

# Constant speed optimal reciprocal collision avoidance

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# Constant Speed Optimal Reciprocal Collision Avoidance

Nicolas Durand

*ENAC*

*nicolas.durand@enac.fr*

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## Abstract

In this article, the Optimal Reciprocal Collision Avoidance (ORCA) algorithm is modified to make it work for speed constrained aircraft. The adaptation of ORCA to aircraft conflict resolution shows that when the speed norm is constrained, aircraft flying within the same speed range with small angle converging trajectories tend to remain on parallel tracks, preventing a resolution of the conflict. The ORCA algorithm is slightly modified to avoid this behavior. In the new algorithm called CSORCA (Constant Speed Optimal Reciprocal Collision Avoidance), the directions of the semi-plane used to calculate the conflict free maneuvers are modified when the relative speed vector is in the semi-circular part of the conflicting area. After explaining the reasons that make the original algorithm fail in the constant speed environment, the modification made on the algorithm is detailed and its impact on a simple example is shown. The new strategy is also compared to an Add-Up strategy close to the Airborne Separation Assurance System (ASAS) strategy found in the literature. Hundreds of fast time simulations are then performed to compare the two versions of the algorithm for different traffic densities in the horizontal plane. In these simulations the speed norm is first constrained. The aircraft can only change direction with a limited turning rate. Simulations with released speed constraints are then performed to compare the behavior of both algorithms in a more general environment. In all the scenarios tested, CSORCA is more efficient than ORCA to solve conflicts.

*Keywords:* ORCA, CSORCA, self-separation, geometrical algorithm, air traffic

control, UAS

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## 1. Introduction

The growing traffic of Unmanned Airborne Systems (UAS) flying in the lower airspace is leading researchers to study self-separation algorithms in order to separate UAS. Self-separation has been widely studied in the case of two UAS Durand et al. (2000); Zeghal (1998); Eby and Kelly (1999); Hoekstra et al. (2002), but dealing with complex situations involving many aircraft (more than 2) has mainly been treated by adding pairwise maneuvers.

In this article, the Optimal Reciprocal Collision Avoidance (ORCA) algorithm van den Berg et al. (2011) is modified to improve its performance for speed constrained aircraft. The modification is tested on different densities in order to show that it improves the performance of the algorithm when the speed norms are constrained without decreasing its performance when speed constraints are relaxed.

Van den Berg et al. proposed an Optimal Reciprocal Collision Avoidance (ORCA) Algorithm van den Berg et al. (2011) using a simple geometric approach in order to guarantee conflict resolution for the next  $\tau$  minutes in an autonomous manner. Each agent is only aware of the surrounding traffic. For multiple conflict situations, the algorithm finds compatible maneuvers that separate aircraft in complex situations in a decentralized way without sequencing maneuvers. This makes it very different from previous decentralized approaches such as sliding forces Zeghal (1998) or ASAS Hoekstra et al. (2002) that simply add up the pairwise maneuvers when dealing with multiple aircraft conflicts.

It was adapted to aircraft separation by Snape et al. in 2010 Snape and Manocha (2010). The algorithm guarantees a conflict free solution for the next  $\tau$  minutes if the agents can change their speed without constraint. In the worst case, when the density gets too high, some agents may completely be stopped before heading back to their destination.

The ORCA algorithm was tested in a previous project Durand and Barnier (2015) and showed some limits when the speed norm was constrained. We showed that when speed norms are similar and strongly constrained, the ORCA

algorithm cannot handle low densities of traffic. Aircraft converging on a point with small angles tend to choose parallel trajectories that do not solve the conflicts but divert them far from their destination. This phenomenon can be avoided by slightly modifying the ORCA algorithm. This modification and its  
35 consequences on a series of tests involving different densities of traffic is the main contribution of this article.

### 1.1. Related Works

Aircraft conflict resolution is still performed by humans in a centralized approach. Air traffic controllers have a global view of the whole situation  
40 and give maneuvers to aircraft to keep a minimum vertical and horizontal separation between them. In the 90s, we showed that conflict resolution was highly combinatorial and could not be globally solved with local optimization methods Durand et al. (1996). We proposed a genetic algorithm to solve multiple aircraft conflicts using a centralized approach with simple maneuvers (similar to  
45 those used by air traffic controllers) and showed that we were able to solve every conflict on real traffic data Durand and Alliot (1997).

Centralized methods can be divided into two main categories. Some methods Krella et al. (1989); Chiang et al. (1997); Hu et al. (2002) use greedy sequential algorithms to optimize trajectories one by one after ranking the aircraft. However, finding an appropriate ordering is challenging Archambault and  
50 Durand (2004). The others, mentioned in the next paragraph, try to find the global optimum without the need to prioritize aircraft.

Using evolutionary computation, our team was the first to address conflict resolution globally Durand et al. (1996). Others later introduced a powerful  
55 Semidefinite Programming approach Oh et al. (1997); Frazzoli et al. (2001), which is also able to handle multiple aircraft in a single scenario. However the solution is only locally optimal and the model requires constant speeds and perfect trajectory prediction. In the early 2000s, a mathematical model using Mixed Integer Linear Programming, which could be solved by CPLEX  
60 and ensured the global optimality of the solution, was proposed Pallottino et al.

(2001, 2002). The model was extended in 3D in 2008, but required constant speed during climbing phases Christodoulou and Kontogeorgou (2008). Uncertainties on the trajectory heading were added to the horizontal model in 2009, but all maneuvers still needed to be executed at the same time (at every optimization  
65 step) Gariel and Feron (2009). The method is powerful but cannot be used for developing a realistic advisory tool for controllers.

More recently, in the horizontal plane, Omer et al. Omer and Farges (2013) proposed a Mixed-Integer Linear Programming approach that takes into account the trajectory recovery. Alonso-Ayuso et al. Alonso-Ayuso et al. (2016) proposed  
70 a Mixed Integer Non Linear Optimization model to deal with the horizontal case and take the trajectory recovery into account. Peyronne et al. Peyronne et al. (2015) introduce a trajectory model using B-Splines and a semi-infinite programming formulation of the constraints. The combinatorial aspect of the resolution of multi-aircraft conflicts is solved with a genetic algorithm like in  
75 Durand et al. (1996). Rey et al. Rey and Hijazi (2017) recently proposed a new complex number formulation and convex relaxations for the centralized problem and showed that it could significantly reduce the resolution time.

In 2013, our team used a Constraint Programming approach to globally solve complex conflicts Allignol et al. (2013). For each aircraft, a number of alternative  
80 trajectories and a matrix of pairwise conflicts were precalculated, taking various uncertainties into account, before the optimization process was performed. This approach separated the problem model from its resolution and was close to the graph model proposed by Lehouillier et al. (2017).

In the 1990s, a wide debate started among the Air Traffic Control communi-  
85 ties on the possibility of eliminating centralized control systems by developing autonomous algorithms for solving conflicts. At that time the motivation was to reduce the cost of air traffic control. The recent perspective of development of Unmanned Airborne Systems in the lower airspace gives a new interest to autonomous approaches and the high traffic demand in the lower airspace Bulusu  
90 et al. (2016) will create complex situations that centralized approaches will probably not be able to handle.



The first effective approach used sliding forces to coordinate maneuvers between aircraft Zeghal (1998). Potential or vortex fields Košecká et al. (1998) as well as a model based on an analogy with electrical particle repulsion Eby and Kelly (1999) were also used. These algorithms took into account an important constraint of aircraft: the speed constraint. During a level flight an aircraft can easily change direction respecting a maximum turning rate but it can barely modify its speed range. For situations involving many aircraft, these three algorithms relied on the principle that forces or potential fields virtually generated by aircraft on each other would add up. There is no guarantee that this principle always leads to a conflict free solution. Eby et al.'s approach Eby and Kelly (1999) inspired the Airborne Separation Assurance System (ASAS) approach developed by Hoekstra et al. Hoekstra et al. (2002). ASAS uses a Modified Voltage Potential (MVP) close to Eby's definition. It was tested in different contexts such as the Mediterranean Free-Flight Ruigrok and Hoekstra (2007). More recently, it was used to model airspace stability and capacity in a decentralized separation context Sunil et al. (2017) and in a Capacity Assessment Model Emmanuel Sunil and Hoekstra (2018).

A geometric optimization approach was proposed by Bilimoria Bilimoria (2000) in 2000 and used in the Future Air traffic management concepts Evaluation Tool (FACET) project D. Bilimoria et al. (2001), but it solved complex situations by using an iterative process. This is also the case of the Free-flight Autonomous and Coordinated Embarked Solver Granger et al. (2001b,a) proposed by Granger et al. in 1999 that used a token allocation strategy to coordinate sequential maneuvers.

In 2007, Wang et al. Hwang et al. (2007) introduced a geometric approach that required a shared knowledge of the whole situation. This strong hypothesis undermines autonomous conflict resolution. In an autonomous context, aircraft are only aware of their surrounding context. In contrast, implementing the same central approach in every aircraft leads to a non autonomous resolution. Leny et al. Le Ny and Pappas (2010) in 2010 also used a geometric approach for scheduling crossing times of aircraft through a metering fix. The model was not

adapted to general situations. Pallottino et al. Pallottino et al. (2007) proposed a model where coordination is ensured by protected stacking areas in which each aircraft can move without conflicting with other aircraft. The areas are headed  
125 to the flight destination. Even if the aircraft is flying at constant speed these protected areas can stop moving and ensure that a solution can be found even in high densities. Schouwenaars et al. Schouwenaars and Feron (2004) used the same kind of approach to plan safe trajectories in a decentralized way. In 2009,  
130 Alam et al. Alam et al. (2009) proposed to use data mining to determine which conflict detection algorithm is more likely to give an accurate answer depending on the traffic situation.

Geometric approaches using automated coordination were first introduced by Van den Berg et al. van den Berg et al. (2008), first with the Reciprocal  
135 Velocity Obstacles and then with the Optimal Reciprocal Collision Avoidance (ORCA) van den Berg et al. (2011) algorithm which is used in the present paper. ORCA was first meant to be used to simulate robots using self-separation logic in a 2D environment. In air traffic control, Snape et al. Snape and Manocha (2010) have extended the model to the vertical dimension without segregating  
140 the horizontal and vertical maneuvers. More recently, Geometric approaches have led to the definition of Solution Space Diagrams that were used to analyze the dynamic of air traffic controller workload d’Engelbronner et al. (2015) and were recently compared to the Modified Voltage Potential approach Balasoorian (2017) showing that the latter seemed to be more efficient in many situations.

## 145 1.2. Outline

Section 2 presents the geometrical algorithm and describes the problem that occurs when two similar speed aircraft which can only change headings converge with a small angle. In section 3, the modification made to the algorithm is detailed and its consequence is shown on simple examples. The new algorithm  
150 is also compared to a maneuver Add-Up strategy for multiple aircraft conflicts close to the ASAS multiple aircraft conflict strategy. Section 4 describes the simulation scenarios used to validate the modification on different traffic densities.

The Add-Up strategy is used to compare the results. Finally the speed norm constraint is relaxed in order to check the behavior of the algorithm for high traffic densities. A conclusion and perspectives are given in section 5.

## 2. Detect and Avoid Model

This section describes the Optimal Reciprocal Collision Avoidance (ORCA) algorithm developed in van den Berg et al. (2011) and its adaptation to the case where speed norms are constrained. First the maneuver model is detailed for an aircraft pair, then the maneuver calculation is explained when more than two aircraft are involved in a conflict.

### 2.1. Constraint Model

Let  $d$  be the target separation distance and  $\tau$  be a look ahead time. In figure 1, let us consider aircraft  $A$  and aircraft  $B$ . Aircraft  $B$ 's position can be represented in the referential of aircraft  $A$ . Consider a circle of radius  $d$  centered at aircraft  $B$ , the two lines issued from position  $A$ , tangent to the circle of radius  $d$  form a cone. If the relative speed  $\vec{v}_r = \vec{v}_A - \vec{v}_B$  lies in this cone, a conflict will occur in the future. A circle of size  $\frac{d}{\tau}$  centered at  $B'$  such that  $\overrightarrow{AB'} = \frac{\overrightarrow{AB}}{\tau}$  defines a zone (in light red) bounded by the bold line in figure 1.  $\vec{v}_r$  lies in this zone if and only if it intersects the disc of size  $\frac{d}{\tau}$  centered in  $B'$ , i.e. if and only if there exists  $\lambda \in [0 : 1[$  such that  $|\lambda\vec{v}_r - \overrightarrow{AB'}| < \frac{d}{\tau}$  or  $|\lambda\tau\vec{v}_r - \overrightarrow{AB}| < d$  which means that A and B are in conflict at time  $\lambda\tau < \tau$ . Consequently a conflict will occur within time  $\tau$  if and only if  $\vec{v}_r$  lies in this light red zone.

If this is the case, then it is necessary to modify speed vectors  $\vec{v}_A$  and  $\vec{v}_B$  in such a way that the resulting relative speed vector  $\vec{v}_r$  is outside of the constraint. Let  $\vec{c}$  be the projection of  $\vec{v}_r$  on the closest edge of the red zone.

The ORCA algorithm is based on the principle that the effort to keep  $\vec{v}_r$  out of the red area should be minimal and shared by the two aircraft. In its original version the necessary minimal speed modification  $\vec{c}$  is shared equally between the two aircraft and defines two semi-planes  $P_{B \rightarrow A}$  and  $P_{A \rightarrow B}$  (in light

green) perpendicular to vector  $\vec{c}$  in which aircraft  $A$  and  $B$  can choose their new speeds.

If the modified speeds  $\vec{v}'_A$  and  $\vec{v}'_B$  are in these semi-planes, the new relative speed  $\vec{v}'_r$  is out of the red zone and the conflict is solved.

185 Indeed, the semi-plane corresponding to the modified speed of aircraft  $A$  is defined by the equation:

$$(\vec{v}'_A - (\vec{v}_A + \frac{\vec{c}}{2})).\vec{c} \geq 0$$

The semi-plane corresponding to the modified speed of aircraft  $B$  is defined by the equation:

$$(\vec{v}'_B - (\vec{v}_B - \frac{\vec{c}}{2})).\vec{c} \leq 0$$

The difference of the two previous equations leads to:

$$(\vec{v}'_r - (\vec{v}_r + \vec{c})).\vec{c} \geq 0$$

190 This equation guarantees that  $\vec{v}'_r$  is outside the truncated cone.

When three or more aircraft are in conflict, each aircraft pair involving aircraft  $A$  defines a semi-plane for aircraft  $A$  (see figure 3) and the new speed for  $A$  is chosen in the intersection of the semi-planes generated by every other aircraft. If the intersection of the semi-planes is empty, every semi-plane is  
 195 equally slightly moved until the intersection exists. This process is described by Van den Berg van den Berg et al. (2011).

This is the main difference with the ASAS approach based on Modified Potential Voltage (MVP). In the (MVP) approach, the maneuvers are calculated for pairs of aircraft and when 2 or more maneuvers are calculated for the same  
 200 aircraft, the resulting maneuver is the sum of each single maneuver.

## 2.2. Possible Heading Range

In this paper, heading changes are the only possible maneuvers for aircraft. The capacity of aircraft to change speed is small. A velocity increase or reduction

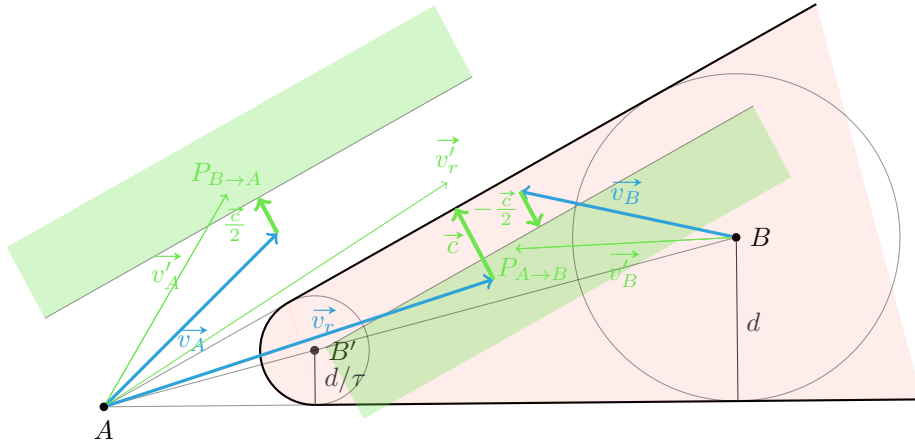


Figure 1: Conflicting aircraft model: a conflict will occur within time  $\tau$  if and only if the relative speed  $\vec{v}_r$  lies in the forbidden zone in red.

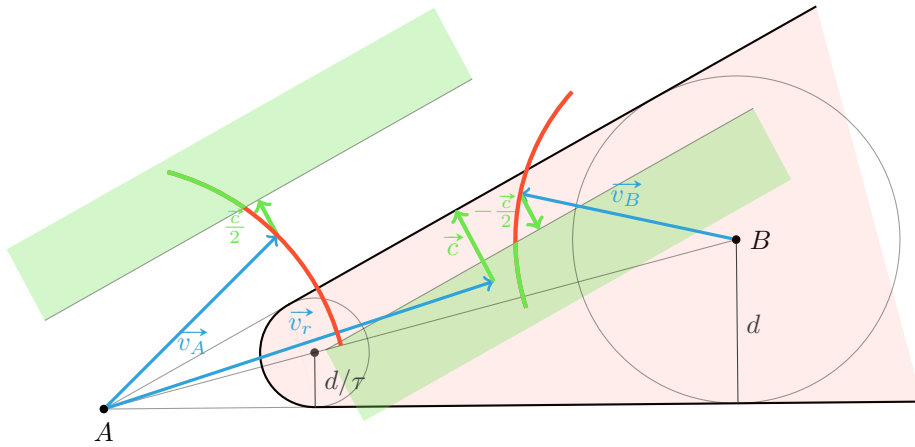


Figure 2: Conflicting aircraft model: the effort is shared by the two aircraft. The new speeds must be chosen on the green arc.

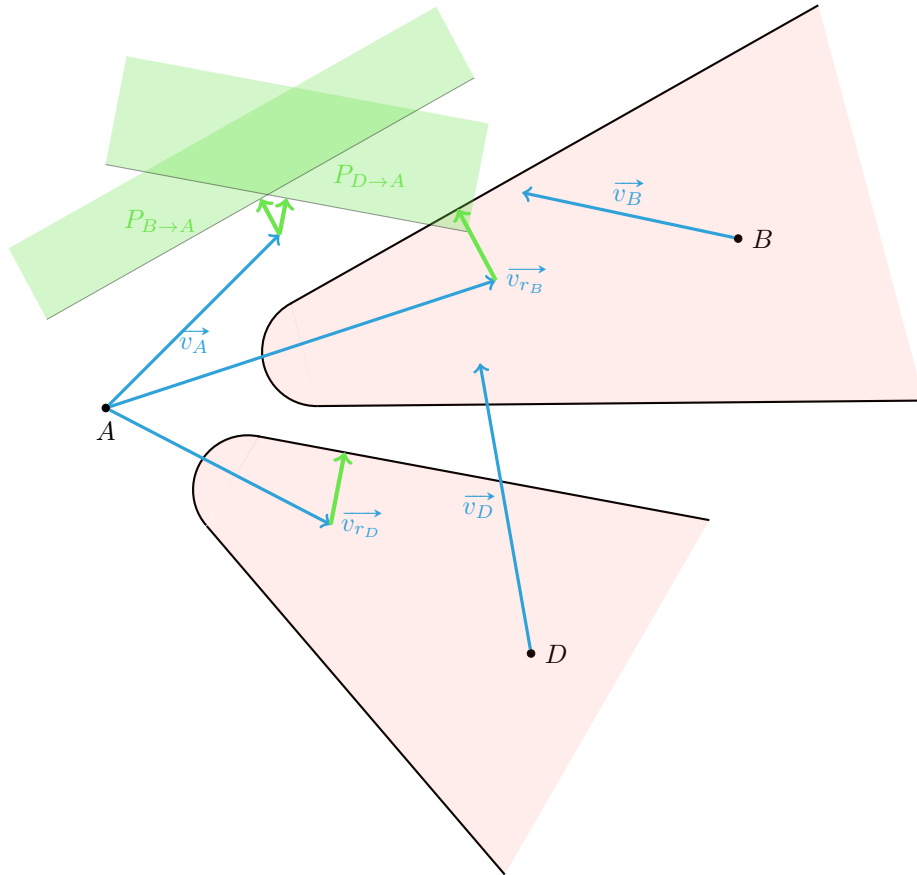


Figure 3: Multi-Conflicting aircraft model: a conflict will occur within time  $\tau$  if and only if the relative speed  $\vec{v}_r$  lies in the forbidden zone in red.

is limited in amplitude and takes time. In the ERASMUS speed control project,  
 205 the speed modification range was set to  $[-6\%; +3\%]$  Bonini et al. (2009).

The norm of the aircraft speeds remains constant throughout the conflict resolution process if and only if:

$$\begin{aligned}\|\vec{v}'_A\| &= \|\vec{v}_A\| \\ \|\vec{v}'_B\| &= \|\vec{v}_B\|\end{aligned}$$

Because of the speed norm constraint, the new vectors must be chosen on the  
 210 green arc (see figure 2). The arc length is limited by the maximum turning rate. In the further experiments, we compare a standard turning rate (3 degrees per second) and a doubled standard turning rate (6 degrees per second). If the time step is set to 3 seconds, the arc range is  $\pm 9$  degrees for the standard turning rate and  $\pm 18$  degrees for the doubled standard turning rate.

215 Aircraft can limit the speed change by choosing the new vectors that give the smallest heading change. This strategy was called “closest” by Allignol et al. Allignol et al. (2017) because the current heading  $h_c$  is modified as little as possible.

In the present case, in order to ensure that aircraft reach their destinations,  
 220 the ideal heading  $h_i$  (the heading toward the destination) is first calculated and the “closest” approach is used considering  $h_i$  instead of the current heading  $h_c$

Algorithm 1 describes the ORCA algorithm. Time is discretized into time steps  $t_s$  (for the experiments  $t_s = 3$  seconds). As long as every aircraft has not reached its destination, every aircraft pair  $(i,j)$  is checked to calculate semi-planes  
 225  $P_{j \rightarrow i}$  and  $P_{i \rightarrow j}$ . For every aircraft  $i$ , the intersection of the  $P_{k \rightarrow i}$  with  $k \neq i$  semi-planes creates a convex constraint surface  $C_i$ . The intersection of  $C_i$  and the arc  $A_i$  of possible aircraft  $i$  turning range defines the arc  $S_i$  of possible conflict free headings. The new current heading  $h_c$  is then chosen as close as possible to the ideal heading  $h_i$  (facing the destination). If the intersection of  
 230 the semi-planes is empty, every semi-plane is equally slightly moved until an intersection appears. The convex  $C_i$  does not guarantee a conflict free maneuver anymore, but it remains close to the conflict free domain.

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**Algorithm 1** ORCA Algorithm

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**Input:**  $t_s, \tau, h_c^i$ **Output:** modified headings  $h_s^i$ **while** every aircraft has not reached destination **do**  **for** every aircraft couple (i,j) **do**    Define the semi-plane constraints  $P_{j \rightarrow i}$  and  $P_{i \rightarrow j}$   **end for**  **for** every aircraft i **do**    Calculate the convex intersection  $C_i$  of every semi-planes  $C_i = \bigcap_{k \neq i} P_{k \rightarrow i}$     Calculate the arc  $A_i$  of possible aircraft  $i$  turning range.    Calculate  $S_i = C_i \cap A_i$     Calculate the ideal heading  $h_i$     Choose the closest heading  $h_s^i$  of  $h_i$  in  $S_i$     Change current heading:  $h_c^i = h_s^i$     Move aircraft  $i$  with current heading  $h_c^i$   **end for**  increase current time  $t = t + t_s$ **end while**

---

### 3. ORCA modification for Constant Speeds

The ORCA algorithm described in the previous section does not solve conflicts  
235 for small angle converging aircraft that can only change headings Durand and  
Barnier (2015). It tends to postpone the conflict outside of time window  $\tau$ .

#### 3.1. Critical cases

Figure 4 represents another conflict in which the relative speed is close to the  
circular part of the cone. When the relative speed vector  $\vec{v}_r$  is close to the circular  
240 part of the cone, the direction of vector  $\vec{c}$  moves the corresponding semi-planes  
for  $\vec{v}_A$  and  $\vec{v}_B$  in directions that are not parallel to one of the cone sides (see  
figure 4). As a result, depending on the initial conditions, the new aircraft speeds  
tend to become parallel instead of choosing directions that solve the conflict.



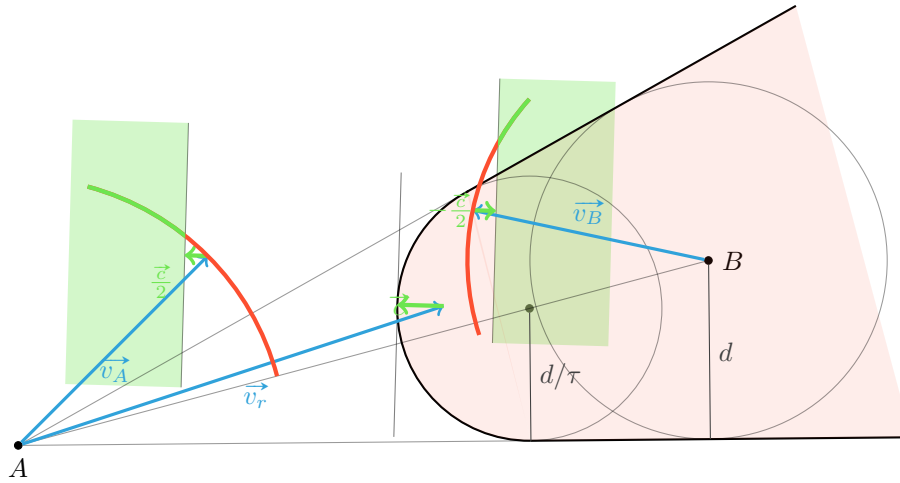


Figure 4: Conflicting aircraft model: in this example  $\vec{c}$  (the projection of  $\vec{v}_r$  outside the forbidden zone) is on the circular part of the red zone.

This is called the horizon effect in game theory. Instead of solving the conflict,  
 245 the algorithm tends to postpone the conflict outside the time window.

When the relative vector is in the circular part of the forbidden cone, it means that the conflict is “just starting” and it is “cheaper” to postpone it than to solve it. This continuous approach of solving the conflict prevents both aircraft from making a resolution decision. Aircraft tend to end-up parallel and  
 250 divert from their destination. This phenomenon is made worse by the speed norm constraint. In figure 5, the two aircraft converging to the center of the figure (red lines) with the same speed are turned in opposite directions and progressively choose parallel speeds. The conflict is solved, because the aircraft are parallel, but the aircraft cannot reach their destinations.

### 255 3.2. Algorithm modification

To deal with the problem described in the previous section, the relative speed vector  $\vec{v}_r$  is projected on the side of the cone even when it is in the circular part of the forbidden zone. In figure 6 aircraft  $A$  and  $B$  are moved to the left as soon as the relative speed  $\vec{v}_r$  gets in the forbidden zone. This helps solve the conflict

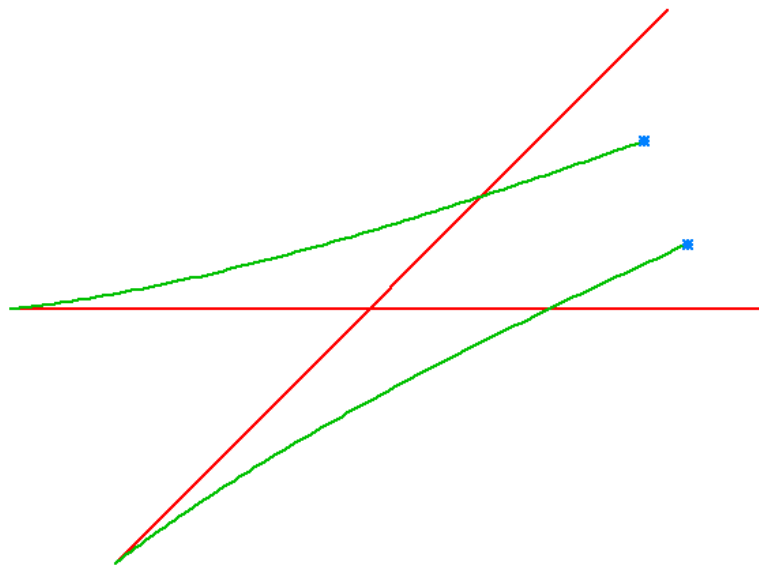


Figure 5: Two aircraft conflict with the original ORCA algorithm. Aircraft are turned opposite directions and fly parallel tracks without reaching their destinations.

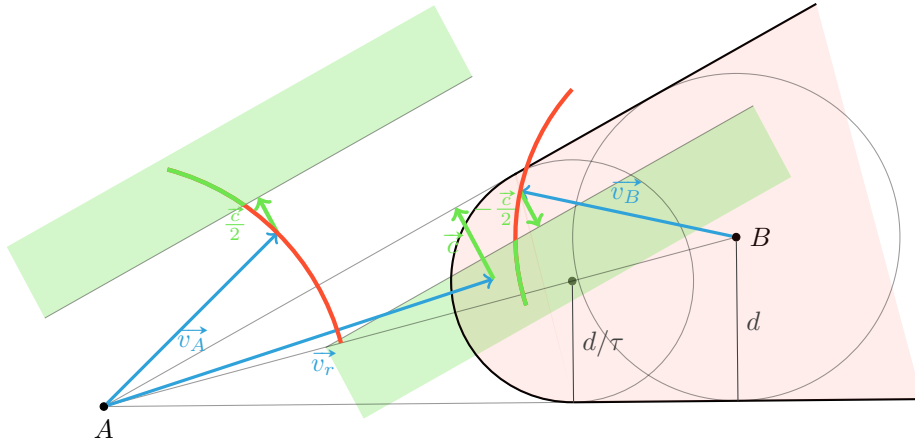


Figure 6: New model: the relative speed  $\vec{v}_r$  is now projected on the closest side of the cone instead of the circular part.

260 and avoids the horizon effect described in the previous section.

In figure 7, the two aircraft converging to the center of the figure (red lines) with the same speed are turned left and can reach their destination while solving the conflict.

The effect of the algorithm can be observed on a bigger example involving 10  
 265 aircraft converging on a point from a circle arc. The converging angle between the aircraft is 6 degrees. They all fly with the same speed and are initially supposed to reach the center of the circle at the same time.

With the ORCA algorithm (see figure 8), the aircraft choose parallel tracks and cannot reach their initial destinations. With the CSORCA modification (see  
 270 figure 9), the aircraft manage to solve their conflicts and reach their destinations.

### 3.3. Comparison with an Add-Up strategy

In the case of two aircraft conflict, the modified ORCA algorithm is very close to the Modified Voltage Potential (MVP) algorithm developed for the ASAS Hoekstra et al. (2002). The MVP algorithm, like CSORCA, uses the  
 275 full cone. The main difference between the modified ORCA and MVP appears for multiple aircraft conflicts. Instead of calculating the intersection of the

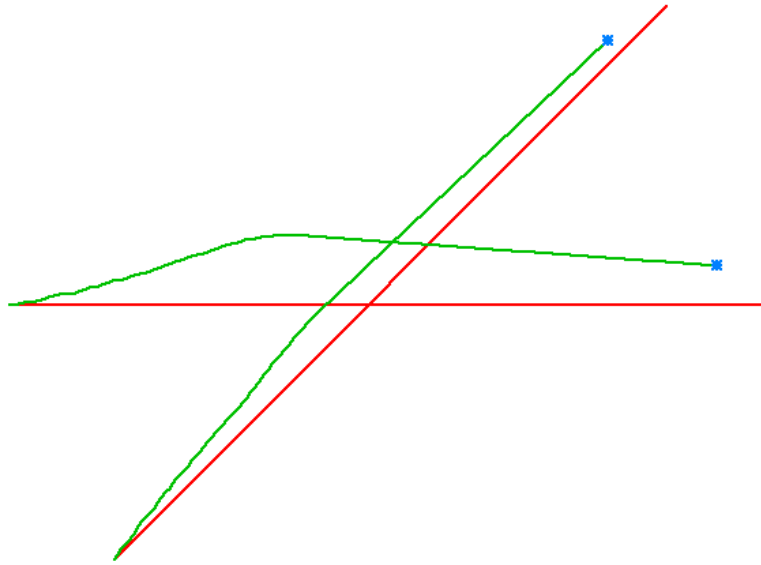


Figure 7: Two aircraft conflict with the modified ORCA algorithm. Both aircraft are turned left and the conflict is easily solved.

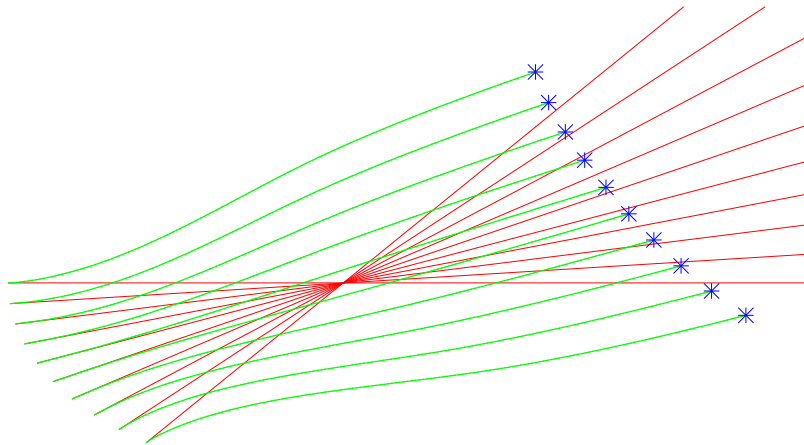


Figure 8: Ten aircraft conflict with the original ORCA algorithm. Aircraft choose parallel tracks.

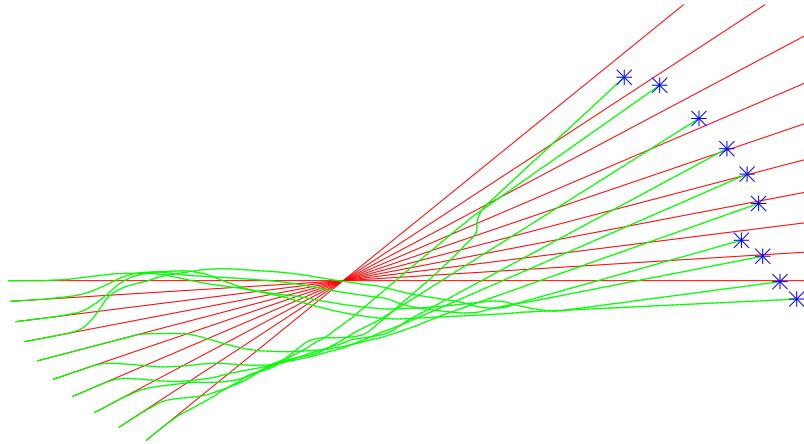


Figure 9: Ten aircraft conflict with the modified ORCA algorithm. Every aircraft reaches its destination without conflict.

semi-planes to define the conflict free heading domain, a speed deviation is calculated for each intruding aircraft and they are Added-Up to determine a resulting speed. When the speed norm is constrained, the heading changes are added-up and the resulting heading is limited by the maximum turning rate.  
 280 This strategy is applied on a 8 aircraft conflict in figure 10. It cannot solve the 10 aircraft example without conflict. It will be called the Add-Up strategy in the next section.

#### 4. Experimental results

285 We showed in Durand and Barnier (2015) that the ORCA algorithm in its initial version is not efficient when the speed norms are constrained. Even with low densities, such as presented in figure 11, the phenomenon presented in part 3.1 prevents aircraft from reaching their destination while staying in the window. In this figure, there are only ten aircraft flying at the same speed in a  $500 \times 500$   
 290 nautical mile square area. With the original ORCA algorithm proposed by Van den Berg et al. van den Berg et al. (2011), small angle converging aircraft tend

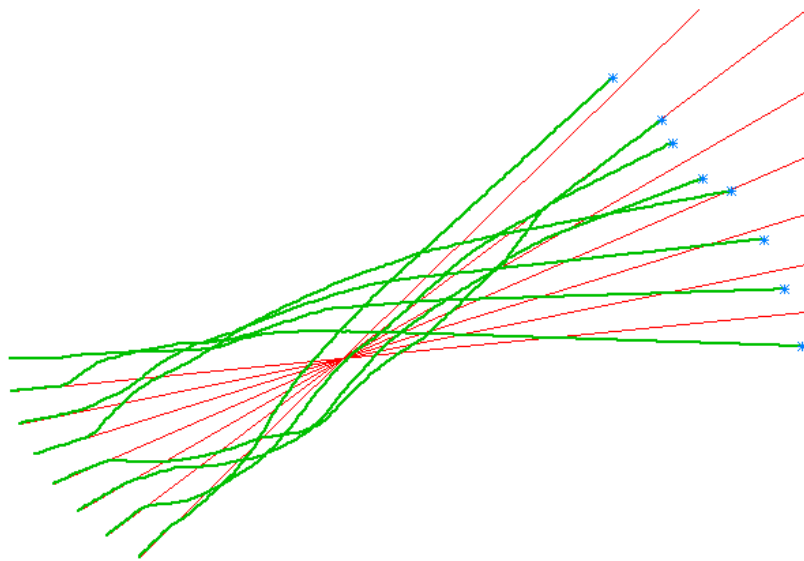


Figure 10: 8 aircraft conflict with the Add-Up Strategy.

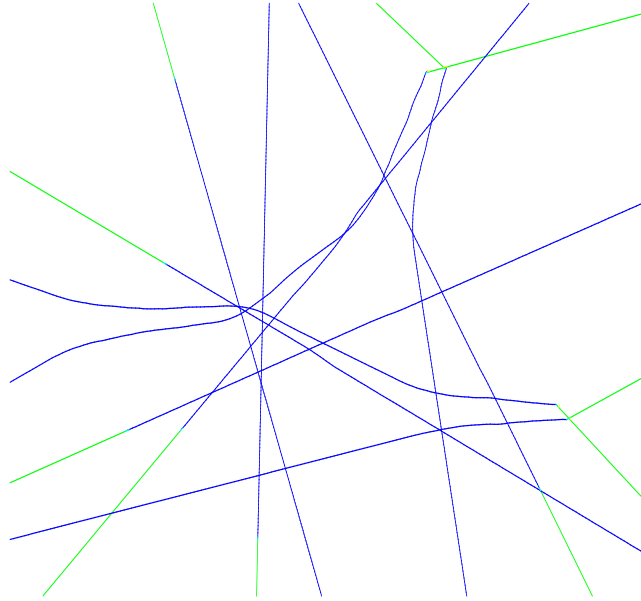


Figure 11: A scenario example with 10 aircraft and no speed norm modification.

to adopt parallel tracks that keep them away from their destination. Figure 12 represents a scenario where 130 aircraft fly in the same  $500 \times 500$  nautical mile square area but can increase or reduce their speeds by 30%. The original ORCA  
 295 algorithm has no difficulty finding a solution for each of the 130 aircraft in this case.

In this section different scenarios are built to show how the modified version of the algorithm (CSORCA) behaves with different traffic densities and speed norm constraints.

300 *4.1. ORCA and CSORCA comparison on various densities and constrained speed norms.*

First the behaviors of ORCA and CSORCA are compared in a speed norm constrained environment to check how it handles different densities. Specifically, it is tested in a  $500 \times 500$  nautical mile sector with varying numbers of aircraft

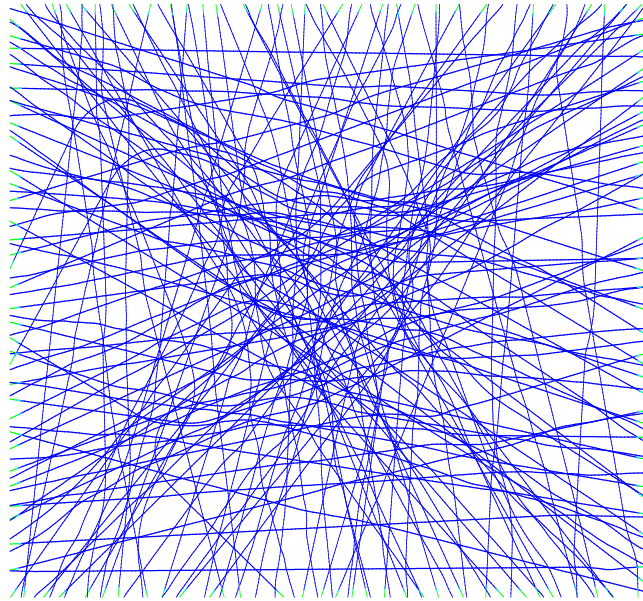


Figure 12: A scenario example with 130 aircraft and 30% speed norm modification.



Nb (number) of aircraft	Total nb of failures	Nb of border violations	Nb of remaining conflicts	Mean delay	Maximum delay
10	4	1	3	1.00172	1.00763
20	7	2	5	1.00477	1.01088
30	26	3	23	1.01159	1.02739
40	32	6	26	1.01979	1.03626
50	53	14	39	1.02708	1.04028
60	70	15	55	1.04270	1.06213
70	81	28	53	1.05514	1.08631
80	89	32	57	1.06571	1.10463
90	99	20	79	1.07024	1.07024

Nb (number) of aircraft	Total nb of failures	Nb of border violations	Nb of remaining conflicts	Mean delay	Maximum delay
10	96	96	0	1.02100	1.07680
20	100	98	2		
30	100	93	7		
40	100	60	40		
50	100	25	75		
60	100	10	90		
70	100	7	93		
80	100	2	98		
90	100	1	99		

Table 1: Total number of failures, due to border violations or remaining conflicts, mean and max delays for 10 to 90 aircraft with no speed norm variation and 3 degrees/second maximum turning rate. Left: CSORCA - Right: ORCA

305  $n$  and different parameters for the turning angle. In the following, low density is defined as 10, 20, 30 aircraft, medium as 40, 50 or 60 aircraft, and high as 70, 80 or 90 aircraft. The density of conflicts is proportional to the square of the density of traffic because the traffic is randomly distributed.

310 The aircraft speeds are set to 1 (speeds are normalized), which means that all aircraft fly at the same speed. This is not totally realistic, but similar results are obtained with nominal speed norms varying by  $\pm 20\%$  and some preliminary tests have shown that the worst situations appear when speed norms are close to each other. The time step  $t_s$  is set to 3 seconds.

315 The parameter  $\tau$  is fixed at  $\tau = 120$  s (i.e. 2 minutes) because it is a good compromise: if the lookahead time is too long, the algorithm is less efficient for high densities because too many conflicts are simultaneously taken into account and the permitted zones often are empty; whereas if  $\tau$  is too short, some conflicts appear too late to be resolved. A sensitivity analysis done with a 3 degrees/second maximum turn rate and no speed norm variation showed that 320 for low densities (up to 30 aircraft) a longer  $\tau$  value can minimize the number of failures (see table 2) but for higher densities,  $\tau = 120$  seconds is the best

Number of aircraft	$\tau = 60$ seconds	$\tau = 90$ seconds	$\tau = 120$ seconds	$\tau = 150$ seconds	$\tau = 180$ seconds
10	10	7	4	3	<b>2</b>
20	31	24	<b>7</b>	15	10
30	71	48	26	23	<b>16</b>
40	79	60	<b>32</b>	34	50
50	94	76	<b>53</b>	67	81
60	98	79	<b>70</b>	82	97
70	100	85	<b>81</b>	99	100
80	100	91	<b>89</b>	100	100
90	100	100	<b>99</b>	100	100

Table 2: Total number of failures, for different values of  $\tau$  for 10 to 90 aircraft with no speed norm variation and 3 degrees/second maximum turning rate. CSORCA

compromise.

100 runs are first executed for different values of  $n$  and a maximum standard turning rate of 3 degrees per second. The speed norms are strictly constrained.

325 For each execution, the origins and destinations of the aircraft are randomly chosen on the edges of the square. The aircraft are separated by at least 3 separation standards (15 NM) at the origin and the destination. A quarter of the traffic goes from the east side to the west side, another quarter from west to east, a quarter from north to south and the last one from south to north.

330 For each scenario, the number of runs which fail because aircraft are “pushed” out of the  $500 \times 500$  nautical mile sector before reaching their destination is counted. The number of runs which do not solve every conflict is also counted. For the remaining runs, the trajectory lengthening (the ratio of the modified trajectory and the original trajectory) is measured.

335 Table 1 gives the experimental results for densities varying from 10 to 90 aircraft with the CSORCA (left) and ORCA (right) algorithm, no speed norm change and a maximum 3 degrees per second turning rate.

The CSORCA algorithm helps solve more conflicts in denser airspace. For  $n = 40$  the number of failures is still close to zero with the CSORCA algorithm  
340 whereas it is already a 100% for 20 aircraft with ORCA algorithm. With

Nb (number) of aircraft	Total nb of failures	Nb of border violations	Nb of remaining conflicts	Mean delay	Maximum delay
10	15	1	14	1.001428	1.03708
20	81	0	81	1.002502	1.003631
30	100	0	100		
40	100	0	100		
50	100	0	100		
60	100	0	100		
70	100	0	100		
80	100	0	100		
90	100	0	100		

Table 3: Add-Up Strategy: Total number of failures, due to border violations or remaining conflicts, mean and max delays for 10 to 90 aircraft with no speed norm variation and 3 degrees/second maximum turning rate.

CSORCA, the number of failures grows with the density and reaches 100% only for high densities involving 90 aircraft.

The Add-Up strategy (see figure 3) is better than the initial ORCA approach but it is unable to solve 30 aircraft problems. The main cause off failure is due  
 345 to a separation violation for multiple aircraft conflicts.

When doubling the maximum turning rate by allowing a maximum 6 degrees  
 per second turning rate, table 4 shows that the number of failures is not reduced  
 except for very low densities for which it drops from 96 to 93. However, the  
 CSORCA algorithm gives much better results, with almost no failures before  
 350 reaching 50 aircraft and a lower rate of failures for higher densities.

When comparing the results in terms of border violation, tables 1 and 4 show  
 that for the ORCA algorithm and low densities, the main reason for failure is  
 due to border violation. This is a consequence of the phenomenon observed in  
 figure 6 where aircraft tend to “push” each other and finally reach the limits of  
 355 the resolution area. When the density increases the number of failures due to  
 separation standard violation increases as well: before reaching the border of  
 the resolution area, aircraft may encounter other trajectories that can lead to a

Nb (number) of aircraft	Total nb of failures	Nb of border violations	Nb of remaining conflicts	Mean delay	Maximum delay	Nb (number) of aircraft	Total nb of failures	Nb of border violations	Nb of remaining conflicts	Mean delay	Maximum delay
10	0	0	0	1.00228	1.01115	10	93	93	0	1.07941	1.17754
20	1	0	1	1.00537	1.01479	20	100	99	1		
30	4	0	4	1.01299	1.03417	30	100	89	11		
40	2	0	2	1.02099	1.03659	40	100	49	51		
50	14	3	11	1.03308	1.05832	50	100	21	79		
60	23	4	19	1.04661	1.07188	60	100	15	85		
70	36	7	29	1.06012	1.08392	70	100	3	97		
80	53	21	32	1.07471	1.11324	80	100	1	99		
90	74	21	53	1.09584	1.12779	90	100	0	100		

Table 4: Total number of failures, due to border violations or remaining conflicts, mean and max delays for 10 to 90 aircraft with no speed norm variation and 6 degrees/second maximum turning rate. Left: CSORCA - Right: ORCA

separation standard violation. For the CSORCA algorithm, both the number of border violations and remaining conflicts increase with the density even though the number of failures is lower when the maximum tuning rate is doubled. The mean delay due to maneuvers increases with the density, which seems logical and when the maximum turning rate is increased, which was also expected.

#### 4.2. ORCA and CSORCA comparison on various densities and small speed norm variations.

In this section the previous tests are reproduced and allow a small speed norm variation to check the behavior of the modified algorithm when the speed norm constraint is relaxed. A 5% speed norm modification around the nominal speed is allowed (see figure 5). With the CSORCA algorithm the number of failures is decreased, even for high densities. With the ORCA algorithm, the number of failures is only significantly decreased for very low densities (10 aircraft). This shows that the modification made on ORCA is compatible with a relaxed speed norm constraint environment.

When the speed norm can be modified by  $\pm 15\%$ , the CSORCA algorithm

Nb (number) of aircraft	Total nb of failures	Nb of border violations	Nb of remaining conflicts	Mean delay	Maximum delay	Nb (number) of aircraft	Total nb of failures	Nb of border violations	Nb of remaining conflicts	Mean delay	Maximum delay
10	0	0	0	1.00061	1.00331	10	72	72	0	1.00666	1.02857
20	0	0	0	1.00168	1.00730	20	99	99	0	1.07514	1.07514
30	0	0	0	1.00382	1.00969	30	97	97	0	1.03329	1.04360
40	1	0	1	1.00664	1.01412	40	100	99	1		
50	4	2	2	1.01179	1.02745	50	100	85	15		
60	14	2	12	1.01634	1.03171	60	100	61	39		
70	12	4	8	1.02170	1.03884	70	100	38	62		
80	20	5	15	1.03050	1.05314	80	100	9	91		
90	47	22	25	1.03485	1.04844	90	100	4	96		

Table 5: Total number of failures, due to border violations or remaining conflicts, mean and max delays for 10 to 90 aircraft with  $\pm 5\%$  maximum speed norm variation and 3 degrees/second maximum turning rate. Left: CSORCA - Right: ORCA

never fails for densities up to 60 aircraft (see figure 6) and even with high  
 375 densities (90 aircraft), the rate of failure is only 10%. With the ORCA algorithm,  
 there is still more than 20% of failure for only 10 aircraft and no solution is  
 found for high densities.

## 5. Conclusion and Future Work

The ORCA algorithm proposed by Vandenberg et al. van den Berg et al.  
 380 (2011) is very efficient for solving conflicts between aircraft that can easily modify  
 their speed ranges. It is a very effective geometric self-separation algorithm that  
 can safely handle complex situations. When several aircraft are involved in a  
 conflict, the projection of the relative speed outside the conflicting area ensures  
 conflict free trajectories for every robot. The more maneuverable a robot is,  
 385 the more options it has to escape the conflicting zone. In the worst cases it  
 can be completely stopped by the ORCA algorithm at its current position to  
 wait for the rest of the traffic to clear the area. This algorithm could be a good  
 candidate for aircraft separation because it ensures a solution for multiple robot  
 conflicts provided that the aircraft are sufficiently maneuverable. The issues

Nb (number) of aircraft	Total nb of failures	Nb of border violations	Nb of remaining conflicts	Mean delay	Maximum delay
10	0	0	0	1.00019	1.00072
20	0	0	0	1.00070	1.00223
30	0	0	0	1.00162	1.00435
40	0	0	0	1.00294	1.00842
50	0	0	0	1.00525	1.01143
60	0	0	0	1.00721	1.01284
70	1	1	0	1.01070	1.02259
80	5	3	2	1.01472	1.02449
90	10	5	5	1.01791	1.03095

Nb (number) of aircraft	Total nb of failures	Nb of border violations	Nb of remaining conflicts	Mean delay	Maximum delay
10	21	21	0	1.00166	1.02927
20	50	50	0	1.00657	1.04444
30	79	79	0	1.01374	1.03287
40	92	92	0	1.02868	1.04121
50	97	97	0	1.02558	1.03069
60	100	100	0		
70	100	100	0		
80	100	89	11		
90	100	67	33		

Table 6: Total number of failures, due to border violations or remaining conflicts, mean and max delays for 10 to 90 aircraft with no  $\pm 15\%$  maximum speed norm variation and 3 degrees/second maximum turning rate. Left: CSORCA - Right: ORCA

390 of the ORCA algorithm in the air traffic control context were first exhibited  
by Durand and Barnier in Durand and Barnier (2015). We showed that the  
speed constraint prevented the algorithm from solving conflicts when aircraft had  
similar constrained speeds. When the speed is constrained, the algorithm loses its  
performance even for low densities. When aircraft with similar speeds converge  
395 with small angles they tend to choose parallel tracks that keep them away from  
their destination. The algorithm enhancement proposed in this article aims at  
solving this issue. By projecting the relative speed outside the conflicting cone as  
soon as the vector enters the forbidden area, aircraft tend to make coordinated  
maneuvers that solve conflicts instead of delaying their resolutions. An example  
400 of two aircraft was introduced to illustrate the behavior of the original ORCA  
algorithm and the CSORCA enhancement. A 10 aircraft example was then  
introduced: the 10 aircraft converged with a 6 degree angle to the center of  
a circle at the same speed. The CSORCA algorithm manages to solve every  
conflict whereas the ORCA approach only delays the resolution by giving parallel  
405 tracks to every aircraft. An Add-Up strategy was compared to the CSORCA  
algorithm but it was only able to deal with 8 aircraft.

ORCA and CSORCA approaches were then compared on random traffic scenarios with various densities and different turning rates. With no speed norm variation and a standard turning rate of 3 degrees per second, ORCA can rarely  
410 handle 10 aircraft whereas CSORCA can still manage 47% of the scenarios involving 70 aircraft. The Add-Up strategy gave better results than the ORCA approach but only for low densities (less than 30 aircraft).

With a 6 degree per second turning rate, ORCA does not behave any better whereas CSORCA can handle 53% of the scenarios involving 90 aircraft. Relaxing  
415 the speed norm by  $\pm 5\%$  slightly improves results with ORCA: 10 aircraft examples are solved in 28% of the scenarios with a 3 degree turning rate. With CSORCA, 80% of the scenarios involving 80 aircraft are solved. Relaxing the speed norm by  $\pm 15\%$  improves results with ORCA even more: 50% of the scenarios involving 20 aircraft are solved with a 3 degree turning rate. With  
420 CSORCA, 90% of the scenarios involving 90 aircraft are solved. These statistical results illustrate the efficiency of the modification made on the initial ORCA algorithm.

The next step will be to add vertical maneuvers to the model and check the behavior of both ORCA and CSORCA when a vertical option is added to  
425 maneuvers. The vertical option will not be combined with a horizontal maneuver in order to comply with current practices in air traffic control. The impact of errors on the information captured by the aircraft should also be carefully studied. Allignol et al. Allignol et al. (2017) have conducted many experiments for a single UAS interfering with real traffic. Results show that performances are  
430 reduced. It would be interesting to adapt the ORCA and CSORCA approaches to take errors into account.

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