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AIR TRAFFIC COMPLEXITY THROUGH LOCAL COVARIANCE IN THE CONTEXT OF LARGE AREAS OF OPERATIONS

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Abstract. *The notion of air traffic complexity has many facets and can be related to workload, which is a perception of a given situation by a human controller or to disorder, which is intrinsic. The present work falls within the second category and aims at computing the level of organization in a neighborhood of a point on the earth. It is based on a two steps approach: in the first one, a smooth time-dependent vector field is inferred from the sampled traffic using local linear models. Since positions are measured on a sphere, some special care must be taken as it is not a vector space. Using the Levi-Civita connection and its associated parallel transport, a local linear model can be defined in the tangent space at any point. The time evolution is captured through the kernel function that take the form of a product with one term being time dependent. In a second phase, the underlying dynamical is characterized at each point using a symmetric positive definite matrix. Thanks to the Riemannian manifold structure of the set of such matrices, a complexity indicator is then defined.*

1 INTRODUCTION

Air traffic is currently facing a shift of paradigm due to changes in consumers' behavior. The drop of air traffic in the last months is going to exhibit a slow recovery after the crisis ends while environmental concerns are gaining increasing importance [1]. The recent reports from ICAO [2] and ACI highlighted that recovery from the crisis will not occur unless the evolution of the demand and the customer confidence in the means of transportation are taken into account [3][4]. More than ever, a reliable, safe, environment friendly and agile air transportation system, should be put in place. To address future challenges, an optimal use of resources is mandatory and can be achieved by dynamically fine-tuning the capacity of Air Navigation Service Providers (ANSP) according to the fluctuation of the demand. Within this frame, a new concept using flows to organize the air traffic emerged and aims at delegating part of the operations tasks to aircraft. It is pushed through the project Flight Centric Air Traffic Control with Airstreams (FC2A), funded by SESAR, whose final goal is to satisfy the initial demand of airlines to fly the most direct trajectory between city pairs while avoiding the high traffic complexity inherent to pure free flight. In this ongoing work, parallel flight lanes are created within a larger tubular volume thus creating a highway-like structure. Due the high level of organization, complexity is significantly low within such a tube, allowing a denser traffic. The medial axis is found using a clustering procedure that extract major flows of traffic and is the most representative flown trajectory within a set of samples. It can be adjusted on the fly to cope with a structural change. To assess the performance of the FC2A concept of operations, both airline oriented KPI and complexity must be considered. While indicators pertaining to the first class are quite well known, the second one is still on area of active research and has several facets. It can be related to workload, which is a perception of a given situation by a human controller or to disorder, which is intrinsic.

A lot of work was dedicated to the controller-centric perspective assuming that the complexity is roughly equivalent to cognitive workload. Within this frame, one of the most widely used complexity measures is the dynamic density [5], that combines several operational indicators, such as the number of maneuvering aircraft, number of level changes, convergence. All these values are used as inputs of a multivariate linear model, or in recent implementations, of a neural network. The tuning of the free parameters of the predictors is made using samples coming from an expertized database of traffic situations. While being quite efficient for assessing complexity values in a given control center, the method suffers two important drawbacks:

- The tuning procedure requires a large number of expertized samples. A costly experiment involving several air traffic controllers must be set up.
- The indicator is only valid within a specific area of the airspace and has to be tuned when moved to another one. Adapting it to different national airspaces is even more demanding as control practices may diverge.

Another way to deal with complexity is through purely geometrical indicators [6][10] that extract salient structural features without referring to the way the traffic is controlled. An obvious benefit is that the same metric may be used everywhere, without the need of a specific tuning. It is also the weak point of the method as the relation with the workload is not direct.

In the present article, the approach taken for the complexity assessment is based on a measure of local disorder [8], thus yielding an indicator pertaining to the last class, and uses some of the concepts described by Le Brigant et al.[9].

This is a work in progress that will ultimately allow the use of deep learning in conjunction with an expertized database to produce a complexity metric with low tuning requirements. As indicated in[9], a by-product is the ability to compute distances between traffic situations, allowing for an efficient indexing in dedicated databases. The paper is structured as follows. In Section 2, the traffic is modeled after a Gaussian random field on the sphere, whose covariance function is estimated on global grid. In Section 3, tools dedicated to the processing of such grids of symmetric positive definite matrices are introduced.

2 LOCAL LINEAR MODELS ON THE SPHERE.

2.1 Models in real vector spaces

Local regression is a common statistical smoothing procedure that possesses several interesting features[10] Starting with a sample set of the form $(x_i, y_i)_{i=1\dots N}$ with $x_i \in R^n, y_i \in R^m$, the local linear model at $x \in R^n$ is the couple $(A_x, v_x), A_x \in M(m,n),\ v_x \in R^n$ that realizes the minimum of the weighted least square problem:

$$\min_{A \in M(m,n), v \in R^m} \sum_{i=1}^{N} \|A(x_i - x) + v - y_i\|^2 K(x, x_i) \tag{1}$$

Where $K: R^n \times R^n \to R^+$ is the so-called kernel function that is most of the time depending only on the distance between its arguments, that is: $K(x,y) = K(\|x-y\|)$, and compactly supported or at least rapidly decreasing. Since K takes only positive values, the initial problem can be turned into a standard least square one:

$$\min_{A \in M(m,n), v \in R^m} \sum_{i=1}^{N} \left\|AK^{1/2}(x, x_i)(x_i - x) + K^{1/2}(x, x_i)(v - y_i)\right\|^2 \tag{2}$$

Or in synthetic form:

$$\min_{A \in M(m,n), v \in R^m} tr\left((AX - V)W(AX - V)^t\right) \tag{3}$$

With: $X = (x_i - x)_{i=1\dots N} \in M(n,N), V = (y_i - v)_{i=1\dots N} \in M(m,N), W = diag\left(K(x, x_i)\right) \in M(N,N)$.

Taking the derivative with respect to A, v and equating to 0, one obtains the matrix normal equations:

$$\begin{aligned} A_x &= VWX^t(XWX^t)^{-1} \\ v_x &= \frac{(Y - AX)W1}{1^tW1} \end{aligned} \tag{4}$$

With: $Y = (y_i)_{i=1\dots N} \in M(m,N), 1 = (1)_{i=1\dots N} \in M(N,1)$

Please note that the expression for v in equation can be written using weighted means as follows.

Putting:

$$\bar{X} = \frac{XW1}{1^tW1}, \bar{Y} = \frac{YW1}{1^tW1}, \tag{5}$$

The normal equations become:

$$\begin{aligned} A_x &= \tilde{Y}W\tilde{X}^t\left(\tilde{X}W\tilde{X}^t\right)^{-1} \\ v_x &= \bar{Y} - A\bar{X} \end{aligned} \tag{6}$$

Where: $\tilde{X} = X - \bar{X}, \tilde{Y} = Y - \bar{Y}$.

Since matrix product and inverse depend smoothly on their arguments, clearly the mappings $x \to A_x, x \to v_x$ have the same regularity as the one of the kernel K, thus showing the smoothing effect of the procedure. Interpreting the local expression $y \to A_x(y - x) + v\_x$ as a first order approximation to an underlying unknown vector field, v_x is its value at x while A_x is its derivative. Most of the time, the kernel K is controlled by a positive real parameter h, called the bandwidth, and its expression is given by:

$$K_h(x,y) = \frac{1}{h} K\left(\frac{\|x-y\|}{h}\right) \tag{7}$$

While automatic procedures can be used to optimally tune h, they rely on an a priori knowledge about the Hessian of the field to be approximated and cannot be used straightforwardly in our problem. Since h has the dimension of a length, it represents the characteristic scale of the phenomenon modelled and can be given a realistic value in the order of 100NM, at least in dense traffic area. For complexity assessment, especially when the flight paths are organized in lanes, it becomes important to capture variations at a larger scale and the effects of the earth curvature cannot be neglected. In fact, an unwanted contribution to complexity will be added due to the intrinsic rotation experienced by the speed vector along the route.

2.2 Local regression on the sphere

Extending the previous approach to a non-Euclidean space is not straightforward as the sampled positions $(x_i)_{i=1\ldots N}$ and speeds $(v_i)_{i=1\ldots N}$ are of different nature. Going back to the original least square problem (Eq.(1)), several points have to be addressed:

1. Definition of a linear vector field on a Riemannian manifold.
2. Comparison of tangent vectors in different tangent spaces.
3. Definition of a kernel compatible with the manifold structure.

Point 2 can be delt with using parallel translation [5] (Ch. 1). Basic notions about connections are summarized below without proofs.

Let M be a differentiable manifold of dimension *n* with tangent bundle TM. Its space of sections is denoted in the sequel by $\Gamma(TM)$.

Definition 1. A connection on *TM* is a mapping $\nabla: \Gamma(TM) \times \Gamma(TM) \to \Gamma(TM)$ such that:

$$\nabla_{fX+Y}Z = f\nabla_X Z + \nabla_Y Z, \nabla_X(fY+Z) = X(f)Y + \nabla_X Z, f \in C^\infty(M,R), X,Y,Z \in \Gamma(TM) \tag{8}$$

A connection on a manifold is a mean to take the derivative of a vector field in the direction of another while ensuring that the result is still in $\Gamma(TM)$.

The action of a connection can be described in coordinates using the so-called Christoffel symbols Γ_{ij}^k. If ∂_i is the i-th canonical section, then, assuming the summing convention on repeated indices:

$$\nabla_X Y = \left(X^i \frac{\partial Y^k}{\partial x_i} + \Gamma_{ij}^k X^i Y^i\right)\partial_k, X = X^i \partial_i, Y = Y^i \partial_i \tag{9}$$

The symbols Γ_{ij}^k account for the reference frame infinitesimal variation while the first term is related to intrinsic coefficients derivatives.

Definition 2. A connection ∇ is said to be without torsion if $\nabla_X Y - \nabla_Y X = [X,Y], X, Y \in \Gamma(TM)$. This is equivalent to the Christoffel symbols being symmetric i.e., $\Gamma_{ij}^k = \Gamma_{ji}^k$.

Definition 3. Let (M,g) be a Riemannian manifold. A connection ∇ on TM is said to be metric if for any vector fields X, Y, Z in $\Gamma(TM)$:

$$X\big(g(Y,Z)\big) = g(\nabla_X Y, Z) + g(Y, \nabla_X Z) \tag{10}$$

The next proposition is particularly important in Riemannian geometry and can be proved using the Koszul formula [6] (p. 25).

Proposition 1 On any Riemannian manifold, it exists a unique metric connection without torsion, called the Levi-Civita connection and denoted by∇^{lc}.

Proposition 2 Let *M* be a differentiable manifold and ∇ a connection on TM. Let $\gamma : [0,1] \to M$ be a smooth path with $\gamma(0) = p, \gamma(1) = q$. For any tangent vector $v \in T_pM$, it exists a unique curve $\tilde{\gamma}: [0,1] \to TM$ such that:

$$\tilde{\gamma}(0) = v, \pi \circ \tilde{\gamma} = \gamma, \nabla_{\dot{\gamma}(t)} \tilde{\gamma}(t) = 0 \tag{11}$$

The tangent vector $\tilde{\gamma}(1)$ is called the parallel translation of *v* at *q*.

Definition 4 A C^1 curve $\gamma : [0,1] \to M$ is said to be a geodesic for a connection ∇ if:

$$\forall t \in]0,1[, \nabla_{\dot{\gamma}(t)} \dot{\gamma}(t) = 0 \tag{12}$$

Shortest paths between pairs of points are geodesics for the connection is ∇^{lc}. In general, not all pairs of points on a manifold can be connected by a geodesic and if possible, it may not be unique. On the two-dimensional sphere the first property is true, and the second property holds for shortest paths unless points are antipodal. In a neighborhood of a point however, the Cauchy-Lipschitz theorem allows to define geodesics given an initial tangent vector.

Proposition 3 Let M be a differentiable manifold and ∇ a connection on TM. Let $p \in M$. It exists a starlike open set $T_pU \subset T_pM$ and a diffeomorphism $exp_p \quad : T_pU \to M$ such for any $v \in T_pU$, $exp\, v = \gamma(1)$ with $\gamma :$ $[0,1] \to M$ the unique geodesic with $\gamma(0) = p, \gamma'(0) = v$.

T_pM being a vector space isomorphic to R^n, it is possible to fix a basis $e_i, i = 1 \dots n$.

Definition 5 Under the assumptions of *Proposition 3*, the normal coordinates at p are the real valued functions $x_i : exp_p\, T_pU \to R\,, i = 1 \dots n$ defined by:

$$x_i\left(\exp_p \sum_{j=1}^{n} t_j e_j\right) = t_i \tag{13}$$

In the Riemannian setting, if the basis $e_i, i = 1 \dots n$ is orthonormal with respect to the Riemannian metric and the connection is ∇^{lc}, normal coordinates are close to Euclidean ones, as indicated in the next proposition.

Proposition 4 Let (M, g) be a Riemannian manifold. Fix a point p on M. For $v \in T_pU$:

$$g_{jk}(\exp v) = \delta_{jk} - \frac{1}{3} g\left(R\left(v, e_j\right)v, e_k\right) + O(\|v\|^3) \tag{14}$$

Where R is the Riemann curvature tensor.
This shows that in normal coordinates at p, the metric is tangent to the Euclidean one at order 2.
Normal coordinates can be used to answer point 1 in the list, through the use of a Taylor expansion.

Proposition 5 Let X be a smooth vector field. The Taylor expansion in direction v up to order 2 of X at p is given by:

$$X\left(\exp_p tv\right) = \tau_p^{\exp_p tv}\left(X(p) + t\nabla_v X(0) + \frac{t^2}{2}\nabla_v^2 X(0)\right) + o(t^2) \tag{15}$$

Where the parallel translation is taken along geodesics.
The proof is quite direct and is a matter of applying the usual Taylor expansion to the field $\tau_{\exp_p tv}^{p} X(\text{exp\_p}\, tv)$ that lives in $T_pM$regardless of the value of t.

The above proposition shows that from an approximation point of view, the equivalent to a linear vector field in a vector space is a linear vector field in Riemannian normal coordinates and yields the required extension to local linear models. Finally, point 3 can be addressed by imposing a constant integral for the kernel, which is the usual requirement in non-parametric statistics.

Definition 6 Let (M, g) be a Riemannian manifold. The Riemannian mesure $\mu_g$is defined on open sets as:

$$\mu_g(U) = \int_U \det g^t(x) g(x) \mathrm{dx} \tag{16}$$

Where x stands for local coordinates (one can check through partitions of unity that the definition is still valid if U spans several charts domains).

Definition 7 Under the assumptions and notation of *Definition 6*, let p be a point in M and Ka kernel function defined on R^n. The kernel function K_q is defined for any point q in the image of exp_p by:

$$K_q(x) = K\left(\exp_p^{-1} x\right) \frac{\left|\det D \exp\left(\exp_p^{-1} q\right)(x)\right|}{d\mu_g(x)} \tag{17}$$

Please note that the factor occurring after the kernel K is the one defined in [1](p. 209).

Gathering things together, local linear smoothers can be defined on a Riemannian manifold provided the exponential map has a large enough domain as follows:

- Given a dataset $(p_i, v_i), i = 1 \dots n$, in *TM* and a fixed-point p in M, use parallel translations $\tau_{p_i}^{p}$ along geodesics to pull back all tangent vectors at the p_i to T_pM.
- Express the points $p_i, i = 1 \dots n$ in normal coordinates at p as $x_i, i = 1 \dots n$.
- Solve a vector linear model with data $\left(x_i, \tau_{p_i}^{p} v_i\right), i = 1 \dots n$ and kernel $K(x_i, x) = K_{p_i}(x)$.

In the case of the sphere, the above model simplifies greatly as parallel translations along geodesics are just rotations and can be computed using elementary linear algebra.

3 COMPLEXITY COMPUTATION

In this section, the dataset $(p_i, v_i), i = 1 \ldots n$ is considered as a set of sampled positions and speeds of aircraft. Using the derivations of the previous section, each point p of the sphere can be associated with a matrix A and a vector v given by a local linear model at p. Complexity is captured by the matrix A that describes the local evolution of the velocities. Several indicators can be derived from it, each dedicated to a specific aspect of the traffic. As the local linear model is a vector field, the trace of A is its divergence and thus gives an information about the density variation in the vicinity of the reference point: if positive, it tends to decrease and if the converse is true, it tends to increase. This makes sense only when the matrix is square, which is nevertheless the case for all the traffic models considered in this study. The results on the ECAC region for September 17th,2018 traffic are presented in the figures below. The interaction distance between aircraft is set to 40NM and the time window is 6:30 to 12:30 GMT. The map on the left represents traffic above FL260 whereas the one on the right represents traffic below FL260.

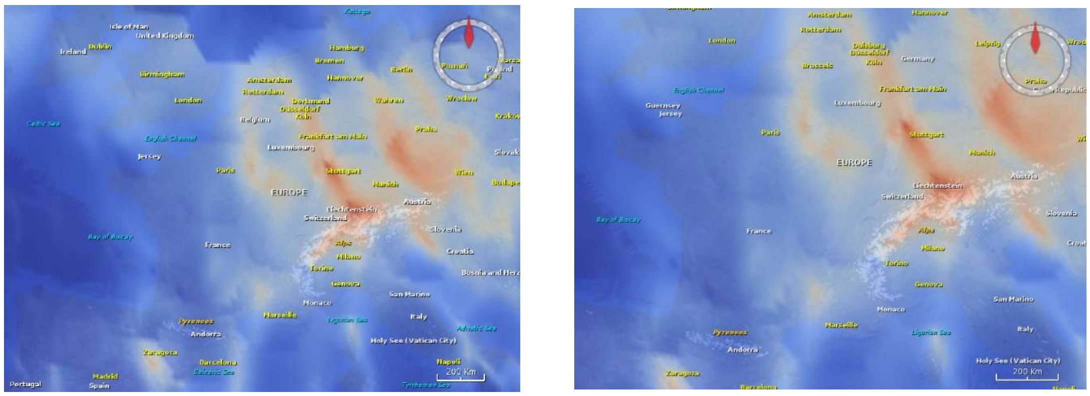

Figure 1. Complexity maps - En-Route (left), low altitude (right)

Similar to the one presented above is the one obtained by finding the lowest eigenvalue of A. If negative, a convergence occurs at the reference point, thus indicating a possible conflict. In conjunction with a density estimate of the traffic, it reveals a potential hot spot for the controller, although the true complexity may be lower than expected in such a situation. In fact converging, but otherwise organized traffic may be easily delt with using a roundabout procedure. The lowest eigenvalue indicator is somewhat more pertinent from the operational standpoint than the trace one, but has a higher computational cost.

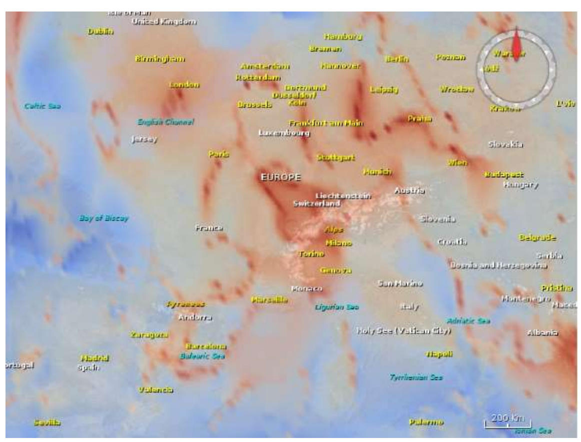

Figure 2. Lowest eigenvalue complexity indicator - En-Route

Finally, a criterion of predictability based on the rate of divergence of neighboring trajectories can be introduced. This criterion has proven to be adequate in many situations. Let $X: S^2 \rightarrow R^2$ be the vector field given by the local linear smoother, i.e. in normal coordinates:

$$X(q) = \tau_p^q(v_p + A_p v) \tag{18}$$

Where v_p, A_p are the parameters of the linear model and $q = \exp_p v$.

An approximate trajectory in a neighborhood of p is thus of the form: $\dot{\gamma}(t) = X\big(\gamma(t)\big), \gamma(0) = p$. Using the variational equation for the derivative of γ with respect to the initial condition, it can be proved that the distance between the original trajectory and a perturbed trajectory originating from $p + \epsilon w$ is at order one in ϵ, given by $\epsilon \exp(tA^tA)\, \|w\|$. From this, it can be concluded that eigenvalues of A^tA les than one are stable directions, where perturbations tend to vanish, while those larger than one indicates a fast divergence, thus a complex situation. The most positive eigenvalues, or zero if all are negative, is thus an indicator of complexity.

Since the problem has low dimension, typically three, the eigenvalues can be found quite fast using Jacobi iteration. The results on the same traffic presented before are given below.

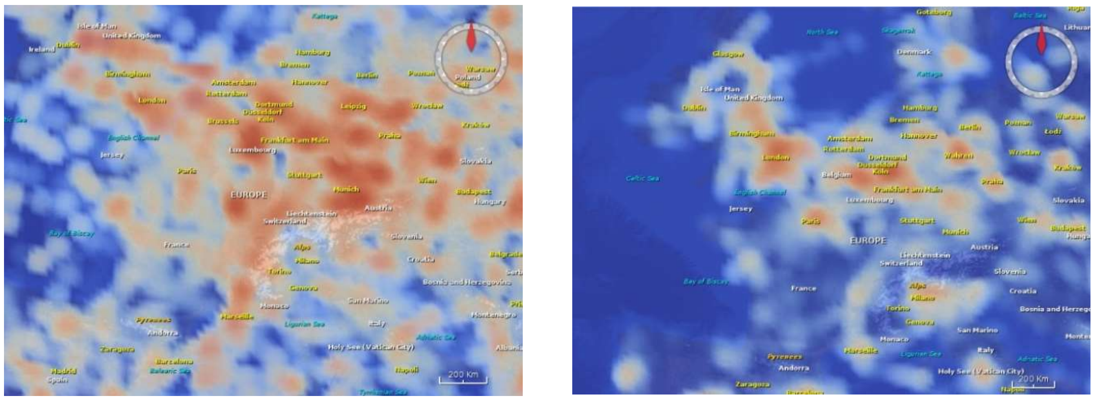

Figure 3. Information geometry complexity indicator - En-Route (left), low altitude (right)

4 CONCLUSION

This work proposes an evolution of the previous approach using Riemannian manifold in the analysis of air traffic samples. The main idea is to seek after a continuous and time dependent vector field model of the traffic. A spacetime cutoff window is applied to limit the domain of influence of a flight. To be able to consider bigger samples of traffic on large areas, calculation is extended to the coverage of the sphere using a mix of pentagonal and hexagonal cells.

As the calculation time is reasonable and the local linear model smoother gives access not only to a continuous version of the sampled traffic vector field but also to its gradient, it is possible to envisage a combined complexity/bundling approach. Designing an iterative algorithm, it could be possible to perform a complexity optimal bundling of the trajectories. It could be also possible to calculate the complexity of a single trajectory to evaluate the best path (i.e. inducing less complexity) between potential alternative trajectories.

It can be also considered that the complexity maps obtained by sampling on an evenly space grid can feed a deep neural network, thus allowing the use of AI techniques to characterize traffic patterns.

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