Pi-Invariant Unscented Kalman Filter for Sensor Fusion

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Jean-Philippe Condomines[1], Cédric Seren[2] and Gautier Hattenberger[3]

***Abstract*— A novel approach based on Unscented Kalman Filter (UKF) is proposed for nonlinear state estimation. The Invariant UKF, named π-IUKF, is a recently introduced algorithm dedicated to nonlinear systems possessing symmetries as illustrated by the quaternion-based mini Remotely Piloted Aircraft System (RPAS) kinematics modeling considered in this paper. Within an invariant framework, this algorithm suggests a systematic approach to determine all the symmetry-preserving terms which correct accordingly the nonlinear state-space representation used for prediction, without requiring any linearization. Thus, based on both invariant filters, for which Lie groups have been identified and UKF theoretical principles, the developed π-IUKF has been previously and successfully applied to the mini-RPAS attitude estimation problem, highlighting remarkable invariant properties. We propose in this paper to extend the theoretical background and the applicability of our proposed π-IUKF observer to the case of a mini-RPAS equipped with an aided Inertial Navigation System (INS) which leads to augment the nonlinear state space representation with both velocity and position differential equations. All the measurements are provided on board by a set of low-cost and low-performance sensors (accelerometers, gyrometers, magnetometers, barometer and even Global Positioning System (GPS)). Our designed π-IUKF estimation algorithm is described in this paper and its performances are evaluated by exploiting successfully real flight test data. Indeed, the whole approach has been implemented onboard using a data logger based on the well-known Paparazzi system. The results show promising perspectives and demonstrate that nonlinear state estimation converges on a much bigger set of trajectories than for more traditional approaches.**

I. INTRODUCTION

The necessary resort to multiple miniaturized low-cost and low-performance sensors integrated into mini-RPAS, which are obviously subjected to hard space requirements or electrical power consumption constraints, has led to an important interest to design nonlinear observers for data fusion, unmeasured systems state estimation and/or flight path reconstruction. Exploiting the capabilities of nonlinear observers allows, by generating consolidated signals, to extend the way mini-RPAS can be controlled while enhancing their intrinsic flight handling qualities.That is why numerous recent research works related to RPAS certification and intergration into civil airspace deal with the interest of highly robust estimation algorithm. Therefore, the development of reliable and performant aided-INS for many nonlinear dynamic systems is an important research topic and a major concern in the aerospace engineering community, e.g.[1].

Among the fundamental methods the Extended Kalman Filter (EKF), and its variant the Multiplicative EKF (MEKF) ([2], [3]), is the most widely used signal processing methodology for RPAS-type systems. Its principle relies firstly on the well known Kalman Filtering (KF) operations applied to a tangent linear kinematic modeling of the UAV and secondly on multiplicative correction terms which preserve the unit norm of the estimated quaternion. The MEKF recovers the poor convergence property of the standard KF in the linear case only at equilibrium points because the resulting process and observation linear matrices depend on the trajectory. Divergence issues may happen in many practical cases. This has led to the development of other filters. Introduced in [4], the Unscented KF (UKF), also known as sigma-point filter, is an efficient linearization free estimation algorithm which determines approximate solutions to discrete or continuous-time nonlinear optimal filtering problems. It has received considerable attention until recent years about its convergence and stability ([5], [6]), its potential applications ([7], [8]), and has been shown that it outperforms the EKF in many cases [9]. More recently, several research works on nonlinear invariant observers have been led and provide a geometrical-based constructive method for designing filters able to estimate dynamical systems state vector while preserving their symmetries [10]. Building upon both invariant frame and output-error, this peculiar kind of observer allows to formulate a state estimation error whose dynamics has a remarkable property: it does not depend on the trajectory followed. Followingly, it can be then employed to determine a set of correction gains. Nonetheless, such an approach can be very tedious and non-systematic for complex dynamical systems. That is why this theory has been coupled recently with a standard EKF-based technique to calculate the multiplicative weighting correction factors ([11], [12]). Unfortunately, the resulting Invariant EKF (IEKF) is based on second-order approximations for the invariant estimation error (cf [14]). To overcome this issue, an approach, primary proposed in [15] and named π-IUKF, also dedicated to nonlinear systems possessing symmetries, has been developed and successfully applied to an attitude estimation problem. The results have highlighted that the π-IUKF had the same estimation invariance properties than the ones obtained with the IEKF. Guided by both invariant filter theory and UKF principles, this algorithm suggests a systematic approach

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[1] Jean-Philippe Condomines is with the Laboratory in Applied Mathematics, Computer Science and Automatics For Air Transport (MAIAA), ENAC University, BP 54005, Toulouse Cedex 4, 31055, France jean-philippe.condomines@recherche.enac.fr

[2]Cédric Seren Researcher with the Department of Control Systems and Flight Dynamics, ONERA-The French Aerospace Lab, BP 74025, Toulouse Cedex 4, 31055, France cedric.seren@onera.fr

[3]Gautier Hattenberger Researcher with the Laboratory in Applied Mathematics, Computer Science and Automatics For Air Transport (MAIAA), ENAC University, BP 54005, Toulouse Cedex 4, 31055, France gautier.hattenberger@enac.fr

to determine all the symmetry-preserving correction terms, associated with a nonlinear state-space representation used for prediction, without requiring any linearization of the differential equations.

In the sequel, §II presents the basics of the modeling adopted to tackle the nonlinear estimation problem of determining the state vector components of a mini-RPAS fitted out with an aided INS. §III presents the theoretical background of our proposed π-IUKF estimation algorithm. Finally, §IV gathers all the results obtained after solving the aided INS estimation problem in real conditions.

II. DYNAMICAL SYSTEM MODELING

The navigation quality is limited by inertial sensors performance specifies by the size, power and cost constraints of the RPAS. To recover navigation accuracy using low-cost aided-INS, it is necessary to use, if possible, additional instruments (e.g. magnetometers, barometer, which are used to increase the heading and position accuracies) and/or nonlinear estimation algorithms to improve the flight handling qualities of the aerial vehicle. The nonlinear state estimation makes use of 3 triaxial sensors plus both GPS and barometric sensor units which deliver a total of 16 scalar measurement signals:

- 3 of them are associated with 3 gyroscopes which provide a measurement of the instantaneous angular velocity vector denoted by $\omega_m \in \mathbb{R}^3$ s.t. $\omega_m = [\omega_{mx}, \omega_{my}, \omega_{mz}]^T$;
- 3 accelerometers give a measurement of the specific acceleration denoted by $a_m \in \mathbb{R}^3$ s.t. $a_m = [a_{mx}, a_{my}, a_{mz}]^T$;
- 3 magnetometers allow to obtain a local measurement of Earth's magnetic field, which is known constant[1] and expressed in the body-fixed frame s.t. $y_B = q^{-1} * B * q$ where $B = [B_x, B_y, B_z]^T$;
- 1 GPS unit measures both position and velocity vectors denoted by $y_X = X \in \mathbb{R}^3$ and $y_V = V \in \mathbb{R}^3$ s.t. vectors $X = [X_x, X_y, X_z]^T$ and $V = [V_x, V_y, V_z]^T$ are used in the observation equations;
- 1 barometric sensor provides a scalar measurement of the altitude s.t. $y_h = X_z$, which is used as a mean to improve the vertical position accuracy.

All the sensors embedded are low-cost ones and so have imperfections. The major error sources in the navigation system are due to: - all the disturbances (noises) that affect all the instruments; - the potential incorrect navigation system initialization (e.g. on magnetometers or barometric sensor); - and the inadequacy between the real local Earth's gravity value and the one used for computation. The largest error is usually a bias instability (expressed respectively in deg/hr for gyros and μg for the accelerometers). All these measurements are obviously corrupted by additive noises for which it appears reasonable to assimilate their stochastic properties to the ones of gaussian processes. Their covariances have been identified in [17] from logged sensor data using the Allan variance method [16]. Assuming a flat non-rotating Earth, the flying rigid body motion of our considered mini-RPAS can be mathematically described s.t:

$$\Sigma \begin{cases} \dot{q} = \frac{1}{2} q * \omega \\ \dot{V} = A + q * a * q^{-1} \\ \dot{X} = V \end{cases}$$

In the first differential equation, symbol $*$ corresponds to the quaternion product and ω represents the angular velocity vector. In the second one, A is the constant gravity vector expressed in the North-East-Down coordinates system, i.e. $A = ge_3$ and a is the specific acceleration vector[2]. It is possible to choose how the previously imperfections can be modeled since some degrees of freedom exist in the modeling. A first-order observability analysis $-$ which can be led analytically $-$ shows that 10 additional unknown constants can be estimated without introducing inobservability. Thus, an additive constant bias vector ω_b is basically considered on the angular velocity vector measurement ω_m. Then, a constant positive scaling factor a_s and another scalar bias h_b are introduced in the estimation scheme. The latter quantity h_b will be used to correct the potential wrong altitude in windy conditions so that the attitude delivered will be hybridized with the GPS. All these sensor imperfections are modeled as gaussian random walks which can be physically interpreted as slowly varying parameters. The Σ modeling now becomes:

$$\mathscr{M}_s \begin{cases} \dot{q} = \frac{1}{2} q * (\omega_m - \omega_b) \\ \dot{V} = A + \frac{1}{a_s} q * a_m * q^{-1} \qquad \text{(process)} \\ \dot{X} = V \\ \dot{\omega}_b = 0 \\ \dot{a}_s = 0 \\ \dot{h}_b = 0 \\ \begin{pmatrix} y_V \\ y_X \\ y_h \\ y_B \end{pmatrix} = \begin{pmatrix} V \\ X \\ X_z - h_b \\ q^{-1} * B * q \end{pmatrix} \qquad \text{(measurement)} \end{cases}$$

where ω_m and a_m can be seen as imperfect and noisy but known measured inputs and $[V, X, B]^T$ as an available imperfect and noisy measured output vector. The nonlinear state space representation corresponding to $\mathscr{M}_s$ can be described in a compact form such as: $\dot{x} = f(x, u)$ and $y = h(x, u)$ where: $x = [q^T, V^T, X^T, \omega_b^T, a_s, h_b]^T, u = [\omega_m^T, a_m^T]^T$ and $y = [y_V^T, y_X^T, y_h, y_B^T]^T$ are the state, input and output vectors respectively.

[1]The magnetic field can be determined from a world magnetic model such as B is equal to [0.5156 0.0570 0.8549] at the local flight coordinates 43.617-43°-37′ (N) and 1.450-1°-27′ (E).

[2]Remark that instantaneous attitude of the flying mini-UAV can be generalized and deduced using the aforementioned standard quaternionial form which provides a global parameterization and avoid the mathematical singularities inherent to Euler angles.

III. Pi-Invariant Unscented Kalman Filter

A. π-IUKF algorithm

Inspired by the theory of continuous-time symmetry preserving observer [10] a novel and original UKF-based approach has been developed in [15] to adress the approximation issue of the invariant EKF without requiring any linearization of the dynamical systems equations. The idea is to exploit the UKF principles within a continuous-time invariant framework to the system considered in this paper. This section presents briefly the main theoretical principles of some research works dealing with dynamical system symmetries, invariant observer and π-IUKF algorithm. Without considering any system description, the theory of invariant observer is formulated using both differential geometry and transformation groups theory presented as following.

Definition 1: A Lie group action $(\theta_g)_{g\in G}$ on a manifold M, with identity θ_e (where e denotes the neutral element of G), s.t. $(g,\xi) \in G\times M \mapsto \phi_g(\xi) \in M$, is a differentiable map which verifies:

- $\theta_e(\xi) = \xi$ for all $\xi \in M$;
- $\theta_{g_2} \circ \theta_{g_1}(\xi) = \theta_{g_2 g_1}(\xi)$ for all $g_1, g_2, \in G$.

From definition 1, it results that $(\theta_g)_{g\in G}$ is a diffeomorphism. In the following, we will consider fully-dimension Lie group actions only s.t. dim(G)=dim(M). In that case, we can identify the group G to the manifold M.

Definition 2: In the case of fully-dimension Lie group actions, we can assimilate the application θ_g to a *left* or *right* multiplication s.t.:

$$\theta_g(\xi) = g\xi = L_g(\xi) \quad \text{or} \quad \theta_g(\xi) = \xi g^{-1} = R_{g^{-1}}(\xi)$$

By analogy, considering the state space representation $\mathscr{M}_s$, where the state (resp. input)(resp. output) vector belongs to an open set $\mathscr{X} \subset \mathbb{R}^n$ (resp. $\mathscr{U} \subset \mathbb{R}^m$) (resp. $\mathscr{Y} \subset \mathbb{R}^p$, $p \leq n$), we defined a set of *right* Lie group transformations acting locally on $\mathscr{X}\times\mathscr{U}\times\mathscr{Y}$ s.t.:

$$\begin{aligned} G\times(\mathscr{X}\times\mathscr{U}\times\mathscr{Y}) &\to (\mathscr{X}\times\mathscr{U}\times\mathscr{Y}) \\ (g,x,u,y) &\mapsto (\varphi_g(x),\psi_g(u),\rho_g(y)) = (X,U,Y) \end{aligned}$$

where $(\varphi_g, \psi_g, \rho_g)$ are 3 local diffeomorphisms parametrized by $g \in G$ where the Lie group G verifies dim(G)=dim($\mathscr{X}$)=n. The above coordinates transformations must be defined such that their respective actions on state, input and output variables leave the whole system dynamics unchanged i.e. $\dot{X} = f(X,U)$ and $Y = h(X,U)$.

Definition 3: Any smooth state/output dynamical system $\dot{x} = f(x,u)$ will be said G-invariant if $\exists(\varphi_g,\psi_g)_{g\in G}$, $\forall(g,x,u) \in G\times(\mathscr{X}\times\mathscr{U})$:

$$f(\varphi_g(x),\psi_g(u)) = D\varphi_g(x)\cdot f(x,u)$$

and G-equivariant if $\exists(\rho_g)_{g\in G}$ acting on $\mathscr{Y}$, $\forall(g,x,u) \in G\times(\mathscr{X}\times\mathscr{U})$:

$$h(\varphi_g(x),\psi_g(u)) = \rho_g(h(x,u)).$$

These last definitions mean that all state and output equations remain explicitly identical.

Based on the Cartan moving frame method, a complete set of n-invariants in G can be constructed by considering the group action ψ_g only.

Definition 4: For any two points $x, c \in G$, there exists $g = \gamma(x) \in G$ such that $L_g(x) = gx = c$ or $R_{g^{-1}}(x) = xg^{-1} = c$. The existence of the moving frame $\gamma(x)$ is guaranteed. In particular, choosing $c = e$ we deduce $\gamma(x) = x^{-1}$.

Definition 5: Consider the change of variable $X = \varphi_g(x)$, $U = \psi_g(u)$ and $Y = \rho_g(y)$, a symmetry-preserving observer reads:

$$\dot{\hat{x}} = f(\hat{x},u) + \sum_{i=1}^{n}(K_i(\mathscr{E}(\hat{x},u,y), I(\hat{x},u))\cdot\mathscr{E}(\hat{x},u,y))w_i(\hat{x}) \quad (1)$$

where the gain matrix K depends on the system's trajectory only through a known complete set of invariant $I(\hat{x},u) = \psi_{\hat{x}^{-1}}(u)$ and on the invariant output error $\mathscr{E} := \rho_{\hat{x}^{-1}}(h(\hat{x},u)) - \rho_{\hat{x}^{-1}}(y)$. $w_i(\hat{x}) := \left[D\varphi_{\gamma(\hat{x})}(\hat{x})\right]^{-1}\cdot\partial/\partial x_i$ is an invariant vector which projects the set of invariant correction terms on each component of $f(\hat{x},u)$ (i.e. the tangent state space). $(\partial/\partial x_i)$ is the i-th canonical vector field of $\mathbb{R}^n$.

The convergence properties of (1) depend on the choice of K and in the way the state estimation error is defined. Instead of considering the usual "linear" state estimation error $\hat{x}-x$, the invariant observer theory defines an invariant state estimation error denoted $\eta(x,\hat{x}) = x^{-1}\hat{x}$ which has invariant properties (as explained in §III-B).

Definition 6: The asymptotic convergence of $\hat{x}$ to x is equivalent to the stability of the invariant state error dynamic which takes the general form:

$$\dot{\eta} = \Upsilon(\eta, I(\hat{x},u)) \quad (2)$$

where Υ is a smooth function. It appears that η depends on the system's trajectory only through the invariant $I(\hat{x},u)$.

Applied to $\mathscr{M}_s$, a direct and analytical observer can be built thanks to (2). Such an approach can be very tedious and non-systematic for more complex dynamical systems than the ones represented by a fully kinematic model. That's why recently, a more general EKF based method dedicated to gains calculation has been developed in [11], [14], [13] and exploits a re-linearization of the invariant state estimation error dynamics. In that way, the IEKF makes use of a "second-order" approximation of (2). Once again, this technique can be very complex to implement due to this re-linearization.

To get free from this tricky re-linearization, it is possible to derive an UKF-based version for the invariant observer given in Eq.(1). To come up with such a solution, our proposed π-IUKF algorithm requires to integrate a compatibility condition which ensures that the measurement prediction update step (abusively expressed as $\hat{Z} = h(\chi,u)$ where χ gathers a fixed number of deterministically chosen sigma-points) is performed without undermining both invariant geometric and signal processing theories. Indeed, when using an UKF-like technique for gains calculation, the measurement update equation must be modified s.t. the new predicted output vector reads: $\hat{z}_\pi = \pi(h(\chi,u))$. The application π projects the invariant output vector $\rho_{\gamma(x)}(h(\chi,u)) = h(e, I(\chi,u))$ associated with $h(\chi,u)$ on the local invariant frame defined by $\omega_i(\chi)$ for sigma-point χ. In order words, the sigma-points of the standard UKF must be projected through the invariant transformation of the observation function on the local invariant frame at χ.

Proposition 1: If such an application can be designed analytically then the gain matrix of Eq.(1) can be calculated on the basis of an UKF technique by sampling the stochastic distributions of the state and output.

Considering the whole state space representation of $\mathscr{M}_s$ and the Lie-group $G = \mathbb{H}_1 \times \mathbb{R}^{11} \ni (q_0 \;\; V_0 \;\; X_0 \;\; \omega_0 \;\; a_0 \;\; h_0) = g$, this compatibility condition π on the invariant output is the following transformation:

$$\begin{pmatrix} \sum_{i=1}^3 (\hat{V} * \hat{q}) e_i \\ \sum_{i=1}^3 (\hat{X} * \hat{q}) e_i \\ < \hat{X} * \hat{q}, e_3 > - \hat{h}_b \\ \sum_{i=1}^3 (B * \hat{q}) e_i \end{pmatrix} = \pi(\rho_{\gamma(\hat{x})}(h(\hat{x}, u)))$$

B. Motivating example

In this section, we illustrate and prove that the proposed algorithm retains the invariance of the problem, and that the error's evolution is independent of the system's trajectory, inheriting the properties of the deterministic continuous-time case [10]. Thus, we consider the non-aided AHRS case where no velocity and position measurements are available in $\mathscr{M}_s$ (no GPS or no Pitot sensors). This example has been also tackled in more details in [15]. To keep the whole non-linear state representation observable given these available informations, the assumption that the linear acceleration $\dot{V}$ remains small is made, i.e. $\dot{V} = 0$. As a result, the specific acceleration vector, expressed in the body-fixed frame, can be approximated by $a = -y_A = -q^{-1} * A * q$ where A is the local Earth's gravity vector. Moreover, a constant positive scaling factor b_s is introduced and will be used to adjust and preserve the unit norm on vector y_B. Based on these explanations, the observer considered in the π-IUKF algorithm takes the following form:

$$\mathscr{O}_{\mathscr{M}_s^-} \begin{cases} \dot{\hat{q}} = \frac{1}{2} \hat{q} * (\omega_m - \hat{\omega}_b) + \sum_{i=1}^{3} (\bar{K}_{Ai}^q \cdot E_A + \bar{K}_{Bi}^q \cdot E_B) e_i * \hat{q} \\ \dot{\hat{\omega}}_b = \sum_{i=1}^{3} \hat{q}^{-1} * (\bar{K}_{Ai}^{\omega_b} \cdot E_A + \bar{K}_{Bi}^{\omega_b} \cdot E_B) * \hat{q} \\ \dot{\hat{a}}_s = \hat{a}_s (\bar{K}_{Ai}^{a_s} \cdot E_A + \bar{K}_{Bi}^{a_s} \cdot E_B) \\ \dot{\hat{b}}_s = \hat{b}_s (\bar{K}_{Ai}^{b_s} \cdot E_A + \bar{K}_{Bi}^{b_s} \cdot E_B) \end{cases}$$

where the invariant output error E is given by :

$$E = \begin{pmatrix} A - \hat{a}_s^{-1} \hat{q} * y_A * \hat{q}^{-1} \\ B - \hat{b}_s^{-1} \hat{q} * y_B * \hat{q}^{-1} \end{pmatrix} = \begin{pmatrix} E_A \\ E_B \end{pmatrix}$$

and the invariant compatibility condition π reads:

$$\begin{pmatrix} a_s \cdot \sum_{i=1}^3 (A * \hat{q}) e_i \\ b_s \cdot \sum_{i=1}^3 (B * \hat{q}) e_i \end{pmatrix} = \pi(\rho_{\gamma(\hat{x})}(h(\hat{x}, u)))$$

Thereby, we consider the invariant state estimation error s.t.:

$$\begin{pmatrix} \eta \\ \beta \\ \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} \hat{q} * q - \mathbf{1} \\ \hat{q} * (\hat{w}_b - w_b) * \hat{q}^{-1} \\ a_s / \hat{a}_s \\ b_s / \hat{b}_s \end{pmatrix}$$

with the invariant state error dynamic ϑ given by:

$$\begin{cases} \dot{\eta} = (\sum_{i=1}^{3} (\bar{K}_{Ai}^q \cdot E_A + \bar{K}_{Bi}^q \cdot E_B) e_i) * \eta - \frac{1}{2} \eta * \beta \\ \dot{\beta} = (\eta^{-1} * I * \eta) \times \beta + \eta^{-1} * \sum_{i=1}^{3} (\bar{K}_{Ai}^{\omega_b} \cdot E_A + \bar{K}_{Bi}^{\omega_b} \cdot E_B) * \eta \\ \dot{\alpha} = -\alpha (\bar{K}_{Ai}^{a_s} \cdot E_A + \bar{K}_{Bi}^{a_s} \cdot E_B) \\ \dot{\gamma} = -\gamma (\bar{K}_{Ai}^{b_s} \cdot E_A + \bar{K}_{Bi}^{b_s} \cdot E_B) \end{cases}$$

The main benefit of the invariant property concerns state estimation error dynamics ϑ. It depends only on the invariant state error $(\eta, \beta, \alpha, \gamma)$ and on the "free" but known complete set of invariants $I(\hat{x}, u)$ and not on the trajectory followed by the system which is a major difference with almost nonlinear filters, such as the standard EKF. As a result, for a system following a (nearly) permanent trajectory defined by $I(\hat{x}, u) = c$, the gain matrix K is proved to converge to fixed and permanent values. In that way, a remarkable property can be pointed out for the non-aided AHRS. Under certain slightly decoupling conditions, we can establish an *autonomous* invariant state error dynamics, i.e. $\dot{\vartheta} = \Upsilon(\vartheta, c)$.

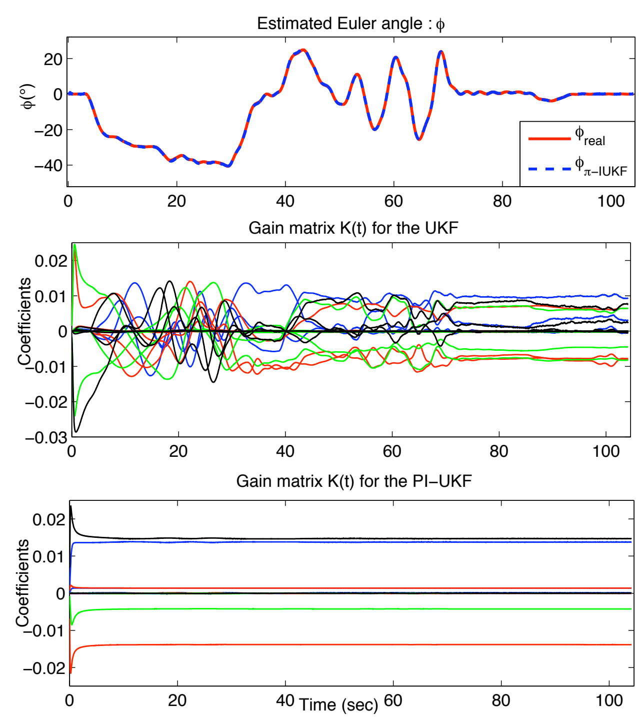

Fig. 1. Simulation: attitude estimation results on the bank angle ϕ and coefficients of the gain matrix K(t): UKF vs. PI-IUKF

We now illustrate the previous explanations by applying the developed π-IUKF algorithm to simulated data used in

[15]. Among the usual Euler estimated attitude angles (ϕ, θ, ψ) deduced from the quaternion, only the back angle ϕ is plotted in Fig. 1. $\phi(t)$ follows a trajectory denoted $\mathscr{T}_\phi$. The π-IUKF correction coefficients become as expected constants after $t < 5$sec whereas those of the UKF have an irregular evolution due to an estimation state error far away from its corresponding equilibrium point. After $t > 75$sec, $\mathscr{T}_\phi$ tends to become a permanent trajectory as well as the UKF gain constant coefficients.

IV. THE AIDED INERTIAL NAVIGATION PROBLEM

We now apply the previous developed theory in §III-A to $\mathscr{M}_s$ in order to exploit the capabilities of the π-IUKF algorithm. First of all, we look for geometric transformations that leave the whole system identical. Considering the Lie-group $G = \mathbb{H}_1 \times \mathbb{R}^{11}$ (where $\mathbb{H}_1$ is the unit quaternions manifold) acting on the whole state space of $\mathscr{M}_s$, the following variable transformations confer to our considered dynamical system the G-invariant and G-equivariant properties s.t. $\forall g = (q_0^T \; V_0^T \; X_0^T \; \omega_0^T \; a_0 \; h_0) \in G$:

$$\varphi_{x_0}\left(x = \begin{pmatrix} q \\ V \\ X \\ \omega_b \\ a_s \\ h_b \end{pmatrix}\right) = \begin{pmatrix} q * q_0 \\ V + V_0 \\ X + X_0 \\ q_0^{-1} * \omega_b * q_0 + \omega_0 \\ a_s \cdot a_0 \\ h_b + h_0 \end{pmatrix}$$

$$\psi_{x_0}\left(u = \begin{pmatrix} \omega_m \\ a_m \end{pmatrix}\right) = \begin{pmatrix} q_0^{-1} * \omega_m * q_0 + \omega_0 \\ a_0 \cdot q_0^{-1} * a_m * q_0 \end{pmatrix}$$

$$\rho_{x_0}\left(y = \begin{pmatrix} y_V \\ y_X \\ y_h \\ y_B \end{pmatrix}\right) = \begin{pmatrix} y_V + V_0 \\ y_X + X_0 \\ y_h - h_0 + < X_0, e_3 > \\ q_0^{-1} * y_B * q_0 \end{pmatrix}$$

These latter are equivalent to time-constant rotations and translations in both earth- and body-fixed frames. Then, if we consider besides the particular case where : $g = x^{-1} = (q^{-1} \;\; -V \;\; -X \;\; (-q * \omega_b * q^{-1}) \;\; a_s^{-1} \;\; -h_b + < X, e_3 >)^T$, we obtain:

$$\varphi_{x^{-1}}(x) = \begin{pmatrix} q * q^{-1} \\ V - V \\ X - X \\ q * \omega_b * q^{-1} - q * \omega_b * q^{-1} \\ a_s \cdot a_s^{-1} \\ h_b + < X, e_3 > - (h_b + < X, e_3 >) \end{pmatrix} = \begin{pmatrix} \mathbf{1} \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = e$$

which is equal to the neutral element e for the local transformation φ_g. Followingly, considering the canonical basis of $\mathbb{R}^3$, $(e_i)_{i \in [1;3]}$, the vectors which define the invariant reference frame (also called invariant natural basis of the tangent state space) are given by the 14 elements which define $w_j(\hat{q}, \hat{V}, \hat{X}, \hat{\omega}_b, \hat{a}_s, \hat{h}_b)$ with $j \in [\![1;14]\!]$ s.t.:

$$\underset{i\in[1;3]}{\begin{pmatrix} e_i * q \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ 0 \\ 0 \end{pmatrix}}, \underset{i\in[1;3]}{\begin{pmatrix} \mathbf{0} \\ e_i \\ \mathbf{0} \\ \mathbf{0} \\ 0 \\ 0 \end{pmatrix}}, \underset{i\in[1;3]}{\begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ e_i \\ \mathbf{0} \\ 0 \\ 0 \end{pmatrix}}, \underset{i\in[1;3]}{\begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ q^{-1} * e_i * q \\ 0 \\ 0 \end{pmatrix}}, \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ a_s \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ 0 \\ h_b \end{pmatrix}$$

Combining all these results, Eq.(1) can be detailed s.t.:

$$\mathscr{O}_{\mathscr{M}_s} \begin{cases} \dot{\hat{q}} = \frac{1}{2}\hat{q} * (\omega_m - \hat{\omega}_b) + \sum_{i=1}^{3} (\bar{K}_{Vi}^{q} E_V + \bar{K}_{Bi}^{q} E_B) e_i * \hat{q} \\ \dot{\hat{V}} = \frac{1}{\hat{a}_s} \hat{q} * a_m * \hat{q}^{-1} + A + \sum_{i=1}^{3} (\sum_{j=1}^{3} (\bar{K}_{V_{ij}}^{V} E_{V_j}) + \bar{K}_{hi}^{V} E_h) e_i \\ \dot{\hat{X}} = \hat{V} + \sum_{i=1}^{3} (\sum_{i=1}^{3} (\bar{K}_{Nx}^{X} E_X) + \bar{K}_{hi}^{X} E_h) e_i \\ \dot{\hat{\omega}}_b = 0 + \hat{q}^{-1} * (\sum_{i=1}^{3} (\bar{K}_{Vi}^{\omega_b} E_V + \bar{K}_{Bi}^{\omega_b} E_B) e_i) * \hat{q} \\ \dot{\hat{a}}_s = 0 + \hat{a}_s (\sum_{i=1}^{3} (\bar{K}_{Vi}^{a_s} E_V + \bar{K}_{h}^{a_s} E_h)) \\ \dot{\hat{h}}_b = 0 + \bar{K}_h^h E_h \end{cases}$$

where the invariant output error E is given by :

$$\begin{pmatrix} E_V \\ E_X \\ E_B \\ E_h \end{pmatrix} = \begin{pmatrix} \hat{V} - y_v \\ \hat{X} - y_x \\ < \hat{X}, e_3 > - \hat{h}_b - y_h \\ B - \hat{q} * y_B * \hat{q}^{-1} \end{pmatrix}$$

A. Ground experiment

We now illustrate the performances reached by the π-IUKF algorithm on the basis of data provided by onboard logged sensor data integrated into the mini-RPAS. All measurements are obviously corrupted by additive measurement noises whose covariance matrices are given in [17]. In order to erase the significant vibration phenomena observed on the recorded data during the flight we chose to process these ones with low pass filters applied to both accelerometers and gyros. The observer is numerically implemented using a fourth Runge-Kutta order integration method sampled at 125hz (sampling frequency of the IMU). Thus, the estimated dynamics $f(\hat{x}, u)$ is integrated at the 125Hz rate to obtain rough predictions of x, while the full observer equations are computed at a lower frequency as soon as a GPS measurement is available (nominally 5Hz).

B. Flight experiment

Due to a lack of space, we briefly compare the flight trajectory of both π-IUKF and UKF algorithms only, using a Differential Global Positioning System (DGPS) unit as the reference. The DGPS receiver, mounted on the nose of the mini-RPAS, runs with its own KF, which provides a centimeter-level precision thanks to a carrier phase difference between several DGPS units. We can observe on Fig. 2

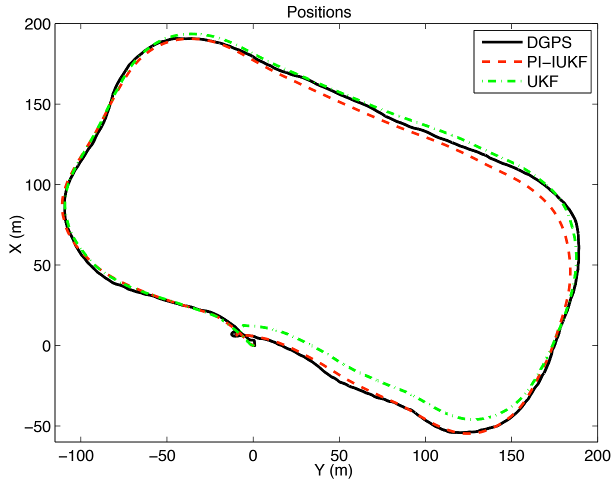

Fig. 2. Experiment: estimated positions UKF vs. π-IUKF

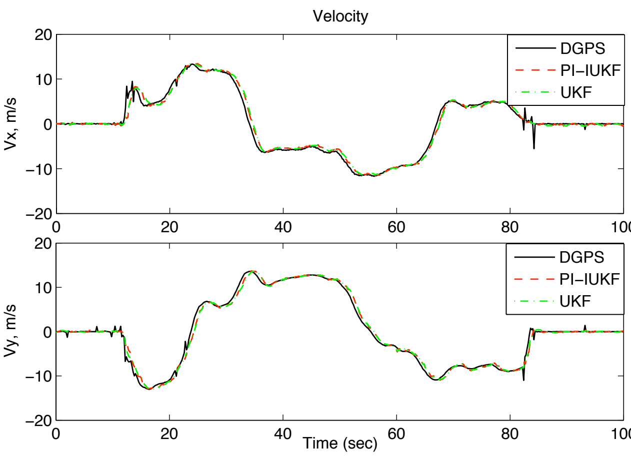

Fig. 3. Experiment: estimated velocities UKF vs. π-IUKF

that the estimation errors on (X,Y) positions between both algorithms and DGPS are small in spite of strong acceleration phases and strong speeds (Fig. 3) for which the GPS has difficulty to correct the gravity model on $\mathcal{O}_{\mathcal{M}_s}$. On Fig. 2, the mini-RPAS takes off (in direction of $X > 0$, $Y < 0$) from the landing pad, hovers making an hippodrome shape and lands (in direction of $X < 0$, $Y > 0$). Performances between both algorithms appear to be quite similar except for the last turn before the landing where the π-IUKF provides more accurate results. The main reason behind this position estimation errors is the unmodeled effect of engine speed changes on sensor noise (i.e. vibration).

V. CONCLUSIONS

This paper has reviewed the theoretical background and the applicability of our proposed π-IUKF to the case of a mini-RPAS equipped with low-cost sensors. In order to address the linearization issue of the invariant IEKF, our algorithm has proved to retain the invariance of the problem and inherits the properties of the deterministic continuous-time case. The hybridation of the standard SR-UKF with the invariant observer theory represents a significant research topic to improve the handling qualities of mini-RPAS. Furthermore, the π-IUKF algorithm has been validated on the basis of real data provided by onboard logged sensor data integrated into a mini-RPAS. A brief comparison with conventional UKF demonstrated that both invariant observer and UKF methodologies is able to provide increased robustness, making them appealing for a variety of practical applications.

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