MINLP in Air Traffic Management: Aircraft Conflict Avoidance

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1 Introduction

Air Traffic Management (ATM) represents a domain of emerging and challenging applications of MINLP. A number of problems arising in ATM lead naturally to optimization problems whose efficient and reliable solution constitutes a key ingredient to ensure air traffic safety [11]. The air-traffic level currently attained in Europe is around tens of thousands of flights per day, and it is expected to furtherly grow on the world scale during the next 20 years. Increasing levels of traffic raises the problem of increasing the capacity of air sectors by better managing the air traffic. This needs increasing the level of automation in ATM, as pointed out in the context of the major projects Single European Sky ATM Research (SESAR) [24] in Europe and Next Generation Air Transportation System (NextGen) [15] in the United States, that are aimed at designing the future ATM systems. Aircraft conflict detection and resolution in en-route flights, and the related problem of conflict-free aircraft trajectory planning, constitute prominent examples of problems that urgently need to be addressed to ensure a higher level of automation in ATM, and consequently more efficiency and safety in air traffic. These problems still deserve investigation from both the identification of suitable mathematical models and the development of efficient and reliable solution methods and algorithms. Mixed-Integer Nonlinear Programming formulations appear particularly suitable, as they allow us to simultaneously consider continuous as well as discrete decision-making variables and modeling the complex nonlinear processes characterizing ATM systems.

In the following, we focus on aircraft conflict detection and resolution for en-route flights. The aim of this chapter is to present and discuss the use of mixed-integer optimization for this real-life application, with an emphasis on MINLP modeling.

Aircraft conflict avoidance is described in Section2 and the main approaches based on MINLP are recalled. In Section 3 the main ingredients and issues of mathematical MINLP modeling are discussed. Section 4 presents a brief overview of solution approaches. Section 5 draws a few conclusions.

2 Aircraft conflict avoidance

Aircraft sharing the same airspace are said to be potentially in *conflict* when they are too close to each other according to their predicted trajectories, i.e., their relative horizontal and vertical distances do not satisfy two given safety-distance constraints. More precisely, the standard separation norms in the en-route airspace are 5 NM horizontally and 1000 ft vertically (1NM (nautical mile)= 1852 m; 1ft (feet)= 0.3048 m). One can then imagine an aircraft as the center of a cylinder of 5 NM radius and 1000 ft height: it is conflict-free if there is no other aircraft entering this protection volume. When a loss of separation occurs, aircraft have to be separated by performing suitable maneuvers. Aircraft conflicts resolution, also referred to as *aircraft deconfliction*, is the problem of providing, starting from an initial configuration given by aircraft trajectories (positions, heading angles), and velocities, a new conflict-free configuration. In general, a selected portion of the airspace is observed over a given time horizon, then the process is restarted on the next time window. We consider the case of en-route cruise flights, at a tactical level, i.e., potential conflicts are resolved a few minutes before the loss of separation potentially occurs.

The main challenge is to propose mathematical formulations that are able to model the complex choices characterizing the target problem without assuming any unrealistic simplifying hypotheses, and that are amenable to be solved by efficient algorithms. The underlying problem is an optimization problem, as conflict avoidance may be performed deviating as little as possible from the original aircraft flight plan, i.e., minimizing the impact of the separation maneuvers on the flight efficiency.

Modeling aircraft conflict avoidance is strictly dependent on the separation maneuver chosen to solve conflicts. The most common way to achieve separation, that represents the separation maneuver usually exploited by air traffic controllers, is based on changing either the trajectory (heading) or the flight level of the aircraft involved in a conflict. Alternatively, conflict avoidance can be performed through aircraft velocity regulation, keeping the predicted trajectories. The European aeronautical project ERASMUS [6] in recent years promoted the idea of *subliminal control*, according to which velocity regulation should be performed in such a way that aircraft speeds are modified only in a small range (namely, from -6% to +3%) around the original speed. A subliminal speed control is considered promising in view of a future, more automated ATM system, thanks to its limited impact on the workload of air traffic controllers.

Various models and solution strategies have been proposed for the underlying optimization problem. A survey up to the year 2000 can be found in [17]. Since then, numerous other approaches have been introduced and mixed-integer linear and nonlinear optimization is appearing a powerful framework for mathematically modeling aircraft deconfliction. Pallottino et al. in [19] and Richards and How [22] in 2002 first proposed geometrical constructions to model aircraft separation using either velocity or trajectory changes. They obtained mixed-integer linear programming (MILP) models that, even though characterized by quite stringent hypothesis, have the advantage to be solved by any state-of-the-art MILP solver. A mixed-integer linear model for conflict resolution by velocity and altitude changes is proposed in [2]. The main drawback of this approach is that altitude changes are not preferred as they are uncomfortable for passengers and fuel consumptive. In [3] a nonlinear model is developed starting from the geometrical construction in [19]. The authors propose a sequential linear solution approach for its solution. The same authors develop a Variable Neighborhood Search heuristic in [4]. In [26] a model is proposed based on speed control and flight-level assignments for conflict resolution over predefined routes, while the authors in [10] propose a MINLP coming from a combination of velocity control and heading angle control methods. In [8] a MINLP model based on velocity changes is proposed, as well as deterministic solution approaches. The speed regulation strategy is also the basis of mixed-integer linear and nonlinear models proposed in [20, 21] for the related problem of minimization of potential conflicts. A number of contributions also come from the related problem of conflict-free trajectory planning (i.e., finding trajectories such that aircraft are separated all along their path, *a priori* in long-term strategical planning or in tactical phases of flights). In [9, 23] mixed-integer models are proposed for trajectory planning problems, that are solved through heuristic approaches. Interesting MINLP problems also come from mixed-integer optimal control models for conflict-free trajectories planning and conflicts resolution, where typical discretization steps lead to the solution of MINLP problems, see e.g. [5].

3 MINLP modeling

As for many real-life applications, aircraft conflict avoidance is quite challenging in that a successful model should exhibit a good trade-off between being adherent to realistic constraints and amenable to be computationally treated.

The focus of this section is on MINLP formulations. We provide an overview of the main formulation elements, pointing out different possible choices and discussing some modeling issues. We mainly refer to the MINLP model in [8], and variants therein.

We assume that aircraft are represented by points moving at constant speed along linear trajectories at the same altitude and are able to change instantaneously their course and speed. Thus, the focus is on the two-dimensional space and only the horizontal separation has to be ensured.

The choice of the separation maneuvers to solve conflicts directly affects the choice of the decision variables. Thus, the main decision variables in a conflict avoidance model may include aircraft velocities, heading angles, or flight levels, according to the chosen separation maneuvers. These variables are in general continuous variables, typically bounded because of operational constraints: aircraft velocity cannot be reduced to zero and it is limited by the aircraft engine, and trajectories presenting sharp turns may be not feasible for the aircraft. Referring to modeling based on velocity regulation [8], we consider, as main decision variables, speed variations q_i for each aircraft $i \in A$, bounded in the interval $[-6\%, +3\%]$ of the original aircraft speed v_i, according to the ERASMUS directives. The actual aircraft speed is then $q_i v_i$. A few authors consider alternatively instants when aircraft cross intersection points as main continuous variables [21]. Integer, and in particular binary, variables usually come from the need to express combinatorial decisions characterizing the problem: typically, the choice among possible scenarios or that of an order for aircraft to perform their maneuver or to arrive at a given point of the airspace. In that sense, the problem can naturally be modeled as a mixed-integer program.

Different cost functions can be identified as the objective of the optimization in the considered framework. In general, the aim is to achieve deconfliction in such a way that aircraft deviate as less as possible from their original flight plan. In modeling based on speed regulation, this corresponds to minimizing the aircraft speed changes; when trajectory changes (heading) are performed, the aircraft flight on a deviated trajectory has to be minimized. This is in turn related to the minimization of time delays due to deconflicting maneuvers. On the other hand, the minimization of fuel consumption, associated with the changes imposed to aircraft to ensure their separation, is important for airline companies and is valued in the context of sustainable environment. In our example using speed regulation, we minimize the speed variations over the set of all aircraft:

$$\min \sum_{i \in A} (q_i - 1)^2. \tag{1}$$

This shows that one can reasonably model the objective using functions with "desirable" properties, like convexity, and in general amenable to be computationally treated. The main difficulties come in fact from modeling the aircraft separation condition, as discussed below.

The constraints that definitely characterize the aircraft conflict avoidance problem are separation constraints. They have to be expressed for pairs of aircraft, so their number rapidly increases with the number n of aircraft. These constraints, in the general form $g(t) \geq 0 \; \forall t$, are in principle nonconvex constraints expressed by relations involving state variables, like positions and velocities, and depending on time t, although they are often reformulated into a different form. The aircraft separation between two aircraft i and j at time t is expressed by the condition

$$||\mathbf{x}^r_{ij}(t)|| \geq d \qquad \forall t \tag{2}$$

where d is the minimum required separation distance (5 NM for en-route flights), $\mathbf{x}^r_{ij}(t) = \mathbf{x}_i(t) - \mathbf{x}_j(t)$ is a vector representing the relative distance between aircraft i and j, and we use the Euclidean norm. Notice that a few already mentioned approaches (see [19]), express separation in a different way, looking at the geometry of the problem when the trajectories are intersecting straight lines. Such approaches require a few hypothesis and lead to "OR" constraints to take into account different possible configurations. In the following, we use (2) to express separation. A reformulation can be provided ([8]) to find an expression not explicitly dependent on the time, and numerically treatable. We can assume that uniform motion laws apply, so that the relative distance of aircraft i and j is expressed as the sum of their relative initial position and the product of their relative speed $\mathbf{v}_{ij}^r$ by the time:

$$\mathbf{x}_{ij}^r(t) = \mathbf{x}_{ij}^{r0} + \mathbf{v}_{ij}^r \; t \qquad \forall t,$$

that, substituting into (2) and squaring, gives

$$\|\mathbf{v}_{ij}^r\|^2 \; t^2 + 2(\mathbf{x}_{ij}^{r0} \cdot \mathbf{v}_{ij}^r) \; t + (\|\mathbf{x}_{ij}^{r0}\|^2 - d^2) \geq 0 \qquad \forall t. \tag{3}$$

The study of (3) and its associated equation can provide interesting insights to model aircraft separation. Let us consider the equation associated with (3). It is a quadratic equation in t. Its graph is a parabola that, as $\|\mathbf{v}_{ij}^r\|^2 > 0$, has a minimum point and opens upward. The discriminant Δ is defined as:

$$\Delta = (\mathbf{x}_{ij}^{r0} \cdot \mathbf{v}_{ij}^r)^2 - \|\mathbf{v}_{ij}^r\|^2(\|\mathbf{x}_{ij}^{r0}\|^2 - d^2) \tag{4}$$

If $\Delta < 0$ there are no solutions of the quadratic equation and aircraft are not in conflict, while if $\Delta \geq 0$ there are two solutions, eventually coincident (entry and exit points from the protection zone). If the two roots t' and t'' are both negative, then the conflict is over (it happened in the past). So, there is no conflict when the discriminant is negative or when the discriminant is positive and the two roots are negative (see [7, 14]). Binary variables and disjunctive constraints can then be introduced to model separation taking into account these possible scenarios.
Also, one can look at the sign of the scalar product $\mathbf{x}_{ij}^{r0} \cdot \mathbf{v}_{ij}^r$ to infer which kind of angle the vectors form. When the scalar product $\mathbf{x}_{ij}^{r0} \cdot \mathbf{v}_{ij}^r$ is negative, then aircraft flying on straight-line trajectories are converging, potentially generating a conflict, while they are diverging when the product is positive. This last condition can be used in conjunction with other ones, like the condition on Δ [7].

Again considering (3), we can observe that, by differentiation, the minimum occurs at:

$$t_{ij}^m = -\frac{\mathbf{v}_{ij}^r \cdot \mathbf{x}_{ij}^{r0}}{\|\mathbf{v}_{ij}^r\|^2}. \tag{5}$$

So, t_{ij}^m represents the time of closest separation for aircraft i and j: it is a worst-case, and aircraft are further apart for times greater than t_{ij}^m. Thus, by substituting into (3), we obtain:

$$\|\mathbf{x}_{ij}^{r0}\|^2 - \frac{(\mathbf{v}_{ij}^r \cdot \mathbf{x}_{ij}^{r0})^2}{\|\mathbf{v}_{ij}^r\|^2} - d^2 \geq 0 \tag{6}$$

that represents a separation condition in the considered time window $(0, T)$. In [8], separation is imposed through this condition (6) when $t_{ij}^m \in (0, T)$, and $\|\mathbf{x}_{ij}^{r0}\| \geq d$, $\|\mathbf{x}_{ij}^{r0} + \mathbf{v}_{ij}^r\ T\| \geq d$ (separation at the initial time and at the time horizon). Through the above transformation, we have obtained a nonlinear expression (6), but not depending on time anymore. Notice that constraint (6) has to be imposed, for each i and j, when $t_{ij}^m > 0$. This condition requires us to introduce a binary variable, and corresponding constraints (y_{ij}, and constraints (11), in the formulation below), to check such condition and impose separation when this variable takes value 1. Finally, we obtain the following MINLP formulation:

$$\begin{aligned}
\min \quad & \sum_{i \in A} (q_i - 1)^2 && && (7)\\
\text{s.t.} \quad & \|\mathbf{v}_{ij}^r\| = \|\mathbf{v}_i q_i - \mathbf{v}_j q_j\| && \forall i, j \in A, i < j && (8)\\
& y_{ij} \left(\|\mathbf{x}_{ij}^{r0}\|^2 - \frac{(\mathbf{v}_{ij}^r \cdot \mathbf{x}_{ij}^{r0})^2}{\|\mathbf{v}_{ij}^r\|^2} - d^2 \right) \geq 0 && \forall i, j \in A, i < j && (9)\\
& t_{ij}^m = -\frac{(\mathbf{v}_{ij}^r \cdot \mathbf{x}_{ij}^{r0})}{\|\mathbf{v}_{ij}^r\|^2} && \forall i, j \in A, i < j && (10)\\
& t_{ij}^m\,(2y_{ij} - 1) \geq 0 && \forall i, j \in A, i < j && (11)\\
& \|\mathbf{x}_{ij}^{r0}\|^2 \geq d^2 && \forall i, j \in A, i < j && (12)\\
& \|\mathbf{x}_{ij}^{r0} + \mathbf{v}_{ij}^r\ T\|^2 \geq d^2 && \forall i, j \in A, i < j && (13)\\
& y_{ij} \in \{0, 1\} && \forall i, j \in A, i < j && (14)\\
& 0.94 \leq q_i \leq 1.03 && \forall i \in A && (15)
\end{aligned}$$

where bounds on q_i take into account the limitation on speed variation for a subliminal control. We remark that the most of constraints, (8) to (14), are on pairs of aircraft, so there are $n(n-1)/2$ of each one of these constraints. The main nonlinearities come from squares and products of continuous variables and of binary and continuous variables, for which linearizations like Fortet's linearization can be computed [13].

It is worth noticing that this kind of model, like the most of the models in the literature, considers only one maneuver per aircraft, and maneuvers performed simultaneously by all aircraft in the airspace to be deconflicted. A more flexible model can be developed, on the one hand allowing different kinds of maneuvers, combining for example velocity and heading changes (see e.g.[10]), and on the other hand modeling different scenarios where aircraft perform their maneuvers at different times instead of simultaneously. Evidently, new (mainly, integer) variables and constraints have to be added to the formulation to accommodate these new model features.

In order to model the problem in such a way that separation maneuvers are not performed simultaneously, we introduce for each aircraft i a time t_i^1 and a time t_i^2 when the aircraft can modify its speed and respectively go back to its original speed. These times represent new (continuous) variables of the problem. As no conditions are imposed on the order of execution of separation maneuvers, and consequently, for each pair of aircraft i,j, on the order of times t^1 and t^2 for i and j, the idea is to handle different possible time configurations for pairs of aircraft. These configurations, for a pair $i, j \in A$, correspond to all possible permutations of t^1 and t^2 for the two aircraft of the pair (six configurations per pair). For example, supposing that aircraft i is the first to start its maneuver (changing its speed from the initial v_i to $v_i q_i$), that then j starts flying with modified speed, and then i ends its maneuver while j is still flying with modified speed, then the sequence of times is $t_i^1, t_j^1, t_i^2, t_j^2$, with $0 \le t_i^1 \le t_j^1 \le t_i^2 \le t_j^2 \le T$. We then introduce, for each pair $i, j \in A$, binary variables z_{ij}^ℓ to identify the configuration ℓ holding ($\ell \in \{1, \dots, 6\}$), i.e., the sequence of times. For example, z_{ij}^1 identifies the first configuration:

$$z_{ij}^1 = \begin{cases} 1 & \text{if } t_i^1 \le t_j^1 \quad \text{and} \quad t_j^1 \le t_i^2 \quad \text{and} \quad t_i^2 \le t_j^2 \\ 0 & \text{otherwise} \end{cases}$$

Then, the idea is to deal with the different time windows, in each configuration, where each aircraft flies either with its original (known) speed or with a changed speed. For example, in the first configuration the time windows are from 0 to t_i^1, from t_i^1 to t_j^1, from t_j^1 to t_i^2, from t_i^2 to t_j^2 and from t_j^2 to T, and aircraft i flies with a modified speed in the second and third time window. As we consider instantaneous speed changes, aircraft speeds are piecewise constant in time windows.

Dealing with time windows means to impose separation conditions like (6) in each time window, for each pair of aircraft, in each possible configuration. This makes the number of variables and constraints significantly growing with respect to the above model. First, constraints have to be added to the model to identify, for each pair $i, j \in A$, which is the current time configuration (i.e., which is the order of times t^1 and t^2 for i and j). Then, for each time window, further constraints express the size of the time window, the initial position and the speed of each aircraft, and finally the separation condition. The reader is referred to [8] for the whole detailed model. Here we focus on constraints expressing time configurations, as they can be formulated in different ways, thus showing the interest of reformulations in mathematical programming formulations [18] in the considered context. Following the definition, the first time configuration for a pair $i, j \in A$ is expressed by:

$$z_{ij}^1(t_i^1 - t_j^1) \leq 0, \qquad z_{ij}^1(t_j^1 - t_i^2) \leq 0, \qquad z_{ij}^1(t_i^2 - t_j^2) \leq 0$$

and similarly for the other five configurations, for each $i, j \in A$. To eliminate the nonlinearities given by the products between continuous and binary variables (t and z respectively), these are easily reformulated using *big M* constraints:

$$\begin{array}{lll}
t_i^1 \leq t_j^1 + M(1 - z_{ij}^1), & t_j^1 \leq t_i^2 + M(1 - z_{ij}^1), & t_i^2 \leq t_j^2 + M(1 - z_{ij}^1) \\
t_j^1 \leq t_i^1 + M(1 - z_{ij}^2), & t_i^1 \leq t_i^2 + M(1 - z_{ij}^2), & t_j^2 \leq t_j^2 + M(1 - z_{ij}^2) \\
t_i^1 \leq t_i^2 + M(1 - z_{ij}^3), & t_i^2 \leq t_j^1 + M(1 - z_{ij}^3), & t_j^1 \leq t_j^2 + M(1 - z_{ij}^3) \\
t_j^1 \leq t_j^2 + M(1 - z_{ij}^4), & t_j^2 \leq t_i^1 + M(1 - z_{ij}^4), & t_i^1 \leq t_i^2 + M(1 - z_{ij}^4) \\
t_i^1 \leq t_j^1 + M(1 - z_{ij}^5), & t_j^1 \leq t_j^2 + M(1 - z_{ij}^5), & t_j^2 \leq t_i^2 + M(1 - z_{ij}^5) \\
t_j^1 \leq t_i^1 + M(1 - z_{ij}^6), & t_i^1 \leq t_j^2 + M(1 - z_{ij}^6), & t_j^2 \leq t_i^2 + M(1 - z_{ij}^6)
\end{array}$$

Taking into account all pairs $i, j \in A$, this gives $18n(n-1)/2$ constraints. Interestingly, the value of constant M, usually difficult to choose, in this case can be chosen using the time horizon T, which represents an upper bound on the length of all time intervals. The above constraints can be furtherly reformulated, again using a *big M* approach where the constant M can be chosen using the time horizon T, but using a different formulation with new variables p_{ij} (one variable for each pair i, j and for each time window) [1]:

$$\begin{array}{rcl}
-M + M(z_{ij}^1 + z_{ij}^3 + z_{ij}^5) & \leq & p_{ij}^1 - t_i^1 \leq M - M(z_{ij}^1 + z_{ij}^3 + z_{ij}^5) \\
-M + M(z_{ij}^2 + z_{ij}^4 + z_{ij}^6) & \leq & p_{ij}^1 - t_j^1 \leq M - M(z_{ij}^2 + z_{ij}^4 + z_{ij}^6) \\
-M + M(z_{ij}^2 + z_{ij}^6) & \leq & p_{ij}^2 - t_i^1 \leq M - M(z_{ij}^2 + z_{ij}^6) \\
-M + M(z_{ij}^1 + z_{ij}^5) & \leq & p_{ij}^2 - t_j^1 \leq M - M(z_{ij}^1 + z_{ij}^5) \\
-M + Mz_{ij}^3 & \leq & p_{ij}^2 - t_i^2 \leq M - Mz_{ij}^3 \\
-M + Mz_{ij}^4 & \leq & p_{ij}^2 - t_j^2 \leq M - Mz_{ij}^4
\end{array}$$

$$
\begin{array}{rcl}
-M + M(z_{ij}^1 + z_{ij}^2) & \leq & p_{ij}^3 - t_i^2 \ \leq \ M - M(z_{ij}^1 + z_{ij}^2) \\
-M + Mz_{ij}^3 & \leq & p_{ij}^3 - t_j^1 \ \leq \ M - Mz_{ij}^3 \\
-M + Mz_{ij}^4 & \leq & p_{ij}^3 - t_i^1 \ \leq \ M - Mz_{ij}^4 \\
-M + M(z_{ij}^5 + z_{ij}^6) & \leq & p_{ij}^3 - t_j^2 \ \leq \ M - M(z_{ij}^5 + z_{ij}^6) \\
-M + M(z_{ij}^1 + z_{ij}^2 + z_{ij}^3) & \leq & p_{ij}^4 - t_j^2 \ \leq \ M - M(z_{ij}^1 + z_{ij}^2 + z_{ij}^3) \\
-M + M(z_{ij}^4 + z_{ij}^5 + z_{ij}^6) & \leq & p_{ij}^4 - t_i^2 \ \leq \ M - M(z_{ij}^4 + z_{ij}^5 + z_{ij}^6)
\end{array}
$$

with

$$0 = p_{ij}^0 \leq p_{ij}^1 \leq p_{ij}^2 \leq p_{ij}^3 \leq p_{ij}^4 \leq p_{ij}^5 = T.$$

Taking into account all pairs $i, j \in A$, this gives $12n(n-1)/2$ constraints, which also give the sizes of time intervals (directly obtained by p_{ij} values), that in the prior formulation requires adding specific constraints.

Summarizing, MINLP formulations for the aircraft conflict avoidance problem are generally characterized by:

- a number of variables and constraints rapidly growing with the number n of aircraft and generally leading to large-scale problems. In some cases, a suitable pre-processing, able to identify pairs of aircraft whose trajectories remain separated regardless of other separation maneuvers in the airspace, can help reducing the size of the problem to be solved;
- integer, and in particular binary, variables, due to the combinatorial nature of the problem. The number of integer variables is strictly related to the generality and flexibility of the chosen model;
- continuous variables often bounded on the basis of operational constraints, that restricts their degree of freedom;
- nonlinearities appearing in the objective(s) and constraints. They are mainly related to modeling separation conditions on the one hand, and to logical choices (using binary variables) on the other hand. Thus, nonlinearities come specially from products of continuous as well as continuous and binary variables, from trigonometric functions when angles have to be decided, and from "OR" constraints. This clearly affects the complexity of the solution process, and reveals the importance of reformulations and of the choice of an appropriate solver.

4 Solution approaches

While a thorough discussion of solution approaches for aircraft deconfliction via MINLP is out of the scope of this chapter, we recall in this section the main approaches and issues related to the numerical solution of the considered application.

Deterministic approaches to compute a global solution, mainly based on Branch-and-Bound methods, are applied e.g. in [19, 2, 10, 8, 5]. On simpler models with linear formulations, solutions are efficiently obtained using state-of-the-art solvers [19, 2]. The global exact solution is evidently more difficult to compute for complex mixed-integer nonlinear models, mainly due to the nonconvexity of the region described by aircraft separation constraints. Note that, as these constraints are indexed on all pairs of aircraft, their number rapidly grows with the number n of aircraft. Furthermore, remark that for the considered application a feasible solution is not guaranteed to exist. This is specially related to the tight bounds that are often imposed on decision variables (speeds, angles) because of aerodynamical and operational limitations, thus restricting the freedom to find a feasible solution. Results are obtained for small to medium-scale instances in moderate computing time [8, 5], using global optimization engines like `COUENNE`. These results demonstrate that the aforementioned MINLP formulations behave reasonably well for the considered application and are amenable to be solved by general-purpose solvers for MINLP. The proposed reformulation of the separation condition to avoid the dependence on time, on the one hand, enables to avoid a time discretization, and on the other hand is flexible enough to potentially be used in different models (e.g., also based on aircraft angle modifications). In other cases, like in [3], the proposed complex model cannot be solved by general-purpose MINLP solvers, and a sequence of linear approximations based on Taylor polynomials is used instead.

Alternatively, some authors resort to heuristics [4, 9, 23], and are able to solve large instances, though obtaining feasible (if any) but not guaranteed global optimal solutions. More specifically, a Variable Neighborhood Search metaheuristic is proposed in [4], while a simulated annealing and, respectively, a genetic algorithm tailored to the problem are used in [9, 23] for the related problem of trajectory planning. Finally, in [8], besides the exact solution of the whole MINLP, a heuristic is proposed, based on exact solutions of subproblems that are represented by clusters of conflicting aircraft. This kind of hybrid solution strategies appears to be promising and deserves further developments.

5 Conclusion

We presented an application of MINLP arising in Air Traffic Management, namely aircraft conflict avoidance for en-route flights. We primarily focused on modeling aspects, specific to the considered application, emphasizing the role of MINLP. It appears that the considered application is challenging and leaves room to further interesting developments using mixed-integer optimization.

Current research is specially addressed to efficiently solving large-scale instances, and to incorporate in the problem formulation objectives which are relevant for a sustainable environment [25], as well as the uncertainty affecting the aircraft motion (see, e.g., [16, 21]).

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