Towards 4D Trajectory Tracking for Transport Aircraft

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Abstract: This paper presents a new approach to perform 4D trajectory tracking for transportation aircraft. As current systems are extensions of 3D guidance with overfly time constraints at some given points and no general control framework has been developed for 4D guidance of a transport aircraft; the main goal of the proposed approach is to introduce a new method based on the inversion of the flight dynamics while avoiding numerical issues. A six degree of freedom model for a wide body transportation aircraft was developed in Matlab to provide a numerical simulation of the proposed approach, showing satisfying results.

Keywords: Trajectory Tracking. Transportation Aircraft. Navigation, Guidance and Control. Automatic control. Non Linear Inversion.

1. INTRODUCTION

As air traffic is predicted to increase dramatically in the upcoming years, new problems and requirements are arising, among those related with the use of 4D trajectories are of most interest. NextGen FAA (2016) and SESAR EUROCONTROL (2016) control projects, where traffic capacity and safety issues are central, adopt the Trajectory Based Operations (TBO) paradigm, which supposes 4D guidance effectiveness. Then, an important enabler is automation, allowing aircraft to follow with more accuracy flight plans characterized by a 4D reference trajectory. It is expected that accurate 4D guidance will improve safety by decreasing the occurrence of near mid-air collisions for planned conflict free 4D trajectories, and then diminish the workload associated to a single flight for air traffic controllers. This leads to propose in this paper a new approach to perform 4D guidance.

Until today no general control framework has been developed for 4D guidance of a transport aircraft and current systems are extensions of 3D guidance with overfly time constraints at some given points. In that case control laws are based on frequency decoupling and different PID control layers with gain scheduling provisions. Some attempts have been performed recently Wahid et al. (October 2014), Wahid et al. (2016) using mainly Non Linear Inversion (NLI). Other authors have proposed to extent the energy-based control approach Lambregts (1983) to 4D guidance Lambregts (1996), Chudy and Rzucidlo (August 2009). However in both cases serious limitations appear.

In this study it is considered that aircraft dynamics are composed of fast dynamics related with the angular attitude of the aircraft and of slow dynamics related with the trajectory followed by the aircraft, referenced at its center of gravity. For modern transportation aircraft with Fly by Wire (FBW) technology, the autopilot is in charge of controlling the aircraft angular attitude, improving flying qualities through stabilization and generating automatic protections in dangerous piloting situations. In this paper, the autopilot will be considered to be an specific device providing in an integrated way these essential functions, so that the attention will be focused on the auto guidance system in charge of controlling the slow flight dynamics with the aim of providing 4D guidance.

The paper is organized as follows, Section 2 provides the developed mathematical model and actuator dynamics. In Section 3 are discussed two previous approaches pointing out their limitations for 4D guidance. Section 4 describes the adopted control structure and Section 5 shows the simulation results of the control approach. Finally, conclusions are given in Section 6.

2. FLIGHT DYNAMICS

2.1 Adopted Frames and Flight Variables

The Earth reference frame is considered in this paper to describe the 4D trajectories. It is assumed that the Earth reference frame is Earth centered and is denoted by $F_E = (O_E, x_E, y_E, z_E)$. Then, a 4D reference trajectory can be defined by three functions associated with the coordinates of the center of gravity of the aircraft supposed to follow it. These functions are parameterized by time: $x_R(t), y_R(t), z_R(t), t \in [t_{init}, t_{end}]$, where t_{init} is the reference flight starting time and t_{end} is the reference flight ending time. A second reference frame, the body frame, is considered to represent the fast dynamics of the aircraft, it is attached to the aircraft c.g. and is defined as $F_B = (C, x_B, y_B, z_B)$. The x_B axis goes from tail to nose of the aircraft while the z_B axis, perpendicular to the latter, points downwards and lies in the symmetry plane of the aircraft, to complete the triad, the direction of y_B can be obtained by the cross product $\boldsymbol{z_B} \times \boldsymbol{x_B}$. Lastly, denoted by $F_W = (C, x_W, y_W, z_W)$, the wind frame is introduced since it allows to represent the aerodynamic actions on the aircraft. This frame has its x_W axis aligned with the aircraft velocity vector relative to the surrounding air mass, i.e. the airspeed (V_a). The difference between x_B and the projection of x_W in the Cx_Bz_B plane, gives birth to the angle of attack (α). Also, the angle created by the projection of V_a in the Cx_By_B plane and the x_B axis, is known as the sideslip angle (β). Physical quantities from the wind frame can be mapped into the body frame by the following rotation matrix:

$$L_{BW} = \begin{bmatrix} c_\alpha c_\beta & -c_\alpha s_\beta & -s_\alpha \\ s_\beta & c_\beta & 0 \\ s_\alpha c_\beta & -s_\alpha s_\beta & c_\alpha \end{bmatrix} \tag{1}$$

In the same tenor, the velocity of the aircraft $V_E = [\dot{x_E}, \dot{y_E}, \dot{z_E}]^T$ expressed in the body frame, is given by $V_B = [u, v, w]^T$. Also, the euler angles will be used to describe the attitude of the aircraft. These angles will be bounded as follows $\phi\{-\pi, \pi\}$; $\theta\{-\frac{\pi}{2}, \frac{\pi}{2}\}$; $\psi\{-\pi, \pi\}$, although limits are never reached during normal operation for transportation aircraft. The rotation matrix from the body to the earth frame considering a rotation around the axes in the order *zyx*, is given by:

$$L_{EB} = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix} \tag{2}$$

Moreover, from the rigid-body equations and other known relations extracted from Etkin and Reid (1996); Stevens and Lewis (1992) it is obtained:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} V_a c_\alpha c_\beta + V_{w_x} \\ V_a s_\beta + V_{w_y} \\ V_a s_\alpha c_\beta + V_{w_z} \end{bmatrix} \tag{3}$$

$$\alpha = arctan\left(\frac{w - V_{w_z}}{u - V_{w_x}}\right) \tag{4a}$$

$$\beta = arcsin\left(\frac{v - V_{w_y}}{V_a}\right) \tag{4b}$$

$$V_a = \sqrt{(u - V_{w_x})^2 + (v - V_{w_y})^2 + (w - V_{w_z})^2} \tag{4c}$$

where $V_{w_{x,y,z}}$ are the wind components in the body frame. The vector $R = [\alpha, \beta]^T$ is defined for simplicity in further equations.

2.2 Attitude Dynamics

Concerning the attitude of the aircraft, the angular rates ($\Omega = [p, q, r]^T$) are produced by the deflection of ailerons, elevator and rudder, denoted by $[\delta_{ail}, \delta_{ele}, \delta_{rud}]^T$. The rotational equations of the aircraft are given by:

$$\dot{\Omega} = I^{-1} M_{ext} - I^{-1} \Omega \times (I\Omega) \tag{5}$$

where $M_{ext} = [L', M, N]^T$ are the rolling, pitching and yawing moments respectively, and I stands for the inertia matrix, in $kg \cdot m^2$:

$$I = \begin{bmatrix} A & 0 & -E \\ 0 & B & 0 \\ -E & 0 & C \end{bmatrix} = \begin{bmatrix} 1,278,369.56 & 0 & -135,588.17 \\ 0 & 3,781,267.79 & 0 \\ -135,588.17 & 0 & 4,877,649.98 \end{bmatrix} \tag{6}$$

Thereby, introducing relations for the aerodynamic moments:

$$\begin{bmatrix} L' \\ M \\ N \end{bmatrix} = \frac{1}{2}\rho S V_a^2 \left(\begin{bmatrix} bC_l \\ \bar{c}C_m \\ bC_n \end{bmatrix} + C_\delta \begin{bmatrix} \delta_{ail} \\ \delta_{ele} \\ \delta_{rud} \end{bmatrix} \right) \tag{7}$$

where

$$C_\delta = \begin{bmatrix} bC_{l_{\delta ail}} & 0 & bC_{l_{\delta rud}} \\ 0 & \bar{c}C_{m_{\delta ele}} & 0 \\ bC_{n_{\delta ail}} & 0 & bC_{n_{\delta rud}} \end{bmatrix} \tag{8}$$

and ρ is the air density, S wing area, b wingspan, $\bar{c}$ mean chord, and the rolling, pitching and yawing aerodynamic coefficients (C_l, C_m, C_n respectively) are denoted by:

$$\begin{bmatrix} C_l \\ C_m \\ C_n \end{bmatrix} = \begin{bmatrix} C_{l_\beta}\beta + C_{l_p}\frac{bp}{2V_a} + C_{l_r}\frac{br}{2V_a} \\ C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}\frac{\bar{c}q}{2V_a} \\ C_{n_\beta}\beta + C_{n_p}\frac{bp}{2V_a} + C_{n_r}\frac{br}{2V_a} \end{bmatrix} \tag{9}$$

Then, equation (5) can be rewritten as in Lombaerts et al. (May-June 2009) like.

$$\dot{\Omega} = \frac{1}{2}\rho V_a^2 S I^{-1}\left(\begin{bmatrix} bC_l \\ \bar{c}C_m \\ bC_n \end{bmatrix} + C_\delta \begin{bmatrix} \delta_{ail} \\ \delta_{ele} \\ \delta_{rud} \end{bmatrix}\right) - I^{-1}\Omega \times (I\Omega) \quad (10)$$

Subsequently, differentiating (4a), (4b), and using (3), (14), (15), an expression rearranged for Ω is obtained:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \quad (11)$$

where

$$\begin{aligned}
H_{11} &= -\left(tan_\beta c_\alpha + \frac{V_{w_y} c_\alpha}{V_a c_\beta}\right) \\
H_{12} &= 1 + \frac{V_{w_x} c_\alpha + V_{w_z} s_\alpha}{V_a c_\beta} \\
H_{13} &= -\left(tan_\beta s_\alpha + \frac{V_{w_y} s_\alpha}{V_a c_\beta}\right) \\
H_{21} &= s_\alpha + \frac{V_{w_z} c_\beta + V_{w_y} s_\alpha s_\beta}{V_a} \\
H_{22} &= \frac{V_{w_z} c_\alpha s_\beta - V_{w_x} s_\alpha s_\beta}{V_a} \\
H_{23} &= -\left(c_\alpha + \frac{V_{w_y} c_\alpha s_\beta + V_{w_x} c_\beta}{V_a}\right)
\end{aligned}$$

$$\begin{aligned}
Q_1 &= \frac{1}{V_a c_\beta}\left(g_1 - \frac{1}{m}\left(L + F_{thr} s_\alpha\right)\right) + \frac{1}{V_a c_\beta}\left(\dot{V}_{w_x} s_\alpha - \dot{V}_{w_z} c_\alpha\right) \\
Q_2 &= \frac{1}{V_a}\left(g_2 + \frac{1}{m}\left(Y - F_{thr} c_\alpha s_\beta\right)\right) \\
&\quad + \frac{1}{V_a}\left(\dot{V}_{w_x} c_\alpha s_\beta - \dot{V}_{w_y} c_\beta + \dot{V}_{w_z} s_\alpha s_\beta\right)
\end{aligned}$$

with

$$g_1 = g\left(c_\alpha c_\theta c_\phi + s_\alpha s_\theta\right)$$

$$g_2 = g\left(c_\beta c_\theta s_\phi + s_\beta c_\alpha s_\theta - s_\alpha s_\beta c_\theta c_\phi\right)$$

which can be written as:

$$\dot{R} = H(R)\,\Omega + Q(R) \quad (12)$$

The rotation speed components are related with the attitude angle rates by the Euler equations given by:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & tg_\theta s_\phi & tg_\theta c_\phi \\ 0 & c_\phi & -s_\phi \\ 0 & \frac{s_\phi}{c_\theta} & \frac{c_\phi}{c_\theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (13)$$

2.3 Guidance Dynamics

An acceleration equation in the body frame is denoted by:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} \frac{1}{m}\left(F_{x_a} + F_{thr}\right) - g s_\theta + rv - qw \\ \frac{1}{m} F_{y_a} + g c_\theta s_\phi + pw - ru \\ \frac{1}{m} F_{z_a} + g c_\theta c_\phi + qu - pv \end{bmatrix} \quad (14)$$

where m is the mass, g gravity, F_{thr} thrust force, and

$$\begin{bmatrix} F_{x_a} \\ F_{y_a} \\ F_{z_a} \end{bmatrix} = L_{BW} \begin{bmatrix} -D \\ Y \\ -L \end{bmatrix} \quad (15)$$

are the aerodynamic forces dependent on Lift (L), Drag (D) and Sideforce (Y), which simultaneously are related with their aerodynamic force coefficients by:

$$\begin{bmatrix} D \\ Y \\ L \end{bmatrix} = \frac{1}{2}\rho S V_a^2 \begin{bmatrix} C_D \\ C_Y \\ C_L \end{bmatrix} \quad (16)$$

Then, an expression in the Earth frame is obtained:

$$\begin{bmatrix} \ddot{x}_E \\ \ddot{y}_E \\ \ddot{z}_E \end{bmatrix} = L_{EB} \begin{bmatrix} F_{x_a} + F_{thr} \\ F_{y_a} \\ F_{z_a} \end{bmatrix} \frac{1}{m} + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \quad (17)$$

Regarding to airspeed in the wind frame, after differentiating (4c), and using (3), (14), (15), it is obtained that:

$$\begin{aligned}
\dot{V}_a = g_3 &+ \frac{1}{m}\left(F_{thr} c_\alpha c_\beta - D\right) + p\left(V_{w_z} s_\beta - V_{w_y} s_\alpha c_\beta\right) \\
&+ q\left(V_{w_x} s_\alpha c_\beta - V_{w_z} c_\alpha c_\beta\right) + r\left(V_{w_y} c_\alpha c_\beta - V_{w_x} s_\beta\right) \\
&- \dot{V}_{w_x} c_\alpha c_\beta - \dot{V}_{w_y} s_\beta - \dot{V}_{w_z} s_\alpha c_\beta
\end{aligned} \quad (18)$$

with

$$g_3 = g\left(-c_\alpha c_\beta s_\theta + s_\beta c_\theta s_\phi + s_\alpha c_\beta c_\theta c_\phi\right)$$

Then, neglecting wind disturbances and taking into account a longitudinal decoupled motion, (18) can be reduced to:

$$\dot{V}_a = \frac{1}{m}\left(F_{thr} c_\alpha - D - mg s_\gamma\right) \quad (19)$$

Introducing the Flight Path Angle ($\gamma = \theta - \alpha$), the vertical motion is given by:

$$\dot{\gamma} = \frac{1}{mV_a}\left(F_{thr} s_\alpha + L - mg c_\gamma\right) \quad (20)$$

Considering that transport aircraft perform through their yaw stabilizer equilibrated turns, the heading rate is given by:

$$\dot{\psi} = \frac{g}{V_a} tg_\phi \quad (21)$$

2.4 Actuator Dynamics

Let a first-order model be adopted for the aerodynamic actuators, writing δ_i^d $(i = ail, ele, rud)$ as the commanded positions of the control surfaces, and δ_i as the current positions of the control surfaces:

$$\dot{\delta}_i = \frac{1}{\xi_i}\left(\delta_i^d - \delta_i\right) \quad (22)$$

where ξ_i are the time-constants. Also, the resultant thrust produced by the engines is supposed to behave as a first-order system, denoted by

$$\dot{F}_{thr} = \frac{1}{\xi_T}\left(F_{thr}^d - F_{thr}\right) \quad (23)$$

where the F_{thr}^d is the desired thrust and F_{thr} the current thrust. Besides, the time-constants of the actuators keep the relation: $\xi_T >> \xi_i$.

3. PROPOSED CONTROL APPROACHES

In this part are briefly discussed two previous control approaches that resulted in the design of 4D guidance devices. The first one is related to energy-based control while the second is related with direct Non Linear Inversion control. Limitations of both methods are pointed out.

3.1 Total Energy Control Approach

In this method, the concept of a potential flight path angle denoted by: $\gamma_p = \gamma + \frac{\dot{V}_a}{g}$ is considered. It indicates the potential path angle that can be achieved by bringing the acceleration to zero by applying elevator until γ becomes γ_p. This potential angle is found to be related with the specific total energy rate of the aircraft, therefore, the approach consists in controlling the behaviour of the energy rate by controlling γ_p using the thrust and elevator. However, the control of this variable is performed with non available plant dynamics while introducing empirically an energy rate distribution variable $\dot{L} = \gamma - \frac{\dot{V}_a}{g}$ to distribute the energy rate between the flight path angle and acceleration. This distribution variable is controlled directly by the elevator, bypassing the fast dynamics and arising difficulties to integrate other autopilot functions. Furthermore, the speed and altitude dynamics in this approach are expected to be identical, bringing to light a problem for some 4D trajectories where this is not true. Finally, no integral term has been proposed to force online position error to zero.

3.2 Direct Non Linear Inverse Control Approach

This method distinguishes between fast and slow dynamics. The variables used to create the link between these dynamics are the components p, q, r of the rotational speed. Therefore, two layers of inversion are considered for this approach. The first regards to the fast dynamics and allows to determinate the necessary position of the actuators (deflections of aileron, elevator and rudder) as functions of the desired angular velocities, based on equations (10),(12),(18), and (22). The second layer takes care of the slow dynamics, and allows to obtain the angular velocities and Thrust required to follow the 4D commanded trajectory.

However, in the second layer, this approach presents as main drawback a singularity when inversion is performed to get the input parameters to the slow dynamics.

This problematic appears after introducing the control inputs $\left[p, q, \dot{F}_{thr}\right]^T$ in the equation of the outputs, which is made by obtaining the jerk vector of the positions in F_E by differentiating (17). The control inputs appear in the matrix $\dot{L}_{EB}$. This is possible thanks to the Euler property and (13), which allows to rewrite $\dot{L}_{EB}$ in terms of L_{EB} and the skew-symmetric matrix of the angular velocities $\left(\dot{L}_{EB} = L_{EB}\tilde{\Omega}\right)$. The resultant equation using the vector $F_{ab} = [F_{x_a} + F_{thr}, F_{y_a}, F_{z_a}]^T$ and considering that the mass rate of change is very small compared to the aircraft total mass, has the form:

$$\begin{bmatrix} x_E^{(3)} \\ y_E^{(3)} \\ z_E^{(3)} \end{bmatrix} = \frac{1}{m} M_{pqT} \begin{bmatrix} p \\ q \\ \dot{F}_{thr} \end{bmatrix} + \frac{1}{m} M_r F_{ab} r + \frac{L_{EB}}{m} \begin{bmatrix} \dot{F}_{x_a} \\ \dot{F}_{y_a} \\ \dot{F}_{z_a} \end{bmatrix} \quad (24)$$

where

$$M_{pqT} = \left[\begin{array}{c|c|c} M_p F_{ab} & M_q F_{ab} & \begin{matrix} c_\theta c_\psi \\ c_\theta s_\psi \\ -s_\theta \end{matrix} \end{array} \right] \quad (25)$$

with

$$M_p = \begin{bmatrix} 0 & c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\psi - s_\phi s_\theta c_\psi \\ 0 & c_\phi s_\theta s_\psi - s_\phi c_\psi & -s_\phi s_\theta s_\psi - c_\phi c_\psi \\ 0 & c_\phi c_\theta & -s_\phi c_\theta \end{bmatrix} \quad (26a)$$

$$M_q = \begin{bmatrix} -c_\phi s_\theta c_\psi - s_\phi s_\psi & 0 & c_\theta c_\psi \\ -c_\phi s_\theta s_\psi + s_\phi c_\psi & 0 & c_\theta s_\psi \\ -c_\phi c_\theta & 0 & -s_\theta \end{bmatrix} \quad (26b)$$

$$M_r = \begin{bmatrix} s_\phi s_\theta c_\psi - c_\phi s_\psi & -c_\theta c_\psi & 0 \\ s_\phi s_\theta s_\psi + c_\phi c_\psi & -c_\theta s_\psi & 0 \\ s_\phi c_\theta & s_\theta & 0 \end{bmatrix} \quad (26c)$$

Note that to develop a control law using NLI or any other model-based-approach relying on equation (24), the inverse of (25) needs to be computed.

In the case in which $\phi \approx \psi \approx 0$, the determinant of this matrix is given by:

$$\begin{aligned} \mid M_{pqT} \mid &= s_\theta F_{y_a} + \left(s_\theta \left(F_{x_a} + F_{thr}\right) - c_\theta F_{z_a}\right)\left(s_\theta F_{z_a}\right) \\ &\quad + c_\theta \left(c_\theta \left(F_{x_a} + F_{thr}\right) + s_\theta F_{z_a}\right) F_{z_a} \end{aligned} \quad (27)$$

and, when considered $\theta \approx 0$, a singularity appears for $F_{x_a} + F_{thr} = 0$. This is the case when the plane is cruising at constant speed. Therefore, taking into account that the cruise phase of a flight is essential, and that the airplane will go through this condition very often, any algorithm using this control approach should be discarded. This result is not surprising, realizing that when a cruise flight with the Euler angles near zero is performed, the matrix L_{EB} will remain constant, so its derivative is expected to tend to zero. Furthermore, it is worth to say that even if the first derivative of the Euler angles along with the Thrust are considered as the control inputs, $\left[\dot{\phi}, \dot{\theta}, \dot{F}_{thr}\right]^T$ for instance, like in Wahid et al. (2016), the corresponding matrix (similar to (25)) will be described by (28) and will also present singularities during the cruise phase.

$$M_{\phi\theta} = \left[M_\phi F_{ab} \middle| M_\theta F_{ab} \middle| \begin{matrix} c_\theta c_\psi \\ c_\theta s_\psi \\ -s_\theta \end{matrix} \right] \tag{28}$$

where

$$M_\phi = M_p \tag{29a}$$

$$M_\theta = \begin{bmatrix} -s_\theta c_\psi & c_\theta s_\phi c_\psi & c_\theta c_\phi c_\psi \\ -s_\theta s_\psi & c_\theta s_\phi s_\psi & c_\theta c_\phi s_\psi \\ -c_\theta & -s_\theta s_\phi & -s_\theta c_\phi \end{bmatrix} \tag{29b}$$

So another method should be searched in order to avoid this singularity.

4. PROPOSED 4D CONTROL APPROACH

The main objective for the proposed 4D trajectory tracking control strategy is to avoid singularity issues while inverting the flight dynamics. This will be made possible by considering three phases:

(1) Control of the longitudinal and lateral motions, θ and ϕ, with stabilisation in yaw ψ. We will consider that thanks to the relation $\alpha = \theta - \gamma$, to command θ when γ is known, is equal to command the AoA. The control of θ and ϕ while stabilizing yaw motion can be performed using classical control techniques Hameduddin and Bajodah (June 2012), Kim and Kim (November 2003), Ali et al. (2010), Mattei and Monaco (October 2014). In this work, the autopilot implemented in the simulations is based on an inversion of the fast dynamics, like in Escamilla-Núñez et al. (January 2017) (see Section 7).
(2) Control of Speed and the Flight Path Angle as functions of AoA and Thrust using the guidance equations (19),(20).
(3) Adjustment of the heading angle given by (21), targeted speed, and desired flight path angle as functions of the positions errors generated by a 4D reference trajectory.

Proposing desired dynamics for the speed $\left(V_a^d\right)$, heading $\left(\psi^d\right)$, and flight path angle $\left(\gamma^d\right)$ as first order linear responses, they are such as:

$$\dot{V}_a = \frac{1}{\tau_V}\left(V_a^d - V_a\right) \tag{30a}$$

$$\dot{\gamma} = \frac{1}{\tau_\gamma}\left(\gamma^d - \gamma\right) \tag{30b}$$

$$\dot{\psi} = \frac{1}{\tau_\psi}\left(\psi^d - \psi\right) \tag{30c}$$

where $\tau_V, \tau_\gamma, \tau_\psi$ are time-constants.
Equations (19), (20), and (21) can be rewritten as

$$\frac{m}{\tau_V}\left(V_a^d - V_a\right) + mgs_\gamma = F_{thr}c_\alpha - D(\rho,\alpha,V_a) \tag{31a}$$

$$\frac{mV_a}{\tau_\gamma}\left(\gamma^d - \gamma\right) + mgc\gamma = F_{thr}s_\alpha + L(\rho,\alpha,V_a) \tag{31b}$$

$$tg^{-1}\left(\frac{V_a}{g\tau_\psi}\left(\psi^d - \psi\right)\right) = \phi \tag{31c}$$

Therefore, in order to obtain the values of the thrust and AoA required to follow their proposed dynamics, we solve the nonlinear set of equations (31a), (31b) for F_{thr} and α, and apply saturations if the results are out of the thrust $\left[F_{thr}^{min}, F_{thr}^{max}\right]$ and angle of attack $\left[\alpha^{min}, \alpha^{max}\right]$ limits. Note that the Jacobian of the right hand side of equations (31a), (31b) is

$$J = \begin{bmatrix} c_\alpha & -\left(F_{thr}s_\alpha + \frac{\partial D(\rho,\alpha,V_a)}{\partial\alpha}\right) \\ s_\alpha & F_{thr}c_\alpha + \frac{\partial L(\rho,\alpha,V_a)}{\partial\alpha} \end{bmatrix} \tag{32}$$

and its determinant

$$\mid J \mid = F_{thr} + \frac{\partial L(\rho,\alpha,V_a)}{\partial\alpha}c_\alpha + \frac{\partial D(\rho,\alpha,V_a)}{\partial\alpha}s_\alpha > 0 \tag{33}$$

assuring that inversion is possible at all times.
The ϕ value required to follow the lateral dynamics, it is obtained directly from (31c) and is also saturated within the limits $\left[\phi^{min}, \phi^{max}\right]$.
Now, to assure that the 4D trajectory error tends to zero after some perturbation, the reference values for V_a, γ, and ψ must be adapted. The proposed adaptive scheme is the following:

$$V_a^d = V_a + \delta V_a \tag{34a}$$

$$\gamma^d = \gamma + \delta\gamma \tag{34b}$$

$$\psi^d = \psi + \delta\psi \tag{34c}$$

where

$$\delta V_a = \frac{1}{\tau_x}\left(x_R - x_E\right) \tag{35a}$$

$$\delta\gamma = \frac{1}{\tau_z}\left(z_R - z_E\right) \tag{35b}$$

$$\delta\psi = \frac{1}{\tau_y}\left(y_R - y_E\right) \tag{35c}$$

and τ_x, τ_y, τ_z are time-constants such that: $\tau_x >> \tau_V$; $\tau_y >> \tau_\psi$; $\tau_z >> \tau_\gamma$.
The term $\delta\psi$ is depicted in figure 1.

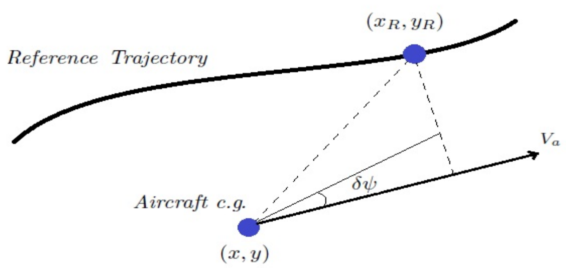

Fig. 1. Change in ψ required to follow the reference trajectory

5. SIMULATION RESULTS

A six degree of freedom model for a wide body transportation aircraft has been developed in Matlab to provide simulation results with the adopted control law. The aircraft parameters were chosen close enough to emulate a Boeing 737-200, flying in an International Standard Atmosphere model with uniform gravity. The time response for engines is 4 s, and for actuators used to move the control surfaces is 50 ms. To obtain the values of the aerodynamic coefficients, two-layer feed-forward neural networks with sigmoid hidden neurons and linear output neurons were trained using data bases obtained from the United States Air Force (USAF) Stability and Control Digital DATCOM (Data Compendium) PDAS (a). This is a Public Domain Aeronautical Software that computes the static stability, control and dynamic derivative characteristics of fixed-wing aircrafts using the methods contained in the USAF Stability and Control DATCOM. Furthermore, the data sets were compared and refined using the JSBSim open source Flight Dynamics Models (FDM) PDAS (b), used in several open source simulators and also employed to drive the motion-base research simulators of many universities worldwide. The training, validation and test sets were 70, 15 and 15 percent respectively of the available data for each aerodynamic coefficient. The training algorithm used was Bayesian Regularization and the number of hidden neurons was selected by trial and error trying to improve the performance as much as possible.

When the control surfaces are jammed at zero degrees, and enough constant Thrust to hold a longitudinal flight is set, the behaviour of the aircraft is a stable, with decreasing phugoid mode with a 100s period.

In order to test the proposed 4D trajectory tracking approach for the longitudinal motion, a 100m amplitude sinusoidal reference trajectory for $z_R(t)$ is proposed. The results are depicted in figure 2. The required thrust is computed and affected by the first order response of its actuator (eq. (23)). Also, the required AoA is computed and then used to calculate a desired pitch angle (using the Flight Path Angle). The commanded $(x_R(t), z_R(t))$ trajectory is followed with a small delay.

For the lateral motion, another 100m sinusoidal reference trajectory for $y_R(t)$ is proposed, the response of the controller is depicted in figure 3. For practical purposes, the initial heading of the airplane is zero degrees.

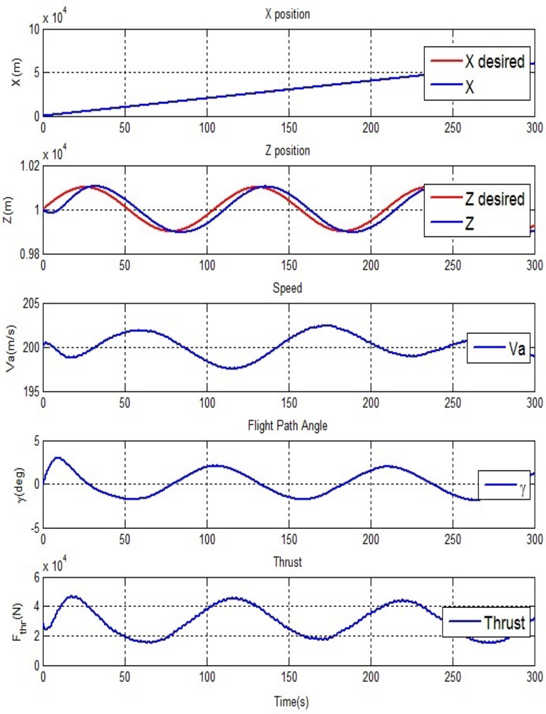

Fig. 2. X, Z desired stand for x_R, z_R. X, Z stand for x_E, z_E.

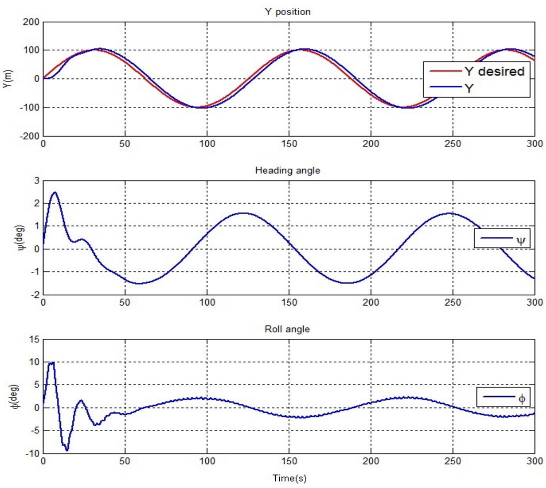

Fig. 3. Y desired stands for y_R. Y stands for y_E. Heading and roll angles are also shown.

6. CONCLUSION

This paper presents a new approach to perform 4D trajectory tracking for transportation aircraft. The proposed approach avoid the numerical difficulties encountered by direct non linear inverse control approaches by limiting inversion in the guidance dynamics. Contrarily to the energy-based methods, frequency decoupling is preserved, avoiding blind setting of control parameters. The proposed approach has been tested through numerical simulation of the flight of a large transportation aircraft, providing satisfying results. Future studies should consider the effect of introducing wind estimates and the establishment of 4D trajectory tracking performance as a parameter in order to improve air traffic management.

7. APPENDIX A. FAST DYNAMICS INVERSION

The jerk vector of the angular velocities is obtained by differentiating one more time equation (10) and including (22). Also, taking into account the aerodynamic moment coefficients dynamics due to their close relation with $Ra = [\alpha, \beta, V_a]$ by differentiating (9), wind information can be included in the model. The control surfaces moment coefficients dynamics are neglected. Yielding:

$$\begin{aligned}\begin{bmatrix}\ddot{p}\\ \ddot{q}\\ \ddot{r}\end{bmatrix} &= \frac{1}{2}\rho S I^{-1}\left\{V_a^2 C_\delta \xi \begin{bmatrix}\delta_{ail}^d - \delta_{ail}\\ \delta_{ele}^d - \delta_{ele}\\ \delta_{rud}^d - \delta_{rud}\end{bmatrix} + V_a^2 C_c \dot{R}a\right.\\ &\left. +2V_a\dot{V}_a\left(\begin{bmatrix}bC_l\\ \bar{c}C_m\\ bC_n\end{bmatrix} + C_\delta\begin{bmatrix}\delta_{ail}\\ \delta_{ele}\\ \delta_{rud}\end{bmatrix}\right)\right\}\\ &+ I^{-1}\left(\frac{1}{4}\rho S V_a C_k - In\right)\dot{\Omega} \end{aligned} \tag{36}$$

where

$$I_n = \begin{bmatrix} -Eq & (C-B)r - Ep & (C-B)q\\ (A-C)r + 2Ep & 0 & (A-C)p - 2Er\\ (B-A)q & (B-A)p + Er & Eq \end{bmatrix}$$

$$\xi = \begin{bmatrix} \frac{1}{\xi_{ail}} & 0 & 0\\ 0 & \frac{1}{\xi_{ele}} & 0\\ 0 & 0 & \frac{1}{\xi_{rud}} \end{bmatrix}$$

$$C_c = \begin{bmatrix} 0 & bC_{l_\beta} & -\frac{b^2}{2V_a^2}(C_{l_p}p + C_{l_r}r)\\ \bar{c}C_{m_\alpha} & 0 & -\frac{\bar{c}^2}{2V_a^2}C_{m_q}q\\ 0 & bC_{n_\beta} & -\frac{b^2}{2V_a^2}(C_{n_p}p + C_{n_r}r) \end{bmatrix}$$

$$C_k = \begin{bmatrix} b^2C_{l_p} & 0 & b^2C_{l_r}\\ 0 & \bar{c}^2C_{m_q} & 0\\ b^2C_{n_p} & 0 & b^2C_{n_r} \end{bmatrix}$$

The vector $\dot{R}a$ can be taken from (12) and (18), and the vector $\dot{\Omega}$ from (10). Consequently a NLI leads to an attitude control input denoted by:

$$\begin{aligned}\begin{bmatrix}\delta_{ail}^d\\ \delta_{ele}^d\\ \delta_{rud}^d\end{bmatrix} &= \frac{1}{V_a^2}\xi^{-1}C_\delta^{-1}\left\{\frac{2I}{\rho S}\begin{bmatrix}\tau_p\\ \tau_q\\ \tau_r\end{bmatrix} - \frac{2}{\rho S}\left(\frac{1}{4}\rho S V_a C_k - In\right)\dot{\Omega}\right.\\ &\left. -2V_a\dot{V}_a\left(\begin{bmatrix}bC_l\\ \bar{c}C_m\\ bC_n\end{bmatrix} + C_\delta\begin{bmatrix}\delta_{ail}\\ \delta_{ele}\\ \delta_{rud}\end{bmatrix}\right) - V_a^2 C_c\dot{R}a\right\} + \begin{bmatrix}\delta_{ail}\\ \delta_{ele}\\ \delta_{rud}\end{bmatrix}\end{aligned} \tag{37}$$

where the wind effects appear in the terms involving $\dot{R}a$, and:

$$\begin{bmatrix}\tau_p\\ \tau_q\\ \tau_r\end{bmatrix} = \begin{bmatrix} -k_1(p - p^d) - k_2(\dot{p} - \dot{p}^d) + \ddot{p}^d\\ -k_3(q - q^d) - k_4(\dot{q} - \dot{q}^d) + \ddot{q}^d\\ -k_5(r - r^d) - k_6(\dot{r} - \dot{r}^d) + \ddot{r}^d \end{bmatrix} \tag{38}$$

For $k_i > 0$ $(i = 1, ...6)$ are gains chosen in order to assure asymptotical convergence of the variables to their desired values. The feasibility of this approach depends on the singularity of C_δ, the matrix involving the aerodynamic coefficients (assumed to be known thanks to experimental data, airflow simulations, or any other method) due to the control surfaces, aspect that can be handled.

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