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A PROPOSAL FOR COLLABORATIVE NAVIGATION IN CORRIDORS

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Abstract

The current study presents the proposal of a collaborative method to take advantage of the available information from surrounding airplanes in a corridor and provide a back-up navigation aid with a performance equivalent to GPS. Here it is considered that an aircraft may use the information of distance between itself and other airplanes provided by TCAS as well as the absolute position information of aircraft made available by ADS-B, creating a dynamic constellation similar to GPS where the satellites are replaced by the surrounding airplanes. A simulation study considering two types of traffic scenarios is developed to access the resulting accuracy of the proposed system.

Introduction

With the expected huge increase in air traffic over the next decades, it is believed that a large proportion of traffic will be organized in main air streams operating under advanced CNS procedures. Many studies and proposals of solution for a future congested ATM are developed and presented in many scientific events and magazines, and one of the most considered is known as "Flow Corridors". In order to enable this type of traffic flow, with minimal or no interference at all of controllers, the airplanes must follow certain traffic specifications of the corridor and be able to maintain self separation and surveillance of the inside traffic. In such environment, self-controlled dense traffic, the navigation should be based on accurate and reliable systems throughout the entire corridor.

The current study aims to present a collaborative method to take advantage of the many measurements from surrounding airplanes to improve their navigation performance. For the purpose of this study, it is understood that collaborative navigation is characterized by the utilization of measurements performed by surrounding aircraft and exchanged to improve the navigation performance of the each of them. Said in another way, an airplane surrounded by

other airplanes should experiment a better navigation performance than an aircraft flying alone.

The main objective of this study is to assess the possibility of using TCAS and ADS-B signals, already made available in commercial aircraft, as means to improve navigation performance. To improve navigation performance, an aircraft may use the information of distance between itself and other airplanes provided by TCAS, since the estimation of the absolute position of each airplane is available (ADS-B), and to create a dynamic constellation similar to GPS where the satellites are replaced by the surrounding airplanes. In this situation, instead of having the ephemeris with the satellite position we have an ADS-B message with the aircraft position and instead of measuring the distance aircraft-satellite using the satellite clock and received message time, we use the TCAS to measure the distance between aircraft. The challenges of this approach are not only the level of uncertainty of each measurement system, but also the lack of synchronization between measurements.

The mathematical model used in this study is similar to the one used for GNSS positioning applications, with a slight difference adopted to fit the proposed method. The concept used by GNSS is the Time of Arrival (TOA) where the time that a signal sent by the transmitter (satellite) to reach the user receiver is taken into account and its multiplication by the speed of light gives the distance between them. This supposes that the information about distance satellite-user and satellite position are synchronized. However in the considered case, all surrounding airplanes send their ADS-B message at different times and even the distance measurement and position signals of a given aircraft are not synchronized. So, to apply a least square technique to provide an additional estimation of position for each aircraft, it is necessary first to synchronize all this information with respect to a unique time line, a reference aircraft time line.

Collaborative Navigation in Corridors

Here the main ATM concepts used in this study are introduced.

Corridors

Many studies and proposals of solution for a future congested ATM are developed and presented in many scientific events and magazines, and one of the most considered is known as "Flow Corridors". Corridors are like long tubes with parallel trajectories in the vertical and horizontal planes, where the aircraft could fly for long distances with minimal interference from the conventional surrounding traffic as Youse et al. describe in [1]. Figure 1 shows a simplified corridor view with two flight levels and two lanes for each level (2 x 2).

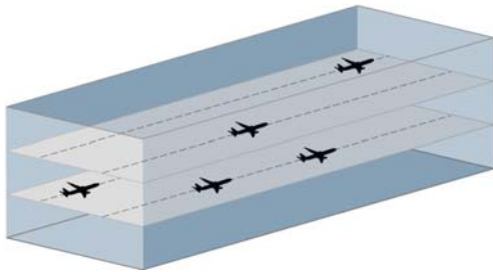


Figure 1. Basic flow corridor

Several characteristics permit to differentiate the corridors from the common route besides their layout:

- The parallel trajectories inside a corridor have all the same direction;
- Separation between airplanes inside the corridor is done by the aircraft itself, without interference of controllers;
- Corridors have entrance and exit points while in conventional routes aircraft can intercept the route where they want.

Flow corridors are not restricted to high flight levels, but their implementation seems to be more natural in this airspace since the corridors appear to be more advantageous for long distance flights.

In order to enable this type of traffic flow, with minimal or no interference at all of controllers, the airplanes must follow certain traffic specifications of the corridor and be able to maintain self separation

and surveillance of the inside traffic. In such environment, dense traffic and self-controlled, the navigation should be based on accurate and reliable systems throughout the entire corridor.

Collaborative Navigation

The study performed by MAIAA Lab about the air streams theory tries to evaluate many aspects of this kind of traffic flow, such as surveillance, corridors rules, transitions between the corridor and the outside airspace and also the navigation performance inside the corridor. Professionals and students are involved in this study led by MAIAA, each one developing a different aspect of the research, and the part intended for my internship is related to collaborative navigation.

Definition: For the purpose of this study, we should understand collaborative navigation as the utilization of measurements performed by surrounding aircraft and exchanged to improve the navigation performance of the each of them. Said in another way, an airplane surrounded by other airplanes should experiment a better navigation performance than an aircraft flying alone.

There are different ways to improve the navigation performance with the available navigation signals. The direction of this study is to assess the possibility of using TCAS and ADS-B signals, already made available in commercial aircraft, as means to improve navigation performance. Some studies using TCAS for improvement of positioning accuracy only for surveillance purposes have been already published [2,3].

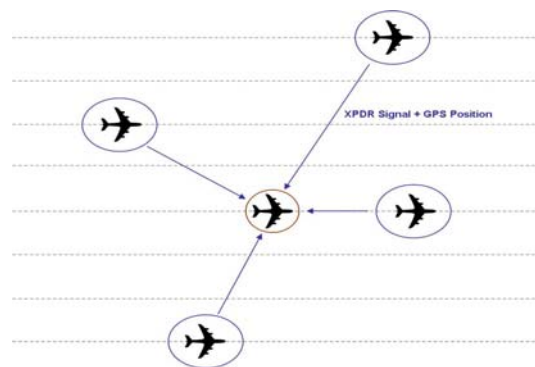


Figure 2. Triangulation with airplanes constellation

To improve navigation performance, an aircraft could use the information of distance between itself and other airplanes provided by TCAS, since the estimation of the absolute position of each airplane is available (ADS-B), and to create a dynamic constellation similar to GPS where the satellites are replaced by the surrounding airplanes (Fig. 2).

In the configuration above, instead of having the ephemeris with the satellite position we have an ADS-B message with the aircraft position and instead of measuring the distance aircraft-satellite using the satellite clock and received message time, we use the TCAS to measure the distance between aircraft. The challenges of this approach are not only the level of uncertainty of each measurement system, but also the lack of synchronization between measurements.

Position Determination Principle

Triangulation with airplanes constellation

Cartesian coordinates are adopted here considering that distances between involved aircraft are very small with respect to the size of Earth. The Euclidean distance between two points in Cartesian coordinates can be expressed as follows:

$$\rho^a = \sqrt{(x^a - x_r)^2 + (y^a - y_r)^2 + (z^a - z_r)^2} \quad a \in A \quad (1)$$

where A is the set of involved aircraft with $|A| = n$. Here, scalar ρ^a is the TCAS measurement between a surrounding aircraft a and the reference aircraft. Here (x^a, y^a, z^a) are the Cartesian coordinates of the surrounding aircraft a and (x_r, y_r, z_r) are the Cartesian coordinates of the reference aircraft which have to be estimated.

There are different nonlinear methods [4,5,6] to solve the pseudo-range equation (1) for multiple surrounding airplanes (at least 3 of them) and to find the reference aircraft position. Among these methods, the linear approximation method appears with particular interest since the linear method is not only easy from the computational side but also brings the into view the concept of Dilution of Precision (DOP). This method produces a mean square error solution by solving iteratively linear approximations of the set of equations (1). The coordinates that present the minimum mean square error value are considered to be the best estimation.

So, let at time t the variations at iteration k be:

$$\underline{\Delta P}_r^{k,t} = \begin{cases} \Delta x_r^{k,t} = \hat{x}_r^{k,t} - \hat{x}_r^{k-1,t} \\ \Delta y_r^{k,t} = \hat{y}_r^{k,t} - \hat{y}_r^{k-1,t} \\ \Delta z_r^{k,t} = \hat{z}_r^{k,t} - \hat{z}_r^{k-1,t} \end{cases} \quad (2)$$

where $\hat{x}_r^{k-1}, \hat{y}_r^{k-1}, \hat{z}_r^{k-1}$ are the current estimates of the position coordinates and $\hat{x}_r^k, \hat{y}_r^k, \hat{z}_r^k$ are the new estimates of the position coordinates. We will write:

$$\underline{P}_r^{k,t} = \begin{bmatrix} \hat{x}_r^{k,t} \\ \hat{y}_r^{k,t} \\ \hat{z}_r^{k,t} \end{bmatrix} \quad k = 1, 2, \dots \quad (3)$$

Let the separation coordinates considered at iteration k with aircraft a be given by:

$$\begin{aligned} X_{r,a}^{k,t} &= x^a(t) - \hat{x}_r^{k-1,t} \\ Y_{r,a}^{k,t} &= y^a(t) - \hat{y}_r^{k-1,t} \\ Z_{r,a}^{k,t} &= z^a(t) - \hat{z}_r^{k-1,t} \end{aligned} \quad a \in A \quad (4)$$

Let $\hat{x}_r(t-h), \hat{y}_r(t-h), \hat{z}_r(t-h)$, $h = 1, 2, \dots$ be the estimates of the position coordinates at times $t-1, t-2, \dots$. Then the initial estimation can be taken as (here $\delta t = 1$ second):

$$\begin{aligned} x_r^{0,t} &= \hat{x}_r(t-1) + (\hat{x}_r(t-1) - \hat{x}_r(t-2)) \cdot \delta t \\ y_r^{0,t} &= \hat{y}_r(t-1) + (\hat{y}_r(t-1) - \hat{y}_r(t-2)) \cdot \delta t \\ z_r^{0,t} &= \hat{z}_r(t-1) + (\hat{z}_r(t-1) - \hat{z}_r(t-2)) \cdot \delta t \end{aligned} \quad (5)$$

The first order approximation of equations (1) at time t for point $\hat{x}_r^{k-1}, \hat{y}_r^{k-1}, \hat{z}_r^{k-1}$ is given by:

$$\rho^a(t) = \rho_{r,a}^{k-1,t} - \frac{X_{r,a}^{k,t}}{\rho_{r,a}^{k-1,t}} \Delta x_r^{k,t} - \frac{Y_{r,a}^{k,t}}{\rho_{r,a}^{k-1,t}} \Delta y_r^{k,t} - \frac{Z_{r,a}^{k,t}}{\rho_{r,a}^{k-1,t}} \Delta z_r^{k,t} \quad a \in A \quad (6)$$

where

$$\rho_{r,a}^{k-1,t} = \sqrt{\begin{aligned} &(x^a(t) - \hat{x}_r^{k-1,t})^2 \\ &+ (y^a(t) - \hat{y}_r^{k-1,t})^2 \\ &+ (z^a(t) - \hat{z}_r^{k-1,t})^2 \end{aligned}} \quad a \in A \quad (7)$$

Considering the approximate equations (6) for the n surrounding airplanes and writing them in matrix format, we can write:

$$\underline{\Delta D}_r^k = H_r^{k,t} \underline{\Delta P}_r^{k,t} \quad (8)$$

with:

$$\underline{\Delta D}_r^k = \begin{bmatrix} \rho^1(t) - \rho_{r,1}^{k-1,t} \\ \rho^2(t) - \rho_{r,2}^{k-1,t} \\ \vdots \\ \rho^n - \rho_{r,n}^{k-1,t} \end{bmatrix} \quad (9)$$

and

$$H_r^{k,t} = \begin{bmatrix} -X_{r,1}^{k-1,t} / \rho_{r,1}^{k-1,t} & -Y_{r,1}^{k-1,t} / \rho_{r,1}^{k-1,t} & -Z_{r,1}^{k-1,t} / \rho_{r,1}^{k-1,t} \\ -X_{r,2}^{k-1,t} / \rho_{r,2}^{k-1,t} & -Y_{r,2}^{k-1,t} / \rho_{r,2}^{k-1,t} & -Z_{r,2}^{k-1,t} / \rho_{r,2}^{k-1,t} \\ \vdots & \vdots & \vdots \\ -X_{r,n}^{k-1,t} / \rho_{r,n}^{k-1,t} & -Y_{r,n}^{k-1,t} / \rho_{r,n}^{k-1,t} & -Z_{r,n}^{k-1,t} / \rho_{r,n}^{k-1,t} \end{bmatrix} \quad (10)$$

where $\underline{\Delta D}_r^k$ can be computed from equation (7) and the TCAS signals. The solution w.r.t. $\underline{\Delta P}_r^{k,t}$ of equations (8) depends on the number of surrounding aircraft:

- If the number of surrounding aircraft is less than 3, this system has no particular solution (infinite number of solutions).

- If there are 3 surrounding aircraft, there is a unique solution when H is invertible.

- If there are more than 3 surrounding aircraft, there is in general no solution. Then, minimizing $\|\underline{\Delta D}_r^k - H_r^{k,t} \underline{\Delta P}_r^{k,t}\|^2$ we get the least square estimation of $\underline{\Delta P}_r^{k,t}$ which is given by:

$$\underline{\Delta \hat{P}}_r^{k,t} = \left(H_r^{k,tT} \cdot H_r^{k,t} \right)^{-1} H_r^{k,tT} \underline{\Delta D}_r^k \quad (11)$$

So the updated position at iteration k will be:

$$\underline{P}_r^{k,t} = \underline{P}_r^{k-1,t} + \underline{\Delta \hat{P}}_r^{k,t} \quad (12)$$

Iterations will be stopped at k^* , either when $k^* = k_{\max}$ or when condition:

$$\left\| \underline{\Delta \hat{P}}_r^{k^*,t} \right\| \leq \varepsilon \quad (13)$$

is satisfied, where ε is a very small positive number.

Theoretical performance evaluation

In the previous subsection we described the iterative method to estimate the position of the reference aircraft, and as in the classical GNSS linear method, it is possible to determine the covariance of the position error to assess the accuracy of the considered navigation process.

For a fixed configuration where $H = H_r^{k^*,t}$ when considering by analogy with GNSS that the pseudo-range errors $\underline{\Delta D} = \underline{\Delta D}_r^{k^*}$ are random variables with Gaussian distributions around zero means [ref], then according to (11), $\underline{\Delta P} = \underline{\Delta \hat{P}}_r^{k^*,t}$ are also Gaussian variables with zero mean. Then:

$$\text{cov}(\underline{\Delta P}) = E \left\{ \underline{\Delta P} \cdot \underline{\Delta P}^T \right\} \quad (14)$$

or

$$\text{cov}(\underline{\Delta P}) = (H^T H)^{-1} H^T E \left\{ \underline{\Delta D} \cdot \underline{\Delta D}^T \right\} H (H^T H)^{-1} \quad (15)$$

When considering that the components of $\underline{\Delta D}$ are statistically independent and are identically distributed, it is possible to write:

$$\text{cov}(\underline{\Delta D}) = \sigma^2 I_3 \quad (15)$$

where σ^2 is a pseudo-range error, then:

$$\text{cov}(\underline{\Delta P}) = \sigma^2 \cdot (H^T H)^{-1} \quad (16)$$

Writing \underline{h}_i the i^{th} column of H , we get:

$$(H^T H)^{-1} = \left[\underline{h}_i^T \underline{h}_j \right]^{-1} = [d_{ij}] \quad (17)$$

Then:

$$\text{cov}(\underline{\Delta P}) = [\sigma^2 d_{ij}] \quad (18)$$

and a position dilution of precision (PDOP) is given by:

$$PDOP = \sigma \sqrt{d_{11} + d_{22} + d_{33}} \quad (19)$$

Time frames and offsets

An important adjustment to be performed to the GPS model to fit with the current study is the removal of the clock offset parameters since here there is no such offset: the transmitter clock is also the receiver clock, since the TCAS generates the

transmission time of interrogation and also the received time of reply.

The time frame of data reception is probably the biggest issue to be solved in the present approach. All the measurements, ADS-B and TCAS, are received by the transponder (XPDR) but not at the same time because the XPDR is not able to receive different sources simultaneously. So, the data received from other airplanes is not synchronized with the TCAS time frame of the reference aircraft. Therefore, the time frame has to comply with the unsynchronized issue and also with the update interval chosen equal to one second.

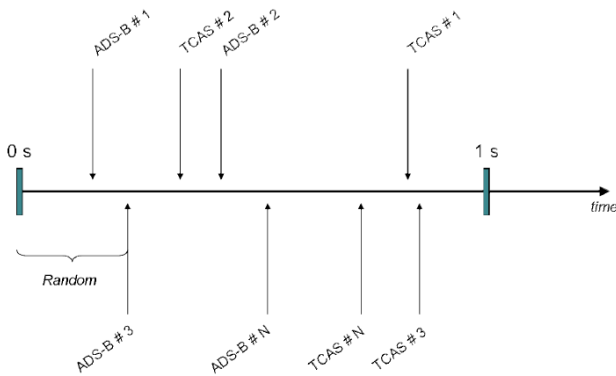


Figure 3. Time frame of received data from surrounding aircraft

All data is received with the same interval but at different times, while to deploy the GNSS model it is necessary to have synchronized information. For GPS systems it is not a problem because the receiver is able to receive many satellites at the same time and the distance measurement is made with the same signal used to transmit the satellite position. This implies that the information about distance satellite-user and satellite position are synchronized. In our case we do not have the same advantage. All surrounding airplanes send their ADS-B message at different times and even the distance measurement and position of a given aircraft are not synchronized. So, to apply the proposed method we have to synchronize all this information with respect to a unique time line, the reference aircraft time line.

TCAS synchronization

Initial simulation trials were made just repeating the distance measurement in the reference time but it

was easy to notice a delay in the final position results. Therefore, it was decided to estimate a correction for TCAS measurements considering the delay between TCAS time and reference time (t_{TCAS}). The method proposed was to use the information of the previous TCAS measurement and estimate the speed that the distance increases or decreases. The result is a scalar value that when multiplied by t_{TCAS} and added to the last TCAS information it produces an estimated distance at reference time:

$$TCAS_k^{sync} = TCAS_k + (TCAS_k - TCAS_{k-1}) / t_{TCAS} \quad (20)$$

Figure 4 displays a situation where the TCAS measures the distance between the reference aircraft and the sur-rounding airplane #7 for the first time ($k=1$) at 0.27 seconds and the second measurement ($k=2$) at 1.27 seconds. The distance values are 4828 m and 4880 m respectively. Thus, to synchronize with the reference time at 2 seconds we need to calculate the TCAS delay ($t_{TCAS} = 1 - 0.27 = 0.73$ s), to multiply it by the difference between both measurements ($4880 - 4828 = 52$ m) and to sum to the last TCAS data to find the estimated distance ($(520.73 + 4880 = 4917.96$ m).

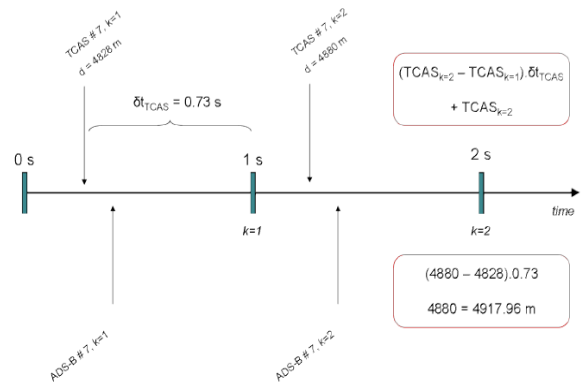


Figure 4. TCAS data synchronization

ADS-B synchronization

For the position data received from ADS-B message we have the same problem of synchronization but the extra information about the velocity vector makes easier the estimation of surrounding airplane positions at reference time. The ADS-B message contains the velocity vector (V_x ; V_y and V_z) of the transmitter aircraft which contains also some uncertainty but still helpful to estimate the

synchronized position. Unlike the TCAS estimation, since we have now this velocity vector it is not necessary to use previous reference aircraft position information to estimate the current position, only the last position and velocity data from the transmitter aircraft. So, to estimate the synchronized position of the transmitter air-craft we have just to multiply the velocity component by the delay time and to add to the respective coordinate:

$$\hat{x}_k = x_k + V_x \cdot t_{ads} \quad (21)$$

Example - The ADS-B message received brings the information that the x coordinate of the airplane #7 is 8945 m and the x component of its velocity is 142 m/s. If t_{ads} is equal to 0.61 seconds, so the estimated x coordinate for synchronized time is 9031.62 m ($8945 + 0.61 \cdot 142 = 9031.62$ m). We shall apply the same for y and z coordinates.

Simulation Study

To simulate these situations, we adopted the following definitions and simplifications:

The reference time line is the clock of the reference aircraft, which starts on zero and has interval update of $t = 1$ second (the main line in figures 3, 4 and 5).

The interval update of all data has the same value, $t = 1$ second also.

For each data, the first received message time is zero plus a random value in seconds. Each data has its own random time start, so there is no order in receiving messages. For example, the TCAS data from airplane #2 could be the first message to be received, the next could be the ADS-B data from airplane #7 and so on.

Two types of scenario were specified: A corridor scenario, the main theory considered in the study, and a scenario named "free route", where all the trajectories do not follow straight lines or same direction.

The main objective to have both scenarios was to be able to compare the performance of the proposed approach in both situations and assess the possible effect caused by the organization of aircraft flows in the corridor scenario.

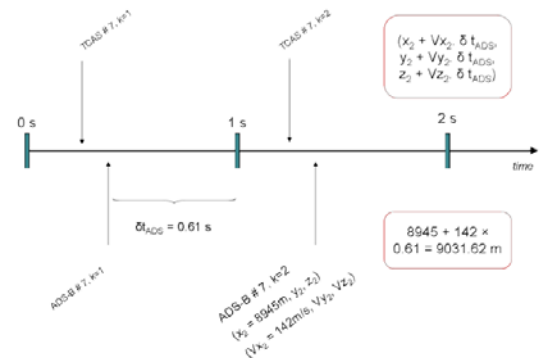


Figure 5. ADS-B data synchronization

Corridor Scenario

To set up a configuration, considering the reference aircraft performing a flight inside a corridor in cruise phase, initially are introduced the adopted minimum allowed separations and spacing between the involved aircraft:

- Minimum longitudinal spacing of 5 NM as performed in dense traffic airspace;
- Vertical separation of 1000 ft as specified by RVSM airspace;
- Horizontal separation of 4 NM adopted in RNP 1.

Using the minimum separation parameters defined above, the next step is to define the corridor layout to be applied in the simulation for different scenarios. The adopted corridor layout has 8 parallel lanes in 3 different flight levels (FL), totaling 24 trajectories inside the corridor, as shown in figure 6 below.

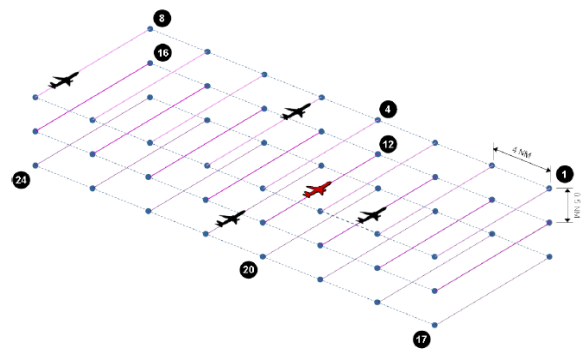


Figure 6. Chosen corridor structure

For codification in the simulation algorithm, the lanes have been identified by numbers taken between

1 and 24. Indexes begin at the right upper corner and going left and from upper to lower lanes. The reference aircraft, which is supposed to perform collaborative navigation, is assigned to lane 12 (the red aircraft in the figure above).

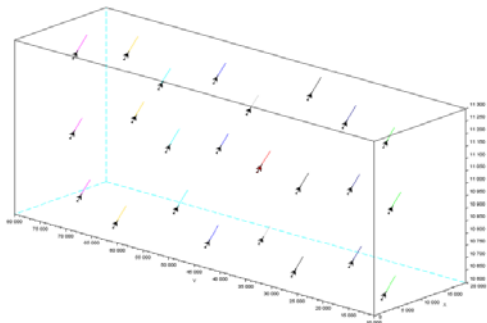


Figure 7. Corridor airspace configuration with 23 surrounding aircraft

Simulations were performed for 8 different traffic corridor configurations, using respectively 23, 20, 17, 14, 12, 11, 8 and 5 surrounding airplanes (see figure 7 for a 23 aircraft scenario). The scenario with 12 airplanes was used to compare with the free route scenario, but it was not taken into account in the corridor configuration tendency study.

Free Route Scenario

This scenario has two main differences comparing to the corridor one:

- the aircraft trajectories are not straight lines but follow arcs with different radius;
- not all airplanes fly the same direction

Since this scenario has been introduced with a comparative purpose, a single configuration with 12 surrounding aircraft has been considered (see figure 8) in the present paper.

Figure 9 shows a comparative picture between the free route and corridor scenario, both with 12 surrounding aircraft.

Simulation results

A post processing of the points estimated in relation to true position of the reference aircraft has been

performed. Figures 10 and 11 show these distributions for the free route and the corridor scenarios respectively.

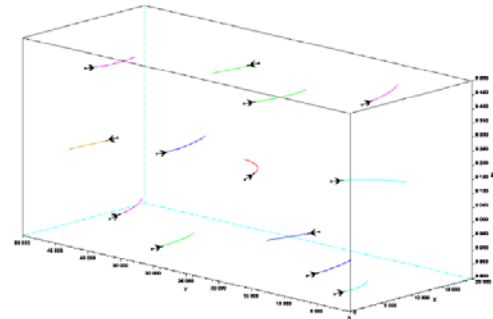


Figure 8. Free Route airspace configuration with 12 surrounding aircraft

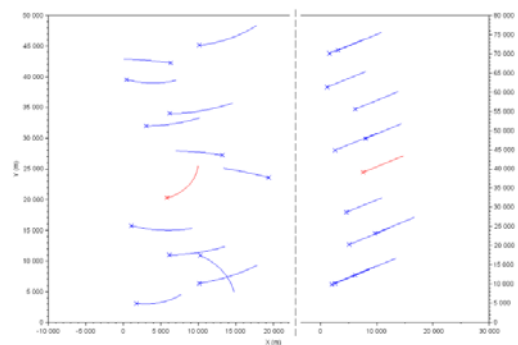


Figure 9. Free route and corridor traffic scenarios

To build the distribution graphics 29000 points have been computed, corresponding to the number of performed simulations (1000) multiplied by the number of points considered in each simulation (29). The flights simulations has been performed during 30 seconds time windows.

It can be observed in figures 10 and 11 that the error distribution does not follow a Gaussian distribution as expected for a GNSS model, so the accuracy with a probability of 95% was got statistically. The obtained accuracy for the corridor scenario is 19.2 meters and 31 meters for the free route scenario. The obtained mean values are 8.5 meters and 12.1 meters respectively.

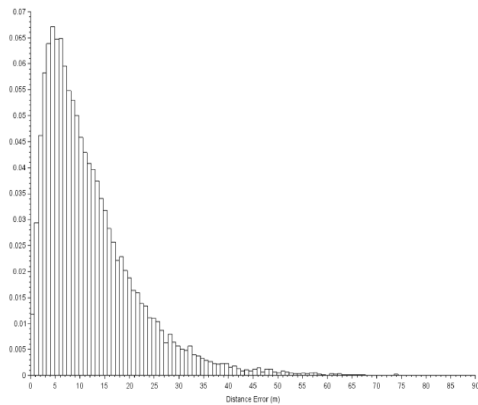


Figure 10 Free route horizontal error distribution

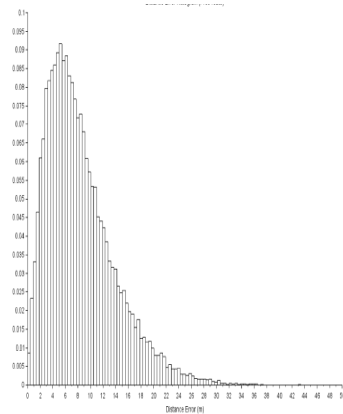


Figure 11. Corridor horizontal error distribution

Figure 12 displays the evolution of the horizontal error distribution when the quantity of surrounding airplanes, represented by the numbers inside the circles, is increased.

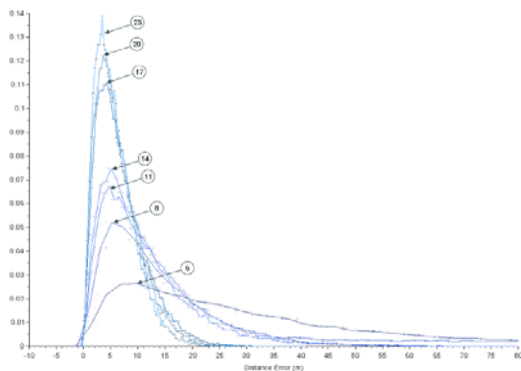


Figure 12. Evolution of error with the number of aircraft

Conclusions

In this communication, we proposed an additional navigation device totally independent of actual on-board navigation systems in a modern transportation aircraft and based on collaboration with surrounding traffic. Then, the design of a backup navigation system to face a failure of a classical navigation system appears feasible. This is very important in high traffic density situations. The accuracy performance of the new system, obtained through collaborative navigation, seems to be comparable to current GNSS systems, when there is enough surrounding traffic.

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