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# A Clustering-Based Algorithm for Aircraft Conflict Avoidance

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**Abstract** Aircraft conflict avoidance is a critical issue in Air Traffic Management, which can be addressed, among others, by means of Mixed Integer Non-Linear Programming (MINLP) techniques.

In this work we introduce a new approach to address, via velocity regulation, the problem of avoiding conflict for a set of aircraft flying in an air sector at cruise flight. Speed variations for all aircraft are to be minimized so that, at any time instant, the horizontal distance between each pair of aircraft is above a threshold security value. The problem can be expressed as a MINLP, solvable with standard MINLP solvers for a small number of aircraft, but intractable for instances of more realistic size. This motivates us the design of a cluster-based procedure.

In our approach, aircraft are clustered into groups, so that within each cluster the aircraft are conflicting, while conflicts do not exist (or they are at least less severe) between aircraft in different clusters. Then a MINLP solver is used sequentially on each cluster to minimally modify the aircraft speeds so that conflicts within the cluster are solved, not creating new conflicts with aircraft in other clusters.

Theoretical convergence of the procedure and preliminary numerical results will be discussed in the talk.

**Keywords:** Clustering, MINLP, Aircraft conflict avoidance, Subliminal speed control

## 1. Introduction

One of the main and most crucial tasks of air traffic control services is continuously monitoring aircraft trajectories, detecting potential *conflicts*, i.e., potential loss of separation between trajectories, and issuing appropriate conflict resolution maneuvers. The increasing air traffic on the world-scale has an immediate impact on air traffic controllers' workload, making it more and more difficult to handle aircraft conflicts. Hence, a higher level of automation in Air Traffic Management and Control urgently needs to be introduced, so that automatic aircraft conflict avoidance procedures have in particular received a growing attention over the past few years.

Mathematical optimization naturally arises in this context, as one usually aims at separating conflicting aircraft while optimizing a selected criterion (e.g., stay as close as possible to the original trajectory, or minimizing delays induced by separation maneuvers). Mixed-Integer Nonlinear Programming is attracting a growing attention for the considered real-world application, as it enables to model the complex nonlinear (nonconvex) aircraft separation constraints while considering mixed variables (continuous variables typically used for aircraft speeds, heading angles, etc., and integer ones typically used for logical choices). First approaches based on mixed-integer optimization date back to 2002 ([5, 8]), and more recently, were proposed in [1, 2, 3, 6, 7].

One can observe that the complexity of the problem under consideration is specially related to the nonlinear nonconvex separation constraints, that are indexed on all pairs of aircraft, and

whose number increases quadratically with the number of aircraft. Hence, exact solution algorithms easily turn to be computational demanding, specially when the number of aircraft considered simultaneously is large. On the other hand, in realistic situations, the airspace section under consideration is a quite large portion of the airspace where generally only small groups of aircraft with close trajectories may potentially be in conflict. This suggest to decompose the problem into smaller subproblems (clusters) of conflicting aircraft. Such an approach was introduced in [3], where however a procedure to create subgroups of aircraft was not proposed.

In the present work, we propose to decompose the overall problem into subproblems involving only a small number of aircraft and to perform conflict avoidance exactly on these subproblems, then combining all the obtained solutions. Subproblems are described through mixed-integer nonlinear programs, and are constructed by clustering aircraft using a suitable dissimilarity measure. Each cluster is solved to optimality, then clusters are modified if conflicts are still present, and the process is repeated until no conflicts occur anymore.

## 2. Problem modeling

Let us consider a set  $A$  of  $n$  aircraft flying during their cruise flight in a given air sector, all at the same flight level. The horizontal separation only has to be satisfied. This means that, at any time instant  $t \geq 0$ , the distance between any pair of aircraft  $i, j$  should not be smaller than the horizontal separation standard  $d$ . Assuming that uniform motion laws apply, the position  $\mathbf{x}_i(t)$  of aircraft  $i$  at time  $t$  is given by

$$\mathbf{x}_i(t) = \mathbf{x}_i(0) + t\mathbf{v}_i, \quad (1)$$

where both the initial position  $\mathbf{x}_i(0)$  and velocity  $\mathbf{v}_i$  are assumed to be known, and the distance  $\|\mathbf{x}_i(t) - \mathbf{x}_j(t)\|$  between  $i$  and  $j$  satisfies

$$\|\mathbf{x}_i(t) - \mathbf{x}_j(t)\|^2 = t^2 \|\mathbf{v}_i - \mathbf{v}_j\|^2 + 2t(\mathbf{v}_i - \mathbf{v}_j) \cdot \mathbf{x}_{ij}^{r0} + \|\mathbf{x}_{ij}^{r0}\|^2, \quad (2)$$

where  $\|\cdot\|$  is the Euclidean norm in the two-dimensional space and  $\mathbf{x}_{ij}^{r0} = \mathbf{x}_i(0) - \mathbf{x}_j(0)$  denotes the relative position at time  $t = 0$  of aircraft  $i$  with respect to  $j$ .

Since, keeping the original trajectories, equation  $\|\mathbf{x}_i(t) - \mathbf{x}_j(t)\| \geq d$  may not be fulfilled i.e., aircraft may be in conflict, suitable separation maneuvers, corresponding to trajectory deviations, have to be carried out. We consider aircraft speed deviations, following the concept of *subliminal* speed control, that was introduced as a promising method to improve traffic congestion while maintaining a low impact on controller's workload [4]. In this framework, we allow each aircraft  $i$  to (slightly) modify its speed from  $\mathbf{v}_i$  to  $q_i\mathbf{v}_i$ , where the speed variations  $q_i$  are bounded in the interval  $[-6\%v_i, +3\%v_i]$ .

Speed variations  $q_i$  for all  $i$ , represent the main decision variables of our nonlinear optimization problem, that is formulated as follows:

$$\begin{aligned} \min \quad & \sum_{i \in A} (q_i - 1)^2 \\ \text{s.t.} \quad & d_{ij}(q_i, q_j) \geq d \quad \forall i, j \in A, i < j \\ & q_{\min} \leq q_i \leq q_{\max} \quad \forall i \in A \end{aligned} \quad (3)$$

where  $q_{\min}$  and  $q_{\max}$  are lower and upper bounds on  $q_i$ , and  $d_{ij}(q_i, q_j)$  represents the distance between aircraft  $i$  and  $j$ . It is a nonlinear nonconvex expression, as detailed below.

## 3. Aircraft clustering-based algorithm

For speed variations  $q_i$ ,  $i \in A$ , the second-degree polynomial function in equation (2) attains its minimum in  $[0, \infty)$  at time instant

$$\max \{0, t_{ij}^m(q_i, q_j)\}, \quad (4)$$

with

$$t_{ij}^m(q_i, q_j) = \frac{-(q_i \mathbf{v}_i - q_j \mathbf{v}_j) \cdot \mathbf{x}_{ij}^{r0}}{\|q_i \mathbf{v}_i - q_j \mathbf{v}_j\|^2}. \quad (5)$$

Hence, by substituting in (2), we have that the separation between aircraft  $i$  and  $j$  is

$$d_{ij}(q_i, q_j) = \begin{cases} \frac{\sqrt{\|\mathbf{x}_{ij}^{r0}\|^2 \|q_i \mathbf{v}_i - q_j \mathbf{v}_j\|^2 - (\mathbf{x}_{ij}^{r0} \cdot (q_i \mathbf{v}_i - q_j \mathbf{v}_j))^2}}{\|q_i \mathbf{v}_i - q_j \mathbf{v}_j\|}, & \text{if } t_{ij}^m(q_i, q_j) \geq 0 \\ \|\mathbf{x}_{ij}^{r0}\|, & \text{else} \end{cases} \quad (6)$$

Equation (6) gives nonlinear nonconvex constraints for problem (3). Furthermore, the condition “if  $t_{ij}^m(q_i, q_j) \geq 0$ ” leads us to introduce for each  $i, j \in A, i < j$ , the binary variables  $y_{ij}$  as

$$y_{ij} = \begin{cases} 1, & \text{if } t_{ij}^m \geq 0 \\ 0, & \text{else,} \end{cases} \quad (7)$$

or, equivalently, as

$$y_{ij} = \begin{cases} 1, & \text{if } (q_i \mathbf{v}_i - q_j \mathbf{v}_j) \cdot \mathbf{x}_{ij}^{r0} \leq 0 \\ 0, & \text{else,} \end{cases} \quad (8)$$

i.e., the constraint

$$((q_i \mathbf{v}_i - q_j \mathbf{v}_j) \cdot \mathbf{x}_{ij}^{r0}) (2y_{ij} - 1) \geq 0. \quad (9)$$

We decompose the overall problem into subproblems involving only a small number of aircraft and perform aircraft conflict resolution exactly on these subproblems, then combining all the obtained results. To do so, clustering is performed to make groups of aircraft. As a dissimilarity measure, we consider the pairwise critical distance for aircraft separation  $d_{ij}(q_i, q_j)$  as defined in (6). The smaller the value of  $d_{ij}(q_i, q_j)$ , the more critical the conflict is between aircraft  $i$  and  $j$ , i.e., the more similar  $i$  and  $j$  are. We propose a hierarchical agglomerative clustering procedure using single linkage (closest distance) to merge groups, and imposing an upper bound  $\sigma$  on each cluster size in order to avoid clusters to be unbalanced in size.

Given aircraft with speeds  $(\bar{q}_i \mathbf{v}_i)_{i \in A}$ , the above clustering procedure yields a list of clusters  $A_1, \dots, A_m$ . Such a list is sequentially inspected, and, for each cluster  $A_c, c = 1, 2, \dots, m$ , an optimization problem is solved, in which the speeds of all aircraft  $j$  outside  $A_c$  is considered to be fixed to  $\bar{q}_j \mathbf{v}_j$ . More precisely, we seek the values  $(q_i)_{i \in A_c}$  as close as possible to 1 so that

1. inside  $A_c$ , all conflicts are solved, i.e.,  $d_{ij}(q_i, q_j) \geq d$  for all  $i, j \in A_c, i \neq j$ ,
2. for a given set of aircraft pairs  $B_c \subset \{(i, j) : i \in A_c, j \notin A_c, d_{ij}(\bar{q}_i, \bar{q}_j) \geq d\}$ , no conflict appears, i.e.,  $d_{ij}(q_i, \bar{q}_j) \geq d$ .

Formally, the subproblem to be addressed for cluster  $A_c$  can be stated as

$$\min \sum_{i \in A_c} (q_i - 1)^2 \quad (10)$$

$$\text{s.t. } d_{ij}(q_i, q_j) \geq d \quad \forall (i, j) \in A_c, i < j \quad (11)$$

$$d_{ij}(q_i, \bar{q}_j) \geq d \quad \forall (i, j) \in B_c \quad (12)$$

$$q_{\min} \leq q_i \leq q_{\max} \quad \forall i \in A_c. \quad (13)$$

Convergence issues and empirical performance on test instances will be presented.

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## References

- [1] A. Alonso-Ayuso, L. F. Escudero, and F. J. Martín-Campo. On modeling the air traffic control coordination in the collision avoidance problem by mixed integer linear optimization. *Annals of Operations Research*, 222:89–105, 2014.
- [2] A. Alonso-Ayuso, L. F. Escudero, P. Olayo, and C. Pizarro. Conflict avoidance: 0-1 linear models for conflict detection & resolution. *TOP*, 21(3):485–504, 2013.
- [3] S. Cafieri and N. Durand. Aircraft deconfliction with speed regulation: new models from mixed-integer optimization. *Journal of Global Optimization*, 58(4):613–629, 2014.
- [4] C. Dupré, D. Bonini, and G. Granger. How erasmus can support an increase in capacity in 2020. In *Proceedings of the 7th International Conference on Computing, Communications and Control Technologies: CCCT 2009*, 2009.
- [5] L. Pallottino, E. M. Feron, and A. Bicchi. Conflict resolution problems for air traffic management systems solved with mixed integer programming. *Intelligent Transportation Systems, IEEE Transactions on*, 3(1):3–11, 2002.
- [6] D. Rey, C. Rapine, V. V Dixit, and S. T. Waller. Equity-oriented aircraft collision avoidance model. *IEEE Transactions on Intelligent Transportation Systems*, 16(1):172–183, 2015.
- [7] D. Rey, C. Rapine, R. Fondacci, and N.-E. El Faouzi. Subliminal speed control in air traffic management: Optimization and simulation. *Transportation Science*, 240–262, 2015.
- [8] A. Richards and J.P. How. Aircraft trajectory planning with collision avoidance using mixed integer linear programming. In *Proceedings of the American Control Conference 2002*, volume 3, pages 1936–1941 vol.3, 2002.