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Validity Domain of the Odunaiya Expression for Computing the Conventional VOR Multipath Error

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Abstract—This work presents a method to determine the validity domain of the static Odunaiya expression for computing the VOR multipath error in the presence of wind turbines. The Odunaiya formula is considered valid when it gives the consistent response compared to a dynamic receiver model. This validity domain is then expressed in terms of the multipath with respect to the direct path. This leads to a geometric criterion that is illustrated.

Index Terms—VOR receiver, Odunaiya, multipath, wind turbines

I. INTRODUCTION

The VHF Omnidirectional Range (VOR) systems are essential for air navigation: they give an aircraft its bearing relative to magnetic North. This information can be impaired by surrounding wind turbines that produce multipaths. The on-board VOR receiver processes signals from unwanted directions, which yields bearing error.

To compute the VOR errors, all the models comprise two steps. Firstly, the multipath parameters are obtained by means of a propagation model [3][4][5]. Secondly, the bearing error is calculated from the multipath characteristics using the analytic expression suggested by Odunaiya [1].

In Odunaiya expression, everything is expected static. However, multipaths change quickly in time as the aircraft moves. Thus, we have proposed a digital VOR receiver to reproduce the dynamic response of a realistic VOR receiver [2], taking into account demodulations and filtering. This model has been tested in dynamic scenarios. It has been observed that Odunaiya formula [1] does not always give the consistent response compared to the digital receiver model.

In this article, we propose to determine the validity domain of the static Odunaiya expression for a conventional VOR.

In section II, we present the static Odunaiya expression and the dynamic VOR receiver model used to calculate the bearing error. An example of a simulation where the Odunaiya expression is not valid is presented. In order to explain this phenomenon, we perform in Section III a spectral analysis on the received intermediate signals. In Section IV, we present a criterion that defines the validity domain of the Odunaiya model. It is illustrated by some examples.

II. VOR RECEIVER MODELS

A. Times series generator

The times series generator is applied to obtain time series of the multipath parameters from a realistic aircraft trajectory, i.e. their amplitude \( a_n \), phase \( \theta_n \), and azimuth \( \varphi_n \). The time step between two consecutive epochs is fixed according to a criterion detailed in [2] to accurately model the multipath variations in space.

B. Deterministic propagation model

At each epoch of the time series, the characteristics of multipaths are computed from an electromagnetic model. The direct signal is calculated using the analytical two-ray method. The multipath signal computation is based on the hybridization of the parabolic equation method (PE) with physical optics (PO) [3]. The model overview is displayed in Figure 1. The multipath characteristics are then inputted into a receiver model to obtain the bearing error.

C. Static VOR Receiver : Odunaiya model

For a CVOR, the Odunaiya error is given by [1]

\[
\varepsilon = \tan^{-1}\left( \frac{\sum_{n=1}^{N} \frac{a_n}{a_0} \cos(\theta_n - \theta_0) \sin(\varphi_n - \varphi_0)}{1 + \sum_{n=1}^{N} \frac{a_n}{a_0} \cos(\theta_n - \theta_0) \cos(\varphi_n - \varphi_0)} \right)
\]

(1)

Note that \( a_0, \theta_0 \) and \( \varphi_0 \) corresponds to the direct signal parameters.

This calculation does not take into consideration the dynamic of a mobile aircraft.

D. Dynamic VOR Receiver

We have developed a digital VOR receiver to assess the impact of multipath variations in time on the bearing error [2]. Its block diagram is given in Figure 2. Our model is based on a I/Q signal generator from the multipath parameters. It is able to reproduce a realistic VOR receiver response, taking demodulations and filtering into...
consideration. For the proposed receiver architecture, a band pass filter is applied to extract the VAR BF signal at 30 Hz and a low pass filter is performed to recover the DC component that contains the azimuth information in the phase detector. $W_{30}$ and $W_{DC}$ correspond to the 3 dB bandwidths of these filters, respectively.

E. Static/Dynamic receivers confrontation

As shown in Figure 3, we consider a CVOR station with a power of 50 W operating at a frequency of 113 MHz. At 1 km from the VOR station, there is a generic wind turbine. The aircraft moves along a circular trajectory with a radius of 4.5 km around the VOR and an altitude of 1 km. The aircraft is motionless for 10 s to ensure the end of the transient state of the receiver. It starts at low velocity until the end of the trajectory where it reaches a speed of 324 km/h. $W_{30}$ and $W_{DC}$ are set to 2 Hz and 1 Hz, respectively.

In Figure 4, we display the instantaneous frequency of the multipath $F_{\text{inst}}$ relatively to the direct path with respect to time. It is calculated from finite differences applied to the phase variation with time. As expected, the Doppler frequency depends on the speed of the aircraft and its position with respect to the VOR station and the wind turbine.

In Figure 5, the VOR errors obtained with the Odunaiya expression and with the VOR receiver model are plotted with respect to time. The receiver response fits with the Odunaiya expression during the first 90 s when there are no dynamic effects.

As the Doppler effect increases, we can observe significant discrepancies between the static Odunaiya expression and the dynamic VOR receiver model. It is due to the fast variations of multipath.

In section III, we clarify this phenomenon by a spectral analysis for a scenario in the presence of a canonical multipath.
III. SPECTRAL ANALYSIS OF THE RECEIVED INTERMEDIATE SIGNALS

We test here the influence on the bearing error of the bandwidth of receiver filters.

We consider a canonical multipath defined by a parameter with fixed relative amplitude, Doppler shift and azimuth during a simulation time of 50 s. Its relative power is set at -20 dB. A relative azimuth of 90° is considered.

A. Case 1 : Doppler shift included in both filter bandwidths

In this case, the Doppler shift of the multipath is assumed to be included in both 3 dB bandwidths of filters. Thus, we consider the following parameters: \( F_{\text{inst}} = 1 \text{ Hz} \), \( W_{30} = 6 \text{ Hz} \), \( W_{\text{DC}} = 3 \text{ Hz} \).

![Frequency Response Spectrum](image1)

As shown in Figures 6a and 6b, we observe that the two filters allow the multipath Doppler shifts at 1 Hz around the direct signal at 30 Hz to pass through. Thus, we observe in Figure 7 that the receiver response does not fit with the Odunaiya expression.

B. Case 2 : Doppler shift is not included in one of both filter bandwidths

In this case, we consider that the Doppler shift of the multipath is not included in one of both filter bandwidths. First, it is included only in the DC low pass filter. To do this, we set: \( F_{\text{inst}} = 3 \text{ Hz} \), \( W_{30} = 6 \text{ Hz} \), \( W_{\text{DC}} = 7 \text{ Hz} \). This case is denoted as 2-a.

![Frequency Response Spectrum](image2)

As shown in Figure 8a, the frequency component \((30 \pm F_{\text{inst}})\) is removed by the 30 Hz band pass filter. Thus, we observe in Figure 9 that the receiver response does not fit with the Odunaiya expression.

Secondly, we consider that the Doppler shift is included only in the 30 Hz band pass filter (case 2-b). Thus, we set the following parameters: \( F_{\text{inst}} = 3 \text{ Hz} \), \( W_{30} = 6 \text{ Hz} \), \( W_{\text{DC}} = 1 \text{ Hz} \).

The 30 Hz band pass filter allows the frequency component \((30 \pm F_{\text{inst}})\) to pass through as shown in Figure 10a. Nevertheless, this component is filtered in the phase comparator. That
explains why in this case the receiver response is different to the one of Odunaiya as shown in Figure 11.

In this section, we present a geometric criterion that defines the validity domain of the Odunaiya model. It is illustrated by some examples for radial and circular aircraft trajectories.

A. Definition

We consider a general configuration plotted in Figure 12. The aircraft moves in the direction $\hat{u}_m$.

\[ \delta f_D = -F_e \frac{V_{Rx}}{c_0} \hat{r}_{Rx} \cdot \hat{u}_m \]  

(2)

with
- $F_e$ the source frequency.
- $V_{Rx}$ the receiver speed.
- $c_0$ the speed of light.
- $\hat{r}_{Rx}$ the VOR-aircraft direction.

Similarly, the Doppler shift generated by the aircraft’s motion relative to the wind turbine is given by

\[ \delta f_D = -F_e \frac{V_{Rx}}{c_0} \hat{r}_{Rx_1} \cdot \hat{u}_m \]  

(3)

with $\hat{r}_{Rx_1}$ the wind turbine-aircraft direction.
If the instantaneous frequency is not included in one of the two filter bandwidths, there is no multipath error. Thus, the Odunaiya model is considered invalid when

$$\left| \delta f^*_D - \delta f^0_D \right| > \frac{\min(W_{30}, W_{DC})}{2}. \tag{4}$$

B. Examples

We consider a VOR station operating at a frequency of 113 MHz. An obstacle is placed on azimuth 0° at 5 km from the VOR station. The minimum value between the two bandwidths of the receiver filters is set at 10 Hz. We consider a speed of 180 km/h. The configuration is assumed in a horizontal plane.

In the first case, we consider radial trajectories with distances up to 50 km from the VOR station. In Figure 13, we observe that the Odunaiya model is invalid in the vicinity of obstacle.

![Fig. 13. Validity domain of the Odunaiya expression (in blue).](image)

We consider in the second case a set of circular trajectories around the VOR station. We observe in Figure 14 that the validity area of Odunaiya expression is reduced when compared to radial trajectories.

![Fig. 14. Validity domain of the Odunaiya expression (in blue).](image)

V. Conclusion

In this paper, a confrontation between the static Odunaiya expression and the dynamic VOR receiver model has been studied. A spectral analysis of the received intermediate signals in a canonical scenario has been presented to explain the limits of the Odunaiya expression. To obtain the validity domain of this static model, a geometric criterion has been presented and illustrated with some examples. We have observed that the Odunaiya expression is no more valid when the instantaneous frequency of multipath due to Doppler effect is filtered inside the digital receiver. Then, the multipaths do not affect the bearing estimation.

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