Generating optimal aircraft trajectories with respect to weather conditions
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Generating optimal aircraft trajectories with respect to weather conditions

**Goal:** Compute optimal routes in Cruise Flight.

**Why?** Increase Air Traffic Capacity and reduce time travel and fuel consumption.

**Assumption:** Constant Air Speed and Constant Flight Level.

**Our methodology : Front Propagation method**

Relied on *Ordered Upwind Algorithm*: technique to track the propagation by solving a partial differential equation known as the *Hamilton-Jacobi equation*:

\[
\begin{align*}
\nabla u \cdot F &= \left( x, \frac{\nabla u}{\|\nabla u\|} \right) = 1, \text{ where } u \text{ is the optimal cost and } F \text{ is the speed of the front in the normal direction} \\
\n\nabla u &= 0 \text{ on the initial point}
\end{align*}
\]

**Principle:**
1. Start at the initial point with the cost \( u = 0 \);
2. Compute the propagation of the front from the initial point corresponding to the minimal cost \( u \);
3. Construct the optimal path by tracing backward.

**One aircraft**

**Without Obstacle**

Speed of the front: \( F = \text{Aircraft Ground Speed} \)

**With Obstacles**

\( F = \text{function(Aircraft Ground Speed, Obstacles)} \)

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Optimal route</th>
<th>Direct route</th>
<th>Wind</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal route</td>
<td>1470</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct route</td>
<td>1498</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Profit:** ≈ 30 seconds for 30 min of flight time; 1.9% of time saved for the trajectory.

**Several aircraft**

**Idea:** Propagate the front in the configuration space of several aircraft.

**Example:** for two aircrafts, state space \( \mathbb{R}^4 - \Delta \) with \( \Delta = \{(x, y_1) = (x_2, y_2)\} \) and \( (x, y_1, x_2, y_2) \in \mathbb{R}^4 \) the coordinates of both aircraft.

**Problem:** Curse of dimensionality.

**Proposed solution:** Use *Approximate Dynamic Programming* methods to compute good approximations and not the exact optimal trajectories.

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