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Abstract

This paper introduces a validation step for inferring atmosphere from RF data in an effort towards an 'refractivity from clutter' (RFC) system. The method retrieves the vertical refractive index distribution point-by-point in the lower troposphere from the measurements sampled in bistatic configuration. The new method utilizes wide-angle parabolic equation (WAPE) solved with the method of split-step wavelet (SSW) as the forward model, which is less computationally costly than the typical method used in the literature. The gradient of the cost function is estimated via the adjoint SSW solver developed by means of the variational adjoint approach. The high-dimensional problem is taken into account via gradient based optimization method. Thus the validation of the gradient is crucial. The accuracy of the new method is validated by comparing the gradient to the method of finite differences.

1. Introduction

Anomalous propagation conditions which are controlled by refractive index variations are responsible for unexpected performances of radar (e.g., radar holes) operating in maritime environment. Performance estimation in these conditions requires prediction of ambient refractive index. 'Refractivity from clutter' (RFC) (e.g., [1], [2]) is a practical method of sensing the environment for the retrieval of the refractive index. The inversion typically suffers from nonlinearity and ill-posedness.

A method of inversion is presented in this paper using tomographic approach, i.e., sensing of the propagated field in bistatic configuration as given in Figure 1. In the context of this study, the current literature [3] considers the narrow angle approximation of parabolic equation (NAPE) with the variational adjoint approach, solved with the method of split-step Fourier (SSF).

This formulation is extended to the use of wide-angle approximation of PE (WAPE), which is more accurate than NAPE and solved with the method of split-step wavelet (SSW) [4], which is less computationally costly than SSF. In this paper, the validation of our method is presented.

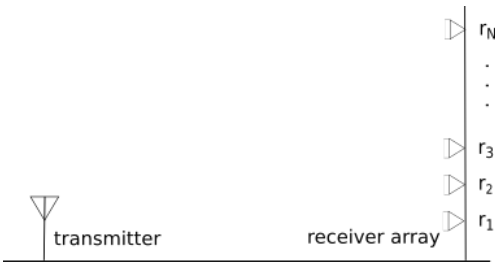


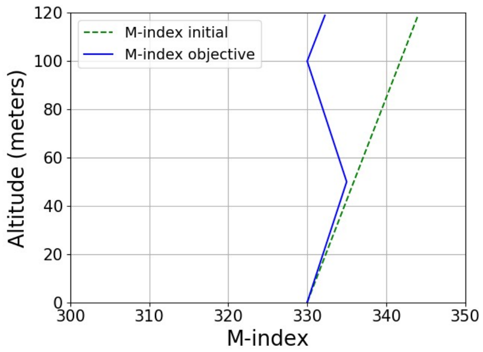
Figure 1. Schematic of bistatic configuration.

2. Simulation Results

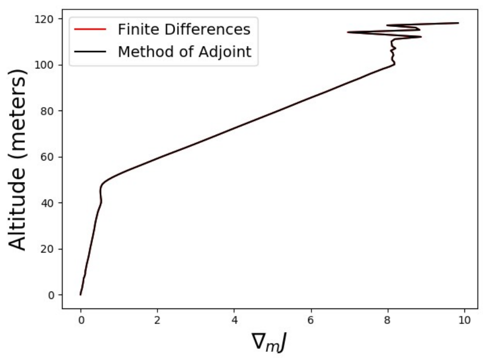
The bistatic configuration composed of a transmitter and an array of receivers in schematized in Figure 1. The initial field is propagated in a medium, whose properties are initialized with standard atmosphere, using SSW until the receiver arrays. The field is sampled there so as to estimate the misfit between the measurement and the samples of SSW corresponding to the initial refractivity distribution. Then the gradient of the misfit with respect to the refractive index can be computed thanks to the variational adjoint formulation [5]. Employing the gradient in an iterative descent algoritm minimizes the misfit and retrieves the ambient refractive index. Thus the validation of the gradient is crucial.

A validation setup is designed using the measurement which is synthetically generated via SSW. The refractivity parameter of the measurement is given in Figure 2-a and labeled as ‘M-index objective’. The initial guess is given by the dashed line in Figure 2-a. The dimensions of the computational domain is [0, R]x[0, Z] = [0, 100 m]x[0, 120 m] discretized by a uniform mesh with cell size of dr=dz=1 m. The source is a complex point source positioned at (r,z)=(-100 km, 25 m) emitting radio wave at 2 GHz at horizontal polarization.

The accuracy of the new method is validated by comparing the gradient with the method of finite differences [5]. The gradients are displayed in Figure 2-b. The curves given by the two different methods perfectly match. The method of adjoint properly estimates the gradient.



(a)



(b)

Figure 2. On the validation of the gradient of the cost function (derived using the adjoint method) using a finite difference approximation. (a) initial guess and objective refractivity profiles, (b) the gradient calculated at r=100 m.

4. Conclusion

A new method of inversion of radar measurements for atmospheric inferring based on wide-angle parabolic equation and variational adjoint approach is presented. The validation by finite differences shows that the gradient is well estimated for bistatic configuration and for propagation in short ranges.

5. Acknowledgements

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6. References

[1] Douvenot R. and Fabbro V. On the knowledge of radar coverage at sea using real time refractivity from clutter. *IET Radar Sonar Navig.*, vol. 4(2), pp.293-301, 2010.

[2] Karimian A., Yardim C., Gerstoft P., Hodgkiss W.S. and Barrios A.E. Refractivity estimation from sea clutter: An invited review. *Radio Sci.*, vol. 46(06), pp.1-16, 2011.

[3] Zhao X.F., Huang S.X. and Du H.D. Theoretical analysis and numerical experiments of variational adjoint approach for refractivity estimation, *Radio Sci.*, vol. 46(01), pp.1-12, 2011.

[4] Zhou H., Douvenot R., and Chabory A., Modeling the long-range wave propagation by a split-step wavelet method, under review, 2019.

[5] Kern M., *Numerical Methods for Inverse Problems*, John Wiley & Sons, 2016