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Sonia Cafieri[1], Frédéric Messine[2] and Ahmed Touhami[1,3]

[1]*ENAC, MAIAA, F-31055 Toulouse, France*
University of Toulouse, IMT, F-31400 Toulouse, France, sonia.cafieri@enac.fr

[2]*LAPLACE-ENSEEIHT-INPT, University of Toulouse, France,* frederic.messine@n7.fr

[3]*Faculty of Sciences and Technologies. Hassan I University, Settat, Morocco,* ahmed.touhami@gmail.com

Abstract In this paper, some improvements of spatial Branch and Bound (sBB) algorithms are discussed to solve aircraft conflict avoidance problems formulated as MINLP. We propose a new quadratic convex relaxation technique based on affine arithmetic. Moreover, a branching strategy is also proposed for the considered problem. Preliminary numerical results validates the proposed approach.

Keywords: Aircraft conflict avoidance, Interval Branch and Bound, Convex relaxation, Affine forms.

1. Introduction

In this work, we deal with the use of deterministic global optimization to solve the aircraft conflict avoidance problem by means of aircraft speed changes. Specifically, we focus on exact global solvers based on Branch and Bound methods. The selected solvers are `Couenne` and `IBBA`, the first based on convex relaxations and the latter based on rigorous interval computations and linear relaxations. Both codes also include interval constraint propagation techniques.

Two aircraft are said *in conflict* when the horizontal distance between them and their altitude distance are smaller than standard safety distances. In this paper, we consider aircraft in their *en route* cruise phase, all at the same altitude, so that only their horizontal distances have to be handled through appropriate *separation constraints*. Aircraft are monitored and suitable separation maneuvers are issued if in the observed time window conflicts may potentially occur. The separation maneuver considered here is aircraft speed deviations, while the directions of motions are kept fixed. Aircraft speed changes may not be able to solve all possible conflict situations, like in the case of two aircraft flying face-to-face; such an approach is however considered very promising to reduce the complexity of air traffic. *Subliminal* speed control is in particular interesting: it is a speed control where aircraft speeds are changed in a very tight range around original speeds, namely between -6% and 3%. In this work we further consider speeds between -12% and 6% of the original speeds as a second range for testing.

The optimization model for aircraft conflict avoidance based on speed changes considered in this work is described in Section 2. In Section 3 we briefly recall on the main characteristics of the two global optimization solvers `Couenne` and `IBBA`. A new convex relaxation based on *affine arithmetic*, to be used within `IBBA`, is proposed in Section 4 for the quadratic convex objective function of the considered model. In Section 5, some numerical tests are discussed. Some conclusions are given in Section 6.

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2. MINLP model

In this section, we recall the main elements of a model developed and described in [1]. It is a MINLP model, where aircraft can change their speed once during the observed time window in order to get a conflict-free configuration. The main decision variables are $q_i, i \in A$ (A being the set of aircraft), representing the aircraft speed variations. For each aircraft i, $q_i = 1$ means that there is no change, $q_i > 1$ means that aircraft i accelerates and $q_i < 1$ that it decelerates. The optimization criterion is $\sum_{i=1}^{n}(q_i - 1)^2$, where n is the number of aircraft. The main difficulty is represented by the separation constraints, for each pair of aircraft i and j:

$$\|x_i(t) - x_j(t)\| \geq d \qquad \forall t \in (0, T), \tag{1}$$

where T is the time horizon, d is the minimum required separation distance (5 Nautic Miles), and $x_i(t)$ is the position of aircraft i. Letting $\mathbf{x}_{ij}^r(t) = x_i(t) - x_j(t)$ be $\mathbf{x}_{ij}^{r0} + \mathbf{v}_{ij}^r\, t$, with $\mathbf{x}_{ij}^{r0}$ the relative initial position of aircraft i and j, and $\mathbf{v}_{ij}^r$ be their relative speed, one obtains $||\mathbf{x}_{ij}^{r0} + \mathbf{v}_{ij}^r\, t||^2 \geq d^2\, \forall t \in (0, T)$, and therefore

$$\|\mathbf{v}_{ij}^r\|^2\, t^2 + 2(\mathbf{x}_{ij}^{r0} \cdot \mathbf{v}_{ij}^r)\, t + (\|\mathbf{x}_{ij}^{r0}\|^2 - d^2) \geq 0 \qquad \forall t \in (0, T).$$

By computing the minimum t_{ij}^m of the above quadratic convex function, and by introducing binary variables y_{ij} to check the sign of t_{ij}^m, following the procedure of Cafieri et al. [1], one can reformulate the constraints above by eliminating the dependence on t. More precisely, the following model $(\mathcal{P})$ is obtained (see [1] for details):

$$(\mathcal{P}) \left\{ \begin{array}{lll} \displaystyle\min_{q_i, t_{ij}^m, y_{ij}} & \displaystyle\sum_{i=1}^{n}(q_i - 1)^2 & \\ s.t. & & \\ & t_{ij}^m \|\mathbf{v}_{ij}^r\|^2 + \mathbf{x}_{ij}^{r0} \cdot \mathbf{v}_{ij}^r = 0, & \forall (i,j) \in \{1, \cdots, n\}^2, i < j \\ & -t_{ij}^m\, (2y_{ij} - 1) \leq 0, & \forall (i,j) \in \{1, \cdots, n\}^2, i < j \\ & -y_{ij} \left(\|\mathbf{v}_{ij}^r\|^2 (\|\mathbf{x}_{ij}^{r0}\|^2 - d^2) - (\mathbf{x}_{ij}^{r0} \cdot \mathbf{v}_{ij}^r)^2 \right) \leq 0, & \forall (i,j) \in \{1, \cdots, n\}^2, i < j \\ & & \\ & q_i \in \mathbf{q_i} = [\underline{\mathbf{q_i}}, \overline{\mathbf{q_i}}], & \forall i \in \{1, \cdots, n\} \\ & t_{ij}^m \in]-\infty, \infty[, & \forall (i,j) \in \{1, \cdots, n\}^2, i < j \\ & y_{ij} \in \{0, 1\}, & \forall (i,j) \in \{1, \cdots, n\}^2, i < j \end{array} \right.$$

Remark 1. *The optimization criterion in $(\mathcal{P})$ is* **quadratic** *and* **convex**.

3. sBB solvers: `Couenne` and `IBBA`

The deterministic global optimization solvers `Couenne` and `IBBA`, that we consider for the present work, are both based on a *spatial Branch-and-Bound (sBB)* method. Its main characteristics include Bissection/Branching techniques and Constraint Propagation techniques (named HC4 or FBBT) [2, 4]. Further characteristics of `Couenne` and `IBBA` are summarized in Table 1. Note that for computing bounds, `Couenne` uses convex relaxations (denoted by $(\mathcal{P}_{conv})$ in Table 1) [2], while `IBBA` uses linear relaxations (denoted by $(\mathcal{P}_{lin}^{AF})$ in Table 1) based on affine and interval arithmetics [3, 5].

Remark 2. `IBBA` *is numerically reliable (because it is mainly based on interval arithmetic).*

Table 1. Main characteristics of Couenne and IBBA

Couenne (Belotti et al.) [2]	IBBA (Messine and Ninin) [3, 5]
- $(\mathcal{P}) \geq (\mathcal{P}_{conv})$	- $(\mathcal{P}) \geq (\mathcal{P}_{lin}^{AF}) + E_r$ (where E_r is a constant)
- Formal Preprocess	- Interval and Affine Arithmetic
- $w_{ij} = x_i x_j$ and $w_{ii} = x_i^2$ with McCormick constraints.	- $x_i = \mathtt{mid}(\mathbf{x_i}) + \mathtt{rad}(\mathbf{x_i})\varepsilon_i$, with $\mathbf{x_i} = [\underline{\mathbf{x_i}}, \overline{\mathbf{x_i}}], \varepsilon_i \in [-1, 1]$.
- Relaxation $\Rightarrow$ + new variables and constraints.	- Relaxation $\Rightarrow$ same nb of variables and constraints.
- Use of IPOPT.	- Use of CPLEX.

4. Quadratic convex relaxation and branching strategy for IBBA

An idea to improve IBBA code is to keep the quadratic convex criterion instead of linearizing it. This yields a new automatic way to make a convex relaxation of problem $(\mathcal{P})$ by using affine arithmetic. More specifically, the constraints, which are mainly concave, are linearized directly using affine arithmetic, and the criterion is just rewritten by employing a change of variables from q_i to $\varepsilon_i \in [-1, 1]$ as $q_i \longrightarrow \mathtt{mid}(\mathbf{q_i}) + \mathtt{rad}(\mathbf{q_i})\varepsilon_i$, where $\mathbf{q_i} = [\underline{\mathbf{q_i}}, \overline{\mathbf{q_i}}]$, $\mathtt{mid}(\mathbf{q_i}) = \frac{\underline{\mathbf{q_i}} + \overline{\mathbf{q_i}}}{2}$ and $\mathtt{rad}(\mathbf{q_i}) = \frac{\overline{\mathbf{q_i}} - \underline{\mathbf{q_i}}}{2}$. Thus, the criterion of problem $(\mathcal{P})$ becomes:

$$\begin{aligned}\sum_{i=1}^{n}(q_i - 1)^2 \quad &\longrightarrow \sum_{i=1}^{n}\left(\mathtt{mid}(\mathbf{q_i}) + \mathtt{rad}(\mathbf{q_i})\varepsilon_i - 1\right)^2 \\ &= \sum_{i=1}^{n}(\mathtt{rad}(\mathbf{q_i}))^2{\varepsilon_i}^2 + 2(\mathtt{mid}(\mathbf{q_i}) - 1)\mathtt{rad}(\mathbf{q_i})\varepsilon_i + (\mathtt{mid}(\mathbf{q_i}) - 1)^2\end{aligned}$$

The quadratic part of the reformulated criterion reads $\varepsilon^T A_\varepsilon \varepsilon$, with A_ε a diagonal matrix having elements $\mathtt{rad}^2(\mathbf{q_i})$; A_ε is a matrix of size $n \times n$.

Remark 3. *A_ε is positive semidefinite and then the criterion reformulated in terms of ε is kept convex. Note that this property is independent on the selected application.*

Proposition 4. *If $\mathtt{rad}(\mathbf{q_i}) \to 0$ then $A_\varepsilon \to 0$, thus the criterion reformulated in ε tends to be linear.*

Another idea to improve IBBA (and possibly Couenne) is related to the branching strategy. We remark that in the constraints of problem $(\mathcal{P})$, the variables t_{ij}^m and y_{ij} can be deduced from variables q_i. Thus, the idea is to branch only on variables q_i, and to use the HC4-constraint propagation technique to automatically reduce bounds on variables t_{ij}^m and y_{ij}.

5. Numerical solutions

We tested IBBA on 5 problem instances, detailed in [1]; the aircraft are positioned around a circle and all of them fly with the same speed $400NM$ towards the center of the circle. In the two following tables, n represents the number of aircraft and r the radius of the circle, and the time window is about 30 minutes. In Table 2, we solve problem $(\mathcal{P})$ by considering a speed variation $q_i \in [0.94, 1.03]$ (subliminal control), while in Table 3 we consider a larger range, $q_i \in [0.88, 1.06]$. In Table 2, we first report the numerical results in terms of computing time obtained in [1] by using Couenne. In the three last columns of Table 2 and Table 3, we provide the results obtained using IBBA in three different versions: (i)IBBA alone, (ii)IBBA using the quadratic convex relaxation detailed in the previous section, (iii)IBBA using the McCormick's linear relaxations on the quadratic convex program. In the three cases, we use the CPLEX software to solve the linear and convex quadratic programs. The number of iterations is also reported in some cases.

We first note that the gain obtained by using the branching strategy discussed above is very important: when this is not used, IBBA behaves not differently from Couenne (that so could

Table 2. Results obtained with `Couenne` and 3 versions of `IBBA`, using $q_i \in [0.94, 1.03]$

n	r	`Couenne` from [1] time (s)	`IBBA` time (s)	`IBBA+Quad` time (s)	`IBBA+McCormick` time (s)
2	1×10^2	0.11	**0.01**	0.19	**0.01**
3	2×10^2	0.98	**0.23**	1.31	**0.23**
4	2×10^2	8.43	0.89	2.70	**0.88**
5	3×10^2	469.86	41.37	80.79	**40.35**
6	3×10^2	46707.03	**395.87** (67287its)	618.54 (**66761its**)	403.03 (66366its)

Table 3. Results obtained with `Couenne` and 3 versions of `IBBA`, using $q_i \in [0.88, 1.06]$

n	r	`IBBA` time (s)/(#its)	`IBBA+Quad` time (s)/(#its)	`IBBA+McCormick` time (s)/(#its)
2	1×10^2	**0.04** / (94)	0.11 / **(89)**	**0.04** / (94)
3	2×10^2	0.43 / (416)	0.74 / **(383)**	**0.41** / (386)
4	2×10^2	4.36 / (2134)	5.82 / **(1915)**	**4.09** / (1930)
5	3×10^2	117.56 / (32151)	136.25 / **(29700)**	**111.57** / (29862)
6	3×10^2	2270.03 / (384552)	2489.33 / **(360233)**	**2188.75** / (361720)

be considerably improved by using this strategy of branching). The impact of the proposed quadratic convex relaxation is actually not very strong on the considered conflict avoidance problem: a reduction in the number of iterations does not correspond to a smaller CPU-time. This is due to the fact that solving a quadratic convex program with `CPLEX` is of course more expansive than solving a linear one. Therefore, for the considered application the use of the McCormick linear relaxation of the quadratic convex problem provides the most efficient results in terms of CPU-time. Note that the main variables q_i vary within very tight bounds, and therefore the quadratic part of the convex relaxation quickly disappears during the computation (A_ε tends to be 0). As a consequence, when the variable ranges are small, the quadratic relaxation is not efficient; with a larger variable range, $[0.88, 1.06]$ (Table 3), the gain in the number of iterations is indeed more important.

6. Conclusion

We have showed that we can obtain promising results using sBB global optimization solvers such as `Couenne` and `IBBA` on an aircraft conflict avoidance model. A suitable branching strategy and a new quadratic convex relaxation based on affine arithmetic, implemented in `IBBA`, associated with a McCormick linear reformulation, enable to significantly improve the efficiency of the solver.

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