Non-linear Analysis and Control Proposal for In-flight Loss-of-control

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Abstract: In-flight loss-of-control (LOC-I) still poses a severe threat to today's commercial aviation. Hence, we review the literature for non-linear analysis and control methods of LOC-I and upset recovery. Using state-of-the-art methods such as continuation theory and reachability estimation, we sketch an analysis of an aircraft's flight envelope in terms of its trim conditions and propose control approaches both within and outside the envelope.

Keywords: Aerospace; UAVs; Navigation, Guidance, and Control; Upset recovery; Application of nonlinear analysis and design; Stability of nonlinear systems; Control of constrained systems; Control of bifurcation and chaos.

1. INTRODUCTION

Over the past three decades, in-flight loss-of-control events (LOC-I) have remained the foremost cause of fatal accidents (Boeing, 2001, 2008, 2016). With a contribution of almost 50 % of fatalities in civil aviation while representing less than a tenth of the total accidents [1], the International Air Transport Association (IATA, 2015a) lists LOC-I as "highest risk to aviation safety." In response, aircraft manufacturers, commercial airlines, and national and international authorities and association have provided procedures and trainings for flight crews in order to tackle—or even avoid—events of LOC-I (Carbaugh et al., 2008; IATA, 2015b).

The Federal Aviation Administration (FAA, 2016) defines LOC-I flight events as deviation from the desired flight condition, "often" leading to upsets characterized by unstable, highly non-linear behaviour of the aircraft aerodynamic system, such as stall, spin, and post-stall rotations (Chambers and Grafton, 1977). Control approaches for upset recovery include throttle-only control (Burcham Jr et al., 1997, 2009; Urnes Sr, 2012) in case of hydraulic failures of the control surfaces, linear-optimal control (Chang et al., 2016), $\mathcal{L}_1$ adaptive control (Xargay et al., 2010), state-based switching control (Engelbrecht et al., 2013), non-linear dynamic inversion (Stepanyan et al., 2016b), and Lyapunov-based control (Engelbrecht, 2016).

Several LOC-I prevention and upset recovery systems were designed (Engelbrecht et al., 2013; Engelbrecht, 2016; Stepanyan et al., 2016a,b; Tekles et al., 2016) for the NASA *generic transport model* (GTM; Jordan et al., 2006) and evaluated in pilot-in-the-loop simulations (Cunningham et al., 2011; Crespo et al., 2012; Richards et al., 2016) and in-flight tests (Gregory et al., 2011). The GTM, a down-scaled model of a typical transport aircraft has been studied exhaustively (Foster et al., 2005; Frink et al., 2016) and provides an open-source six-degree-of-freedom model for MATLAB/Simulink (NASA, 2016).

For the analysis of non-linear regimes, two disparate methods have recently been applied: *bifurcation* and *reachability* analysis. The first has been developed from the mathematical continuation and bifurcation theory to a state-of-the-art analysis tool for trim conditions and periodical orbits of the non-linear aircraft dynamics (cf. Caroll and Mehra, 1982; Jahnke, 1990; Goman et al., 1997; Kwatny et al., 2013; Engelbrecht, 2016) for more than thirty years now. The second, on the other hand, is a relatively new technique based on hybrid system theory, where sub-sets of the state space are evolved over time, determining possible violations of predefined constraints (e.g., Lombaerts et al., 2013; McDonough et al., 2014; McDonough and Kolmanovsky, 2016). A particular form of reachability analysis is the computation of control-invariant sub-sets, or *safe sets* (Lygeros, 2004; Tedrake et al., 2010; Chakraborty et al., 2011).

The CONVEX thesis [2] aims to contribute to LOC-I handling by design, implementation, and flight-test evaluation of non-linear upset recovery for micro air vehicle (MAV) while benefiting from the experimental gestalt of an MAV. In this paper, we present and propose first steps towards non-linear upset recovery control, including but not limited to formal definitions and a brief introduction to continuation and bifurcation theory; a non-linear analysis

[1] Counting both fatal and non-fatal accidents.

[2] *Command based On Non-linear piloting to bring an aerial Vehicle back in its flight Envelope* $\mathcal{X}_\mathcal{E}$; PhD thesis of the first author.

of trim conditions within, but undesired equilibria and periodic orbits beyond the flight envelope; and control strategies for both stable flight and upset recovery.

2. DEFINITIONS

The flight dynamics of an aircraft are commonly given by a system of first-order differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{X} = \boldsymbol{f}(\boldsymbol{X}, \boldsymbol{U}) \tag{1}$$

of the states $\boldsymbol{X}$ and inputs $\boldsymbol{U}$: the *state vector* at time t is then given by

$$\boldsymbol{X} = [\, V, \gamma, \chi,\, \theta, \phi, \psi,\, p, q, r,\, x_g, y_g, z_g \,]\,,^T \tag{2}$$

where V, γ, χ representing the aircraft's velocity (flight-path), θ, ϕ, ψ the aircraft's attitude with respect to the normal earth-fixed axes, p, q, r the aircraft's angular rates with respect to the body-fixed axes, and x_g, y_g, z_g the aircraft's position in the normal earth-fixed reference system; the control inputs to the aircraft are further given by the *input vector*

$$\boldsymbol{U} = [\, \eta, \xi, \zeta,\, T \,]\,,^T \tag{3}$$

with elevator, aileron, and rudder deflections η, ξ, ζ and thrust T.

The *state space* is the subset of all possible states,

$$\mathcal{X} \subseteq \mathbb{R}^n, \tag{4}$$

and we define the *flight envelope* as set of desired states $\mathcal{X}_\mathcal{E} \subset \mathcal{X}$ as well as the set of *viable control inputs*, $\mathcal{U} \subset \mathbb{R}^m$.

Finally, we have the (controlled) *flow* of the system (1) as

$$\boldsymbol{\phi}(\boldsymbol{X}_0, \boldsymbol{u}(\cdot)\,, t) = \boldsymbol{x}(t) \tag{5}$$

for $\boldsymbol{u} : t \mapsto \boldsymbol{U}$ and $\boldsymbol{x}(\cdot)$ is solution to the initial value problem $\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t)\,, \boldsymbol{u}(t))$ with $\boldsymbol{x}(0) = \boldsymbol{X}_0$.

The *safe set* of $\mathcal{X}$ is then the largest control-invariant set, *i.e.*

$$\mathcal{X}^{\text{safe}} = \{\boldsymbol{X} \in \mathcal{X} \,|\, \exists \boldsymbol{u}(\cdot) \in \mathcal{U}.\, \forall t \geq 0.\, \boldsymbol{\phi}_{\boldsymbol{u}}(\boldsymbol{X}, t) \in \mathcal{X}_\mathcal{E}\,\}\,. \tag{6}$$

Given an initial state $\boldsymbol{X}_0$ outside the flight envelope, *i.e.* $\boldsymbol{X}_0 \in \mathcal{X} - \mathcal{X}_\mathcal{E}$, upset recovery is formally given as the task *to find a control law* $\boldsymbol{u} : \mathbb{R} \to \mathcal{U}$ such that for a $t_R > 0$, $\boldsymbol{\phi}(\boldsymbol{X}_0, \boldsymbol{u}(\cdot)\,, t) \in \mathcal{X}_\mathcal{E}$ for all $t > t_R$. Candidate upset recovery approaches can be evaluated by the time of recovery t_R, the initial set of states $\mathcal{X}_0 \subseteq \mathcal{X} - \mathcal{X}_\mathcal{E}$ which can be recovered by the control law $\boldsymbol{u}(\cdot, \cdot)$ in time $t_R \leq \hat{t}_R$, and the undesired region of the state-space intersected by the controlled flow, that is $\boldsymbol{\phi}(\boldsymbol{X_0}, \boldsymbol{u}(\cdot)\,, [0, t_R]) \subseteq \mathcal{X} - \mathcal{X}_\mathcal{E}$.

3. CONTINUATION AND BIFURCATION

Crawford (1991) relates a bifurcation point to a *significant change in the dynamics of a system*. Here, given a dynamic system similar to (1)

$$\dot{\boldsymbol{X}} = \boldsymbol{f}(\boldsymbol{X}, \boldsymbol{\mu})\,, \tag{7}$$

where $\boldsymbol{X}$ denotes the state vector again and $\boldsymbol{\lambda}$ the *continuation parameters*, which may include state variables, control inputs, system parameters, and external influences (Kwatny et al., 2013). Recalling that any point $(\boldsymbol{X}^*, \boldsymbol{\mu}^*)$ is an equilibrium if and only if

$$\boldsymbol{f}(\boldsymbol{X}^*, \boldsymbol{\mu}^*) = \boldsymbol{0}, \tag{8}$$

it is furthermore a *bifurcation point* if at least one real eigenvalue λ—or complex-conjugated pair—crosses the imaginary axis, *i.e.* $\Re\,\lambda(\boldsymbol{X}^*, \boldsymbol{\mu}^*) = 0$. By continuation of the parameters $\boldsymbol{\mu}$, bifurcation analysis discusses creation, vanishing, and changes of stability of the branches of equilibria of (7) as function of $\boldsymbol{\mu}^*$.

4. TRIM CONDITION ANALYSIS

In Kwatny et al. (2013), the longitudinal trim conditions of the GTM have been analyzed. By assuming a considerably damped pitch motion, that is $q = 0$, the system dynamics of $\boldsymbol{f}$ are restricted to speed V and flight-path angle γ as states, elevator η and thrust T as inputs, and the angle of attack α as output. A trim condition is given by $(V^*, \gamma^*, \eta^*, T^*)$ if and only if

$$\boldsymbol{f}_{V,\gamma}(V^*, \gamma^*, \eta^*, T^*) = \boldsymbol{0}. \tag{9}$$

As obtained from Fig. 1, for speeds greater than a certain speed V' there are two trim conditions at low and high angle of attack, respectively. While there are no trim conditions for $V < V'$, at $V = V'$ the trim conditions diminish to a single one. In other words, for flights slower than V' there are no conditions, and thus no angle of attack, to maintain trimmed flight. Recall that is just the definition of stall, *i.e.* $V' \equiv V_{\text{Stall}}$, and the stall speed varies with the flight-path angle γ. As for $V > V_{\text{Stall}}(\gamma)$ there are two branches of trim conditions, the condition at $V_{\text{Stall}}(\gamma)$ is a bifurcation point and maneuverability of the system is lowered (Berg and Kwatny, 1997; Kwatny et al., 2013).

While the limits of elevator deflection and thrust obviously restrict the achievable trim conditions, we define without loss of generality the flight envelope around the set of (viable) trim conditions,

$$\mathcal{X}^{\text{trim}} = \big\{(V, \gamma) \in \mathcal{X} \,\big|\, \exists\, (\eta, T) \in \mathcal{U}.\, \boldsymbol{f}_{V,\gamma}(V, \gamma, \eta, T) = \boldsymbol{0}\,\big\}\,. \tag{10}$$

Hence, the stall trim conditions constitute a boundary of the flight envelope.

5. LQR SAFE SET ANALYSIS

The system can be linearized at a reasonable large number of trim conditions. Thus, one can easily derive a set of linear controllers for stable flight in the flight envelope.

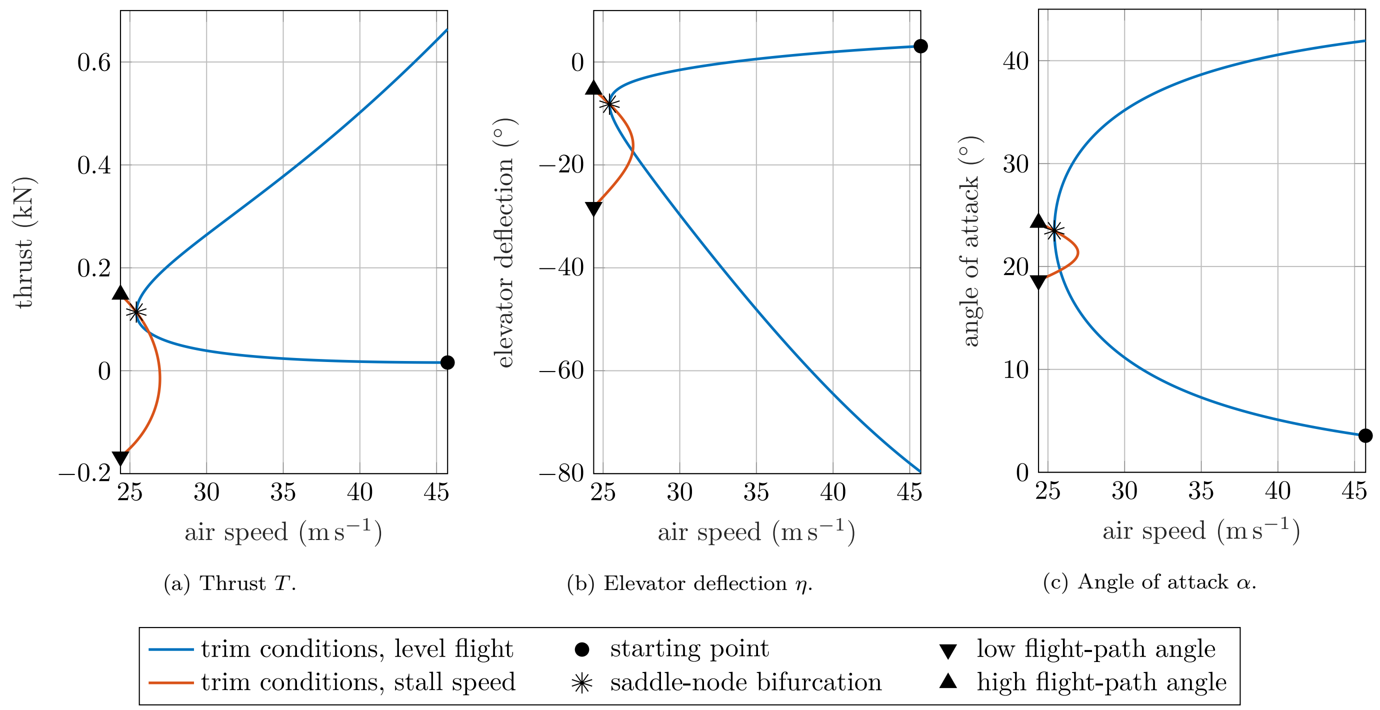
Let $\mathbf{K}_i$ be a linear-optimal regulator (LQR) and $\mathbf{S}_i$ the corresponding solution to the algebraic Riccati equation for a linearization of $\boldsymbol{f}_{V,\gamma}$ around a trim condition $(\boldsymbol{X}_i^*, \boldsymbol{U}_i^*)$. We can employ $\mathbf{S}_i$ for a quadratic Lyapunov-candidate function (Tedrake et al., 2010)

$$\mathcal{V}_i = \frac{1}{2}\bar{\boldsymbol{X}}^T \mathbf{S}_i \bar{\boldsymbol{X}} > 0, \qquad \bar{\boldsymbol{X}} \neq \boldsymbol{0}, \tag{11}$$

$\bar{\boldsymbol{X}} = \boldsymbol{X} - \boldsymbol{X}_i^*$, to have $\mathcal{X}_i^{\text{stable}} = \big\{\boldsymbol{X} \in \mathcal{X} \,\big|\, \mathcal{V}_i\big(\bar{\boldsymbol{X}}\big) \leq \rho_i\,\big\}$ with $\rho_i > 0$ being a stable neighbourhood of $\boldsymbol{X}_i^*$ if and only if

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{V}_i = \bar{\boldsymbol{X}}^T \mathbf{S}_i \boldsymbol{f}\big(\boldsymbol{X}_i^* + \bar{\boldsymbol{X}}, \boldsymbol{U}_i^* - \mathbf{K}_i \bar{\boldsymbol{X}}\big) < 0 \tag{12}$$

for all $\boldsymbol{X} \in \mathcal{X}_i^{\text{stable}} - \{\boldsymbol{X}_i^*\}$ (Slotine and Li, 1991). Thus, $\mathcal{X}_i^{\text{stable}}$ is safe in the sense of (6).



(a) Thrust T. (b) Elevator deflection η. (c) Angle of attack α.

Fig. 1. Trim conditions obtained for level flight and varying speed, with respect to angle of attack, elevator deflection, and thrust. At stall speed, a saddle-node bifurcation occurs (✳) which depends on the respective flight-path angle (——).

Introducing a polynomial, positive semi-definite *Lagrange multiplier* $h \in \mathbb{K}[\boldsymbol{X}]$ Tedrake et al. reduce (12) to a sum-of-squares problem (Parillo, 2003):

$$\bar{\boldsymbol{X}}^T \mathbf{S}_i \boldsymbol{f} + h(\bar{\boldsymbol{X}}) \left(\rho_i - \mathcal{V}_i(\bar{\boldsymbol{X}})\right) \leq -\epsilon \left\|\bar{\boldsymbol{X}}\right\|_2^2, \tag{13}$$

where $\|\cdot\|_2$ denotes the $\mathcal{L}_2$-norm and $\epsilon > 0$. Here, $\rho_i - \mathcal{V}_i(\cdot)$ equals the signed distance to $\partial\mathcal{X}_i^{\text{stable}}$ and with $h(\cdot) \geq 0$ for all $\bar{\boldsymbol{X}} \neq 0$ compensates for non-negative derivatives of $\mathcal{V}_i$ outside the stable neighbourhood. Hence finding a Langrange multiplier proofs the stability of $\boldsymbol{X}_i^*$ in $\mathcal{X}_i^{\text{stable}}$ by the linear controller $\mathbf{K}_i$.

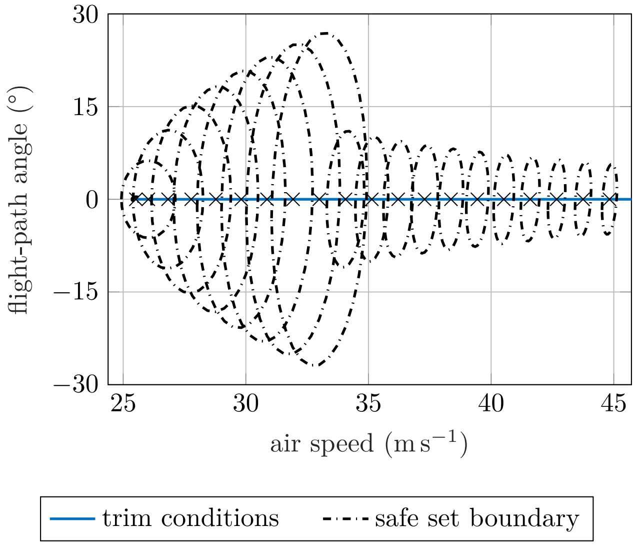


Fig. 2. Safe sets of LQR control at selected trim conditions (×) at level flight and low angle of attack.

6. PROPOSED CONTROL APPROACH

Fig. 2 shows the computed safe sets for linear-quadratic optimal regulators at some trim points of low angle of attack. With safe switching between two or more trim conditions ensured by reachability analyis (cf. McDonough and Kolmanovsky, 2016), we take the flight envelope as union of the safe sets.

Outside the flight envelope, and beyond the stall speed in particular, control of the aircraft may be restricted by limited control effectiveness (Kwatny et al., 2013) and non-linear modes like periodic orbits. In order to tackle these, we propose a further, non-linear analysis; eventually, we will develop a selecting approach of suitable trim conditions *on the boundary* of the flight envelope and a flight control law for recovery from an upset condition to the respective trim condition.

Except for Xargay et al. (2010), uncertainties of the underlying aerodynamic model or in the outputs are not considered in the literature reviewed. Hence a first step towards an upset recovery law is to estimate the effects of uncertainties to the flight envelope as defined in (10).

7. CONCLUSION

In this paper, we have reviewed recent LOC-I prevention and upset recovery approaches, various linear and non-linear control methods, and analysis techniques such as bifurcation and reachability. We have then formally defined an aircraft dynamic system, its state space, and the flight envelope. By considering the non-linear analysis of the generic transport model by Kwatny et al. (2013) we have exemplary shown the results of bifurcation theory and continuation and discussed the outcomes.

Along the branch of level flight trim conditions, we have derived linear-quadratic optimal regulators (LQR) around selected trim conditions and computed the safe sets as stable neighbourhoods of the trim conditions for the linear controlled system. We thus have proposed a control approach based on the bifurcation analysis and reachability.

As argued, further analysis of the non-linear dynamics are required; in particular, uncertainties need to be taken into account before any control law can be designed and implemented to an MAV. We also expect further analysis to give insights on suitable recovery approaches which are able to control the aircraft outside the flight envelope.

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Appendix A. PHUGOID DYNAMICS

In this paper, we were discussing the phugoid dynamics of the GTM adapted from Kwatny et al. (2013),

$$\begin{cases} \dot\alpha = q - \dot\gamma, \\ \dot V = \dfrac{1}{m}\left(T\cos\alpha - \dfrac{1}{2}\rho S V^2 C_D(\alpha,\eta,q) - mg\sin\gamma\right), \quad \dot\gamma = \dfrac{1}{mV}\left(T\sin\alpha + \dfrac{1}{2}\rho S V^2 C_L(\alpha,\eta,q) - mg\cos\gamma\right), \\ \dot q = \dfrac{1}{I_y}\left(l_t T + \dfrac{1}{2}\rho S c_a V^2 C_m(\alpha,\eta,q) + \dfrac{1}{2}\rho S V^2 C_Z(\alpha,\eta,q)\left(x_{\text{cg}}^{\text{ref}} - x_{\text{cg}}\right) + \dfrac{1}{2}\rho S V^2 C_X(\alpha,\eta,q)\left(z_{\text{cg}}^{\text{ref}} - z_{\text{cg}}\right)\right), \end{cases} \tag{A.1}$$

where S and c_a are wing area and aerodynamic mean chord; C_D, C_L, C_X, C_Z, C_m are the aerodynamic coefficients of drag, lift, force body x-axis, force body z-axis, and moment body y-axis, respectively, as functions of angle of attack, elevator deflection, and pitch rate; $x_{\text{cg}}^{\text{ref}}, z_{\text{cg}}^{\text{ref}}, x_{\text{cg}}, z_{\text{cg}}$ are the reference and actual position of the center of gravity with respect to x and z; and l_t is the engine's displacement along the z-axis.

A.1 Restricted longitudinal model

To reduce the number of states, we consider the phugoid motion to be damped—either *a priori* or by a suitable damping system in inner-loop—and $q \equiv 0$ (Kwatny et al., 2013). For the trim condition we then get

$$M = I_y \dot q = 0 \tag{A.2}$$

in addition to $\dot V = \dot\gamma = 0$ at $(V^*, \gamma^*, \eta^*, T^*, \alpha^*)$.

A.2 Linear control approach

For design and analysis of linear control, we can assume an inner controller of the angle of attack and neglect the effect of the elevator to the lift and drag coefficients. We thus get the phugoid dynamics around a trim condition

$$\begin{cases} \dot{\tilde V} = \dfrac{1}{m}\left(\tilde T\cos\tilde\alpha - \dfrac{1}{2}\rho S\tilde V^2 C_D(\tilde\alpha,\eta^*) - mg\sin\tilde\gamma\right), \\ \dot{\tilde\gamma} = \dfrac{1}{m\tilde V}\left(\tilde T\sin\alpha + \dfrac{1}{2}\rho S\tilde V^2 C_L(\tilde\alpha,\eta^*) - mg\cos\tilde\gamma\right), \end{cases} \tag{A.3}$$

where $\tilde T, \tilde\alpha$ are inputs to the inner system.

A.3 Aerodynamic coefficients

Using the MATLAB *Curve fitting toolbox*, the aerodynamic coefficients of the generic transport model in the body axis system has been fitted to the polynomials

$$\begin{aligned} C_X(\alpha,\eta) = &-0.0186 + 0.2413\alpha - 0.0135\eta \\ &+ 1.4957\alpha^2 + 0.1849\alpha\eta - 0.0941\eta^2 \\ &- 7.4482\alpha^3 - 0.2617\alpha^2\eta + 0.2723\alpha\eta^2 \\ &+ 4.5867\alpha^4 + 0.0628\alpha^3\eta - 0.2583\alpha^2\eta^2 \end{aligned} \tag{A.4}$$

$$\begin{aligned} C_Z(\alpha,\eta) = &-0.0418 - 5.2246\alpha - 0.4420\eta \\ &+ 3.6670\alpha^2 + 0.0866\alpha\eta - 0.2135\eta^2 \\ &+ 8.7973\alpha^3 + 0.5947\alpha^2\eta - 0.0499\alpha\eta^2 \\ &- 6.8839\alpha^4 - 0.3686\alpha^3\eta + 0.3358\alpha^2\eta^2 \end{aligned} \tag{A.5}$$

$$\begin{aligned} C_m(\alpha,\eta) = &+0.1866 - 1.5743\alpha - 1.7199\eta \\ &+ 2.1987\alpha^2 + 0.0662\alpha\eta - 0.6995\eta^2 \\ &- 4.4762\alpha^3 + 3.1578\alpha^2\eta + 0.1736\alpha\eta^2 \\ &+ 2.2467\alpha^4 - 1.7551\alpha^3\eta + 0.7337\alpha^2\eta^2 \end{aligned} \tag{A.6}$$

and the lift and drag coefficients can be calculated by rotation,

$$C_L(\alpha,\eta) = -C_Z(\alpha,\eta)\cos\alpha + C_X(\alpha,\eta)\sin\alpha, \tag{A.7}$$

$$C_D(\alpha,\eta) = -C_Z(\alpha,\eta)\sin\alpha - C_X(\alpha,\eta)\cos\alpha, \tag{A.8}$$

from body to air-path axis system.