realizations (see (2)).

When solving this problem, the investment will be considered safe if:

$$\pi^{L*}([\theta_l^i], l \in \{1,\cdots,L_i\}, i \in I) > \pi^{D*}([\theta_l^i], l \in \{1,\cdots,L_i\}, i \in I) \quad (15)$$

A risk level, r, can be attached to the solution:

$$r = \begin{cases} 0 \quad if \quad \pi^{L*} - \pi^{D*} > 0 \\ 100 \cdot (\pi^{L*} - \pi^{D*}) / (2\,\pi^{D*}) \quad if \quad \pi^{L*} - \pi^{D*} < 0 \le \pi^{L*} \\ 100 \cdot (1 - (\pi^{L*} + \pi^{D*}) / (2\,\pi^{D*})) \quad if \quad \pi^{L*} < 0 \le \pi^{L*} + \pi^{D*} \\ 100 \quad if \quad \pi^{L*} + \pi^{D*} < 0 \end{cases} \quad (16)$$

VI. Case Study and Fuzzy Dual Dynamic Programming Approach

For the numerical illustration the case of a regional airport expected to gain an international position has been considered. Mean potential passenger demand is supposed to double every eight years with an initial traffic of 300,000 passengers per year while mean cargo potential demand is supposed to double every five years. The airport has been supposed to be managed under a BOT agreement (Build – Operate – Transfer) over a period of thirty years. In this situation, the BOT project financing involves a private entity which has received a concession from the public sector to finance, design, construct, and operate the complex of airport infrastructure facilities, according to the concession contract. The financial risk of the concessionaire is to not be able to recover its investment, operating and maintenance expenses in the project. In this type of situation, the project proponent is facing a significant amount of risk that needs to be assessed and mitigated.

The project is composed of three main phases:

1) An initial phase where the existing runway and terminal are renewed.
2) A second phase where airport air traffic control tower and related equipment are upgraded, the length of the runway is augmented while passenger and cargo terminals capacities are increased.
3) A third phase where a new runway and a new passengers and cargo terminals are built.

In each phase, a preplanning of airside and landside facilities is necessary. It has been supposed that no new land acquisition is necessary to perform the proposed plan. The fuzzy dual formalism has allowed considering three scenarios with respect to each type of demand (low, medium and high).

This has led to a planning problem with 20 decision variables including timing and size of subprojects resulting in a set of rather small scale optimization problems.

In this case, to solve problem (13), (3), (4), and (14) with (2), Dynamic Programming has been considered since as stated in [9] "Dynamic Programming is a mathematical technique for making a sequence of interrelated decisions, providing a systematic procedure for determining the optimal combination of resources". Dynamic Programming builds an optimal solution step by step by considering that any partial solution up to any intermediate stage must be optimal to that stage to be a candidate part for the global solution (Bellman principle). So, only optimal ways to reach each possible state at each stage are maintained in the search process. The only mathematical condition for applicability of Dynamic Programming is the separation of the objective function and of the level constraints with respect to the decision variables, and then Dynamic Programming may produce an exact solution through a rather efficient computational process even for combinatorial problems. Many different approaches to make use of Dynamic Programming (direct or reverse Dynamic programming) and extensions (stochastic Dynamic Programming, Fuzzy Dynamic programming) have been developed to face different characteristics of sequential decision making. Fuzzy dual programming has been introduced recently [10] to provide a general framework for dealing with uncertainty approached through the fuzzy dual formalism. The paradigm of Dynamic Programming has been extended to this situation by adopting the comparison operators between fuzzy dual numbers detailed in the appendix.

VII. Conclusions

This communication after analyzing the long term airport planning problem has developed a new approach for airport investment risk assessment. This approach takes explicitly into account the degree of uncertainty in the prediction of activity levels while proposing milestones for the different stages of the project for minimizing risk. Uncertainty is represented through fuzzy dual theory which allows limiting problem complexity as well as the computational burden of its solution. Here risk management is performed using a fuzzy dual extension of dynamic programming and the applicability of the proposed approach is discussed through a case study.

Appendix

A set of fuzzy dual numbers is defined as the set $\tilde{\Delta}$ of numbers of the form $a + \varepsilon.b$, where a is the primal part and b is the dual part of the fuzzy dual number, $\forall a \in R, \forall b \in R^+$. ε represents the unity pure dual number. A fuzzy dual number loses both its dual and fuzzy attributes if b equals zero. The lower and upper bounds of $a + \varepsilon.b$ are given by $B^{low}(a + \varepsilon.b) = a - b$ and $B^{high}(a + \varepsilon.b) = a + b$.

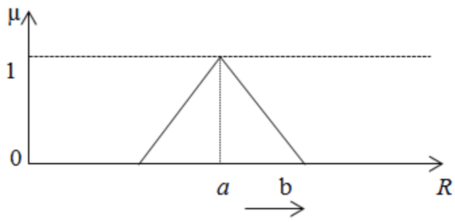

Fig. 1 Graphical representation of a triangular fuzzy number

The pseudo norm of a fuzzy dual number is given by $\|a + \varepsilon.b\| = |a| + \rho.b \in R^+$, where $\rho > 0$ is the shape parameter. The shape parameter is given by $\rho = (1/b)\int_{-b}^{+b} \mu(u)du$, where μ is the membership function. The following properties of the pseudo norm are maintained no matter the values the shape parameters take:

$$\forall a + \varepsilon.b \in \tilde{\Delta} : \|a + \varepsilon.b\| \geq 0 \tag{17}$$

$$\forall a \in R, \forall b \in R^+ \ \|a + \varepsilon.b\| = 0 \Rightarrow a = b = 0 \tag{18}$$

$$\|(a + \varepsilon.b) + (\alpha + \varepsilon.\beta)\| \leq \|a + \varepsilon.b\| + \|\alpha + \varepsilon.\beta\| \ \forall a, \alpha \in R, \forall b, \beta \in R^+ \tag{19}$$

$$\|\lambda.(a + \varepsilon.b)\| = \lambda.\|a + \varepsilon.b\| \ \forall a \in R, \forall b, \lambda \in R^+ \tag{20}$$

Partial orders between fuzzy dual numbers can be introduced using the above pseudo norm. The strong partial written $\check{\geq}$ can be defined over $\tilde{\Delta}$ by:

$$\forall a_1 + \varepsilon.b_1, a_2 + \varepsilon.b_2 \in \tilde{\Delta} : a_1 + \varepsilon.b_1 \check{\geq} a_2 + \varepsilon.b_2 \Leftrightarrow a_1 - \rho.b_1 \geq a_2 + \rho.b_2 \tag{21}$$

The weak partial order written $\hat{\geq}$ can be defined over $\tilde{\Delta}$ by:

$$\begin{gathered}\forall a_1 + \varepsilon.b_1, a_2 + \varepsilon.b_2 \in \tilde{\Delta} : \|a_1 + \varepsilon.b_1\| \hat{\geq} \|a_2 + \varepsilon.b_2\| \\ \Leftrightarrow a_2 + \rho b_2 > a_1 - \rho b_1 \ and \ a_1 - \rho b_1 > a_2\end{gathered} \tag{22}$$

The fuzzy equality between two fuzzy dual numbers, symbolized by $\cong$, is defined:

$$\begin{gathered}\forall a_1 + \varepsilon.b_1, a_2 + \varepsilon.b_2 \in \tilde{\Delta} : \|a_1 + \varepsilon.b_1\| \cong \|a_2 + \varepsilon.b_2\| \\ \Leftrightarrow a_2 \in [a_1 - \rho.b_1, a_1 + \rho.b_1] \text{ and } a_1 \in [a_2 - \rho.b_2, a_2 + \rho.b_2]\end{gathered} \tag{23}$$

Then any two fuzzy dual numbers can be ranked as either strongly different, weakly different or rather equal and a fuzzy ranking can be established between them as well as max and min operators over subsets of $\tilde{\Delta}$.

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