Replacing the singlet spinor of the EPR-B experiment with two single-particle spinors

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Abstract

Recently, for spinless non-relativistic particles, Norsen[1], Norsen, Marian and Oriols[2] show that in the de Broglie-Bohm interpretation it is possible to replace the wave function in the configuration space by single-particle wave functions in physical space. In this paper, we show that this replacment of the wave function in the configuration space by single-particle functions in the 3D-space is also possible for particles with spin, in particular for the particles of the EPR-B experiment, the Bohm version of the Einstein-Podolsky-Rosen experiment.

I. INTRODUCTION

A major difficulty of wave function interpretation of N particles in quantum mechanics is its definition in a 3N-dimensional configuration space. Since the Solvay Conference in 1927, de Broglie and Schrödinger considered the wave function of N particles introduces by Schrödinger in the 3N-dimensional configuration space as fictitious and proposed to replace it by N single-particle wave functions in 3D-space:

"It appears to us certain that if one wants to *physically* represent the evolution of a system of N corpuscles, one must consider the propagation of N waves in space, each N propagation being determined by the action of the N-1 corpuscles connected to the other waves. Nevertheless, if one focusses one's attention only on the corpuscles, one can represent their states by a point in configuration space, and one can try to relate the motion of this representative point to the propagation of a fictitious wave Ψ in configuration space. It appears to us very probable that the wave

$$\Psi = a(q_1, q_2, ..., q_n) cos \frac{2\pi}{h} \varphi(t, q_1, ...q_n),$$

a solution of the Schrödinger equation, is only a fictitious wave, which in the *Newtonian approximation*, plays for the representative point of the system in configuration space the same role of pilot wave and of probability wave that the wave Ψ plays in ordinary space in the case of a single material point." de Broglie[3] (cited by Norsen[1])

"This use of the q-space [configuration space] is to be seen only as a mathematical tool, as it is often applied also in the old mechanics; ultimately... the process to be described is one in space and time." Schrödinger[4] (cited by Norsen et al.[2] p.26)

But, as noted by Norsen et al.[2], this program to replace the wave function in a 3N-dimensional configuration space by N single-particle wave functions was prematurely-abandoned. It was recently re-opened by Norsen[1], Norsen, Marian and Oriols[2]. For spinless non-relativistic particles, these authors show that it is possible in the de Broglie-Bohm pilot-wave theory to replace the wave function in the configuration space by N single-particle wave functions in physical space[2]. These N wave functions in 3D-space are the N *conditional wave functions* of a subsystem introduced by Dürr, Goldstein and Zanghi[6,7]. For a N-particle wave function $\Psi(x_1, x_2, ..., x_N, t)$, the N conditional wave functions are:

$$\Psi_1(x,t) = \Psi(x, x_2, ..., x_N, t)|_{x_2=X_2(t);x_N=X_N(t)}$$

$$\Psi_2(x,t) = \Psi(x_1, x, ..., x_N, t)|_{x_1=X_1(t);x_N=X_N(t)}$$

$$\Psi_N(x,t) = \Psi(x_1, ..., x_{N-1}, x, t)|_{x_1=X_1(t);x_{N-1}=X_{N-1}(t)}$$

where $X_i(t)$ is the position of the particle i at time t in the Bohmian mechanics. The evolutions of these positions $X(t) = \{X_1(t), X_2(t), ..., X_N(t)\}$ are given by the guidance formula:

$$\frac{dX_i(t)}{dt} = \frac{\hbar}{m_i} Im \frac{\nabla_i \Psi}{\Psi}|_{\mathbf{x}=\mathbf{X}(t)} \equiv \frac{\hbar}{m_i} Im \frac{\nabla \Psi_i}{\Psi_i}|_{x=X_i(t)}$$

We discuss in[8] the pertinence of this passage from the configuration space to physical space.

The aim of this paper is to show that this replacement of the wave function in the configuration space by single-particle functions in the 3D-space is also possible for particles with spin, in particular for the particles in the singlet state of the EPR-B experiment, the Bohm version of the Einstein-Podolsky-Rosen experiment.

To realize this decomposition of a pair of entangled atoms into two states, one for each atom, we consider a two-step version of the EPR-B experiment and we use an analytic expression of the wave function. The explicit solution, obtained via a complete integration of the two-body Pauli equation *over time and space* for the two-step version of the EPR-B experiment, is presented in section 2.

In section 3, we show how, in the de Broglie-Bohm interpretation, we can replace the singlet spinor of the two-step version of the EPR-B experiment by two single-particle spinors.

II. EXPLICIT SOLUTION OF THE SPINOR IN CONFIGURATION SPACE FOR THE TWO-STEP VERSION OF EPR-B EXPERIMENT

Fig.1 presents the Einstein-Podolsky-Rosen-Bohm experiment. A source S creates in O pairs of identical atoms A and B, but with opposite spins. The atoms A and B split following the y-axis in opposite directions, and head towards two identical Stern-Gerlach apparatuses $\mathcal{A}$ and $\mathcal{B}$.

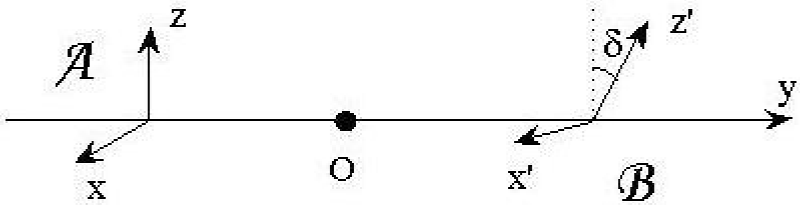

FIG. 1: Schematic configuration of the EPR-B experiment.

The electromagnet $\mathcal{A}$ "measures" the A spin in the direction of the z-axis and the electromagnet $\mathcal{B}$ "measures" the B spin in the direction of the z'-axis, which is obtained after a rotation of an angle δ around the y-axis.

We assume, at the moment of the creation of the two entangled particles A and B, that each of the two particles A and B has an initial wave function $\Psi_0^A(\mathbf{r}_A, \theta_0^A, \varphi_0^A)$ and $\Psi_0^B(\mathbf{r}_B, \theta_0^B, \varphi_0^B)$ with spinors that are opposite spins; for example

$$\Psi_0^A(\mathbf{r}_A, \theta_0^A, \varphi_0^A) = f(\mathbf{r}_A)\left(\cos\frac{\theta_0^A}{2}|+_A\rangle + \sin\frac{\theta_0^A}{2}e^{i\varphi_0^A}|-_A\rangle\right) \tag{1}$$

and

$$\Psi_0^B(\mathbf{r}_B, \theta_0^B, \varphi_0^B) = f(\mathbf{r}_B)\left(\cos\frac{\theta_0^B}{2}|+_B\rangle + \sin\frac{\theta_0^B}{2}e^{i\varphi_0^B}|-_B\rangle\right) \tag{2}$$

with $\theta_0^B = \pi - \theta_0^A$, $\varphi_0^B = \varphi_0^A - \pi$, where $\mathbf{r} = (x, z)$, $f(\mathbf{r}) = (2\pi\sigma_0^2)^{-\frac{1}{2}}e^{-\frac{x^2+z^2}{4\sigma_0^2}}$ and where $|\pm_A\rangle$ (resp. $|\pm_B\rangle$) are the eigenvectors of the spin operators $\widehat{s}_{z_A}$ (resp. $\widehat{s}_{z_B}$) in the z-direction pertaining to particle A (B): $\widehat{s}_{z_A}|\pm_A\rangle = \pm(\frac{\hbar}{2})|\pm_A\rangle$ (resp. $\widehat{s}_{z_B}|\pm_B\rangle = \pm(resp.\frac{\hbar}{2})|\pm_B\rangle$). We treat the dependence on y classically: speed $-v_y$ for A and v_y for B.

Note that we represent a particle with spin by a spinor with a spatial extension (1) and not from a simplified wave function without spatial extension:

$$\Psi_0^A(\mathbf{r}_A, \theta_0^A, \varphi_0^A) = \left(\cos\frac{\theta_0^A}{2}|+_A\rangle + \sin\frac{\theta_0^A}{2}e^{i\varphi_0^A}|-_A\rangle\right) \tag{3}$$

In the usual textbooks on quantum mechanics[9–12], the spatial extension of the spinor is not taken into account and its spatial integration in the Pauli equation is not possible. We lose any possibility of taking the spin evolution into account during the measurement[13,14].

Then the Pauli principle tells us that the two-body wave function must be antisymmetric; it is written:

$$\Psi_0(\mathbf{r}_A, \theta_A, \varphi_A, \mathbf{r}_B, \theta_B, \varphi_B) = \Psi_A^0(\mathbf{r}_A, \theta_A, \varphi_A)\Psi_B^0(\mathbf{r}_B, \theta_B, \varphi_B) - \Psi_A^0(\mathbf{r}_B, \theta_B, \varphi_B)\Psi_B^0(\mathbf{r}_A, \theta_A, \varphi_A)$$

i.e. $\Psi_0(\mathbf{r}_A,\theta_A,\varphi_A,\mathbf{r}_B,,\theta_B,\varphi_B) = f(\mathbf{r}_A)f(\mathbf{r}_B)[(\cos\frac{\theta_A}{2}|+_A\rangle + \sin\frac{\theta_A}{2}e^{i\varphi_A}|-_A\rangle)(\cos\frac{\theta_B}{2}|+_B\rangle + \sin\frac{\theta_B}{2}e^{i\varphi_B}|-_B\rangle) - (\cos\frac{\theta_B}{2}|+_A\rangle + \sin\frac{\theta_B}{2}e^{i\varphi_B}|-_A\rangle)(\cos\frac{\theta_A}{2}|+_B\rangle + \sin\frac{\theta_A}{2}e^{i\varphi_A}|-_B\rangle)]$, and after calculation we obtain:

$$\Psi_0(\mathbf{r}_A,\theta_A,\varphi_A,\mathbf{r}_B,\theta_B,\varphi_B) = -e^{i\varphi_A}f(\mathbf{r}_A)f(\mathbf{r}_B)(|+_A\rangle|-_B\rangle - |-_A\rangle|+_B\rangle)$$

which is the same as the singlet state (4), factor-wise:

$$\Psi_0(\mathbf{r}_A,\mathbf{r}_B) = \frac{1}{\sqrt{2}}f(\mathbf{r}_A)f(\mathbf{r}_B)(|+_A\rangle|-_B\rangle - |-_A\rangle|+_B\rangle) \tag{4}$$

Note that our initial singlet wave function (4) has a spatial extension contrary to the usual wave function (5), which is a simplifield function without spatial extension:

$$\Psi_0(\mathbf{r}_A,\mathbf{r}_B) = \frac{1}{\sqrt{2}}(|+_A\rangle|-_B\rangle - |-_A\rangle|+_B\rangle). \tag{5}$$

And this spatial extension is essential to solve the Pauli equation in space!

The wave function $\Psi(\mathbf{r}_A,\mathbf{r}_B,t)$ of the two identical particles A and B, which is electrically neutral and with magnetic moments μ_0, subject to magnetic fields $\mathbf{B}^{\mathcal{A}}$ and $\mathbf{B}^{\mathcal{B}}$, admits 4 components $\Psi^{a,b}(\mathbf{r}_A,\mathbf{r}_B,t)$ in the basis $|\pm_A\rangle$ and $|\pm_B\rangle$and verifies the two-body Pauli equation[22] p. 417:

$$i\hbar\frac{\partial\Psi^{a,b}}{\partial t} = \left(-\frac{\hbar^2}{2m}\Delta_A - \frac{\hbar^2}{2m}\Delta_B\right)\Psi^{a,b} + \mu B_j^{\mathcal{A}}(\sigma_j)_c^a\Psi^{c,b} + \mu B_j^{\mathcal{B}}(\sigma_j)_d^b\Psi^{a,d} \tag{6}$$

with the initial conditions:

$$\Psi^{a,b}(\mathbf{r}_A,\mathbf{r}_B,0) = \Psi_0^{a,b}(\mathbf{r}_A,\mathbf{r}_B) \tag{7}$$

where the σ_j are the Pauli matrixes and where the $\Psi_0^{a,b}(\mathbf{r}_A,\mathbf{r}_B)$ correspond to the singlet state (4).

We take as numerical values those of the Stern-Gerlach experiment with silver atoms[13,23]. For a silver atom one has $m = 1,8\times10^{-25}$ kg, $v_y = 500$ m/s , σ_0=10^{-4}m. For the electromagnetic field $\mathbf{B}$, $B_x = B'_0x$; $B_y = 0$ and $B_z = B_0 - B'_0z$ with $B_0 = 5$ Tesla, $B'_0 = \left|\frac{\partial B}{\partial z}\right| = -\left|\frac{\partial B}{\partial x}\right| = 10^3$ Tesla/m over a length $\Delta l = 1\ cm$. The time in the magnetic field is $\Delta t = \frac{\Delta l}{v_y} = 2\times10^{-5}s$. The screen that intercepts atoms is at a distance $D = 20\ cm$ (time $t_D = \frac{D}{v_y} = 4\times10^{-4}$s) from the exit of the magnetic field.

One of the difficulties of the interpretation of the EPR-B experiment is the existence of two simultaneous measurements. By doing these measurements one after the other, the interpretation of the experiment will be facilitated. That is the purpose of the two-step version of the experiment EPR-B studied below.

A. First step: Measurement of A spin

In the first step we make, on a couple of particles A and B in a singlet state, a Stern and Gerlach "measurement" for atom A, then in the second step a Stern and Gerlach "measurement" for atom B. It is the experiment first proposed in 1987 by Dewdney, Holland and Kyprianidis[20].

Consider that at time t_0 the particle A arrives at the entrance of electromagnet $\mathcal{A}$. $\triangle t$ is the duration of the crossing electromagnet $\mathcal{A}$ and t is the time after the $\mathcal{A}$ exit. The wave function can be calculated, from the wave function (4), term to term in basis $[|\pm_A\rangle, |\pm_B\rangle]$. After this exit of the magnetic field $\mathcal{A}$, at time $t_0+\triangle t+t$, the wave function (4) becomes[13–15]:

$$\begin{aligned}\Psi(\mathbf{r}_A, \mathbf{r}_B, t_0 + \triangle t + t) &= \frac{1}{\sqrt{2}} f(\mathbf{r}_B) \\ &\times \left(f^+(\mathbf{r}_A, t)|+_A\rangle|-_B\rangle - f^-(\mathbf{r}_A, t)|-_A\rangle|+_B\rangle \right)\end{aligned} \quad (8)$$

with

$$f^{\pm}(\mathbf{r}, t) \simeq f(x, z \mp z_\triangle \mp ut) e^{i(\frac{\pm muz}{\hbar} + \varphi^{\pm}(t))} \quad (9)$$

and

$$z_\Delta = \frac{\mu_0 B_0'(\Delta t)^2}{2m} = 10^{-5} m, , \quad u = \frac{\mu_0 B_0'(\Delta t)}{m} = 1 m/s.$$

The atomic density $\rho(z_A, z_B, t_0 + \Delta t + t)$ is found by integrating $\Psi^*(\mathbf{r}_A, \mathbf{r}_B, t_0 + \triangle t + t)\Psi(\mathbf{r}_A, \mathbf{r}_B, t_0 + \triangle t + t)$ on x_A and x_B:

$$\begin{aligned}\rho(z_A, z_B, t_0 + \Delta t + t) &= \left((2\pi\sigma_0^2)^{-\frac{1}{2}} e^{-\frac{(z_B)^2}{2\sigma_0^2}} \right) \\ &\times \left((2\pi\sigma_0^2)^{-\frac{1}{2}} \frac{1}{2} \left(e^{-\frac{(z_A - z_\Delta - ut)^2}{2\sigma_0^2}} + e^{-\frac{(z_A + z_\Delta + ut)^2}{2\sigma_0^2}} \right) \right).\end{aligned} \quad (10)$$

We deduce that the beam of particles A is divided into two, while the B beam of particle stays whole.

Moreover, we note that the space quantization of particle A is identical to that of an untangled particle in a Stern and Gerlach apparatus: the distance $\delta z = 2(z_\Delta + ut)$ between the two spots N^+ (spin $+$) and N^- (spin $-$) of a family of particles A is the same as the distance between the two spots N^+ and N^- of a particle in a classic Stern and Gerlach experiment[13]. We finally deduce from (10) that:

- the density of A is the same, whether particle A is entangled with B or not,

- the density of B is not affected by the "measurement" of A.

Only spins are involved. We conclude from (8) that the spins of A and B remain opposite throughout the experiment[15].

B. Second step: "Measurement" of B spin.

After a first step of a Stern and Gerlach "measurement" on the A atom between t_0 and $t_0 + \triangle t + t_D$, the second step comprises a Stern and Gerlach "measurement" on the B atom with an electromagnet $\mathcal{B}$ forming an angle δ with $\mathcal{A}$ between $t_0+\triangle t+t_D$ and $t_0+2(\triangle t+t_D)$.

At time $t_0+\triangle t+t_D$, the wave function in configuration space is given by (8). Immediately after the "measurement" of A, still at time $t_0 + \triangle t + t_D$, if the A measurement is $\pm$, the conditionnal wave function of B is:

$$\Psi_{B/\pm A}(\mathbf{r}_B, t_0 + \triangle t + t_D) = f(\mathbf{r}_B)|\mp_B\rangle. \tag{11}$$

To measure B, we refer to the basis $|\pm'_B\rangle$ where $|\pm'_B\rangle$ are the eigenvectors of the spin operators $\widehat{s}_{z'_B}$ in the z'-direction pertaining to particle B. We note $\mathbf{r'} = (x', z')$. So, after the measurement of B, at time $t_0 + 2(\triangle t + t_D)$, the conditional wave functions of B are:

$$\Psi_{B/+A}(\mathbf{r}'_B, t_0 + 2(\triangle t + t_D)) = \cos\frac{\delta}{2} f^+(\mathbf{r}'_B, t_D)|+'_B\rangle + \sin\frac{\delta}{2} f^-(\mathbf{r}'_B, t_D)|-'_B\rangle, \tag{12}$$

$$\Psi_{B/-A}(\mathbf{r}'_B, t_0 + 2(\triangle t + t_D)) = -\sin\frac{\delta}{2} f^+(\mathbf{r}'_B, t_D)|+'_B\rangle + \cos\frac{\delta}{2} f^-(\mathbf{r}'_B, t_D)|-'_B\rangle. \tag{13}$$

We therefore obtain, in this two-step version of the EPR-B experiment, the same results for spatial quantization and correlations of spins as in the EPR-B experiment.

III. THE TWO SINGLE-PARTICLE SPINORS FOR THE TWO-STEP VERSION OF THE EPR-B EXPERIMENT

We assume, at moment of the creation of the two entangled particles A and B, that each of the two particles A and B has an initial wave function $\Psi^A_0(\mathbf{r}_A, \theta^A_0, \varphi^A_0)$ and $\Psi^B_0(\mathbf{r}_B, \theta^B_0, \varphi^B_0)$ with spinors which are opposite spins given by equations (1) and (2), with $\theta^B_0 = \pi - \theta^A_0$, $\varphi^B_0 = \varphi^A_0 - \pi$.

In the de Broglie-Bohm interpretation, we assume therefore that the intial position of the particle A is known $(x^A_0, y^A_0 = 0, z^A_0)$ as well as the particle B ($x^B_0 = x^A_0$,$y^B_0 = y^A_0 = 0$,$z^B_0 = z^A_0$).

A. Step 1: Measurement of A spin

Equation (8) shows that the spins of A and B remain opposite throughout step 1. Equation (10) shows that the densities of A and B are independent; for A equal to the density of a family of free particles in a classical Stern Gerlach apparatus, whose initial spin orientation has been randomly chosen; for B equal to the density of a family of free particles.

The spin of a particle A is oriented gradually following the position of the particle in its wave into a spin + or −. The spin of particle B follows that of A, while remaining opposite.

In equation (8) particle A can be considerd independent of B. We can therefore give it the wave function

$$\Psi^A(\mathbf{r}_A, t_0+\triangle t+t) = \cos\frac{\theta_0^A}{2} f^+(\mathbf{r}_A,t)|+_A\rangle + \sin\frac{\theta_0^A}{2} e^{i\varphi_0^A} f^-(\mathbf{r}_A,t)|-_A\rangle \tag{14}$$

which corresponds to a free particle in a Stern Gerlach apparatus and whose initial spin is given by $(\theta_0^A, \varphi_0^A)$. For an initial polarization $(\theta_0^A, \varphi_0^A)$ and an initial position (z_0^A), we obtain, in the de Broglie-Bohm interpretation[24], an evolution of the position $(z_A(t))$ and of the spin orientation of A $(\theta^A(z_A(t),t))$[13]. In the interval $[t_0, t_0+\Delta t]$, we obtain:

$$\begin{aligned} \frac{dz_A}{dt} &= \frac{\mu_0 B_0' t}{m} cos\theta(z_A,t) \\ \text{with}\quad \tan\frac{\theta(z_A,t)}{2} &= \tan\frac{\theta_0}{2} e^{-\frac{\mu_0 B_0' t^2 z_A}{2m\sigma_0^2}} \end{aligned} \tag{15}$$

with the initial condition $z_A(t_0) = z_0^A$; and in the interval $t_0+\Delta t+t$ $(t \geq 0)$:

$$\begin{aligned} \frac{dz_A}{dt} &= u\frac{\tanh(\frac{(z_\Delta+ut)z_A}{\sigma_0^2}) + \cos\theta_0}{1+\tanh(\frac{(z_\Delta+ut)z_A}{\sigma_0^2})\cos\theta_0} \\ \text{and}\quad \tan\frac{\theta(z_A(t),t)}{2} &= \tan\frac{\theta_0}{2} e^{-\frac{(z_\Delta+ut)z_A}{\sigma_0^2}}. \end{aligned} \tag{16}$$

Fig.2 presents in x0y a set of 10 particles A trajectories of which initial characteristics $(\theta_0^A, \varphi_0, z_0^A)$ have been randomly chosen: θ_0^A and φ_0^A with a uniform law and z_0^A with the Gauss law $N(0,\sigma_0)$. The spin orientation $\theta(z_A(t),t)$ is represented by arrows.

We can see that the final orientation, obtained after the decoherence time t_D, will depend on the initial particle position z_0^A in the wave packet and on the initial angle θ_0^A of the atom magnetic moment with the z axis.

The case of particle B is different. B follows a rectilinear trajectory with $y_B(t) = v_y t$, $z_B(t) = z_0^B$ and $x_B(t) = x_0^B$. By contrast, the orientation of its spin is driven by the orientation of the spin of A: $\theta^B(t) = \pi - \theta(z_A(t),t)$ and $\varphi^B(t) = \varphi(z_A(t),t) - \pi$.

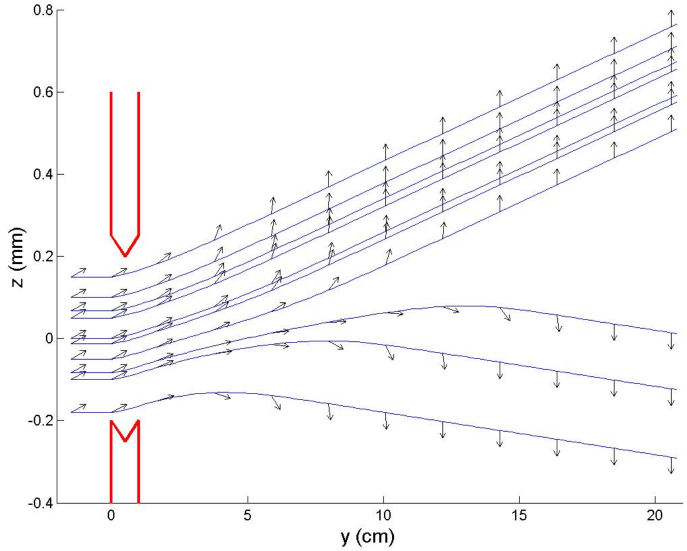

FIG. 2: 10 silver atom trajectories after the A electro-magnet; Arrows represent the spin orientation $\theta(z_A(t),t)$.

We can then associate the wave function:

$$\Psi^B(\mathbf{r}_B, t_0+\triangle t+t) = f(\mathbf{r}_B)\left(\cos\frac{\theta^B(t)}{2}|+_B\rangle + \sin\frac{\theta^B(t)}{2}e^{i\varphi^B(t)}|-_B\rangle\right). \tag{17}$$

During the first step, the singlet spinor in configuration space (8) can be replaced by the two single-particle spinors given by Equations (14) and (17).

B. Step 2: "Measurement" of B spin

Until time $t_0+\triangle t+t_D$, we are in step 1. Immediately after the "measurement" of A at the time $t_0+\Delta t+t_D$, if the A measurement is $\pm$, the conditional wave function of B is given by (11).

Then particle B is in position (x_0^B, z_0^B). We are exactly in the case of a particle in a Stern and Gerlach magnet $\mathcal{B}$ which is at an angle δ in relation to $\mathcal{A}$. To measure the spin of B, we refer to the basis $|\pm'_B\rangle$. So, after the measurement of B, at time $t_0+2(\triangle t+t_D)$, the conditional wave functions of B are given by (12) and (13), and we again find the quantum correlations.

IV. CONCLUSION

We have show that it is possible to replace the singlet spinor in configuration space (8) by the two single-particle spinors given by Equations (14) and (17).

From the wave function of two entangled particles, we have determined spins, trajectories and also a wave function for each of the two particles.

In this interpretation, the quantum particle has a local position like a classical particle, but it has also a non-local behaviour through the singlet wave function.

As we saw in step 1, the non-local influence in the EPR-B experiment only concerns the spin orientation, and not the motion of the particles themselves. This is a key point in the search for a physical explanation of non-local influence.

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1 Norsen, T.: The theory of (Exclusively) local Beables. Foundations of Physics, 40, 1858-1884 (2010).

2 Noren,T., Marian,D., Oriols,X.: Can the wave function in configuration space be replaced by single-particle wave function in physical space?. quant-ph/1410.3676v1 (2014).

3 de Broglie,L.: La Nouvelle Dynamique des quanta. In: Electrons et Photons, Rapports du Cinquième Conseil de Physique Solvay, Paris, Gauthier-Villard, 118-119 (1928). Translated in [5], The new dynamics of quanta, 385.

4 Schrödinger,E.: La mécanique des ondes. In: Electrons et Photons, Rapports du Cinquième Conseil de Physique Solvay, Paris, Gauthier-Villard, 118-119 (1928). Translated in [5], The mechanics of waves, 447.

5 Bacciagaluppi,G., Valentini,A.: Quantum Theory at the Crossroads, Cambridge University Press (2009)).(quant-ph/060918)

6 Dürr,D., Goldstein,S., Zanghi,N.,: Quantum equilibrium and the Origin of Absolute Uncertainty. Journal of Statistical Physics, 67, 843-907 (1992).

7 Dürr,D., Goldstein,S., Zanghi,N.: Quantum equilibrium and the role of operators as observables in quantum theory. Journal of Statistical Physics, 116, 959-1055 (2004).

[8] Gondran,M., Gondran,A.:Whereby we can replace the wave function in the configuration space? in preparation.

[9] Feynman,R.P., LeightonR.B., Sands,M.: The Feynman Lectures on Physics. Addison-Wesley, New York (1965).

[10] Cohen-Tannoudji,C., Diu,B., Laloë,F.: Quantum Mechanics. (Wiley, New York (1977).

[11] Sakurai,J.J.: Modern Quantum Mechanics. Addison-Wesley (1985).

[12] Le Bellac,M.: Quantum Physics. Cambridge University Press (2006).

[13] Gondran,M., Gondran,A.: A complete analysis of the Stern-Gerlach experiment using Pauli spinors. quant-ph/05 1276 (2005).

[14] Gondran,M., Gondran,A., Kenoufi,A.: Decoherence time and spin measurement in the Stern-Gerlach experiment, AIP Conf. Proc. 1424, 116-120 (2012).

[15] Gondran,M., Gondran, A.: A new causal interpretation of the EPR-B Experiment. AIP Conf. Proc. 1508, 370 (2012), (quant-ph/0806.4577v2)

[16] Bohm,D., Aharonov,Y.: Discussion of experimental proofs for the paradox of Einstein, Rosen and Podolsky. Phys. Rev.108, 1070 (1957).

[17] Einstein, A., Podolsky,B., Rosen,N.: Can quantum mechanical description of reality be considered complete?. Phys. Rev. 47,777-780 (1935).

[18] Bell,J. S.: On the Einstein Podolsky Rosen Paradox. Physics 1, 195 (1964).

[19] Bell,J.S.: Speakable and Unspeakable in Quantum Mechanics. Cambridge University Press (1987).

[20] Dewdney,C., Holland,P.R., Kyprianidis,A.: A causal account of non-local Einstein-Podolsky-Rosen spin correlations. J. Phys. A: Math. Gen. 20, 4717-32 (1987).

[21] Dewdney,C., Holland,P.R., Kyprianidis,A., Vigier,J.P.: Nature, 336, 536-44 (1988).

[22] Holland,P.R.: The quantum Theory of Motion. Cambridge University Press (1993).

[23] Cohen-Tannoudji,C., Diu,B., Laloë,F.: Quantum Mechanics, Wiley, New York (1977).

[24] Bohm,D., Hiley,B.J.: The Undivided Universe. Routledge, London and New York (1993).