Air Traffic Flow Management under Uncertainty in Terminal Maneuvering Area

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***Abstract*—In air traffic control system, the Terminal Maneuvering Area (TMA) is one of the most complex areas in which flight operations are easily influenced by inevitable uncertainties such as inaccurate aircraft performance, navigation accuracy, pilot operations. This research addresses the scheduling problem in the TMA under uncertainty and aims to improve the safety and efficiency of air traffic at a tactical level. The uncertainty is managed by introducing probabilities to the temporal information at specific points for each flight. Flight by flight conflict is then measured with probability on each designated point taking all the possible arrival times into consideration. By minimizing the total probability of conflict in the network, appropriate safety margins can be imposed. A meta-heuristic simulated annealing optimization algorithm is proposed, and the solution is obtained based on the real flight data of 2 hours in the Paris Charle-De-Gaulle (CDG) airport. A simulation is conducted to verify the performance of the proposed model while considering the deterministic model as a baseline case. Both the candidate solutions are disturbed in terms of the TMA entry time and the arrival times on specific points of each flight are conditionally deviated from the predicted ones. Final results show the advantage of the proposed model in absorbing conflicts while experiencing uncertainty.**

***Keywords*—Terminal Maneuvering Area, Uncertainty, Probabilistic model, Simulated Annealing**

I. Introduction

According to the ICAO long term traffic forecasts, the global passenger traffic will grow at 4.6 percent annually until 2032. With sustainable growth, air traffic efficiency and safety become a prior issue to be considered, thus a better-performing decision support system adapted to the reality gains much more interest.

In the domain of air traffic management, one of the areas with high complexity is the Terminal Maneuvering Area (TMA). As the connection between the airport and the enroute segment, a high volume of air traffic will be converged in the TMA, which leads to extra complexity. Recently, the arrival management support system is broadly promoted for air traffic management. The systems depend on predictions to provide decisions. Thus prediction accuracy plays an important role in ensuring air traffic safety. However, the time variation during the operation due to uncertainties could lead to potential conflicts between aircraft. This will reduce the credibility of the guidance from the decision support tools. Consequently, air traffic controllers still need to schedule the flights according to their experience and intuition, since these systems provide information without considering the inevitable operation perturbations. Due to this fact, we believe that further systems are required to take the prediction errors into account and the scheduling of aircraft in TMA needs to be considered with multiple elements such as uncertainty, safety, and efficiency at the same time.

With a well awareness of the effects of uncertainty, growing attention towards the researches that cope with prediction error is paid. In this field, Murça *et al.* [1] presented a time prediction model to evaluate the prediction error, which is applied in the deterministic optimization problem as a measurement of robustness parameter. Xue *et al.* [2–4] conducted a series of studies to detect the performance of the scheduling method by applying uncertainty in different cases. Moreover, studies on air traffic flow management incorporating the probabilistic theory with temporal information on the waypoints of different sectors also provide a viewpoint. Related studies [5–8] are mainly based on the enroute segment in which the congested sector is detected based on a probabilistic count. Fuzzy logic and probabilistic approach are described and compared in [9], which provide methods for the circumstances where no rich information is available. Ng, K. K. H. *et al.* [10] used the min-max regret approach to study the uncertainty in the runway segment by considering the optimal solution for the worst cases.

Regarding the complexity of the TMA, safety has been considered as the most striking issue in our study, which makes the priority of this work to be the elimination of flight by flight conflicts. In this paper, the time variations propagated during the flight status change or inaccurate aircraft performance are focused. To take this inevitable uncertainty into consideration, predicted arrival times on specific points are considered as random variables with associated probability distributions. This model combines the probability theory and the optimization process to derive a more robust solution for the arrival segment which can finally hedge against potential risks caused by the nearly violated separation between flights encountering various of uncertainties. To examine the performance of the proposed model, the optimized scheduling solutions of the proposed model and deterministic model are both obtained. Then we introduce the possible time variations to the timestamps for each flight, and statistic analysis is then presented.

This paper is organized as follows: First, the background and related research are stated. Section II describes the opti-

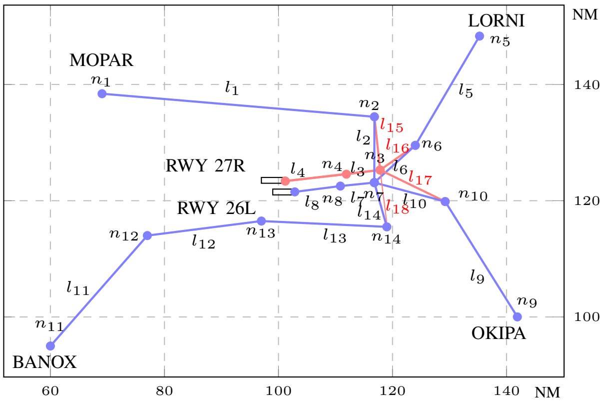

Fig. 1. Arrival route structure.

mization problem and the established model, the constraints and objective are included as well. Section III introduces the optimization approach that we use in our study. In Section IV, a case study is conducted and a simulation is proposed to verify the performance of the proposed model where the result from the deterministic model is considered as a baseline case. Section V displays the results and Section VI concludes the whole paper.

II. Mathematical formulation

A. Notations

In this paper, we focuse on the arrival flights in the TMA and runways. A network abstraction is introduced based on the arrival route structure. Taking the Paris-Charle de Gaulle (CDG) as an example, Fig. 1 displays its TMA route structure, the routes are abstracted as a graph($\mathcal{N}, \mathcal{L}$) that comprises:

- $\mathcal{L} = \{l_1, ..., l_{18}\}$: link set in the network. A link is a partial of arrival trajectory without direction change.
- $\mathcal{N} = \{n_1, ..., n_{14}, r_1, r_2\}$: node set which is composed of the TMA entry points, the connection points of each two links and runway thresholds.

Each TMA entry point corresponds to two routes specified by different runway assignments. Each route is defined by a succession of nodes and links. For a random flight f, $\mathrm{u}_f = \mathrm{u}_f^n \cup \mathrm{u}_f^l \cup \mathrm{u}_f^r$ represents its following route that consists of node set, link set and the landing runway.

Necessary flight information is also given. Assume that we have a set of arrival flights $\mathcal{F} = \{1, ..., F\}$, the original obtained information of each flight ($f \in \mathcal{F}$) is:

- C_f: Wake turbulence category.
- E_f: Entry node of the TMA.
- T_f^o: Initial RTA (Required Time of Arrival) of entering the TMA through corresponding entry node.
- V_f^o: Initial speed of entering the TMA through entry node.
- R_f^o: Initial assigned landing runway.

Regarding the flight operation in TMA, we assume that the speed of each flight decreases at a constant deceleration until the Final Approach Fix (FAF) and then remains constant till the threshold of the corresponding runway. Final speeds for flights are set as 110kt 130kt and 150kt for small, medium and large aircraft respectively.

B. Decision variables

The probabilistic model that we propose contains three types of decision variables: TMA entry time, TMA entry speed, and landing runway.

1) Entry time: In the TMA, flights can adjust the arrival time to TMA by changing the en-route speed or using an alternative route. A discrete time interval denoted as ΔT is used as one unit to measure the total entry time range. The maximum tardiness and earliness are represented as $\Delta T_{\max}$ and $\Delta T_{\min}$ which are composed of multiple of ΔT. Therefore, for a random flight f, a time slot decision variable $t_f \in \mathcal{T}_f$ has a flexible range of:
$\mathcal{T}_f = \{T_f^o + j\Delta T | \Delta T_{\min}/\Delta T \leqslant j \leqslant \Delta T_{\max}/\Delta T, j \in \mathbb{Z}\}$,
where j denotes the interest of time slot deviation from the initial arrival time. Due to the reality, the entry time range is set as $\Delta T_{\max} = 30$mins and $\Delta T_{\min} = -5$mins.

2) Entry speed: Entry speed is managed similarly as the TMA entry time. For an arrival flight f, an entering speed decision variable $v_f \in \mathcal{V}_f$ has a constraint of:
$\mathcal{V}_f = \{V_f^{\min} + j\Delta_f^v | j \in \mathbb{Z}, |j| \leqslant (V_f^{\max} - V_f^{\min})/\Delta_f^v\}$.
In this study, we set $V_f^{\min} = 0.9V_f^o$, $V_f^{\max} = 1.1V_f^o$ and $\Delta_f^v = 0.01V_f^o$.

3) Runway: Landing runway assignment decision is represented by $r_f \in \{r_1, r_2\}$, two landing runways are assigned for arrival flights to improve the efficiency during the peak hour of operation.

To summarize, the decision variable vector associated with the flight set $\mathcal{F}$ is denoted by $\mathbf{x}$, and we have:

$$\mathbf{x} = (\mathbf{t}, \mathbf{v}, \mathbf{r})$$

where $\mathbf{t}$, $\mathbf{v}$ and $\mathbf{r}$ denote the TMA entry time set, entry speed set and landing runway set, respectively.

C. Uncertainty management

In real operations, aircraft may not be able to follow their assigned trajectories with high precision due to the inevitable uncertainties, while with the known information such as departure time, operation speed, etc., the time for arriving at a route point is able to be predicted deterministically. However, prediction error that has a close connection with the predicted time can not be ignored. In our study, uncertainty is modeled by attaching the prediction error to the predicted time, that is to say, the arrival time for a certain point is considered as a random variable which follows normal distribution with predicted time as the mean value. Besides, the further the predicted time is with regard to the current time, the higher is the prediction error. Therefore, the range of prediction error around the predicted time is calculated by multiplying a parameter δ and the difference between predicted arrival time and current time T_C.

Suppose a flight f overflies a specific point i, the predicted arrival time $t_f^{p,i}$ is obtained through deterministic computation. The random variable that taking $t_f^{p,i}$ and the prediction error

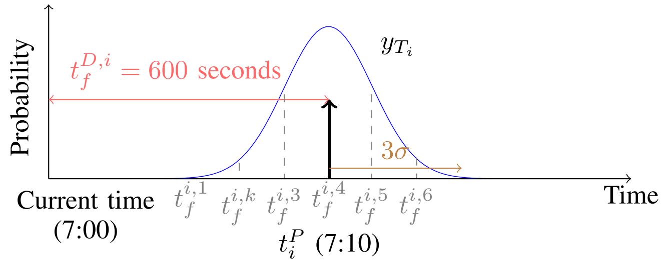

Fig. 2. Uncertainty management scheme with gaussian distribution.

into account is denoted as T_f^i which follows a probability distribution of $\mathcal{N}(\mu, \sigma^2)$. The mean value $\mu = t_f^{p,i}$ and the standard deviation $\sigma = \delta \cdot t_f^{D,i}$, where $t_f^{D,i} = t_f^{p,i} - T_C$. One characteristic of gaussian distribution is about 99.7% of values are within three standard deviations, thus the support range of the random variable is restricted with $[\mu - 3\sigma, \mu + 3\sigma]$.

Knowing that δ should be determined regarding the time prediction error of airborne flights in the TMA or about to enter the TMA. A related research states that under normal instructions, the typical standard deviation for flight is 30 seconds at 20 minutes before arrival with a total transition time of approximately two hours [11]. Considering that δ is not only regarded as a parameter fitted for the complete possible time range but also provides a reference for safety margins between aircraft in the later optimization process, hence an increase of 20 seconds for standard deviation corresponds with the look-ahead-time of each 20 minutes is decided, which yields $\delta = 0.0167$.

In our study, the random variables are discretized by a time interval of $\Delta = 10$ seconds starting from its lower bound $\mu - 3\sigma$. The probability distribution of a random variable will have $K = \lceil 2 \cdot 3\sigma/\Delta \rceil$ of time intervals and the lowest time of each interval constitutes the set of potential values of the associated random variable. The probability of the k-th interval $[t_f^{i,k}, t_f^{i,k} + \Delta)$ is assigned to the component value $t_f^{i,k}$ of T_i^k can be given:

$$P(T_i = t_f^{i,k}) = \int_{t_f^{i,k}}^{t_f^{i,k}+\Delta} y_{T_i}(x)dx \tag{1}$$

where y_{T_i} is the probability density function of the random variable T_i.

Fig. 2 illustrates an example of managing the predicted time error for an aircraft f that arrives at a certain point i. We assume the current time is 7:00 and the predicted time for aircraft f to arrive at the designated point is 7:10. The possible value set of the discrete random variable T_f^i is denoted as $\{t_f^{i,1} = 25770, t_f^{i,2} = 25780, t_f^{i,3} = 25790, t_f^{i,4} = 25800, t_f^{i,5} = 25810, t_f^{i,6} = 25820\}$ with associated probabilities computed from (1).

D. Conflict evaluation model

Based on the previous study [12], in which the conflicts are evaluated on specific points in a deterministic way to verify whether there are separation violations between consecutively operated aircraft, current research incorporates the uncertainty into the model and the conflicts are measured with probability instead of quantity.

The conflict detection is conducted independently using the arrival time of detection points such as link entry points, link exit points, node entry points and node exit points for each pair of chosen aircraft. With the implementation of the uncertainty management scheme, the arrival times of detection points are regarded as random variables that follow the gaussian distribution. As each random variable has a set of possible values and associated probabilities, in the case of measuring the probability of conflict for a pair of randomly chosen aircraft at a specific point, the combinations of possible arrival times in the set of random variables are enumerated. If the combination of two values violate the separation requirement, the probability are summed up as the probability of conflict for the two aircraft at this detection point.

The conflict detection is dedicated to three aspects. The longitudinal separation requirements are applied to links, the lateral separation is ensured at nodes, finally, flight sequence is recorded and measured at the link entry and exit points. Separation requirements at runway thresholds are equally applied. The details are stated as follows.

1) Link conflict detection: For each link $l = (u, v)$, such that u and v represent the link entry and exit point. To make sure a link is conflict free, the separation between each randomly chosen pair of aircraft f and g that consecutively fly through a link should guarantee the minimum wake turbulence separation $D_{f,g}$ associated with the aircraft categories that are introduced in Tab.I. However, the provided separation requirements are given in distance, while the separation information of the aircraft is given in time. In order to have consistency in the unit for the measurement of conflict, we transfer the distance separation to time separation by dividing the $D_{f,g}$ by the speed. The calculation can be written as $\tau_{f,g} = D_{f,g}/\min(v_f^i, v_g^i)$, in which v_f^i and v_g^i are the current speeds of aircraft f and aircraft g when they pass the detection point i. The calculation of $\tau_{f,g}$ can assure that we have a more restricted time separation requirement, since $\tau_{f,g}$ is inversely proportional to the lower speed of the two aircraft.

As shown in Fig. 3, we suppose that aircraft f is the predecessor and aircraft g is the successor. The arrival time of each aircraft at link entry point and exit point of link l are considered as random variables $T_f^{l_\mu}$ and $T_g^{l_\mu}$ and $T_f^{l_\nu}$ and

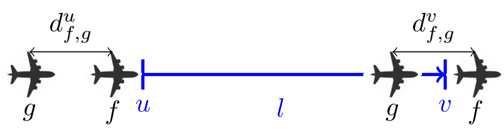

Fig. 3. Link conflict detection.

$T_g^{l_\nu}$. It is worth mentioning that in reality, most of the link exit points are the link entry points of the consecutive link. However, l_1, l_5, l_9, l_{13} have two consecutive links. For these links, two consecutively operated flights on the previous link might diverge to different links, so the conflict detection at link exit points is necessary, thus the link exit points are included as detection points.

The probability of conflict of flight f and g on link l is given as:

$$P_{\mathcal{L}_e} = \sum_{T_f^{l_\mu}=t_f^{l_\mu,1}}^{t_f^{l_\mu,K}} \sum_{T_g^{l_\mu}=t_g^{l_\mu,1}}^{t_g^{l_\mu,K}} P(T_g^{l_\mu} - T_f^{l_\mu} < \tau_{f,g}) + \sum_{T_f^{l_\nu}=t_f^{l_\nu,1}}^{t_f^{l_\nu,K}} \sum_{T_g^{l_\nu}=t_g^{l_\nu,1}}^{t_g^{l_\nu,K}} P(T_g^{l_\nu} - T_f^{l_\nu} < \tau_{f,g}) \tag{2}$$

Note that the arrival flights have different entry speeds and deceleration rates, thus the distance between each pair of flights is usually changed. It is possible that the minimum distance between two aircraft locates in the middle of the link and violates the separation requirements. Based on this situation, the minimum distance investigation in the TMA for aircraft operated on the same route is conducted.

We suppose that two aircraft f and g achieve their minimum distance $d_{f,g}^M$ at time $t_{f,g}^M$ based on a deterministic computation. The most important concern is how to manage the possible variation of the proceeded distance in the TMA for each flight so as to estimate the probability of conflict around the minimum distance. Since we don't have a direct approach, we can refer to the uncertainty management scheme by using $t_{f,g}^M$ as the reference time to deduce a random variable denoted as $T_{f,g}^M$ which yields the probability distribution of $\mathcal{N}(t_{f,g}^M, (\delta(t_{f,g}^M - T_C))^2)$.

In order to find the minimum distance between f and g taking into account the uncertainty, the possible proceeded distances of flight f and g are calculated by enumerating the possible values of $T_{f,g}^M$. As the probabilities are assigned to the set of values of $T_{f,g}^M$, each proceeded distance that pertaining to a specific value will be endowed with a probability. Let's define $\iota_f = d_f(T_{f,g}^M, \mathbf{x})$, where ι_f is the proceeded distance of f in the TMA which depends on the travel time that starts from the the TMA entry time and ends at the $T_{f,g}^M$ and other decision variables. Similarly, the proceeded distance in TMA of flight g can be defined as: $\iota^g = d^g(T_{f,g}^M, \mathbf{x})$. Consequently, the conflict probability of two flights can be calculated as follows:

$$P_{\mathcal{L}_{min}} = \sum_{T_g^M=t_f^{M,1}}^{t_f^{M,K}} \sum_{T_g^M=t_g^{M,1}}^{t_g^{M,K}} P(|d^f(T_f^M, \mathbf{x}) - d^g(T_g^M, \mathbf{x})| < D_{f,g}) \tag{3}$$

2) Flight position shift detection: The position shift is detected by comparing the sequence order of flights that get in and get out of a link. If there exists a position shift for a flight on a link, we consider this as a conflict. Regarding the probability distribution of arrival time on a detection point, the sequence is very sophisticated to determine for the case that two flights arrive at a point with an overlap of possible arrival times. However, the constraint for ensuring no longitude conflict on link dedicates to separating every two aircraft with required separation, and the overlap of arrival times for each two aircraft at a certain point is forced to be eliminated. Therefore, sequence orders of entering and exiting each link are determined based on the predicted arrival times of all the flights. The total number of position shifts of all the flights will be counted as sequence changing conflicts which is denoted by $C_{\mathcal{L}}$.

TABLE I
MINIMUM WAKE TURBULENCE SEPARATION REQUIREMENTS, IN NM.

Categories		Leading Aircraft, f		
		Heavy	Medium	Light
Trailing Aircraft, g	Heavy	4	3	3
	Medium	5	3	3
	Light	6	5	3

3) Node evaluation: On links, flight wake turbulence separation is guaranteed. While the intersection of two links might construct an angle and the horizontal separation requirement of flights have a chance to be violated. The nodes are defined as a small scope of airspace with a radius of 2.2NM to ensure the minimum separation between two consecutive flights [13]. A conflict is detected when two aircraft stay in one node simultaneously. Suppose that we have two aircraft: f is the predecessor and g is the successor, passing through node n. The node entry time of g and node exiting time of f can be represented as random variables $T_g^{n,In}$ and $T_f^{n,Out}$ respectively. The random variables follow the gaussian distribution and the probability of conflict of aircraft f and g on node n is computed by:

$$P_{\mathcal{N}} = \sum_{T_f^{n,Out}=t_f^{n_{out},1}}^{t_f^{n_{out},K}} \sum_{T_g^{n,In}=t_g^{n_{In},1}}^{t_g^{n_{In},K}} P(T_f^{n,Out} > T_g^{n,In}) \tag{4}$$

4) Runway evaluation: Suppose that we have aircraft f and aircraft g landing on the same runway r consecutively,

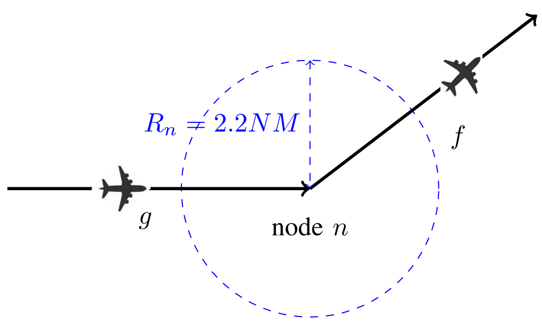

Fig. 4. Node configuration.

TABLE II
RUNWAY SEPARATION REQUIREMENTS, IN SECONDS.

Categories		Leading Aircraft f		
		Heavy	Medium	Light
Trailing Aircraft, g	Heavy	96	60	60
	Medium	157	69	69
	Light	207	123	82

the required time separations on the runway between f and g is denoted as $\tau^r_{f,g}$, which is given in Tab. II according to the aircraft categories. Taking uncertainty into consideration, the time for aircraft f and g to enter the landing runway is considered as random variables denoted by T^r_f and T^r_g. The probability of conflict of f and g on the landing runway is formulated as:

$$P_{\mathcal{R}} = \sum_{T^r_f = t^{r,1}_f}^{t^{r,K}_f} \sum_{\substack{T^r_g = t^{r,1}_g, \\ r_f = r_g}}^{t^{r,K}_g} P(T^r_g - T^r_f > \tau^r_{f,g}) \tag{5}$$

E. Objective function

The objective of this model is to minimize the total probability of conflicts. Therefore, the corresponding objective function to be minimized is the following equation:

$$G(\mathbf{x}) = \sum_{\substack{f,g \in \mathcal{F} \\ f \neq g}} [\sum_{\mathrm{u}_f = \mathrm{u}_g} P_{\mathcal{L}_m} + \sum_{l \in \mathrm{u}^l_f \cap \mathrm{u}^l_g} P_{\mathcal{L}_e} + \sum_{n \in \mathrm{u}_f \cap \mathrm{u}_g} P_{\mathcal{N}} + P_{\mathcal{R}} + C_{\mathcal{L}}] \tag{6}$$

III. SOLUTION APPROACH

Simulated annealing is a meta-heuristic method to approximate global optimization in a large search space for an optimization problem. It is used to find an approximate global optimum other than a precise local one in a fixed amount of time. This is a method involving heating and controlled cooling that mimics the process of metal annealing. The notion of slow cooling implemented in the simulated annealing algorithm is interpreted as a slow decrease in the probability of accepting worse solutions as the solution space is explored. At each time step, the algorithm randomly selects a solution close to the current one, measures its quality, and then decides to move to it or to stay with the current solution depending on whether the new solution is better or worse than the current one. The iteration will be carried out for multiple times. After the iterations on current temperature, the next temperature is decreased by multiplying a temperature reduction coefficient. As the temperature decreases from the initial temperature T_{ini} to 0, the probability for the algorithm to accept a new worse solution decreases drastically. The steps mentioned above are repeated until the objective function reaches 0, or the temperature is lower than 10^{-5}.

Tab. III displays the used parameters. The probability acceptance is defined by the condition listed above, $G(\mathbf{x})_c$ and $G(\mathbf{x})_n$ represent the current value of objective function and the value of the neighborhood solution respectively and T_c denotes the current temperature.

TABLE III
PARAMETERS FOR SIMULATED ANNEALING ALGORITHM.

Parameter	Value
Iteration for each temperature	200
Geometrical temperature reduction coefficient	0.99
Initial temperature T_{ini}	1
Probability acceptance of worse solution	$\exp(\frac{G(\mathbf{x})_c - G(\mathbf{x})_n}{T_c})$

Fig. 5 illustrates the conflicts evolution along with the optimization process in which we can see, as the temperature decreases, the total probability of conflicts reduces intensively at the beginning. While gradually the data become stable and the objective function maintain a value.

As the look ahead time goes further from the current time, the ranges of the random variables increase, which requires a larger separation between each pair of flight to ensure the conflict detection between a pair of flight equals 0. However, with a high density of air traffic, it is impossible to separate all the aircraft accordingly with respect to their large range of possible arrival times, thus it is normal that the probability of conflict is not optimized to 0.

IV. CASE STUDY AND ROBUSTNESS VERIFICATION

A. Application to the Paris CDG Airport

Actual flight data of 2 hours from 07:00-09:00 on 18th February 2016 in the Paris CDG airport, which is the peak hour of flight arrival is applied in our model. In this airport, there are two runway configurations: west-flow (26L, 27R — 26R, 27L) and east-flow (09L, 08R — 09R, 08L). This paper focuses on the more frequently used west-flow configuration. The data have indicated all the time information associated with this research. The categories of aircraft are mostly large and medium. Tab. IV provides the number of flights that operated from different entry nodes and the number

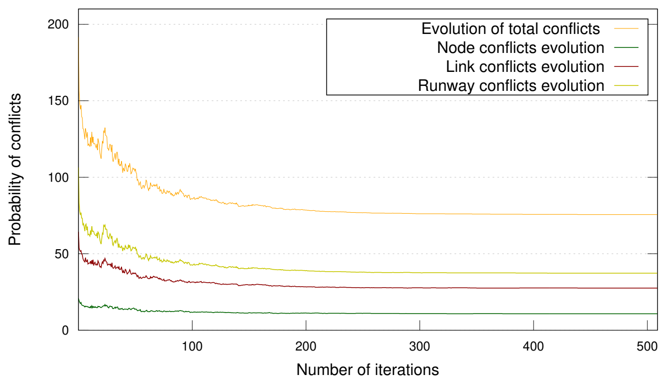

Fig. 5. Conflicts evolution of different resources.

TABLE IV
TRAFFIC FLOW DISTRIBUTION OF THE CASE STUDY.

EntryNode	Number	Medium	Heavy
MOPAR	19	14	5
LORNI	22	22	0
OKIPA	17	17	0
BANOX	11	10	1

of different categories of aircraft are listed. The overall process is run on a 2.50GHz core i7 CPU, under Linux operating system PC with Java code.

B. Robustness verification

In order to verify the performance of the proposed model, a simulation involving the time perturbations during the aircraft operations is conducted under deterministic condition. The arrival times of a flight at specific points are all fixed variables and conflicts are counted by quantity. For each flight, perturbations are applied to the TMA entry time, and the following timestamps for these flights are conditionally derived. The simulation will repeat the experiments for 2000 times so as to diverse the scenarios. Each experiment is evaluated with perturbed timestamps of flights and the generated conflicts are recorded. In the experiment, perturbation is the time deviated from the predicted arrival time.

For a randomly chosen flight f, its overflying detection points are marked sequentially with the number q. $q = 1$ indicates the first evaluation point (TMA entry point), where the arrival time is selected based on probability chance that can be obtained according to the uncertainty management scheme specified in Sec II-C. If $q > 1$, the arrival time for the following detection points always depends on the arrival time of the previous point. The conditional probability of the flight arriving at the $q+1$-th evaluation point given that it arrives at q-th evaluation point at t_q is stated as:

$$T_f^{q+1} | T_f^q = t_q \sim N(t_q + T_f^{l_q}, (\delta(T_f^{l_q})^2)$$

where T_f^q and T_f^{q+1} are a random variables that denotes the arrival time of q-th and $q+1$-th evaluation point for flight f and t_q is an arbitrary variable chosen with respect to the range of T_f^q, $T_f^{l_q}$ is the predicted transition time on the link that connects the q-th and $q+1$-th detection point. Same parameter δ is used to determine the variation range of the perturbation.

The baseline case in our study is the optimized solution drawn from the deterministic model which has no uncertainty involved. The final results are presented as follows.

V. RESULTS

A. Preliminary results

Before the robustness verification, the obtained optimized solutions are analyzed. In our research, both the probabilistic evaluation and the deterministic evaluation are established based on the current network. The main difference is the probabilistic model introduces extra safety margin in a probabilistic based way, thus analyses and comparisons regarding three kinds of resources in separation performances are investigated for the two models.

1) Flights separation on nodes: The node entry time of the succeeding aircraft and the node exit time of the preceding aircraft are used for the conflict detection and the difference between them is defined as the time separation for the two aircraft on node. Intuitively speaking, the larger is the time separation between the two flights, the lower is the risk of encountering conflict under the uncertainty. Therefore, the separation distributions of both models are displayed to verify the effectiveness of the probabilistic model.

In Fig. 6, the upper figure shows the time separation between aircraft on nodes based on the optimized schedule of both models. If unpredictable time perturbation appears for flights, the aircraft which have a small separation with other aircraft always have a higher chance to experience safety issues. Thus in our analysis, the separation range is set from 0 to 400 seconds with a scale of 10 seconds on x-axis. The y-axis shows the number of flight pairs passing through all nodes. The flight pairs represent each two aircraft that consecutively overfly a node. An aircraft can be a predecessor in the flight pair or a successor in another pair. In the figure, the blue columns and yellow columns indicate the separation distribution of the probabilistic model and the deterministic model respectively. Clearly, in the deterministic model, there is a higher proportion of separations located in the range of 10

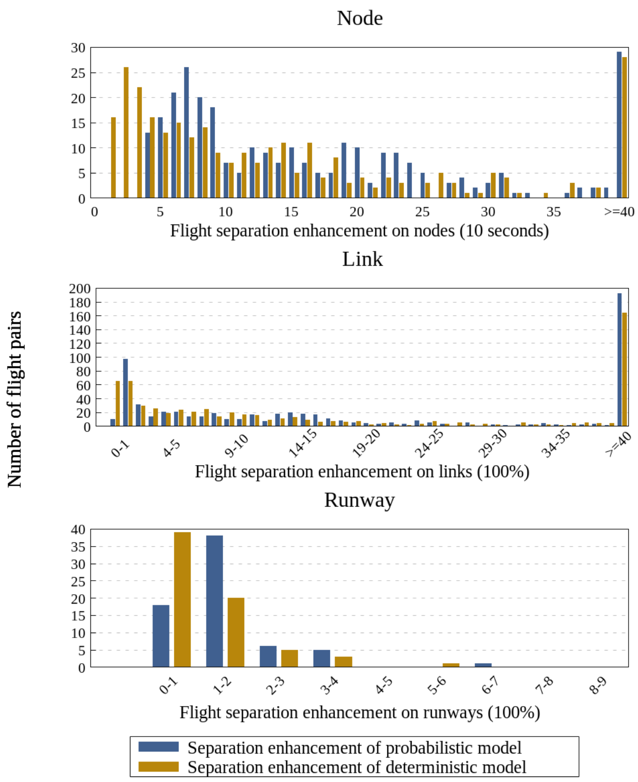

Fig. 6. Resource separation enhancement analysis

seconds to 30 seconds, while the time separation distribution of blue columns locates towards bigger separation times.

2) Flight separation on links: Link conflict detection considers the minimum wake turbulence separation between two flights, thus we introduce a separation measurement criteria of

$$\frac{(S_{\text{real}} - S_{\text{required}})}{S_{\text{required}}} \cdot 100\%$$

for links, where S_{real} denotes the real separation of two consecutively operated aircraft on link detection points, S_{required} is the associated required wake turbulence separation. The middle figure of Fig. 6 depicts the distribution of the link separation criteria with a scale of 100% in the x-axis. Similar to node, the y-axis indicates the flights pairs on all the links detection points. The most important difference between the results of deterministic model and probabilistic model is the number of flight pairs locates in the groups of 0-100% and 100%-200%. The discrepancy states that the probabilistic model tends to separate the flights with a larger distance while the deterministic model just meets the basic separation requirements.

3) Flight separation on landing runways: We take the same measurement criteria as in links for the runway analysis while S_{required} is the runway separation requirement. The bottom figure in Fig. 6 illustrates the flight separation distribution in the runway threshold. A result can be concluded due to the fact that in the two smallest separation groups, the probabilistic model shows a significant improvement in increasing the safety margins with a higher proportion of the number of flight pairs.

B. Results of simulation test

In addition to investigating the separation performance, the results of the simulation provide an intuitive observation of the robustness of the proposed model comparing the baseline case. Fig. 7 shows conflict distributions for both cases in terms of the frequency of appearance. The blue columns represent the simulation result of the probabilistic model and the yellow ones are the simulation result of the deterministic model. The configurations resemble the normal distributions, where the distribution of deterministic case has a bigger mean and span, while the distributions of probabilistic case are thinner and with a rather small mean value.

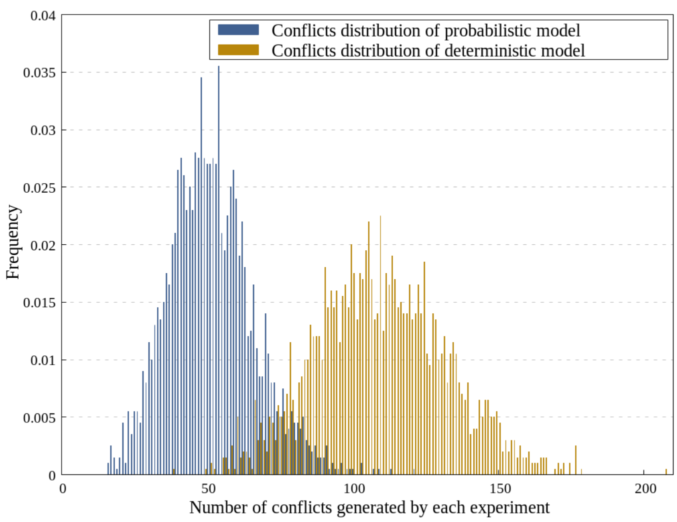

Fig. 7. Total conflict distributions.

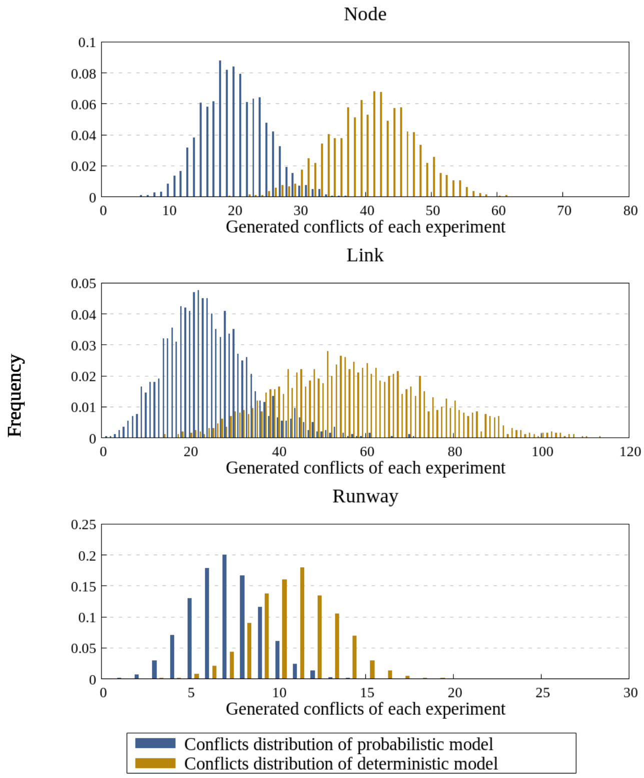

Fig. 8. Conflicts distributions of link, node and runway.

Extra attention is paid to each kind of resource. The conflict distributions generated from the simulation for nodes, links and runways are listed in Fig. 8. The blue columns indicate the results using the optimized solution of the probabilistic model and the yellow columns show the results derived from the deterministic model. Generally speaking, on each resource, the conflict distributions of the probabilistic model always have smaller average values as well as smaller spans, which means that the proposed model has a higher resilience to hedge against time uncertainties in generating fewer conflicts.

VI. Conclusion

This paper presents a methodology that combines probabilistic theory and optimization to tackle the flight scheduling problem in the TMA under uncertainty. As the TMA is one of the most complicated areas, the objective is to ensure safety. Conflict detection is carried out on the detection points including links, nodes and runways based on associated separation requirements.

The uncertainty is managed by incorporating the prediction error into the predicted arrival time of a specific point. Therefore, the timestamps for a flight while passing through all the detection points can be considered as a set of random variables. As we know, the prediction error propagates along with the look-ahead time, thus we consider that the prediction error has a proportional relationship with the predicted arrival time. Moreover, a probability massive function is assigned for the random variable which is derived based on a normal distribution with the predicted arrival time as the mean value, prediction error as three times of the standard deviation. Through enumerating the possible values of the random variables that are involved in a conflict detection, the probability of conflicts on a specific point for the current flight pair can be derived. Then, the total probability of conflict can be obtained.

Regarding the complexity of this problem, simulated annealing is implemented as the solution algorithm that provides an efficient way to get an approximate optimal resolution. A simulation is proposed to verify the robustness of the model by implementing time perturbations on each flight. The results show that the flight schedule generated by the deterministic model is very sensitive to the temporal variation, while the optimized solution of the probabilistic model proves the proposed model is more robust in dealing with the time uncertainty by absorbing a large amount of potential conflicts.

Considering the probabilistic model in this paper, several details still need to be improved: First, the parameters that define the probability distribution are chosen empirically, while with different parameters, the performance of the proposed model might behave differently. Second, the model mainly tackles the minor uncertainties which to some extent can vary diversely in different operational segments, thus different scenarios and areas need to be tested. For future work, the corresponding improvements will be taken into consideration based on the aforementioned points. Besides, it is important to consider the interests of other stakeholders such as delay, fuel cost.

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